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ORIGINAL ARTICLE

Portfolio optimization with covered calls

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Abstract

Covered calls are traditionally formed as an overlay on an existing portfolio. Our analysis suggests that covered calls formed in two steps by first optimizing underlying equity positions and then selecting call overwriting weights are not risk-return optimal in general. We introduce an optimization framework which simultaneously selects underlying asset positions and call options to sell to form risk-return optimal covered call portfolios. Call option market prices form a critical part of the expected return and risk expressions. Variance of the return, semivariance of the return, and conditional value-at-risk are used as risk measures. The model was first tested by forming covered call portfolios composed of three indices and then by forming portfolios using 92 U.S. equities. We find that selling call options not only reduces risk, but when selected optimally can also benefit the expected return.

Keywords Covered call · Portfolio optimization · Mean–variance optimization · Conditional value-at-risk · Semivariance · Options

Introduction

A covered call strategy is formed by holding an asset and selling call options on this asset. In exchange for potential up-side liability the seller receives the call premium. The strategy is attractive during bearish periods where there is unlikely to be any liability while the premium provides a small buffer in down-side cases. Covered calls may offer reduced risk versus the long equity position while not meaningfully compromising return.

The Chicago Board Options Exchange introduced the S&P 500 BuyWrite Index in 2002 to track the performance of a covered call formed on the S&P 500 Index. Whaley Whaley (2002), Feldman and Roy (2004), and Callan Associates (2006) all found that the BuyWrite Index offered a much higher Sharpe ratio than a simple long position in the S&P 500 Index. McIntyre and Jackson McIntyre and Jackson (2007) also found that covered calls have strong empirical performance in a broad range of settings. Board et al. (2000) found that covered calls are preferable to their underlying equity portfolio from a utility perspective. Authors consistently conclude that it is best to form covered call strategies using short-dated options, i.e., one month to maturity options [see Whaley (2002), Figelman (2008), and Maidel and Sahlin (2010)].

There has been little research regarding optimization and options relevant to covered calls. Alexander et al. (2006) examined the well posedness of CVaR and VaR optimization for a portfolio of derivatives under broad conditions. They showed that the widely used formulation of Rockafellar and Uryasev (2000) is ill posed for a portfolio of options under some conditions. The optimization of covered calls introduces many constraints making the problem dissimilar to that studied in Alexander et al. We find that results from our methodology are stable when a sufficiently large number of simulations is used. Zymler et al. (2011) formulated a worst-case optimization for a portfolio including European options. Worst-case optimization is not suitable for covered call portfolios where

risk.

Many authors assessed covered call strategies using a wide range of strike prices and tested heuristics of selecting them. Yang (2011) proposed a dynamic strategy which alternates between selling at-the-money calls and at-the-money puts. As puts are sometimes sold, the Yang (2011) strategy is outside the scope of a covered call; however, we note that when calls are sold, it is at a fixed moneyness level, i.e., at-the-money. He et al. (2014) and Hill et al. (2006) explored

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covered calls of different fixed moneyness levels. Hill et al. (2006) and its extension Che and Fung (2011) tested a dynamic strategy which selects strike prices by targeting a probability of maturing in-the-money depending on current market movements. Although selecting the moneyness level or strike price is important in determining the potential liability at maturity, these studies failed to consider the effect of the options' market prices. Figelman (2008) and Diaz and Kwon (2017) both showed that options' market prices are a critical component when modeling the expected return of a covered call strategy. Furthermore all afore mentioned dynamic strategies explored covered calls where the underlying asset was always fully overwritten using a single call option. In bullish market conditions or if the options' market prices are relatively low, it could be optimal to only partially overwrite the underlying equity or not to overwrite at all.

Diaz and Kwon (2017) developed an optimization framework to select calls (i.e., strike prices, since the maturity date was fixed at one month) to sell to form a covered call strategy on a single long asset position. The framework allows multiple call options of different strike prices to be sold on a single asset position and also includes the possibility of having no overwriting on part or all of the asset position. They found that to produce a risk-return optimal covered call on one asset it is sometimes necessary to partially overwrite with call options of different strike prices.

The framework of Diaz and Kwon (2017) is limited to optimizing a covered call overlay on an existing equity position, and the previously mentioned studies also only examined covered calls using a single equity position. Although there are a number of single position (i.e., index based) covered call funds, there are also many covered call funds which hold positions in a large number of individual securities. These include the Bank of Montreal US High Dividend Covered Call Fund [see Bank of Montreal (2018)], the BlackRock Enhanced Capital and Income Fund [see BlackRock (2018)], and the Manulife Covered Call US Equity Fund [see Manulife Manulife (2018)]. For these types of funds we propose an integrated approach where underlying asset positions and overwritten call options are simultaneously optimized.

In our proposed approach we extend the framework of Diaz and Kwon (2017) to optimize a portfolio of covered calls. Our model provides a prescriptive method of constructing covered call portfolios based on risk-return optimality using variance, semivariance, and CVaR as risk measures. As in Diaz and Kwon (2017) the market prices of the options are used in modeling the expected return. Unlike the original framework which only optimized option positions the new framework optimizes the overwritten call options and simultaneously selects the underlying asset positions. One question we seek to answer is whether knowledge of the available call options affects the optimal equity positions. Our model also explores covered calls of a very general form which includes the possibility of holding multiple assets, selling multiple call options of different strike prices on each asset, and having all or part of the underlying asset weights without overwriting. Previous studies examined covered calls using a single asset (an index) typically fully overwritten with a single call option. Another question we seek to answer is whether risk-return efficient covered call portfolios exploit the general form we propose, i.e., with partial overwriting and overwriting using different strike prices on a single asset.

The remainder of the paper is laid out as follows: in Sect. 2 the general framework is developed and presented, Sect. 3 examines the optimal call overwriting policy from an analytical perspective, Sect. 4 details implementation of the model, Sect. 5 analyzes sample results when using three indices as the underlying, Sect. 6 analyzes how the model scales to solving portfolios with 92 US large-cap equities and several hundred options as it might be used in a practical setting, and Sect. 7 analyzes possible improvements to performance versus conventional covered call strategies.

Methodology

Suppose an investor wishes to create a covered call portfolio using up to n available assets. The investor has W total wealth and spends w_j wealth on each asset j. If the current price of asset j is S_{0j}, then the investor purchases $x_j = w_j/S_{0j}$ units of asset j. We assume no shortselling of the underlying assets. To form a covered call the investor can sell up to x_j total units of n_j available one-month European call options for asset j. We assume all options under consideration have the same maturity date. Letting p_{jl} denote the number of units of call option l sold for asset j, the relationships are summarized below.

$$
\begin{aligned}
W &= \sum_{j=1}^{n} w_j \\
x_j &= \frac{w_j}{S_{0j}} \qquad \text{for } j = 1, \dots, n \\
\sum_{l=1}^{n_j} p_{jl} &\leq x_j \qquad \text{for } j = 1, \dots, n
\end{aligned}
$$

These constraints are linear since the market price of the assets is known. Assuming the premiums from shorted calls are invested in the risk-free rate, at the maturity date in T days the investor's wealth is given by:

$$
W_T = \sum_{j=1}^{n} x_j (S_{Tj} + D_j) + \sum_{j=1}^{n} \sum_{l=1}^{n_j} p_{jl} (C_{jl} e^{r_f T} - \max(S_{Tj} - k_{jl}, 0))
$$

where D_j is the value of any dividend paid out by asset j grown at the risk-free rate, C_{jl} and k_{jl} are the current market price and strike price of call option l for asset j, respectively, r_{f} is the risk-free rate, and S_{Tj} is the price of asset j at the maturity date. Without loss of generality we can set the total wealth W to 1. We then interpret w_j as the proportion of each dollar of wealth invested into asset j, x_j as the number of units of asset j purchased for each dollar of wealth, and p_{jl} as the number of units of call option l for asset j sold for each dollar of wealth. We assume that the total wealth is large enough such that w_j, x_j, and p_{jl} are effectively continuous. The return of the portfolio at the maturity date is given by:

$$\begin{aligned} r = \frac{W_T}{W} - 1 = &\sum_{j=1}^{n} x_j(S_{Tj} + D_j) \\ &+ \sum_{j=1}^{n}\sum_{l=1}^{n_j} p_{jl}(C_{jl}\mathrm{e}^{r_{\mathrm{f}}T} - \max(S_{Tj} - k_{jl}, 0)) - 1 \end{aligned}$$

Risk-return optimization finds the optimal tradeoff between the expected value of r and some risk measure of the distribution of r. Deriving analytical expressions for these quantities would require multiple integrals over the joint distribution of the underlying assets. Whether such integrals would be tractable depends on the assumed underlying joint distribution and the chosen risk measure. Even if solutions were obtainable, the resulting expressions would likely be impractical from an optimization perspective. For example, Figelman (2008) derived analytical expressions for the expected return, variance of the return, and semivariance of the return for a covered call on a single asset by assuming a log-normal distribution for returns. The resulting expressions are highly nonlinear and convoluted, and they are only relevant if a log-normal distribution is assumed. Instead of seeking analytical expressions we propose using the sample expected return and sample risk based on a number of simulated scenarios. From an optimization perspective the simulated scenarios are constants, thus we can readily input values produced using any distribution or method which we like.

If we simulate values of S_{Tj} for all assets j then the return is linear in the decision variables x and p. Thus, for each simulation i we can capture the return in this scenario, r_i, using linear constraints:

$$\begin{aligned} r_i = &\sum_{j=1}^{n} x_j(S^i_{Tj} + D_j) + \sum_{j=1}^{n} \\ &\sum_{l=1}^{n_j} p_{jl}(C_{jl}\mathrm{e}^{r_{\mathrm{f}}T} - \max(S^i_{Tj} - k_{jl}, 0)) - 1 \quad \text{for } i = 1, \dots, N \end{aligned}$$

The sample expected return is given by a linear constraint, where N is the total number of simulations:

$$\mathbb{E}(r) = \frac{1}{N}\sum_{i=1}^{N} r_i$$

The sample variance is given by:

$$\text{risk} = \sigma_r^2 = \frac{1}{N-1}\sum_{i=1}^{N}(r_i - \mathbb{E}(r))^2$$

This formulation has a quadratic objective, which with our linear constraints can be solved rapidly even for a large number of simulations.

The sample semivariance below the risk-free rate is given by:

$$\begin{aligned} &\text{risk} = \text{semivariance} = \frac{1}{N}\sum_{i=1}^{N} z_i^2 \\ &z_i \le r_i - (\mathrm{e}^{r_{\mathrm{f}}T} - 1) \quad \text{for } i = 1, \dots, N \\ &z_i \le 0 \quad \text{for } i = 1, \dots, N \end{aligned}$$

The latter sets of constraints ensure that $z_i = \min(r_i - (\mathrm{e}^{r_{\mathrm{f}}T} - 1), 0)$, i.e., that positive deviations are set to zero. As for variance, this leads to a quadratic optimization problem. Though we choose the risk-free rate as the target, a different reference point can readily be substituted, e.g., $\mathbb{E}(r)$ to optimize the sample semivariance below the expected return.

The sample CVaR_α of the simulated returns can be optimized using the following formulation from Rockafellar and Uryasev (2000):

$$\begin{aligned} &\min \quad q + \frac{1}{N(1-\alpha)}\sum_{i=1}^{N} z_i \\ &z_i \ge -r_i - q, \quad \text{for } i = 1, \dots, N \\ &z_i \ge 0, \quad \text{for } i = 1, \dots, N \end{aligned}$$

The objective function provides the sample CVaR_α. Alexander et al. (2006) show that the Rockafellar and Uryasev (2000) formulation may be ill posed for portfolios of options. In particular, there may be multiple optimal or near-optimal solutions when the feasible set is defined only by the budget constraint and an expected return constraint. Our own formulation has a vastly different feasible set and produces stable results.

Using the expressions presented thus far, we have formulations of risk-return covered call optimization for variance, semivariance, and CVaR. Risk is minimized while the expected return is constrained to exceed a target:

Variance optimization:

$$\underset{w,x,p,\mathbb{E}(r),r}{\text{minimize}} \quad \frac{1}{N-1}\sum_{i=1}^{N}(r_i - \mathbb{E}(r))^2$$

Semivariance optimization:

$$\underset{w,x,p,\mathbb{E}(r),r,z}{\text{minimize}} \quad \frac{1}{N}\sum_{i=1}^{N} z_i^2$$

CVaR optimization:

$$\underset{w,x,p,\mathbb{E}(r),r,z,q}{\text{minimize}} \quad q + \frac{1}{N(1-\alpha)}\sum_{i=1}^{N} z_i$$

Common constraints:

$$\begin{aligned}
& w_j \geq 0 \quad \text{for } j = 1, \dots, n \\
& p_{jl} \geq 0 \quad \forall\, j, l \\
& \sum_{j=1}^{n} w_j = 1 \\
& \frac{w_j}{S_{0j}} - x_j = 0 \quad \text{for } j = 1, \dots, n \\
& \quad - x_j + \sum_{l=1}^{n_j} p_{jl} \leq 0 \quad \text{for } j = 1, \dots, n \\
& \sum_{j=1}^{n}\sum_{l=1}^{n_j} p_{jl}(C_{jl}\mathrm{e}^{r_{\mathrm{f}}T} - \max(S_{Tj}^i - k_{jl}, 0)) \\
& \quad + \sum_{j=1}^{n} x_j(S_{Tj}^i + D_j) - r_i = 1 \quad \text{for } i = 1, \dots, N \\
& \mathbb{E}(r) - \frac{1}{N}\sum_{i=1}^{N} r_i = 0 \\
& \quad - \mathbb{E}(r) \leq -r_{\text{target}}
\end{aligned}$$

Additional constraints for semivariance optimization:

$$\begin{aligned}
z_i - r_i &\leq -(\mathrm{e}^{r_{\mathrm{f}}T} - 1) \quad \text{for } i = 1, \dots, N \\
z_i &\leq 0 \quad \text{for } i = 1, \dots, N
\end{aligned}$$

Additional constraints for CVaR optimization:

$$\begin{aligned}
-r_i - z_i - q &\leq 0 \quad \text{for } i = 1, \dots, N \\
z_i &\geq 0 \quad \text{for } i = 1, \dots, N
\end{aligned}$$

All constraints are linear. Additionally, the CVaR objective function is linear. The variance and semivariance objectives are convex, quadratic functions.

It must now be noted that the chosen risk measures are not without limitations. Generally, since the sale of call options limits return on the up-side, the return distribution of a covered call strategy has a significant negative skew. Therefore variance of the return is not an appropriate risk measure for a covered call investor who has a preference toward skewness. Nonetheless we have included variance due to its ubiquity in asset management. Figelman (2008) suggested using semivariance to focus on down-side cases and address the problem of skewness. One problem with semivariance is that it is defined relative to an arbitrary point, e.g., semivariance below the risk-free rate. A better choice still may be CVaR which is widely used in asset management and simply averages returns in the worst scenarios. It is intuitive to understand and also addresses the problem of skewness.

Even semivariance and CVaR are limited risk measures in that they only capture preferences of the lower tail end of the return distribution. Since we are considering a portfolio composed of covered calls on numerous assets with varied correlations, it isn't clear what distribution the resulting portfolio's return will have. A covered call portfolio's return distribution is likely highly irregular with high-order moments involved. An investor could very well have preferences regarding higher moments beyond just the left tail portion of the return distribution. In this case CVaR and semivariance would not suffice. One possible solution may be to use a utility function and maximize the expected utility of the return rather than optimize the risk-return tradeoff, a topic which we leave for future work. Later we discuss a structured solution policy which results from all risk measures under consideration. Though exact solutions may differ, we expect that the structured policy also exists when using other risk measures or maximizing an expected utility.

Policy analysis

When performing risk-return optimization it is common practice to solve for portfolios along the entire risk spectrum. At one extreme this involves finding the largest expected return target, r_{target}, for which the problem remains feasible. This is done by maximizing the expected return and ignoring the portfolio's risk level.

Under the constraints listed in Sect. 2, it is possible to analytically derive the optimal policy to maximize the expected return. Consider the expected return:

$$\mathbb{E}(r) = \sum_{i=1}^{N}\frac{1}{N} r_i$$

$$\mathbb{E}(r) = \sum_{i=1}^{N}\frac{1}{N}\left(\sum_{j=1}^{n}\sum_{l=1}^{n_j} p_{jl}(C_{jl}\mathrm{e}^{r_{\mathrm{f}}T} - \max(S_{Tj}^i - k_{jl}, 0)) + \sum_{j=1}^{n} x_j(S_{Tj}^i + D_j) - 1\right)$$

Substituting x_j with its defining expression:

$$\mathbb{E}(r) = \sum_{i=1}^{N} \frac{1}{N}\left(\sum_{j=1}^{n}\sum_{l=1}^{n_j} p_{jl}(C_{jl}e^{r_f T} - \max(S_{Tj}^i - k_{jl}, 0)) + \sum_{j=1}^{n} w_j \frac{(S_{Tj}^i + D_j)}{S_{0j}} - 1\right)$$

Rearranging the summation order and simplifying:

$$\mathbb{E}(r) = \sum_{j=1}^{n}\sum_{l=1}^{n_j} p_{jl}\left(C_{jl}e^{r_f T} - \sum_{i=1}^{N}\frac{1}{N}(\max(S_{Tj}^i - k_{jl}, 0))\right) + \sum_{j=1}^{n} w_j \frac{\left(\sum_{i=1}^{N}\frac{1}{N}(S_{Tj}^i) + D_j\right)}{S_{0j}} - 1$$

For a sufficiently large number of simulations N the sample averages are approximately equal to expectations:

$$\mathbb{E}(r) = \sum_{j=1}^{n}\sum_{l=1}^{n_j} p_{jl}\big(C_{jl}e^{r_f T} - \mathbb{E}(\max(S_{Tj} - k_{jl}, 0))\big) + \sum_{j=1}^{n} w_j \frac{(\mathbb{E}(S_{Tj}) + D_j)}{S_{0j}} - 1$$

The difference between the expected call payout at the maturity date and the call price grown at the risk-free rate is the call risk premium (CRP) as defined in Figelman (2008):

$$\begin{aligned} \mathrm{CRP}_{jl} &= \mathbb{E}(\max(S_{Tj} - k_{jl}, 0)) - C_{jl}e^{r_f T} \\ \mathrm{CRP}_{jl} &= \sum_{i=1}^{N}\frac{1}{N}\max(S_{Tj}^i - k_{jl}, 0) - C_{jl}e^{r_f T} \\ \mathbb{E}(r) &= \sum_{j=1}^{n}\sum_{l=1}^{n_j} p_{jl}(-\mathrm{CRP}_{jl}) + \sum_{j=1}^{n} w_j \frac{(\mathbb{E}(S_{Tj}) + D_j)}{S_{0j}} - 1 \end{aligned} \tag{1}$$

Now consider the problem of maximizing the expected return; we may ignore variables and constraints related to modeling risk:

$$\begin{aligned} \underset{w,p}{\text{maximize}} \quad & \mathbb{E}(r) = \sum_{j=1}^{n}\sum_{l=1}^{n_j} p_{jl}(-\mathrm{CRP}_{jl}) + \sum_{j=1}^{n} w_j \frac{(\mathbb{E}(S_{Tj}) + D_j)}{S_{0j}} - 1 \\ \text{subject to} \quad & \sum_{j=1}^{n} w_j = 1 \\ & w_j \geq 0 \quad \text{for } j = 1, \dots, n \\ & p_{jl} \geq 0 \quad \forall\, j, l \\ & \sum_{l=1}^{n_j} p_{jl} \leq \frac{w_j}{S_{0j}} \quad \text{for } j = 1, \dots, n \end{aligned} \tag{2}$$

Since we are only interested in maximizing the expected return, the problem above is applicable to all risk measures.

Consider the problem of selecting p for fixed asset weights w. The expression involving w in the objective function is then constant and can be ignored:

$$\begin{aligned} \underset{p}{\text{maximize}} \quad & \sum_{j=1}^{n}\sum_{l=1}^{n_j} p_{jl}(-\mathrm{CRP}_{jl}) \\ \text{subject to} \quad & p_{jl} \geq 0 \quad \forall\, j, l \\ & \sum_{l=1}^{n_j} p_{jl} \leq \frac{w_j}{S_{0j}} \quad \text{for } j = 1, \dots, n \end{aligned}$$

Since for $j \neq k$ the variables p_{jl} and p_{kl} do not share any constraints and do not interact in the objective function, the problem above can be decomposed into n subproblems. We can examine the subproblem of finding p_{jl} for each asset j individually:

$$\begin{aligned} \underset{p}{\text{maximize}} \quad & \sum_{l=1}^{n_j} p_{jl}(-\mathrm{CRP}_{jl}) \\ \text{subject to} \quad & p_{jl} \geq 0 \quad \text{for } l = 1, \dots, n_j \\ & \sum_{l=1}^{n_j} p_{jl} \leq \frac{w_j}{S_{0j}} \end{aligned}$$

The solution to the subproblem is evident. We find option l which has the lowest CRP among the available options for asset j. If the CRP of this option is negative, the solution is to set p_{jl} to its maximum value of w_j/S_{0j} and all other variables to 0. If all options have a positive CRP, then the solution is to set all variables to zero, i.e., no call overwriting on asset j. The value of the objective function at optimality is therefore:

$$\left(\underset{p}{\text{maximize}} \quad \sum_{l=1}^{n_j} p_{jl}(-\mathrm{CRP}_{jl})\right) = \frac{w_j}{S_{0j}}\big(-\min(\mathrm{CRP}_{j1}, \dots, \mathrm{CRP}_{jn_j}, 0)\big)$$

For convenience, we use V_j to denote the minimum of the CRP of all available options for asset j and 0:

$$\begin{aligned} \min(\mathrm{CRP}_{j1}, \dots, \mathrm{CRP}_{jn_j}, 0) &= V_j \\ \left(\underset{p}{\text{maximize}} \quad \sum_{l=1}^{n_j} p_{jl}(-\mathrm{CRP}_{jl})\right) &= \frac{w_j}{S_{0j}}(-V_j) \end{aligned} \tag{3}$$

We know that for any given value of w_j we should follow the optimal policy for p_{jl} derived above and obtain the return benefit shown in (3). Therefore, the objective of problem (2) becomes:

$$
\begin{aligned}
\underset{w}{\text{maximize}} \quad \mathbb{E}(r) &= \sum_{j=1}^{n}\sum_{l=1}^{n_j} p_{jl}(-\mathrm{CRP}_{jl}) + \sum_{j=1}^{n} w_j \frac{(\mathbb{E}(S_{Tj}) + D_j)}{S_{0j}} - 1 \\
&= \sum_{j=1}^{n} \frac{w_j}{S_{0j}}(-V_j) + \sum_{j=1}^{n} w_j \frac{(\mathbb{E}(S_{Tj}) + D_j)}{S_{0j}} - 1 \\
&= \sum_{j=1}^{n} w_j \frac{(\mathbb{E}(S_{Tj}) + D_j - V_j)}{S_{0j}} - 1
\end{aligned} \tag{4}
$$

If w is only constrained by the budget constraint $\sum w_j = 1$, upper bound constraints $w_j \leq U_j$, and no shortselling, then to maximize expected return we place as much weight as possible into w_j which has the largest coefficient in (4) until it reaches its upper bound, then place as much weight as possible into w_j which has the second largest coefficient in (4), and so on until the budget constraint is binding. The coefficient of w_j in (4) assumes that after w has been decided the optimal policy for p_{jl} will be followed, i.e., for each asset j we look for the option with the lowest CRP as defined in (1); if the lowest CRP is negative, then the option is sold in the maximum amount (w_j/S_{0j}), if the lowest CRP is positive, we do not overwrite calls on asset j.

Many arguments can be made that CRP values ought to be positive. Figelman (2008) assumed that CRP values should be positive if the underlying equity has a positive equity risk premium. McIntyre and Jackson (2007) show that from a Black–Scholes perspective holding a call is equivalent to holding some portion of the underlying equity, which presumably ought to have a positive risk premium. A long call option position is risky, so a purchaser may expect to be compensated for accepting that risk. However, a short call position is also extremely risky for the seller who may incur potentially unlimited losses. It isn't clear why a seller would be willing to accept this risk without expecting a risk premium themself. Market prices contain some evidence that option prices may favor sellers. It has been empirically observed that the implied volatility by assuming that option prices follow the Black–Scholes model is frequently higher than the realized volatility of the underlying asset. This suggests that from a Black–Scholes perspective, i.e., if assets have log-normally distributed returns, options may be overpriced. Figelman (2009) argued that the volatility premium still existed in the presence of non-normality. If an option is overpriced, its CRP would be negative and there would be an expected return benefit for a seller and an expected loss for a buyer. It is important to note that the call risk premiums as computed in this methodology in (1) are dependent on the inputted scenarios. As such, the particular method used to produce the scenarios could also lead to erratic CRP estimates.

Though we have derived the optimal policy for the risk-neutral goal of maximizing the expected return, it is not clear what the optimal policy is for a risk-averse investor. Regardless of the asset positions, we expect the optimal minimum risk portfolios to involve selling at-the-money (ATM) options, or selling in-the-money (ITM) options if they are under consideration. Further in-the-money options reduce variance and down-side risk by providing larger premiums and also reduce variance by more severely restricting upside gain.

Implementation

We test the methodology by using the S&P 500 Index, the Morgan Stanley Capital International (MSCI) Europe, Australasia and Far East (EAFE) Index, and the MSCI Emerging Markets (EM) Index as underlying assets. We form covered call portfolios by selling call options on these assets. Following the findings of other authors, we consider only options with a one month maturity date which tend to have the highest implied volatility. We consider only options which are ATM or out-of-the-money (OTM) at the time the optimization is performed. In practice an investor may wish to include ITM options as they may be appropriate to sell given a bearish outlook. Historical data are limited since exchange-traded European call options for the EAFE and EM indices were only introduced by the Chicago Board Options Exchange (CBOE) in April 2015. Call option data for May 2015 to August 2015 were obtained from OptionMetrics. Data for September 2015 to May 2016 were obtained via Bloomberg. As bid-ask spreads were often large, we used the average of the best bid and best ask prices as the market price of an option; in practice an investor should be careful to use values which they believe best predict the execution price. The optimization is conducted once a month on the day which the previous month's options expired. The 30 day US Treasury Bill rate is used as the risk-free rate.

In order to use the methodology we must simulate asset prices at maturity. According to Figelman (2009), the expected return of far out-of-the-money options may be poorly estimated by using standard geometric Brownian motion versus a more realistic process. Diaz and Kwon (2017) utilized a stochastic volatility with correlated jumps (SVCJ) model for stock returns and found that optimal covered call positions were similar when using the SVCJ model and classic geometric Brownian motion. We use geometric Brownian motion for the purposes of examining the structure of covered call portfolios.

We simulate asset values at maturity by simulating multivariate standard normal variables, $Z^i = (Z_1^i, Z_2^i, Z_3^i)$, with correlation equal to that of the log-returns of the three underlying assets. These values are then used to simulate asset prices at maturity via geometric Brownian motion, for asset j and scenario i:

Table 1 Drift and volatility rates used for simulation

Asset	Annualized return (%)	Annualized volatility (%)	Annual dividend rate (%)
S&P 500	6.5	16	2.2
EAFE	6.1	15.5	3.4
EM	7.3	18	2.7

Dividend rates used in optimization

$$S^i_{Tj} = S_{0j}e^{(\mu_j - 0.5\sigma_j^2)T + \sigma_j\sqrt{T}Z^i_j}$$

Values of μ and σ used reflect the long-term return and volatility of the three assets, summarized in Table 1. While we have used these simulations for illustrative purposes, in practice an investor should simulate asset prices which they believe best predict the behavior of the underlying assets; a wealth of literature already exists in this topic.

We optimize using $N = 15{,}000$ simulations which we find is sufficiently large to produce consistent results. All problems were solved using CPLEX. Optimization of CVaR is a linear program and utilized the CPLEX simplex method. Optimization of semivariance and variance are quadratic programs and utilized the CPLEX quadratic simplex method.

Results

Sample results are shown in Fig. 1 for the month of August 2015; optimal solutions showed complex structures as in Fig. 1 for most months tested. We find that production of efficient frontiers of 50 points generally takes 2 to 3 min for variance and CVaR optimization, and 5 to 6 min for semivariance optimization.

As expected, the maximum return portfolio is the same across all risk measures for any given month. The result in Fig. 1 for the maximum return portfolio matches our prediction from Sect. 3. All options for the S&P 500 and EAFE have positive CRP estimates; thus, from an expected return perspective it is optimal not to overwrite either of these assets. The 13% OTM option has the lowest estimated CRP among all EM options, and this estimate is negative. The greatest expected return for an EM position therefore occurs when the position is fully overwritten using this call option. Since a long EM position overwritten with the 13% OTM call has the greatest expected return, the optimal solution from an expected return perspective is to put as much weight as possible into this combination. This matches the empirical result seen in Fig. 1.

The optimal mixes for all three risk measures prominently feature EM. It has often been noted that risk-return optimization using point estimates for the risk and expected return produces concentrated portfolios which are sensitive to the input values. Because EM has a slightly higher expected total return in Table 1, it has a very large portfolio weight at higher expected return targets. One way to combat this sensitivity is to perform resampled optimization, for example as in the Resampled Efficient Frontier in Chapter 6 of Michaud and Michaud (2008). In short, we produce multiple optimal mixes using resampled estimates of the data in Table 1 and blend the resulting mixes. The resampling and blending procedures are described in detail in "Appendix 1".

An alternative to Michaud resampling is distributionally robust stochastic optimization (DSRO, see Gao and Kleywegt (2016) for details). In standard stochastic optimization a quantity (e.g., risk) is optimized over a distribution of random variables (e.g., asset returns). In DSRO a quantity is optimized over the worst-case distribution from a set of distributions. Michaud resampling blends the solutions from optimizing over a set of distributions one distribution at a time. Compared to Michaud resampling DSRO is conceptually more complex and difficult to implement, this may preclude its usefulness in a practical asset management setting. Furthermore there is an existing precedent for the use of Michaud resampling; it is widely used and well known in portfolio management. Because Michaud resampling optimizes using a range of distributions, it can provide insight into optimal solutions across a wide range of input parameters.

The resulting mix from Michaud resampling for August 2015 is shown in Fig. 2. We observe in Fig. 2 that all minimum risk portfolios continue to overwrite exclusively with ATM options. This is intuitive since ATM options provide the largest premiums to counter down-side losses and the largest liabilities against up-side gain (given that ITM options are not under consideration). These facts do not change when resampling since the premiums and strike prices are fixed. However, the blended maximum return portfolio differs substantially from the concentrated portfolio in Fig. 1. Whereas previously EM had the highest total expected return leading to a concentrated position, there are resampling cases where the S&P 500 or EAFE offer the highest expected total return. This leads to some resampling cases where the maximum return portfolio has a concentrated position in S&P 500 or EAFE. When blended these lead to the maximum return weights seen in the resampled optimal mix; approximately 0.46 in EM, 0.28 in S&P 500, and 0.26 in EAFE. These weights can be interpreted as the probability that each asset (and its corresponding optimal call overwrite) has the highest expected total return according to the view reflected in our estimates in Table 1 and our degree of confidence implicit in the resampling scheme.

Aside from the underlying asset weights, the resampling of the expected return and risk also creates diversity in the

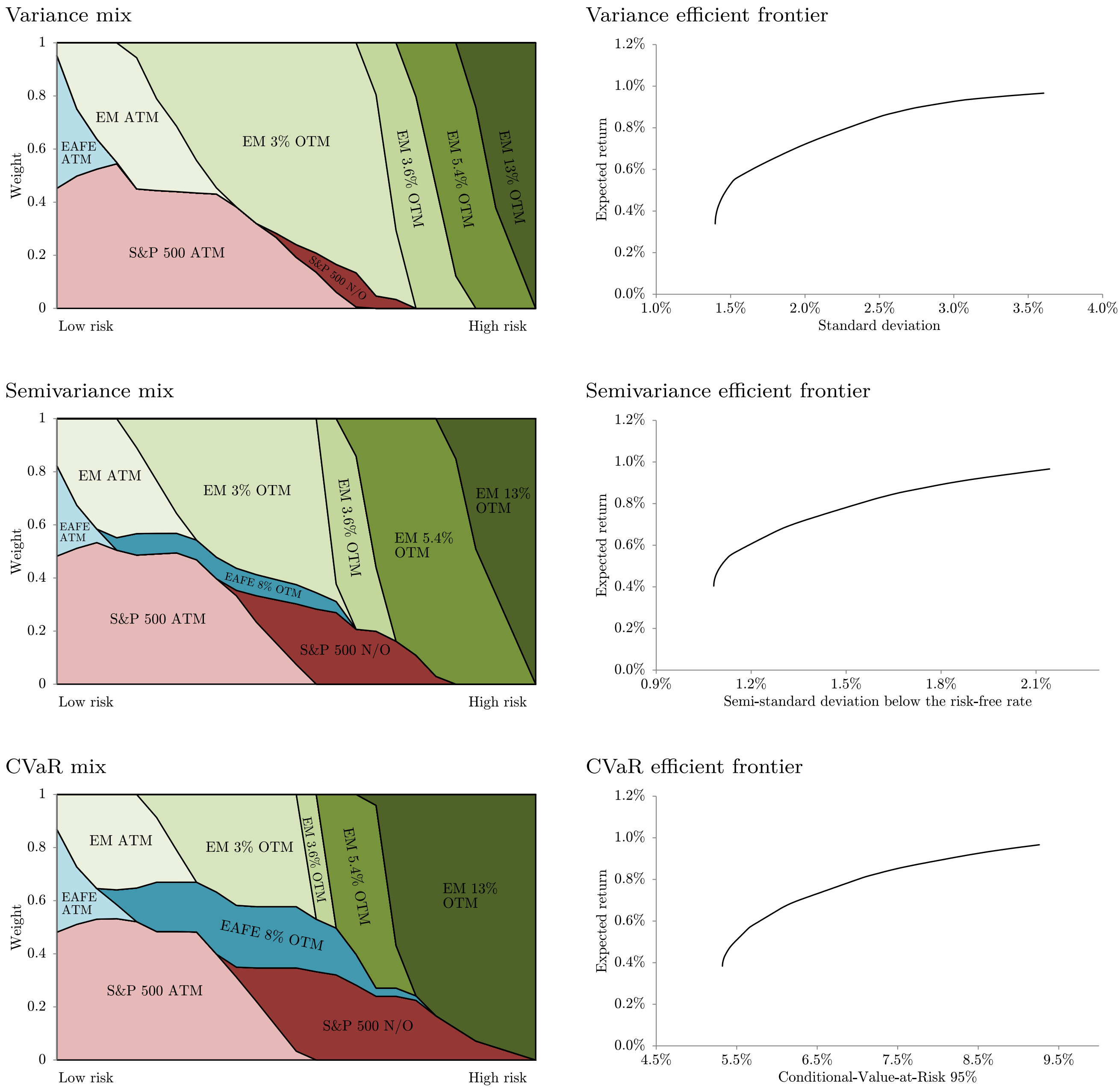


Fig. 1 Sample optimal mixes and frontiers for different risk measures. Areas simultaneously indicate asset weights and call overwriting weights. "N/O" indicates that an asset was held but some of its units were not overwritten. All values are monthly

optimal call overwriting positions. The resampled parameters cause changes to the expected call payoffs at maturity, these in turn effect changes to the CRP estimates. Thus optimal call positions differ between resampling iterations. We still observe that some call options are never sold. This is likely because other options consistently offered more attractive risk-return tradeoffs.

The results of Michaud resampling demonstrate that optimal covered call portfolios are sensitive to assumptions about the expected return and volatility of the underlying assets. Another assumption we have made is that the underlying asset prices follow geometric Brownian motion. We test the sensitivity of the results to the shape of the underlying distributions by rerunning the results from Fig. 2 assuming a GARCH(1,1) model for the assets' volatilities (see "Appendix 2" for details). When using geometric Brownian motion with constant volatilities the resulting distribution of log-returns is of course normal with no skewness and kurtosis of 3. When using the GARCH driven volatilities skewness remains negligible

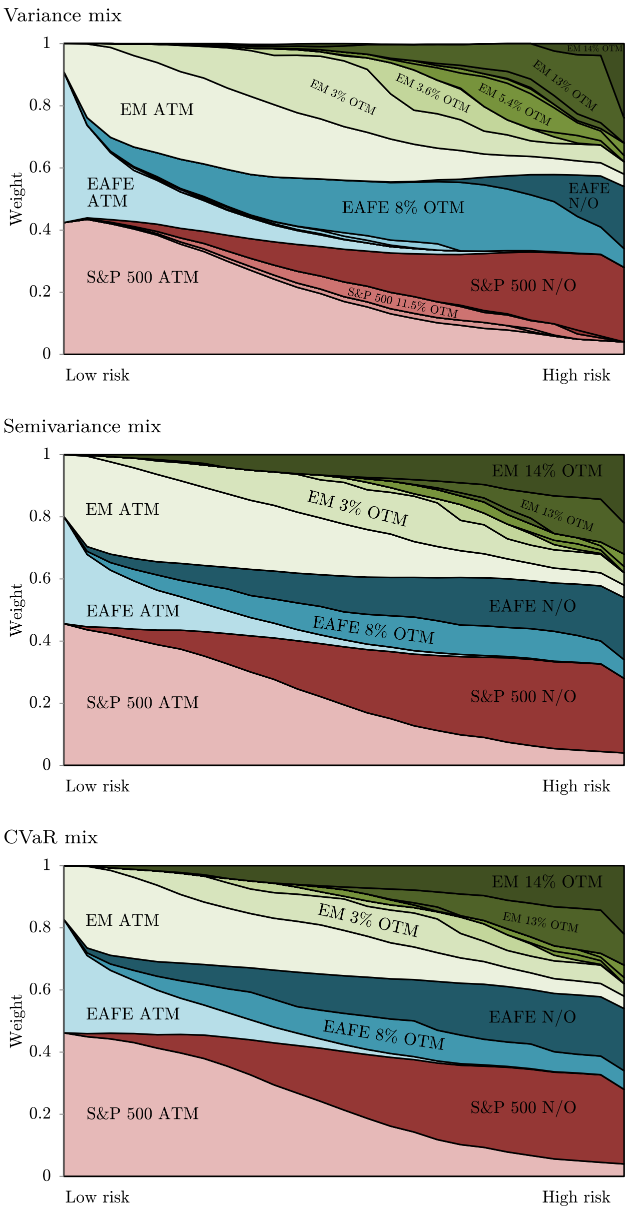


Fig. 2 Sample optimal mixes for different risk measures using Michaud resampling. Areas simultaneously indicate asset weights and call overwriting weights. “N/O” indicates that an asset was held, but some of its units were not overwritten

but substantial kurtosis is produced; the kurtosis of the resulting distributions of log-returns of the S&P 500, EAFE, and EM indices are 3.8, 3.3, and 4.1, respectively. The resulting optimal mixes shown in Fig. 3 differ moderately from those in Fig. 2. It appears that the results are less sensitive to changes in kurtosis than to changes in the first or second moments. Another possibility is that geometric Brownian motion with GARCH volatility is not sufficiently irregular to accurately model the higher moments of the underlying asset returns.

Scaling

A commercial covered call fund may have many more than three assets. To examine how the formulation scales we test it on individual equities. For the test we select the largest 5, 10, 20, 40, 60, and 92 equities in the S&P 500 Index by market capitalization. Since exchange-traded options on these assets are only offered with American exercise style, data for European options on these assets is unavailable. As a proxy we use American option prices for these assets; the call price is taken as the highest closing bid. To avoid arbitrage the price of an American style option must always be higher than any immediate payout from exercising it. Thus American options are rarely executed before their maturity date and can readily be used as a substitute to European options in a covered call strategy.

The expected returns and volatilities of the equities were estimated using 1 year of historical data leading up to the beginning of the sample period. We use these estimates only to analyze the structure of optimal covered calls. In practice a manager should carefully estimate the distributions which they believe best reflect the behavior of assets for the time period under consideration; an abundance of literature exists on this subject. Simulations are produced as in Sect. 4. We vary the number of simulations from 100 to 12,800. As the number of assets and simulations increases, the times to produce efficient frontiers grow as shown in Fig. 4. Although semivariance solution times become extremely large, variance and CVaR solution times remain viable.

Sample solution results for the month of August 2015 are shown in Fig. 5. The 92 assets had a total of 702 call options available to sell, for comparison the S&P 500, EM, and EAFE indices had a total of 72 call options available to sell in the same month. Using point estimates without further constraints again leads to concentrated positions, and we again use Michaud resampling to optimize portfolios of the 92 assets. As in Sect. 5 this leads to an enormous variety of assets and call options in the optimal portfolios, but this time the number of positions may be problematic. In the worst case one of the variance optimized portfolios holds 52 equities and sells 230 different call options, far too many option positions for practical use. However, many positions are very small; over 95% of the wealth is overwritten with just 65 options. Thus some rounding may be all that is needed to produce a portfolio with a practical number of positions while still being nearly optimal.

In Figs. 5 and 6 we observe a structured policy where to minimize risk all assets are fully overwritten with ATM options. At this end of the risk spectrum we ought to simply seek the options with the lowest strike prices and thus the highest market prices, the other portion of the CRP

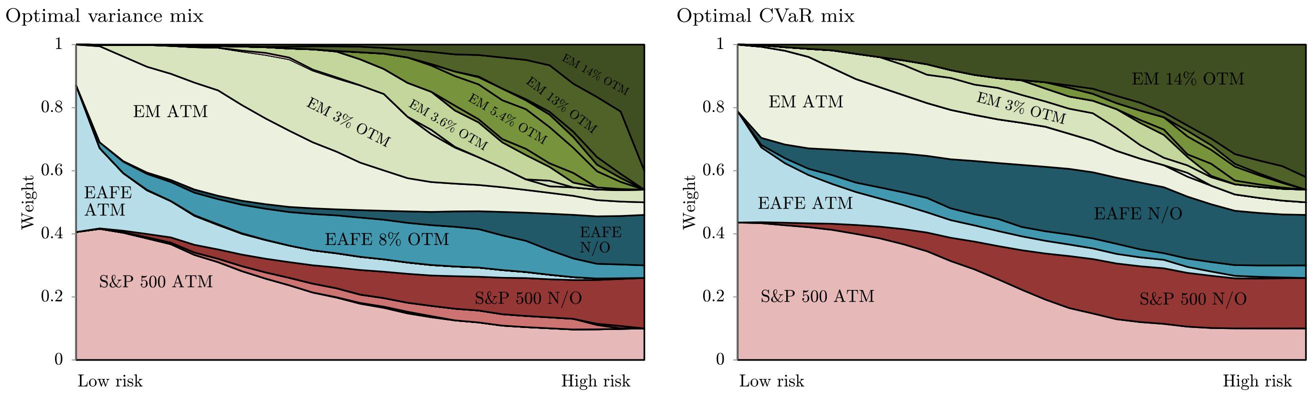


Fig. 3 Sample optimal mixes for different risk measures using Michaud resampling and GARCH volatility. Areas simultaneously indicate asset weights and call overwriting weights. "N/O" indicates that an asset was held but some of its units were not overwritten. Results for semivariance are omitted as they closely resemble those of CVaR

estimate (the expected call payoff) is irrelevant. As risk averseness decreases the amount of wealth overwritten drops, and the options sold have higher strike prices, lower expected payoffs, and likely lower CRP estimates. Finally, a risk-neutral investor should seek to sell only the options with the lowest estimated CRP values (those thought to be the most overpriced). The maximum return portfolio still features 64% of all wealth overwritten. This indicates that CRP estimates were negative across a wide range of expected return and volatility inputs. We also note that the optimal portfolios frequently hold equity weight without overwriting, and sell multiple options with different strike prices on a single asset. Previous studies had typically assumed that the equity position is fully overwritten using a single call option.

In all cases tested we found that semivariance and CVaR optimized portfolios were nearly identical, unsurprising since they are both down-side risk measures. We recommend using CVaR instead since using semivariance requires longer solution times and offers no clear advantage. When using a point estimate for the expected return and volatility, we find that over 10,000 simulations are needed to produce consistent results. However, when using resampling we find 2000 simulations per each of the 50 resampling iterations is sufficiently large to produce consistent results. Since the solution time scales rapidly as a function of the number of simulations this means that solving for stable resampled frontiers is faster than solving for stable results using point estimates. I.e., for a given risk measure, solving for one frontier using 15,000 simulations is far slower than solving for 50 frontiers using 2000 simulations each, even when these frontiers are not optimized in parallel.

Performance

In practice covered calls are sometimes formed as an overlay on an existing equity portfolio. However, a covered call portfolio constructed by optimizing the underlying asset positions then applying a short call overlay is not risk-return optimal in general. The sample results in Fig. 7 suggest that to form a risk-return optimal covered call portfolio it is necessary to simultaneously optimize the call overwriting weights and the underlying asset positions. The top row of Fig. 7 displays optimal equity mixes when call options are not under consideration. A covered call overlay could then be applied to these fixed portfolios. The resulting covered call strategies from this two-step process differ substantially from the optimal mixes shown in the middle and bottom rows of Fig. 7 where equity and option positions were optimized simultaneously. When call options are not included, the optimal portfolio weights are well diversified over the available assets. However, using data from December 2014 when calls are included in the optimization much of the equity weight shifts to a handful of assets. In this particular month these assets had ATM call options trading at relatively high market prices, thus, selling these calls and holding the assets created a very attractive covered call position. In order to form risk-return optimal covered call portfolios, asset positions must sometimes be selected to exploit the market prices of available call options. This is only possible when call positions and underlying asset positions are optimized simultaneously. It isn't necessarily the case that the presence of call options dramatically alters the optimal equity positions. The equity positions of the optimal covered call portfolios in June 2015 resemble the positions when equities were optimized alone.

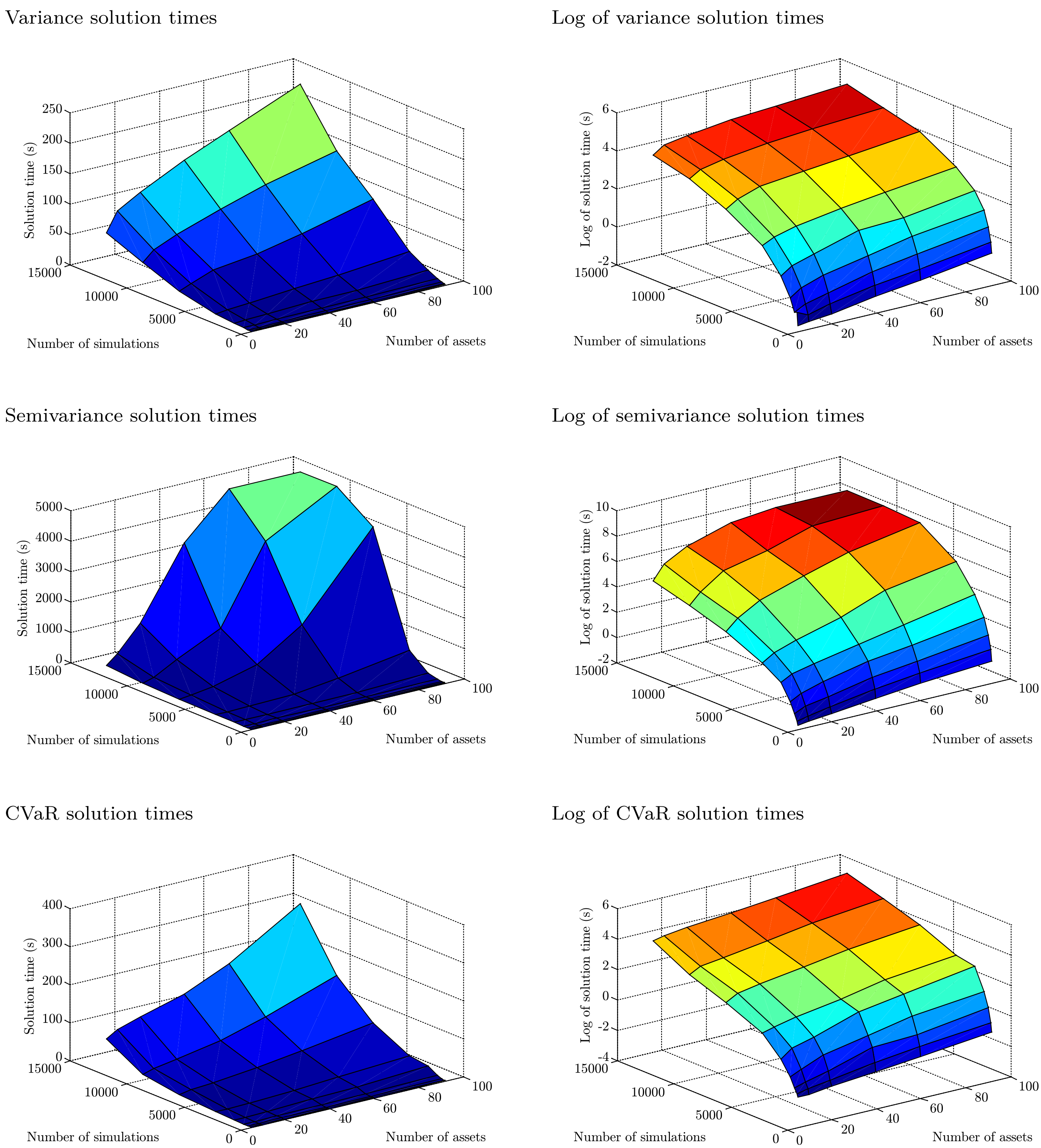


Fig. 4 Solution times for different risk measures varying the number of assets and simulations. Data points represent the mean time to produce an efficient frontier of 50 points for 9 trial months. Solution times were capped at 5000 s

We can also compare the performance of the optimized covered call portfolios versus the optimal equity only portfolios with call overlays applied. We form three conventional call strategies by taking the optimal equity portfolios and fully overwriting it with ATM calls, 5% OTM calls, and 10% OTM calls in each month. The actual moneyness of options sold varies around these targets since the offered strike prices have discrete increments. In each of the nine sample months we select the optimized covered call and conventional covered call portfolios which provide the

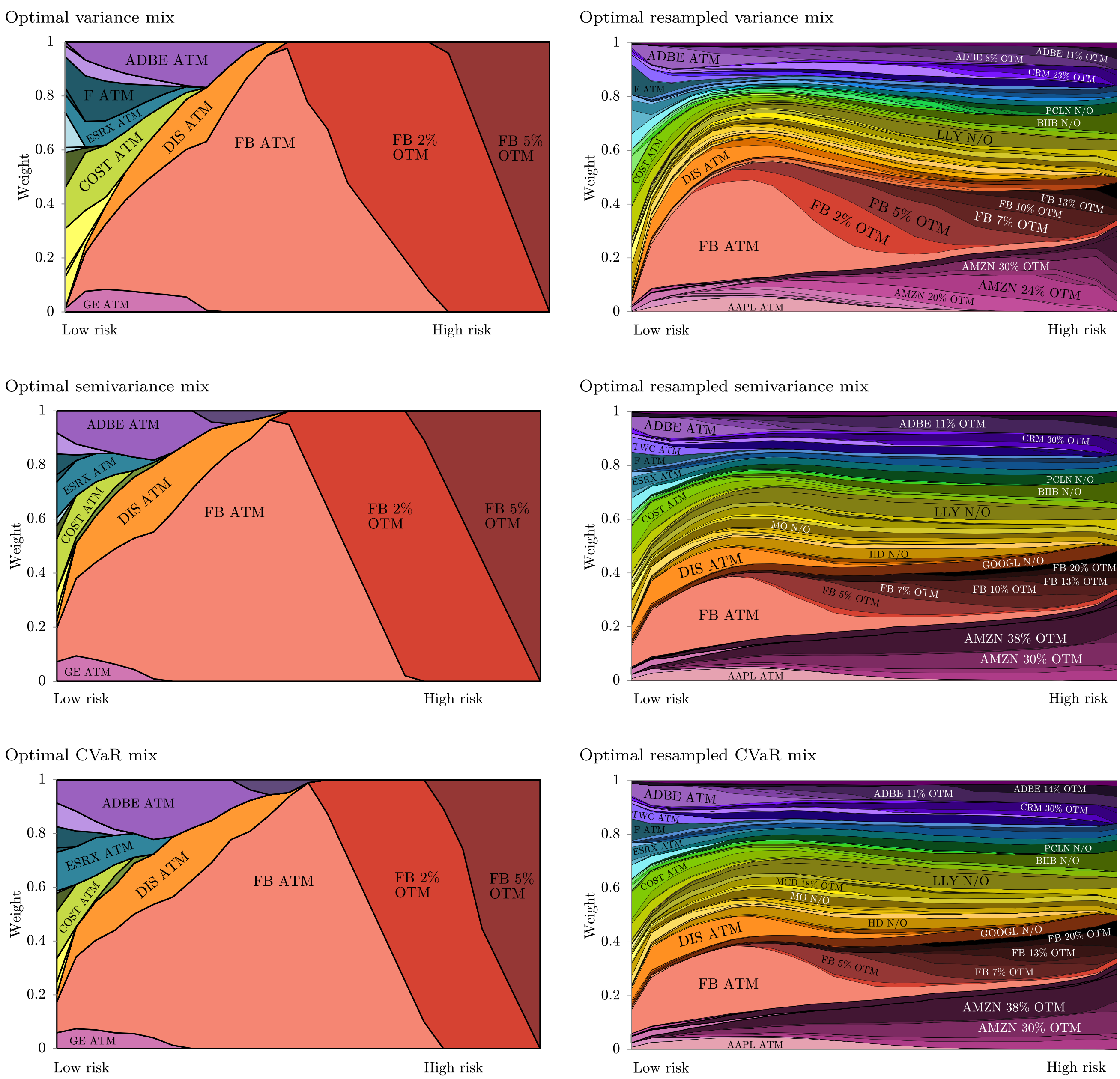
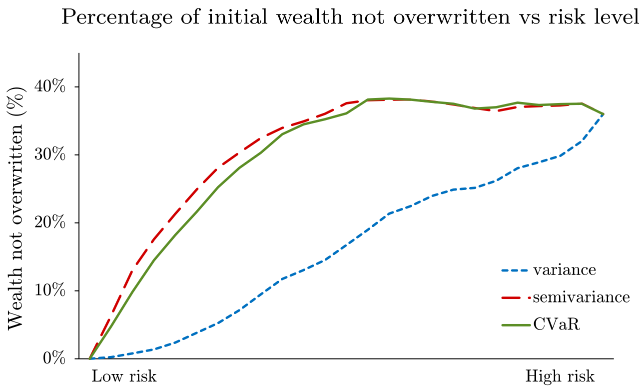


Fig. 5 Optimal portfolios mixes using 92 large-cap US equities in August 2015. Major positions are labeled. Left without resampling, right with Michaud resampling. Areas simultaneously indicate asset weights and call overwriting weights. "N/O" indicates that an asset was held but some of its units were not overwritten

maximum in-sample expected return and evaluate their out-of-sample realized return at maturity one month later. The total returns over the nine month sample period are displayed in Table 2. For reference, we have also included the total return of the equity positions of the optimized covered call portfolios and the total return of the optimized equity portfolio without a covered call overlay. About 4% of the difference in total return between the optimized and conventional covered calls is accounted for by the difference in equity positions, the remainder of the differences are due to the option positions. The conventional overlays which targeted a level of moneyness all produced significant liabilities. Covered calls are not usually thought to improve returns except perhaps on a risk adjusted basis. However, as Figelman (2008) notes and as we demonstrated in Sect. 3, a short call option's contribution to the expected return is given by the negative of its CRP. The CRP is defined as the expected liability at maturity less the price of the option grown at the risk-free rate until maturity. Forming a covered call overlay based on a fixed moneyness level ignores the information



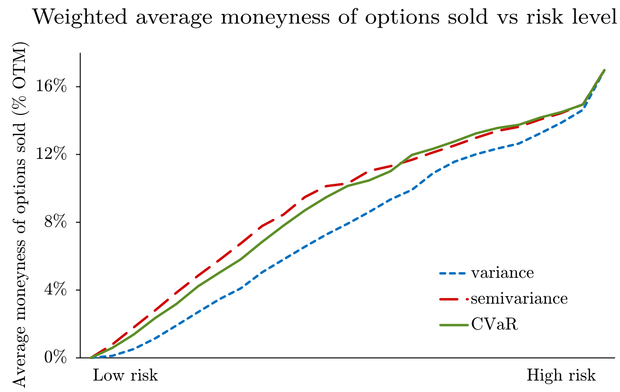


Fig. 6 Both charts display information relating to the resampled portfolios in Fig. 5. The left chart displays the percentage of wealth without call option overwriting as a function of portfolio risk. The right chart displays the average moneyness of options sold as a function of portfolio risk. The average moneyness is weighted by the amount of wealth overwritten by each option

provided by the options' market prices. This could lead to selling options which generally have unfavorable prices and could reduce the expected and realized returns as in Table 2. In contrast, our optimization model uses the option market prices as an input, and to maximize returns sells options that are suspected to have a negative CRP based on the views implicit in the simulation scheme. We see in Table 2 that the option positions in the optimal covered call strategy added about 80 basis points of total return versus the underlying equity position over the nine month sample period. This provides evidence that calls with negative call risk premiums exist, and that selling call options can provide an increase to expected return if selected carefully. Over the nine month period, the optimal covered call portfolios sell options over a wide range of moneyness. This suggests that negative CRP estimates do not occur at a specific moneyness level. Though we have only tested the maximum expected return portfolios, the optimization model ought to significantly improve out-of-sample risk-return efficiency by identifying options which offer a disproportionately large reduction to risk via their selling premium (the market price) versus their potential liability. Additional data and more extensive testing is required to understand the out-of-sample risk and return characteristics of optimized covered call portfolios versus conventional covered calls.

Conclusion

Covered call portfolios formed by overlaying short option positions on an existing portfolio are not optimal in general, and conventional strategies which sell options at a fixed level of moneyness fail to consider the impact of the options' market prices. Our risk-return optimization framework is able to simultaneously select underlying asset positions and call option overwriting weights to produce optimal covered call portfolios. An essential part of the model is the use of the options' market prices in order to assess risk and return. When optimizing variance and semivariance, the model is quadratic, while when optimizing CVaR the model is linear. Thus the models are easy to implement and can be solved by a wide range of commercial solvers. We find that the model scales without major difficulty to portfolios using 92 assets and over 500 call options. In all cases we find that solutions from optimizing semivariance closely resemble solutions from optimizing CVaR. Optimal portfolio mixes generally exhibit a structured policy where a sufficiently risk-averse investor should only overwrite using ATM options and as they become less risk averse should sell further OTM options and gradually decrease the amount of wealth being overwritten. We find that optimal portfolios often involve holding equity positions without call overwriting and overwriting an equity position with more than one call option. This contrasts previous studies which typically considered covered calls composed of a single asset fully overwritten with a single call option. In many cases we find that the solution of resampled frontiers involves a significant amount of overwriting even when attempting to maximize the expected return. This suggests that there are some call options which may be overpriced according to a wide range of views. Future work could test the out-of-sample risk and return of optimized covered call portfolios over extended periods of time and in different equity markets, and compare this to conventional covered call portfolios and alternate methodologies. Future work could also use less stylized scenario distributions to model the effects of higher order moments in the underlying asset returns and optimize a utility function to capture preferences along the portfolio's entire return distribution.

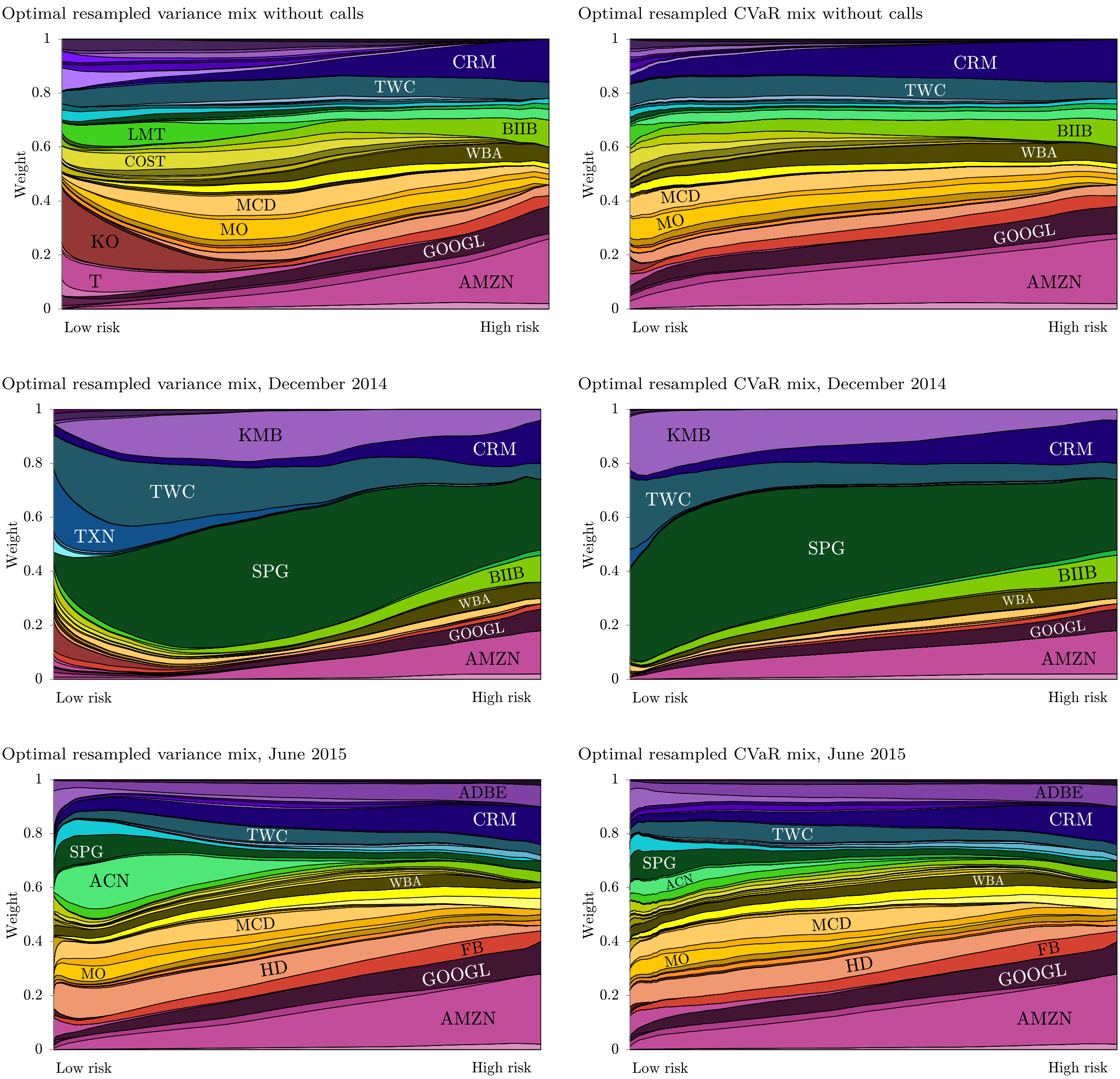


Fig. 7 Optimal equity mixes using 92 large-cap US equities. Major positions are labeled. Top row optimized without options available, middle and bottom rows optimized using the options available in December 2014 and June 2015, respectively. Note that for clarity call overwriting weights are not shown, only the underlying equity weights are displayed. Results for semivariance are omitted as they closely resemble those of CVaR

Table 2 Realized returns of sample portfolios

Strategy	Optimized covered call	Optimized covered call, equity only	ATM overlaid	5% OTM overlaid	10% OTM overlaid	Optimized equity, no overlay
Total return	28.8%	28.0%	9.9%	15.9%	18.5%	24.1%

Overlay portfolios are short call positions overlaid on the optimized equity portfolio. Total return is the sum of realized monthly returns over the nine month sample period

Appendix 1

Here we describe the Michaud resampling procedure as used in Sect. 5. The procedure comprises the following steps:

1. Using geometric Brownian motion and the point estimates in Table 1, $S = 252$ daily returns are simulated.
2. The sample expected return and covariance matrix of the simulations produced in step 1 are computed.
3. The sample statistics from step 2 are used to produce $N = 2000$ simulations of the asset prices at maturity.
4. For the chosen risk measure, the formulation from Sect. 2 and the simulations from step 3 are used to produce an efficient frontier of 50 points. The corresponding portfolio positions are stored.
5. Steps 1 to 4 are performed a total of $L = 50$ times.
6. The stored portfolio positions are blended by averaging over the L resampling iterations.

Though we have used geometric Brownian motion, in practice an investor can substitute a stochastic process which they believe describes the underlying assets.

The quantity S should be carefully selected to reflect the investor's degree of confidence in their point estimates. A higher value reflects a higher degree of confidence and will lead to estimates in step 2 which are closer to the point estimates used in step 1. A lower value reflects a lower degree of confidence and will lead to estimates in step 2 which have a wider spread. An investor should look at the spread of estimates outputted from step 2 and confirm that they match their degree of confidence regarding the point estimate used in step 1. We use $S = 252$ which reflects a relatively low degree of confidence. This produces a broad range of resampled expected returns and covariances and consequently produces blended portfolios which contain positions in a wide range of assets and options.

When formulating in step 4 we use the dividend rates from Table 1 since they are known in advance and certain. Efficient frontiers are obtained by first solving for the minimum risk portfolio and the maximum expected return portfolio. We then optimize for 48 expected return targets equally spaced between the expected return of the minimum risk portfolio and the maximum expected return.

The number of resampling iterations, L, should be sufficiently large to produce consistent results. However, each of the L iterations must solve for an efficient frontier, thus too large a number can be prohibitively slow despite the fact that the iterations can be computed in parallel. We choose $L = 50$ which we find produces consistent results.

From each of the L frontiers we select the portfolio with the lowest expected return target, i.e., the minimum risk portfolio. The first blended portfolio is created by averaging these L portfolio positions. From each of the L frontiers we then select the portfolio with the next lowest expected return, averaging these positions produces the second blended portfolio. And so on until the last blended portfolio is the average of the L maximum expected return positions.

While we have applied a simple blending procedure, Michaud and Michaud (2008) use utility functions to select portfolios to average. A number of other blending procedures also exist in the literature. Further guidelines for Michaud resampling can be found in Michaud and Michaud (2008).

Appendix 2

Here we describe the use of GARCH volatilities to produce results found in Fig. 3 in Sect. 5. Details about GARCH models can be found in Bollerslev (1986). We fit a GARCH(1,1) model on the volatility of monthly log-returns of the S&P 500, EAFE, and EM indices using data from 1988 to 2016. In step 3 of the procedure described in "Appendix 1" the resampled mean and correlation matrix from step 2, and the fitted GARCH models are used to produce simulations of the asset returns at maturity. Each simulated scenario is used to drive the GARCH volatilities in the subsequent scenario. The simulated returns are then rescaled so that their variance matches the resampled variance from step 2. This ensures that the first and second moments do not differ whether using geometric Brownian motion with static volatilities or with the GARCH volatility models; the resulting simulation distributions differ only in higher order moments. The remainder of the procedure from "Appendix 1" is unchanged.

References

Alexander, S., T.F. Coleman, and Y. Li. 2006. Minimizing CVaR and VaR for a portfolio of derivatives. *Journal of Banking and Finance* 30 (2): 583–605.

Bank of Montreal. 2018. BMO US High Dividend Covered Call ETF Fact Sheet. March 2018. https://www.bmo.com/gam/ca/advisor/products/etfs?fundUrl=/fundProfile/ZWH. Accessed 2 Jan 2019.

BlackRock. 2018. Enhanced Capital and Income Fund Fact Sheet. March 2018. https://www.blackrock.com/investing/literature/fact-sheet/cii-enhanced-capital-and-income-fund-factsheet-us09256a1097-us-en-individual.pdf. Accessed 2 Jan 2019.

Board, J., C. Sutcliffe, and E. Patrinos. 2000. The performance of covered calls. *The European Journal of Finance* 6 (1): 1–17.

Bollerslev, T. 1986. Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics* 31 (3): 307–327.

Callan Associates. 2006. An Historical Evaluation of the CBOE S&P 500 BuyWrite Index Strategy. October 2006. www.cboe.com/micro/bxm/Callan_CBOE.pdf.

Che, S.Y.S., and J.K.W. Fung. 2011. The performance of alternative futures buy-write strategies. *Journal of Futures Markets* 31 (12): 1202–1227.

Diaz, M., and R.H. Kwon. 2017. Optimization of covered call strategies. *Optimization Letters* 11 (7): 1303–1317.

Feldman, B.E., and D. Roy. 2004. Passive Options-Based Investment Strategies: The Case of the CBOE S&P 500 BuyWrite Index. ETFs and Indexing, No. 1 (2004), 72–89.

Figelman, I. 2008. Expected return and risk of covered call strategies. *The Journal of Portfolio Management* 34 (4): 81–97.

Figelman, I. 2009. Effect of non-normality dynamics on the expected return of options. *The Journal of Portfolio Management* 35 (2): 110–117.

Gao, R., and A.J. Kleywegt. 2016. Distributionally Robust Stochastic Optimization with Wasserstein Distance. arXiv preprint arXiv:1604.02199 (2016).

He, D.X., J.C. Hsu, and N. Rue. 2014. Option Writing Strategies in a Low Volatility Framework. Working paper, April 2014. www.ssrn.com/abstract=2433676. Accessed 2 Jan 2019.

Hill, J.M., V. Balasubramanian, K. Gregory, and I. Tierens. 2006. Finding alpha via covered index writing. *Financial Analysts Journal* 62 (5): 29–46.

Maidel, S., and K. Sahlin. 2010. Capturing the Volatility Premium Through Call Overwriting. Russell Research, December 2010. www.cboe.com/micro/Buywrite/Russell-2010-12-VolatilityPremiumthroughCallOverwriting.pdf. Accessed 2 Jan 2019.

Manulife. 2018. Manulife Covered Call U.S. Equity Fund Fact Sheet. March 2018. https://funds.manulife.ca/api/mutual/pdf/240e-en-us. Accessed 2 Jan 2019.

McIntyre, M.L., and D. Jackson. 2007. Great in practice, not in theory: An empirical examination of covered call writing. *Journal of Derivatives & Hedge Funds* 13 (1): 66–79.

Michaud, R.O., and R.O. Michaud. 2008. *Efficient Asset Management: A Practical Guide to Stock Portfolio Optimization and Asset Allocation*. Oxford: Oxford University Press.

Rockafellar, R.T., and S. Uryasev. 2000. Optimization of conditional-value-at-risk. *Journal of Risk* 2: 21–42.

Whaley, R.E. 2002. Return and risk of CBOE buy write monthly index. *The Journal of Derivatives* 10 (2): 35–42.

Yang G.Z. 2011. Buy-Write or Put-Write: An Active Index Writing Portfolio to Strike it Right. Working paper, May 2011. www.ssrn.com/abstract=1827363. Accessed 2 Jan 2019.

Zymler, S., B. Rustem, and D. Kuhn. 2011. Robust portfolio optimization with derivative insurance guarantees. *European Journal of Operational Research* 210 (2): 410–424.

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