Assignment 1 / 25146286

Q1 (a)

Throughput = # completed jobs / obsersetime

$$X = \frac{C(0)}{T} = \frac{676 \text{ jobs}}{90 \times 60 s} = 0.125 \text{ jobs/s}$$

The utilization = CPU Busy time / obsersetime

$$U_{cpu} = \frac{B_{cpu}}{T} = \frac{4729 s}{90 \times 60 s} = 0.87$$

$$U_{disk} = \frac{B_{disk}}{T} = \frac{3565 s}{90 \times 60 s} = 0.67$$

Service demind of device j = utilisation of device j / throughput

$$D_{(cpu)} = \frac{U_{cpu}}{X(0)} = \frac{0.87}{0.125} = 6.96$$

$$D_{(disk)} = \frac{U_{disk}}{X(0)} = \frac{0.67}{0.125} = 3.76$$

(b) Yes, it is possible.

According to (a), we have $D_{(j)} = \frac{U_{(j)}}{X_{(0)}}$, thus, $D_{(j)}$ is proportial to $U_{(j)}$. There also have $U_{(j)} = \frac{B}{T}$, thus $U_{(j)}$ is proportial to B.

Above all, we conclude that $D_{(j)}$ is proportial.

For this system, T and X_0 ($X_0 = \frac{C(0)}{T}$) are given.

So we can determine the bottleneck of system by comparing busytime of CPU and disk.

(c) From (a), we know

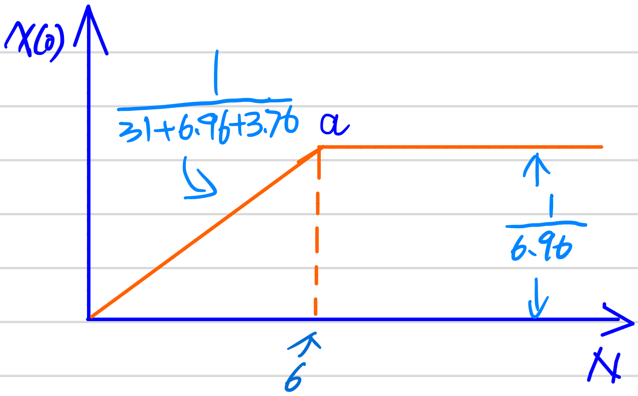
$$D_{cpu} = 6.96$$

$$D_{disk} = 3.7$$

From lecture, we know for this system:

$$X_0 \le \min\left\{\frac{1}{\mathrm{Max}\, D(j)}, \frac{N}{Z+\sum_{j=1}^{k} D(j)}\right\} \quad (Z = \text{mean thinking time})$$

Draw a grab:



— The x-axis of point a $= \frac{31+6.96+3.76}{6.96} = 6 < 30$

— Thus, when there are 30 interactive users and thinking time per job is 31 seconds, the system throughput is $1/6.96 = 0.144$.

(d) M – interactive users

M_{avg} – mean # busy users

N_{avg} – average # jobs in system.

Accounting Little' Law:

$M_{avg} = Z * X_0$

$N_{avg} = R * X_0$ (Z mean think time, R mean response time)

$M = M_{avg} + N_{avg}$

Above all. Response time $R = \frac{M}{X_0} - Z = \frac{30}{0.144} - 31 = 177.33\,s$

Q2

(a) There are given $\lambda = 20$, $\mu_1 = 10$, $\mu_2 = 15$

From lecture, we have utilisation $U = \frac{\lambda}{\mu}$.

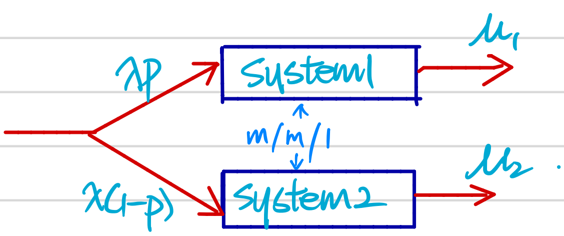
system1 Utilisation $U_1 = \frac{p\lambda}{\mu_1}$

system2 Utilisation $U_2 = \frac{(1-p)\lambda}{\mu_2}$

when $U_1 = U_2 \Rightarrow \frac{p\lambda}{\mu_1} = \frac{(1-p)\lambda}{\mu_2} \Rightarrow p = 0.4$

(b) From (a), $U_1 = U_2 = \lambda p / \mu_1 = \frac{20 \times 0.4}{10} = 0.8$

Draw a graph for this system:



For m/m/1. Response time $T = \frac{\rho}{\lambda(1-\rho)} = \frac{1}{\mu - \lambda}$ where $\rho = \frac{\lambda}{\mu}$

system1, response time $T_1 = \frac{1}{\mu_1 - \lambda p} = \frac{1}{2} s$

system2, response time $T_2 = \frac{1}{\mu_2 - \lambda(1-p)} = \frac{1}{3} s$

whole system response time $T = pT_1 + (1-p)T_2 = 0.4 \times \frac{1}{2} + 0.6 \times \frac{1}{3} = 0.4 s$

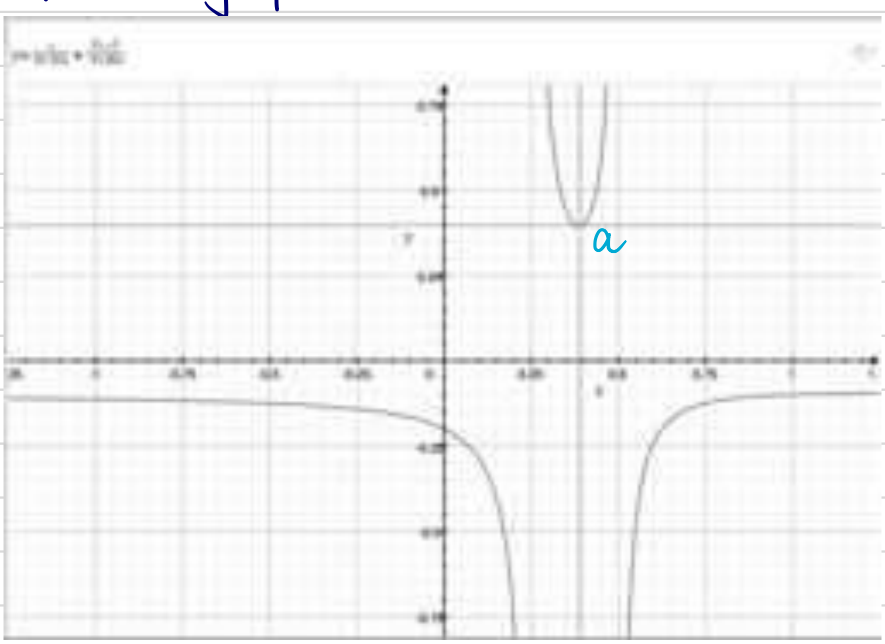
(c) From (b) mean response time $T = pT_1 + (1-p)T_2$. where $T_1 = \frac{1}{\mu_1 - \lambda p}$,

$T_2 = \frac{1}{\mu_2 - \lambda(1-p)}$

$$\therefore \quad T = \frac{p}{\mu_1 - \lambda p} + \frac{1-p}{\mu_2 - \lambda(1-p)}$$

$$\Rightarrow T = \frac{p}{10 - 20p} + \frac{1-p}{20p - 5}$$

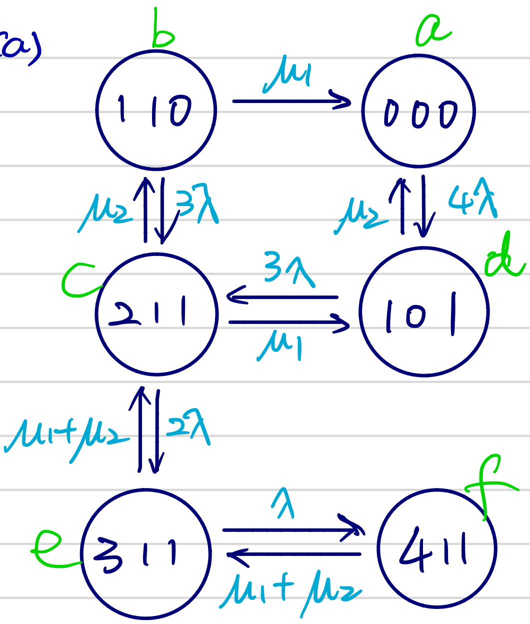
Draw a graph:



point a = (0.3895, 0.3951)

So when $p = 0.3895$, the mean response time is smallest possible.

Q3 (a)



machine fail rate $\lambda = \frac{1}{900}$

leader repair rate $\mu_1 = \frac{1}{60}$

trainee repair rate $\mu_2 = \frac{1}{90}$

— From question, the status goes from all machines working into 1 machine failed, the trainee will work on repaired. So the state (0 0 0) cannot chang to (1 1 0), but (1 1 0) can chang to (0 . 0 . 0).

$\left.\begin{array}{l}(1\ 1\ 0) \to (0\ 0\ 0) \\ (2\ 1\ 1) \to (1\ 0\ 1)\end{array}\right\}$ leader repaired machine. μ_1

$\left.\begin{array}{l}(2\ 1\ 1) \to (1\ 1\ 0) \\ (1\ 0\ 1) \to (0\ 0\ 0)\end{array}\right\}$ trainee repaired machine μ_2

$\left.\begin{array}{l}(3\ 1\ 1) \to (2\ 1\ 1) \\ (4\ 1\ 1) \to (3\ 1\ 1)\end{array}\right\}$ leader or train repaired machine, $\mu_1+\mu_2$

$(0 . 0\ 0) \to (1\ 0\ 1)$ one of four machines failed, 4λ, other state change similarly

(b)

(a) $\mu_1 P(1,1,0) + \mu_2 P(1,0,1) = 4\lambda P(0,0,0)$

(b) $\mu_1 P(1,1,0) + 3\lambda P(1,1,0) = \mu_2 P(2,1,1)$

(c) $(\mu_2 + 2\lambda + \mu_1) P(2,1,1) = 3\lambda P(1,1,0) + 3\lambda P(1.0.1) + (\mu_1+\mu_2) P(3,1,1)$

(d) $\mu_1 P(2,1,1) + 4\lambda P(0,0,0) = (3\lambda + \mu_2) P(1,0,1)$

(e) $2\lambda P(2,1,1) + (\mu_1+\mu_2) P(4,1,1) = (\mu_1+\mu_2) P(3,1,1) + \lambda P(3,1,1)$

(f) $\lambda P(3,1,1) = (\mu_1+\mu_2) P(4,1,1)$

(g) $P(0,0,0) + P(1,1,0) + P(2,1,1) + P(1,0,1) + P(3,1,1) + P(4,1,1)$

(c) computing by programing

$P(0,0,0) = 0.5918$ $\quad$ $P(1,0,1) = 0.3081$

$P(1,1,0) = 0.0313$ $\quad$ $P(3,1,1) = 0.0073$

$P(2,1,1) = 0.0611$ $\quad$ $P(4,1,1) = 0.0004$

(d) Prob[At least 3 machines are available] =

$$P(0,0,0) + P(1,0,1) + P(1,1,0) = 0.5918 + 0.3081 + 0.0313$$
$$= 0.9312$$

(e) mean # failed machines $= 1 \times P(1,0,1) + 1 \times P(1,1,0) + 2 \times P(2,1,1) + 3 \times P(3,1,1)$

$$+ 4 \times P(4,1,1) = 0.3081 + 0.0313 + 0.0611 \times 2 + 0.0073 \times 3 + 0.0004 \times 4$$
$$= 0.4851$$

(f) MTTR = Queueing time for repair + actual repair time

Actually, MTTR is equal to mean response time.

Using Little's Law

$$MTTR = R_{avg} = \frac{N_{avg}}{X}$$

where throughput X can be calculated by

$$\overline{X}_f = \sum_{k=0}^{M-1} (M-k) \lambda P(k)$$
$$= 4\lambda P(0,0,0) + 3\lambda P(1,1,0) + 3\lambda P(1,0,1) + 2\lambda P(2,1,1)$$
$$+ \lambda P(3,1,1)$$

Calculating by programing

$$MTTR = \frac{N}{X} = 82.8499 s$$