



单变量预测对数据的预处理过程中，常规的组成 Patch 的方式忽略了时间序列的周期性，按周期分段再组合成 Patch 的方式，则考虑了周期性，先用 FFT 求出平均周期，每隔一个周期取一小段数据，组合成 Patch

### 求平均周期 $\epsilon$

$$\alpha = 2|\text{FFT}(D)|[: \lfloor N/2 \rfloor]/N \in \mathbb{R}^{\lfloor N/2 \rfloor}$$

$$\zeta = \Phi n[: \lfloor N/2 \rfloor]/N \in \mathbb{R}^{\lfloor N/2 \rfloor}$$

$$I_k = \text{index}(\text{top}_k(\alpha)) \in \mathbb{R}^k$$

$$\alpha_k = \alpha[I_k] = 2|\text{FFT}(D)|[: \lfloor N/2 \rfloor][I_k]/N \in \mathbb{R}^k$$

$$\zeta_k = \zeta[I_k] = \Phi n[: \lfloor N/2 \rfloor][I_k]/N \in \mathbb{R}^k$$

令

$$\lambda_\alpha = |\text{FFT}(D)|[: \lfloor N/2 \rfloor][I_k] \in \mathbb{R}^k$$

$$\lambda_\zeta = n[: \lfloor N/2 \rfloor][I_k] \in \mathbb{R}^k$$

则有

$$\alpha_k = 2\lambda_\alpha/N$$

$$\zeta_k = \Phi \lambda_\zeta/N$$

$$\phi = \text{Mean}(\zeta, \text{weights}) = \frac{\alpha^T \zeta}{\alpha^T \alpha} = \frac{\Phi \lambda_\alpha^T \lambda_\zeta}{2\lambda_\alpha^T \lambda_\alpha} \in \mathbb{R}$$

$$\tau = 1/\phi = \frac{2\lambda_\alpha^T \lambda_\alpha}{\Phi \lambda_\alpha^T \lambda_\zeta} = \frac{2T \lambda_\alpha^T \lambda_\alpha}{\lambda_\alpha^T \lambda_\zeta} \in \mathbb{R}$$

$$\epsilon = \lfloor \tau/T \rfloor = \lfloor \frac{2\lambda_\alpha^T \lambda_\alpha}{\lambda_\alpha^T \lambda_\zeta} \rfloor \in \mathbb{Z}$$

## 分段

$T$ 为原始数据的采样周期,  $\Phi$ 为采样频率,  $\Phi = 1/T$

段长 $L$

段数 $M$

$$P = ML$$

段定义 $m(i, j) = D[i + j\epsilon : i + j\epsilon + L] \in \mathbb{R}^{L \times C}, j \in [0, M - 1]$

$$X_S(i) = \text{Concat}_{j=0}^{M-1} \{m(i, j)\} \in \mathbb{R}^{ML \times C} = \mathbb{R}^{P \times C}$$

最后一个段为 $m(i, M - 1) = D[i + (M - 1)\epsilon : i + (M - 1)\epsilon + L]$

$$Y_S(i) = D[i + (M - 1)\epsilon + L : i + (M - 1)\epsilon + L + F] \in \mathbb{R}^{F \times C}$$

$$S[i] = X_S(i), Y_S(i)$$

因为 $Y_S(i), i + (M - 1)\epsilon + L + F < L(D)$

$$L(S) = L(D) - (M - 1)\epsilon - L - F + 1$$

## 试验

以下是在ETTh1数据集预测OT列时,  $M$ 从1到4的损失迭代图 ( $M = 1$ 时, 按周期分段和常规方法等价), 蓝色为按周期分段方法, 黄色为常规 (不分段) 方法



