

1. Motivation

- **Spiked matrix models** provide a canonical framework for studying the problem of recovering a low-dimensional structure from high-dimensional noise.
- Two canonical examples:

(1) Spiked Wigner model: we observe

$$\mathbf{Y} = \frac{\lambda}{\sqrt{n}} \mathbf{x} \mathbf{x}^\top + \mathbf{Z}.$$

Here \mathbf{x} is normalized such that $\|\mathbf{x}\| \approx \sqrt{n}$ and \mathbf{Z} is an $n \times n$ Wigner matrix.

(2) Spiked Wishart (covariance) model: we observe

$$\mathbf{Y} = \frac{\sqrt{\lambda}}{\sqrt{n}} \mathbf{x} \mathbf{u}^\top + \mathbf{Z}.$$

Here \mathbf{x} is normalized such that $\|\mathbf{x}\| \approx \sqrt{n}$, $\mathbf{u} \in \mathbb{R}^N$ is a Gaussian vector sampled independently from $\mathcal{N}(0, \mathbb{I}_N)$, and \mathbf{Z} is an $n \times N$ matrix with i.i.d. standard normal entries.

- Spectral algorithm exhibit celebrated **BBP-transition**.
- Proved to be statistically optimal for “simple” and “dense” priors but not for “sparse” priors.
- Conjectured to be computationally optimal for any priors under mild conditions (supported by evidences in the low-degree polynomial framework).
- **Correlated spiked matrices models** serve as simple playground for the theoretical study of **multi-modal learning**, with the premise that jointly analyzing multiple correlated datasets can yield more powerful inferences than processing each one in isolation [KZ25].
- In our work [Li25+], we consider the algorithmic aspects of inference from a pair of random matrices with correlated spikes.

2. Mathematical settings

- **Correlated spiked Wigner model:** we observe

$$\mathbf{X} = \frac{\lambda}{\sqrt{n}} \mathbf{x} \mathbf{x}^\top + \mathbf{W}, \quad \mathbf{Y} = \frac{\mu}{\sqrt{n}} \mathbf{y} \mathbf{y}^\top + \mathbf{Z}.$$

Here \mathbf{x}, \mathbf{y} are correlated spikes with norm $\|\mathbf{x}\|, \|\mathbf{y}\| \approx \sqrt{n}$ and correlation $\langle \mathbf{x}, \mathbf{y} \rangle \approx \rho \|\mathbf{x}\| \|\mathbf{y}\|$, and \mathbf{Z}, \mathbf{W} are $n \times n$ Wigner matrices.

- **Correlated spiked Wishart model:** we observe

$$\mathbf{X} = \frac{\sqrt{\lambda}}{\sqrt{n}} \mathbf{x} \mathbf{u}^\top + \mathbf{W}, \quad \mathbf{Y} = \frac{\sqrt{\mu}}{\sqrt{n}} \mathbf{y} \mathbf{v}^\top + \mathbf{Z}.$$

Here \mathbf{x}, \mathbf{y} are correlated spikes with norm $\|\mathbf{x}\|, \|\mathbf{y}\| \approx \sqrt{n}$ and correlation $\langle \mathbf{x}, \mathbf{y} \rangle \approx \rho \|\mathbf{x}\| \|\mathbf{y}\|$, $\mathbf{u}, \mathbf{v} \in \mathbb{R}^N$ are sampled from $\mathcal{N}(0, \mathbb{I}_N)$, and $\mathbf{W}, \mathbf{Z} \in \mathbb{R}^{n \times N}$ are noise matrices with i.i.d. standard normal entries.

- Two natural inference tasks:
 - Detection: deciding whether (\mathbf{X}, \mathbf{Y}) is sampled from the law of correlated spiked model or is sampled from the law of pure noise matrices.
 - Recovery: recovering the planted (correlated) spikes (\mathbf{x}, \mathbf{y}) from the observation (\mathbf{X}, \mathbf{Y}) .
- Widely-used and well-studied methods:

(1) The Canonical Correlation Analysis (CCA) method [BHPZ19, MY23, BG23+]: extract information of the spikes from the MANOVA matrix

$$(\mathbf{X} \mathbf{X}^\top)^{-\frac{1}{2}} (\mathbf{Y} \mathbf{X}^\top) (\mathbf{Y} \mathbf{Y}^\top)^{-\frac{1}{2}}.$$

(2) The Partial Least Squares (PLS) method [MZ25+]: extract information of the spikes from the sample-cross-covariance matrix

$$\mathbf{S} = \mathbf{X} \mathbf{Y}^\top.$$

3. Our algorithmic results

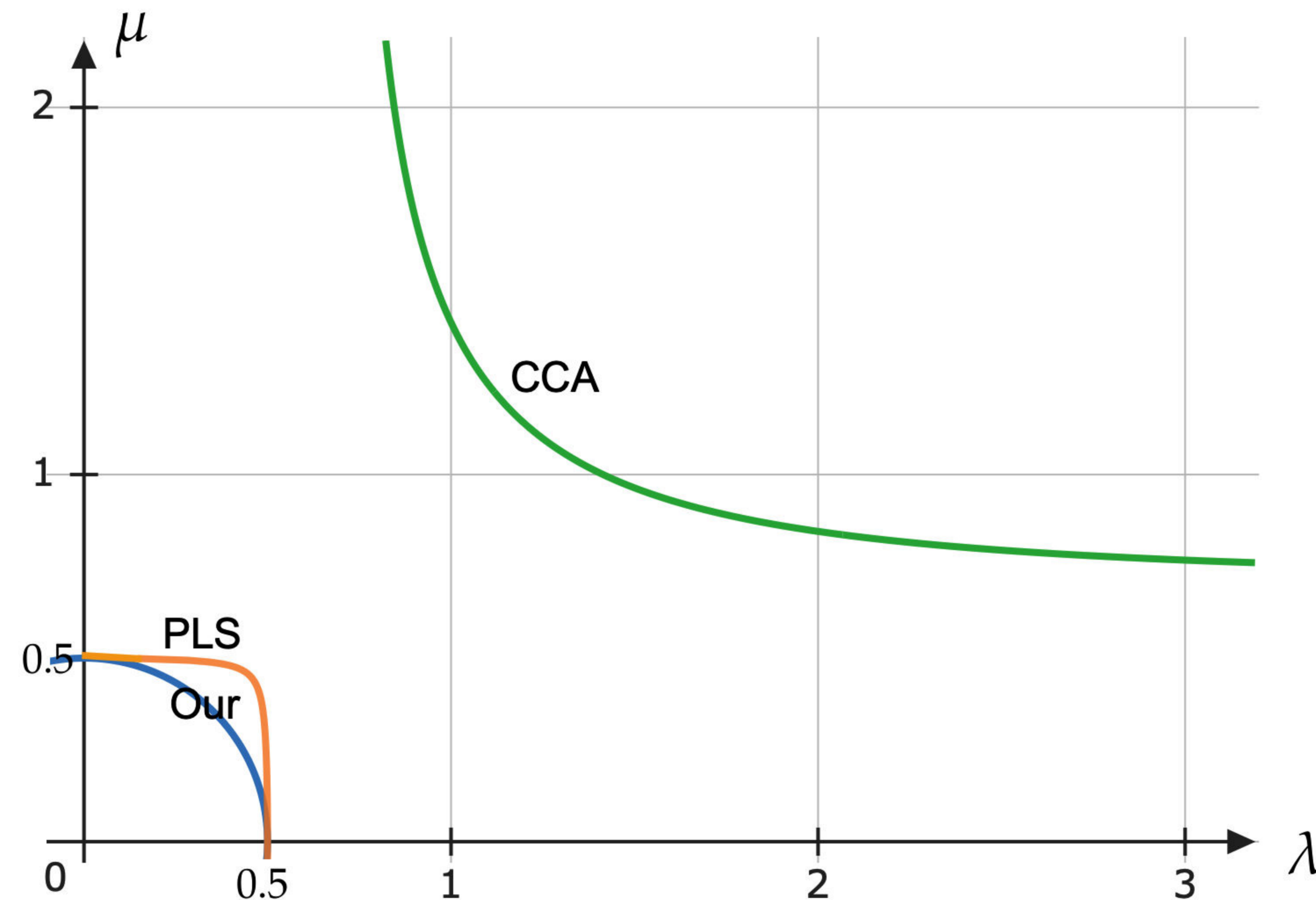
Our focus is twofold:

- Construct an **efficient algorithm** for the detection/recovery problems.
- Provide matching **computational lower bounds** that suggests that our algorithm has optimal performance among all polynomial-time algorithms.

Theorem 1 (L.25+). Define $F(\lambda, \mu, \rho, \gamma) = \max \left\{ \frac{\lambda^2}{\gamma}, \frac{\mu^2}{\gamma}, \frac{\lambda^2 \rho^2}{\gamma - \lambda^2 + \lambda^2 \rho^2} + \frac{\mu^2 \rho^2}{\gamma - \mu^2 + \mu^2 \rho^2} \right\}$.

- (1) For the correlated spiked Wigner model, suppose that $F(\lambda, \mu, \rho, 1) > 1$. Then, there exists two algorithms $\mathcal{A}, \mathcal{A}'$ with polynomial running time such that \mathcal{A} (respectively, \mathcal{A}') takes \mathbf{X}, \mathbf{Y} as input and achieves strong detection (respectively, weak recovery).
- (2) For the correlated spiked Wishart model, suppose that $\frac{n}{N} = \gamma$ for some $\gamma = \Theta(1)$ and $F(\lambda, \mu, \rho, \gamma) > 1$. Then, there exists two algorithms $\mathcal{A}, \mathcal{A}'$ with polynomial running time such that \mathcal{A} (respectively, \mathcal{A}') takes \mathbf{X}, \mathbf{Y} as input and achieves strong detection (respectively, weak recovery).

- Shows that an algorithm can leverage the correlation between the spikes to detect and estimate the signals even in regimes where efficiently recovering either \mathbf{x} from \mathbf{X} alone or \mathbf{y} from \mathbf{Y} alone is believed to be computationally infeasible.
- Outperforms the PLS/CCA method (see the figure below for a comparison).



(Phase diagram in the (λ, μ) plane illustrating the thresholds for our method (blue), the PLS method [MZ25+] (orange), and the CCA method [BHPZ19, MY23, BG23+] (green). Here we take $\gamma = 0.25$ and $\rho = 0.99$.)

- Intuition: spectral method correspond to “counting cycles” in a single matrix.
- Since we have two matrices (with correlated signals), instead we can **count “decorated cycles”**. Specifically, we consider cycles where each edge is “decorated” according to whether it is taken from \mathbf{X} or \mathbf{Y} .
- Technical contribution: introducing a delicate **weighting scheme**, where each decorated cycle is assigned a weight based on its specific combinatorial structure.
- Using the techniques of color-coding, such cycle counts can be approximated efficiently.

4. Computational lower bound

Theorem 2 (L.25+). (1) For the correlated spiked Wigner model, suppose that $F(\lambda, \mu, \rho, 1) < 1$. Then all algorithms based on low-degree polynomials fails to achieve even detection.

(2) For the correlated spiked Wishart model, suppose that $\frac{n}{N} = \gamma$ for some $\gamma = \Theta(1)$ and $F(\lambda, \mu, \rho, \gamma) < 1$. Then, all algorithms based on low-degree polynomials fails to achieve even detection.

- Provide evidence for computational hardness under the framework of **low-degree polynomial algorithms**, which is believed to be a useful proxy of all efficient algorithms since it captures many powerful algorithmic approaches such as subgraph counts, spectral method, approximate message passing, etc.
- Theorems 1 and 2 strongly suggests that $F(\lambda, \mu, \rho, 1) = 1$ (respectively, $F(\lambda, \mu, \rho, \gamma) = 1$) is the computational threshold for the correlated spiked Wigner (respectively, Wishart) model.
- Main approach:
 - Using standard tools for the **Gaussian additive model** (see [KWB22]) to bound the optimal “advantage” of degree- D polynomials by the “replica overlap” $\langle x, x' \rangle$ and $\langle y, y' \rangle$ (here (x, y) and (x', y') are independent spike pairs).
 - Intuition: $(\langle x, x' \rangle, \langle y, y' \rangle)$ behaves like a pair of Gaussian variables (U, V) with mean 0, variance 1 and correlation ρ^2 .
 - Use Lindeberg’s interpolation to argue such Gaussian approximation is indeed valid.

References

- [BHPZ19] Z. Bao, J. Hu, G. Pan, and W. Zhou. Canonical correlation coefficients of high-dimensional Gaussian vectors: Finite rank case. *Annals of Statistics*, 2019.
- [BG23+] A. Bykhovskaya and V. Gorin. High-dimensional canonical correlation analysis. arXiv:2306.16393.
- [KWB22] D. Kunisky, A. Wein, and A. Bandeira. Notes on computational hardness of hypothesis testing: Predictions using the low-degree likelihood ratio. *Mathematical Analysis, its Applications and Computation*, 2022.
- [Li25+] Z. Li, The algorithmic phase transition in correlated spiked models. **arXiv:2511.06040**.
- [MY23] Z. Ma and F. Yang. Sample canonical correlation coefficients of high-dimensional random vectors with finite rank correlations. *Bernoulli*, 2023.
- [KZ25] C. Keup and L. Zdeborová. Optimal threshold and algorithms for multi-model learning in high dimensions. *Journal of Statistical Mechanics: Theory and Experiment*, 2025.
- [MZ25+] P. Mergny and L. Zdeborová. Spectral thresholds in correlated spiked models and fundamental limits of partial least squares. arXiv:2510.17561.