SLAM 文献库

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  - G. Grisetti, R. Kummerle, C. Stachniss, and W. Burgard, "A Tutorial on Graph-Based SLAM," *IEEE Intell. Transport. Syst. Mag.*, vol. 2, no. 4, pp. 31–43, 2010, doi: 10.1109/MITS.2010.939925.

  - R. Kummerle, G. Grisetti, H. Strasdat, K. Konolige, and W. Burgard, "G^{2}o: A general framework for graph optimization," in *2011 IEEE International Conference on Robotics and Automation*, Shanghai, China: IEEE, May 2011, pp. 3607–3613. doi: 10.1109/ICRA.2011.5979949.

- **Observability and Consistency**

  - C. Jauffret, "Observability and fisher information matrix in nonlinear regression," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 43, no. 2, pp. 756–759, 2007, doi: 10.1109/TAES.2007.4285368.

  - Z. Zhang, G. Gallego, and D. Scaramuzza, "On the Comparison of Gauge Freedom Handling in Optimization-Based Visual-Inertial State Estimation," *IEEE Robot. Autom. Lett.*, vol. 3, no. 3, pp. 2710–2717, Jul. 2018, doi: 10.1109/LRA.2018.2833152.

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    - J. Mustaniemi, J. Kannala, S. Särkkä, J. Matas, and J. Heikkilä, "Inertial-Based Scale Estimation for Structure from Motion on Mobile Devices," Aug. 11, 2017, *arXiv*: arXiv:1611.09498. doi: 10.48550/arXiv.1611.09498.

    - T. Qin and S. Shen, "Robust initialization of monocular visual-inertial estimation on aerial robots," in *2017 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, Vancouver, BC: IEEE, Sep. 2017, pp. 4225–4232. doi: 10.1109/IROS.2017.8206284.

  - *Visual Front End*

    - J. Klippenstein and H. Zhang, "Performance evaluation of visual SLAM using several feature extractors," in *2009 IEEE/RSJ International Conference on Intelligent Robots and Systems*, St. Louis, MO, USA: IEEE, Oct. 2009, pp. 1574–1581. doi: 10.1109/IROS.2009.5354001.

    - J. Hartmann, J. H. Klussendorff, and E. Maehle, "A comparison of feature descriptors for visual SLAM," in *2013 European Conference on Mobile Robots*, Barcelona, Catalonia, Spain: IEEE, Sep. 2013, pp. 56–61. doi: 10.1109/ECMR.2013.6698820.

  - *Stereo Camera*

    - L. Wang, Y. Xu, and S. Shen, "VINS-Multi: A Robust Asynchronous Multi-camera-IMU State Estimator," May 23, 2024, *arXiv*: arXiv:2405.14539. doi: 10.48550/arXiv.2405.14539.

## Exactly Sparse Extended Information Filters for Feature-based SLAM

M. R. Walter, R. M. Eustice, and J. J. Leonard, "Exactly Sparse Extended Information Filters for Feature-based SLAM," *The International Journal of Robotics Research*, vol. 26, no. 4, pp. 335–359, Apr. 2007, doi: 10.1177/0278364906075026.

作者给出了在边际概率和条件概率中协方差矩阵和信息矩阵的表达的总结

$$p\left(\boldsymbol{\alpha}, \boldsymbol{\beta}\right) = \mathcal{N}\left(\begin{bmatrix} \boldsymbol{\mu}_\alpha \\ \boldsymbol{\mu}_\beta \end{bmatrix}, \begin{bmatrix} \Sigma_{\alpha\alpha} & \Sigma_{\alpha\beta} \\ \Sigma_{\beta\alpha} & \Sigma_{\beta\beta} \end{bmatrix}\right) = \mathcal{N}^{-1}\left(\begin{bmatrix} \boldsymbol{\eta}_\alpha \\ \boldsymbol{\eta}_\beta \end{bmatrix}, \begin{bmatrix} \Lambda_{\alpha\alpha} & \Lambda_{\alpha\beta} \\ \Lambda_{\beta\alpha} & \Lambda_{\beta\beta} \end{bmatrix}\right)$$

	MARGINALIZATION $p\left(\boldsymbol{\alpha}\right) = \int p\left(\boldsymbol{\alpha}, \boldsymbol{\beta}\right) d\boldsymbol{\beta}$	CONDITIONING $p\left(\boldsymbol{\alpha} \mid \boldsymbol{\beta}\right) = p\left(\boldsymbol{\alpha}, \boldsymbol{\beta}\right) / p\left(\boldsymbol{\beta}\right)$
COVARIANCE FORM	$\boldsymbol{\mu} = \boldsymbol{\mu}_\alpha$ $\Sigma = \Sigma_{\alpha\alpha}$	$\boldsymbol{\mu}' = \boldsymbol{\mu}_\alpha + \Sigma_{\alpha\beta}\Sigma_{\beta\beta}^{-1}(\boldsymbol{\beta} - \boldsymbol{\mu}_\beta)$ $\Sigma' = \Sigma_{\alpha\alpha} - \Sigma_{\alpha\beta}\Sigma_{\beta\beta}^{-1}\Sigma_{\beta\alpha}$
INFORMATION FORM	$\boldsymbol{\eta} = \boldsymbol{\eta}_\alpha - \Lambda_{\alpha\beta}\Lambda_{\beta\beta}^{-1}\boldsymbol{\eta}_\beta$ $\Lambda = \Lambda_{\alpha\alpha} - \Lambda_{\alpha\beta}\Lambda_{\beta\beta}^{-1}\Lambda_{\beta\alpha}$	$\boldsymbol{\eta}' = \boldsymbol{\eta}_\alpha - \Lambda_{\alpha\beta}\boldsymbol{\beta}$ $\Lambda' = \Lambda_{\alpha\alpha}$

当需要删除一个变量时，直接丢弃变量和对应的测量值,会使系统损失一部分信息。更合适的做法是使用边际概率,将丢弃变量所携带的信息通过剩余变量的隐式表达来保留。在边缘化的操作中，稀疏的信息矩阵慢慢变得稠密，原本不相关的变量慢慢变得相关。
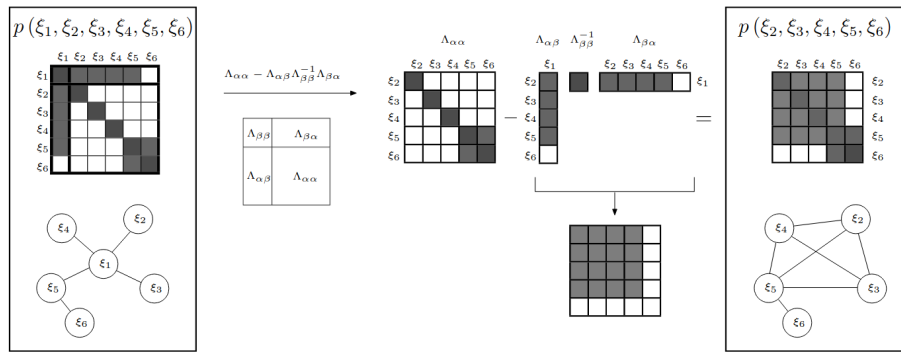
A Tutorial on Graph-Based SLAM

G. Grisetti, R. Kummerle, C. Stachniss, and W. Burgard, "A Tutorial on Graph-Based SLAM," *IEEE Intell. Transport. Syst. Mag.*, vol. 2, no. 4, pp. 31–43, 2010, doi: 10.1109/MITS.2010.939925.

SLAM问题被定义为：在给定初状态和一系列输入与观测的基础上，找到有最大似然概率的轨迹和地图。其中常见的地图表达包括 multilevel surface maps, point clouds 和 occupancy grids.

$$p(\mathbf{x}_{1:t}, \mathbf{m} | \mathbf{z}_{1:t}, \mathbf{u}_{1:t}, \mathbf{x}_0)$$

在图优化SLAM中，待估计的变量被定义为节点，而待估计变量构成的空间约束以残差形式构成图的边。图优化SLAM由此被解耦为从原始传感器数据构成图（前端）和在约束中找到有最大似然概率的参数（后端）两个问题。本文假设前端很完美地找到观测的对应关系，待优化的图被很好地建立起来的基础上，讨论后端图优化的基本技术。

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \sum_{(i,j)\in\mathcal{C}} \mathbf{e}_{ij}^T \Omega_{ij} \mathbf{e}_{ij}$$

其中残差利用一阶雅可比近似

$$\mathbf{e}_{ij}(\check{\mathbf{x}}_i + \Delta\mathbf{x}_i, \check{\mathbf{x}}_j + \Delta\mathbf{x}_j) = \mathbf{e}_{ij}(\check{\mathbf{x}} + \Delta\mathbf{x}) \approx \mathbf{e}_{ij} + \mathbf{J}_{ij}\Delta\mathbf{x}$$

总损失函数有

$$
\begin{aligned}
\mathbf{F}(\check{\mathbf{x}} + \Delta\mathbf{x}) &= \sum_{(i,j)\in\mathcal{C}} \mathbf{e}_{ij}(\check{\mathbf{x}} + \Delta\mathbf{x})^T \Omega_{ij} \mathbf{e}_{ij}(\check{\mathbf{x}} + \Delta\mathbf{x}) \\
&\approx \sum_{(i,j)\in\mathcal{C}} (\mathbf{e}_{ij} + \mathbf{J}_{ij}\Delta\mathbf{x})^T \Omega_{ij} (\mathbf{e}_{ij} + \mathbf{J}_{ij}\Delta\mathbf{x}) \\
&= \sum_{(i,j)\in\mathcal{C}} \underbrace{\mathbf{e}_{ij}^T \Omega_{ij} \mathbf{e}_{ij}}_{c_{ij}} + 2\underbrace{\mathbf{e}_{ij}^T \Omega_{ij} \mathbf{J}_{ij}}_{\mathbf{b}_{ij}}\Delta\mathbf{x} + \Delta\mathbf{x}^T \underbrace{\mathbf{J}_{ij}^T \Omega_{ij} \mathbf{J}_{ij}}_{\mathbf{H}_{ij}}\Delta\mathbf{x} \\
&= c + 2\mathbf{b}^T\Delta\mathbf{x} + \Delta\mathbf{x}^T\mathbf{H}\Delta\mathbf{x}
\end{aligned}
$$

当上式关于 $\Delta\mathbf{x}$ 的导数为 $\mathbf{0}$ 时

$$\mathbf{H}\Delta\mathbf{x}^* = -\mathbf{b}$$

进一步论文给出了一种定义在流形上的误差项

$$\mathbf{e}_{ij}(\mathbf{x}) = (\mathbf{z}_{ij}^{-1} \oplus (\mathbf{x}_i^{-1} \oplus \mathbf{x}_j))_{[1:6]} = \check{\mathbf{e}}_{ij} + \tilde{\mathbf{J}}_{ij}\Delta\tilde{\mathbf{x}}$$

$$
\begin{aligned}
\tilde{\mathbf{J}}_{ij} &= \begin{bmatrix} \cdots & \left.\dfrac{\partial\mathbf{e}_{ij}(\check{\mathbf{x}}\boxplus\Delta\tilde{\mathbf{x}})}{\partial\Delta\tilde{\mathbf{x}}_i}\right|_{\Delta\tilde{\mathbf{x}}=0} & \cdots & \left.\dfrac{\partial\mathbf{e}_{ij}(\check{\mathbf{x}}\boxplus\Delta\tilde{\mathbf{x}})}{\partial\Delta\tilde{\mathbf{x}}_j}\right|_{\Delta\tilde{\mathbf{x}}=0} & \cdots \end{bmatrix} \\
&= \begin{bmatrix} \cdots & \left.\dfrac{\partial\mathbf{e}_{ij}(\check{\mathbf{x}})}{\partial\check{\mathbf{x}}_i}\cdot\dfrac{\check{\mathbf{x}}_i\boxplus\Delta\tilde{\mathbf{x}}}{\partial\Delta\tilde{\mathbf{x}}_i}\right|_{\Delta\tilde{\mathbf{x}}=0} & \cdots & \left.\dfrac{\partial\mathbf{e}_{ij}(\check{\mathbf{x}})}{\partial\check{\mathbf{x}}_j}\cdot\dfrac{\check{\mathbf{x}}_j\boxplus\Delta\tilde{\mathbf{x}}}{\partial\Delta\tilde{\mathbf{x}}_j}\right|_{\Delta\tilde{\mathbf{x}}=0} & \cdots \end{bmatrix}
\end{aligned}
$$

其中增量算子 $\oplus, \boxplus$ 为

$$\mathbf{x}_i \oplus \mathbf{x}_j = \begin{bmatrix} \mathbf{q}_i(\mathbf{t}_j) \\ \mathbf{q}_i \otimes \mathbf{q}_j \end{bmatrix}$$

$$\mathbf{x}_i \boxplus \Delta\tilde{\mathbf{x}}_i = \mathbf{x}_i \oplus \begin{bmatrix} \Delta\tilde{\mathbf{t}}_i \\ \Delta\tilde{\mathbf{q}}_i \\ \sqrt{1 - ||\Delta\tilde{\mathbf{q}}_i||^2} \end{bmatrix}$$

$\mathbf{G^2o}$: A general framework for graph optimization

R. Kummerle, G. Grisetti, H. Strasdat, K. Konolige, and W. Burgard, "G^{2}o: A general framework for graph optimization," in *2011 IEEE International Conference on Robotics and Automation*, Shanghai, China: IEEE, May 2011, pp. 3607–3613. doi: 10.1109/ICRA.2011.5979949.

基本上沿续了 **A Tutorial on Graph-Based SLAM** 的思路，补充了一些工程上的实现和实验的结果：使用阻尼法 $(\mathbf{H} + \lambda\mathbf{I})\Delta\mathbf{x}^* = -\mathbf{b}$ 求解高斯牛顿，利用信息矩阵自伴随的特性用舒尔补快速求解 $\Delta\mathbf{x}^*$

Observability and Consistency

Observability and fisher information matrix in nonlinear regression

C. Jauffret, "Observability and fisher information matrix in nonlinear regression," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 43, no. 2, pp. 756–759, 2007, doi: 10.1109/TAES.2007.4285368.

给出了三种可观性的定义

在 θ_0 处简单可观 **Simply Observable**

$$\forall \theta \in \mathbb{R}^d, \theta \neq \theta_0 \Rightarrow h(\theta) \neq h(\theta_0)$$

全局可观 **(Simply) Observable**

$$\forall \theta, \theta' \in \mathbb{R}^d, \theta \neq \theta' \Rightarrow h(\theta) \neq h(\theta')$$

在 θ_0 处局部可观 **Locally Observable**

$$\exists U_{\theta_0} \subset \mathbb{R}^d \text{ (open subset containing } \theta_0)$$
$$\forall \theta \in U_{\theta_0}, \theta \neq \theta_0 \Rightarrow h(\theta) \neq h(\theta_0)$$

那么对于测量模型 $X = h(\theta) + \epsilon$ 而言，如果噪声的支持集 $p_\epsilon = \mathbb{R}^n$ 并且它的信息矩阵在 θ_0 处非奇异，或等价地 $\nabla_\theta h(\theta_0) \nabla_\theta^T h(\theta_0)$ 非奇异，那么该无噪声系统在 θ_0 处局部可观

On the Comparison of Gauge Freedom Handling in Optimization-Based Visual-Inertial State Estimation

Z. Zhang, G. Gallego, and D. Scaramuzza, "On the Comparison of Gauge Freedom Handling in Optimization-Based Visual-Inertial State Estimation," *IEEE Robot. Autom. Lett.*, vol. 3, no. 3, pp. 2710–2717, Jul. 2018, doi: 10.1109/LRA.2018.2833152.

视觉惯性系统存在四自由度不可观（平移和绕重力方向的旋转），高斯牛顿法求解线性方程时，奇异矩阵H不满秩（存在四维的零空间），求解稳定性较差。这些额外的规范自由度 Gauge Freedom 必须被妥善处理。作者对比了三种常见的处理策略以及它们对应的效果

Gauge Fixation 选择将第一帧的摄像机位姿固定，在全局优化中不更新。这等价于在全局优化时将第一帧摄像机位姿对应的残差的雅可比矩阵设为零矩阵。

Gauge Prior 选择为第一帧位姿加上定义在流形上的正则化惩罚项 $||\mathbf{r}_0^P||_{\Sigma_0^P}^2 = ||(\mathbf{p}_0 - \mathbf{p}_0^0, \Delta\phi_{0z})||_{\Sigma_0^P}^2$

Free Gauge 方法则不增加先验约束，而是在优化时使用阻尼牛顿法，使 $\epsilon\mathbf{I} + \mathbf{H}$ 正定

作者接下来讨论了 **Gauge Prior** 中权重矩阵的选择的策略，并对比了三种方法的效果的差异。在权重矩阵等于协方差构成的单位阵 $\boldsymbol{\Sigma}_0^P = \boldsymbol{\sigma}_0^2\mathbf{I}$ 时，$||\mathbf{r}_0^P||_{\Sigma_0^P}^2 = w^P||\mathbf{r}_0^P||^2, w^P = \frac{1}{\sigma_0^2}$。当 $w^P \to \infty$ 时，**Gauge Prior** 变成 **Gauge Fixation**，当 $w^P = 0$ 时，**Gauge Prior** 变成 **Gauge Free**

权重 w 对均方误差、迭代次数、收敛时间的影响都不大，并且在超过某个阈值后，三者都趋于平稳
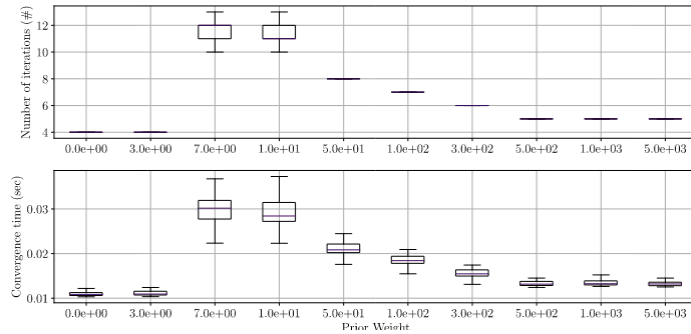
作者进一步实验讨论这种反常的收敛时间的增加。对于非常大的先验权重 10^8，该算法会降低重新投影误差，同时保持先验误差几乎等于零。相反，对于较小的先验权重（例如 $50 \sim 500$），优化算法会在前两次迭代中降低重新投影误差，但代价是增加先验误差。然后优化算法花费许多次迭代来微调先验误差，同时保持重新投影误差较小（沿轨道移动），因此计算时间会增加。
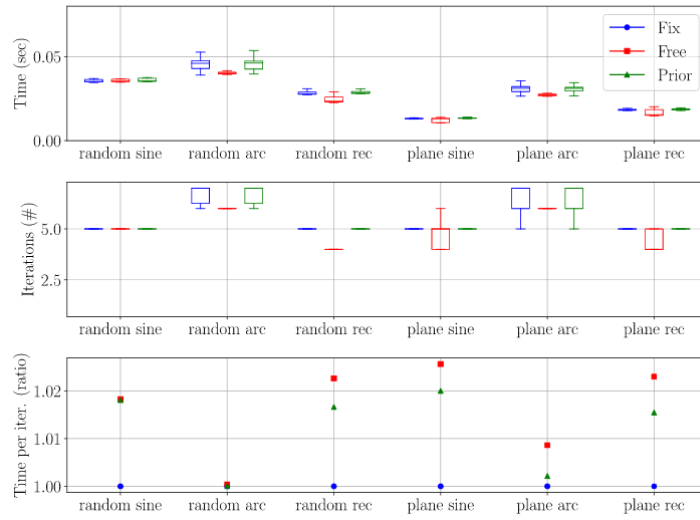
总的来说，三种方法对精度的影响并不显著。在权重合适的情况下，**Gauge Prior** 方法与 **Gauge Fixation** 几乎有相同的表现，而 **Free Gauge** 方法由于需要更少的迭代次数来收敛，消耗的时间会较少

Visual SLAM
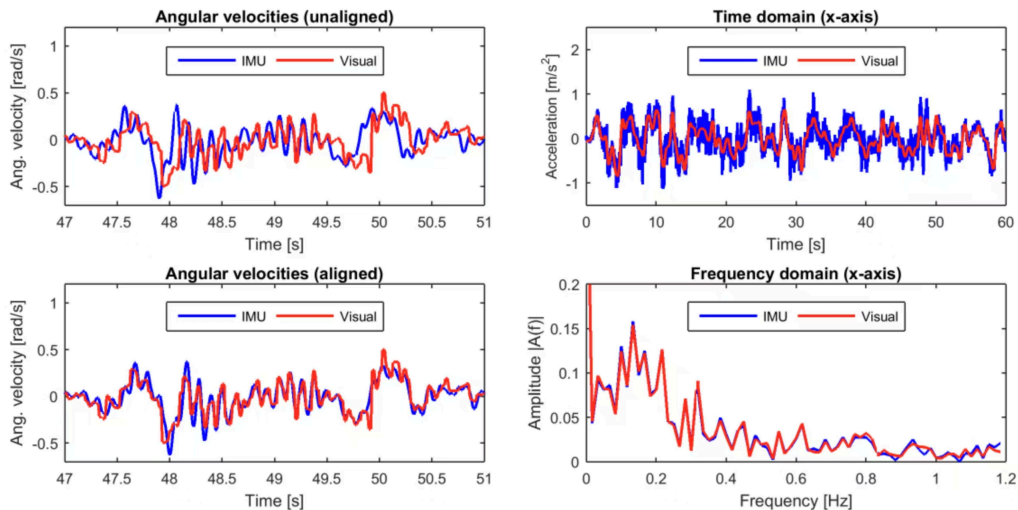
Initialization

Inertial-Based Scale Estimation for Structure from Motion on Mobile Devices

J. Mustaniemi, J. Kannala, S. Särkkä, J. Matas, and J. Heikkilä, "Inertial-Based Scale Estimation for Structure from Motion on Mobile Devices," Aug. 11, 2017, *arXiv*: arXiv:1611.09498. doi: 10.48550/arXiv.1611.09498.

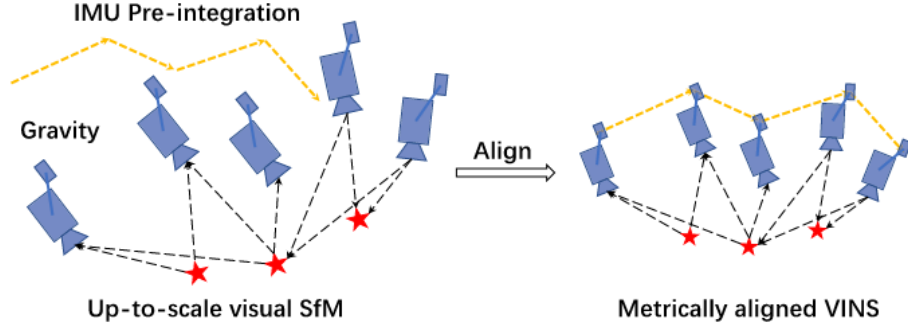利用IMU原始信息而不是积分信息完成对齐工作，避免误差在预积分过程中被累计，开源了MATLAB代码。

通过图像计算相机外参数，利用B样条曲线插值得到角速度和线性加速度，再与IMU测量的原始角速度加速度构建联合损失函数。同时在频域上针对IMU噪声特性，引入高通滤波和低通滤波抑制噪声。

Robust initialization of monocular visual-inertial estimation on aerial robots

利用松耦合的方式对齐IMU与视觉

IMU测量模型有

$$\hat{\boldsymbol{\omega}} = \boldsymbol{\omega}^b + \mathbf{b}^g + \mathbf{n}^g$$
$$\hat{\mathbf{a}} = \mathbf{R}_{bw}(\mathbf{a}^w + \mathbf{g}^w) + \mathbf{b}^a + \mathbf{n}^a$$

预积分

$$\boldsymbol{\alpha}_{b_i b_j} = \iint_{t\in[i,j]} \mathbf{R}_{b_i b_t}(\hat{\mathbf{a}}^{b_t} - \mathbf{b}^a)\mathrm{d}t^2$$
$$\boldsymbol{\beta}_{b_i b_j} = \int_{t\in[i,j]} \mathbf{R}_{b_i b_t}(\hat{\mathbf{a}}^{b_t} - \mathbf{b}^a)\mathrm{d}t$$
$$\mathbf{q}_{b_i b_j} = \int_{t\in[i,j]} \mathbf{q}_{b_i b_t} \otimes \begin{bmatrix} 0 \\ \frac{1}{2}(\hat{\boldsymbol{\omega}}^{b_t} - \mathbf{b}^g) \end{bmatrix}\mathrm{d}t$$

有一阶泰勒展开近似

$$\boldsymbol{\alpha}_{b_i b_j} \approx \hat{\boldsymbol{\alpha}}_{b_i b_j} + \mathbf{J}^{\alpha}_{b^a}\delta\mathbf{b}^a + \mathbf{J}^{\alpha}_{b^g}\delta\mathbf{b}^g$$
$$\boldsymbol{\beta}_{b_i b_j} \approx \hat{\boldsymbol{\beta}}_{b_i b_j} + \mathbf{J}^{\beta}_{b^a}\delta\mathbf{b}^a + \mathbf{J}^{\beta}_{b^g}\delta\mathbf{b}^g$$
$$\mathbf{q}_{b_k b_{k+1}} \approx \hat{\mathbf{q}}_{b_k b_{k+1}} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\mathbf{J}^{\mathbf{q}}_{b^g}\delta\mathbf{b}^g \end{bmatrix}$$

SfM给出视觉约束，¯表示非米制单位

$$\mathbf{q}_{c_0 b_k} = \mathbf{q}_{c_0 c_k} \otimes \mathbf{q}_{bc}^{-1}$$
$$s\bar{\mathbf{p}}_{c_0 b_k} = s\bar{\mathbf{p}}_{c_0 c_k} - \mathbf{R}_{c_0 b_k}\mathbf{p}_{bc}$$

估计外参数 $\mathbf{q}_{bc}$

对于相邻时刻 $k, k+1$ 的 IMU 旋转积分 $\mathbf{q}_{b_k b_{k+1}}$ 和视觉测量 $\mathbf{q}_{c_k c_{k+1}}$

$$\mathbf{q}_{b_k c_{k+1}} = \mathbf{q}_{bc} \otimes \mathbf{q}_{c_k c_{k+1}} = \mathbf{q}_{b_k b_{k+1}} \otimes \mathbf{q}_{bc}$$

$$([\mathbf{q}_{c_k c_{k+1}}]_R - [\mathbf{q}_{b_k b_{k+1}}]_L)\mathbf{q}_{bc} = \mathbf{0}$$

$[\cdot]_L, [\cdot]_R$ 为四元数的左乘矩阵和右乘矩阵，$[\cdot]_\times$ 为反对称矩阵

$$[\mathbf{q}]_L = q_\omega \mathbf{I} + \begin{bmatrix} 0 & -\mathbf{q}_v^T \\ \mathbf{q}_v & [\mathbf{q}_v]_\times \end{bmatrix}$$

$$[\mathbf{q}]_R = q_\omega \mathbf{I} + \begin{bmatrix} 0 & -\mathbf{q}_v^T \\ \mathbf{q}_v & -[\mathbf{q}_v]_\times \end{bmatrix}$$

$$[\boldsymbol{\omega}]_\times = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

将多个时刻的 IMU 预积分和视觉测量累计，即可得到关于 $\mathbf{q}_{bc}$ 的超定方程组

$$\begin{bmatrix} [\mathbf{q}_{c_0 c_1}]_R - [\mathbf{q}_{b_0 b_1}]_L \\ [\mathbf{q}_{c_1 c_2}]_R - [\mathbf{q}_{b_1 b_2}]_L \\ \vdots \\ [\mathbf{q}_{c_n c_{n-1}}]_R - [\mathbf{q}_{b_n b_{n-1}}]_L \end{bmatrix} \mathbf{q}_{bc} = \mathbf{0}$$

利用奇异值分解可解得 $\mathbf{q}_{bc}$

估计陀螺仪偏置

标定得到 $\mathbf{q}_{bc}$ 后，利用旋转约束，可估计处陀螺仪偏置

$$\delta \mathbf{b}^g = \arg\min_{\delta \mathbf{b}^g} \sum_{k \in \mathcal{B}} \left\| \left\lfloor \mathbf{q}_{c_0 b_{k+1}}^{-1} \otimes \mathbf{q}_{c_0 b_k} \otimes \mathbf{q}_{b_k b_{k+1}} \right\rfloor_{\text{xyz}} \right\|^2$$

$\mathcal{B}$ 为所有关键帧的集合，其中目标函数取最小值 0 时

$$\mathbf{q}_{c_0 b_{k+1}}^{-1} \otimes \mathbf{q}_{c_0 b_k} \otimes \mathbf{q}_{b_k b_{k+1}} = \begin{bmatrix} \mathbf{1} \\ \mathbf{0} \end{bmatrix}$$

又由旋转预积分一阶泰勒近似

$$\mathbf{q}_{b_k b_{k+1}} \approx \hat{\mathbf{q}}_{b_k b_{k+1}} \otimes \begin{bmatrix} 1 \\ \frac{1}{2} \mathbf{J}_{b^g}^{\mathbf{q}} \delta \mathbf{b}^g \end{bmatrix}$$

联立得到

$$\begin{bmatrix} 1 \\ \frac{1}{2} \mathbf{J}_{b^g}^{\mathbf{q}} \delta \mathbf{b}^g \end{bmatrix} = \hat{\mathbf{q}}_{b_k b_{k+1}}^{-1} \otimes \mathbf{q}_{c_0 b_k}^{-1} \otimes \mathbf{q}_{c_0 b_{k+1}}$$

考虑虚部

$$\mathbf{J}_{b^g}^{\mathbf{q}} \delta \mathbf{b}^g = 2 \left\lfloor \hat{\mathbf{q}}_{b_k b_{k+1}}^{-1} \otimes \mathbf{q}_{c_0 b_k}^{-1} \otimes \mathbf{q}_{c_0 b_{k+1}} \right\rfloor_{\text{xyz}}$$

进一步可以构建正定方程组，通过 Cholesky 分解求解 $\delta \mathbf{b}^g$

$$(\mathbf{J}_{b^g}^{\mathbf{q}})^T \mathbf{J}_{b^g}^{\mathbf{q}} \delta \mathbf{b}^g = 2(\mathbf{J}_{b^g}^{\mathbf{q}})^T \left\lfloor \hat{\mathbf{q}}_{b_k b_{k+1}}^{-1} \otimes \mathbf{q}_{c_0 b_k}^{-1} \otimes \mathbf{q}_{c_0 b_{k+1}} \right\rfloor_{\text{xyz}}$$

解得 $\delta \mathbf{b}^g$ 后，重新计算预积分项 $\hat{\boldsymbol{\alpha}}_{b_k b_{k+1}}, \hat{\boldsymbol{\beta}}_{b_k b_{k+1}}, \hat{\mathbf{q}}_{b_k b_{k+1}}$

初始化速度、重力和尺度因子

所有我们希望估计的变量包括

$$\mathcal{X}_I = \begin{bmatrix} \mathbf{v}_0^{b_0} & \mathbf{v}_1^{b_1} & \cdots \mathbf{v}_n^{b_n} & \mathbf{g}^{c_0} & \mathbf{s} \end{bmatrix}^T$$

由 1.3 式和 2.1 式，得到

$$\boldsymbol{\alpha}_{b_k b_{k+1}} = \mathbf{R}_{b_k c_0} \left(s(\bar{\mathbf{p}}_{c_0 b_{k+1}} - \bar{\mathbf{p}}_{c_0 b_k}) + \frac{1}{2} \mathbf{g}^{c_0} \Delta t_k^2 - \mathbf{R}_{c_0 b_k} \mathbf{v}_k^{b_k} \Delta t_k \right)$$

$$\boldsymbol{\beta}_{b_k b_{k+1}} = \mathbf{R}_{b_k c_0} (\mathbf{R}_{c_0 b_{k+1}} \mathbf{v}_k^{b_k} + \mathbf{g}^{c_0} \Delta t_k - \mathbf{R}_{c_0 b_k} \mathbf{v}_k^{b_k})$$

整理方程得到

$$\hat{\mathbf{z}}_{b_{k+1}}^{b_k} = \begin{bmatrix} \hat{\boldsymbol{\alpha}}_{b_k b_{k+1}} - \mathbf{p}_{bc} + \mathbf{R}_{b_k c_0} \mathbf{R}_{c_0 b_{k+1}} \mathbf{p}_{bc} \\ \hat{\boldsymbol{\beta}}_{b_k b_{k+1}} \end{bmatrix} = \mathbf{H}^k \mathcal{X}_I^k + \mathbf{n}^k$$

其中

$$\mathcal{X}_I^k = \begin{bmatrix} \mathbf{v}_k^{b_k} & \mathbf{v}_{k+1}^{b_{k+1}} & \mathbf{g}^{c_0} & s \end{bmatrix}^T$$

$$\mathbf{H}_{b_{k+1}}^{b_k} = \begin{bmatrix} -\mathbf{I}\Delta t_k & \mathbf{0} & \frac{1}{2}\mathbf{R}_{b_k c_0}\Delta t_k^2 & \mathbf{R}_{b_k c_0}(\bar{\mathbf{p}}_{c_0 b_{k+1}} - \bar{\mathbf{p}}_{c_0 b_k}) \\ -\mathbf{I} & \mathbf{R}_{b_k c_0}\mathbf{R}_{c_0 b_{k+1}} & \mathbf{R}_{b_k c_0}\Delta t_k & 0 \end{bmatrix}$$

进而可以转化为最小二乘问题求解

$$\mathcal{X}_I = \arg\min_{\mathcal{X}_I} \sum_{k\in\mathcal{B}} ||\hat{\mathbf{z}}_{b_{k+1}}^{b_k} - \mathbf{H}_{b_{k+1}}^{b_k}\mathcal{X}_I^k||^2$$
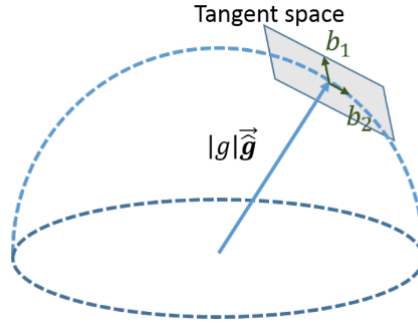
同样可以通过 Chologky 分解求得

重力向量优化

在重力模长已知的情况下，重力向量实际自由度为 2，可以利用球面坐标进行参数化

$$\hat{\mathbf{g}}^{c_0} = ||g||\left(\frac{\tilde{\mathbf{g}}^{c_0}}{||\tilde{\mathbf{g}}^{c_0}||} + w_1\mathbf{b}_1 + w_2\mathbf{b}_2\right)$$

$\tilde{\mathbf{g}}^{c_0}$ 为 2.15 中求得的重力向量，记 $\frac{\tilde{\mathbf{g}}^{c_0}}{||\tilde{\mathbf{g}}^{c_0}||}$ 为 $\hat{\bar{\mathbf{g}}}^{c_0}$

可以通过如下方式找到一组基底垂直于 $\hat{\mathbf{g}}^{c_0}$

$$\mathbf{b}_1 = \begin{cases} \hat{\bar{\mathbf{g}}}^{c_0} \times [1,0,0]^T, & \hat{\bar{\mathbf{g}}}^{c_0} \neq [1,0,0]^T \\ \hat{\bar{\mathbf{g}}}^{c_0} \times [0,0,1]^T, & \text{otherwise} \end{cases}$$
$$\mathbf{b}_2 = \hat{\bar{\mathbf{g}}}^{c_0} \times \mathbf{b}_1$$

将 2.17 式代入 2.15 式，得到

$$\mathcal{X}_I^k = \begin{bmatrix} \mathbf{v}_k^{b_k} & \mathbf{v}_{k+1}^{b_{k+1}} & \mathbf{w}^{c_0} & s \end{bmatrix}^T$$

$$\mathbf{H}_{b_{k+1}}^{b_k} = \begin{bmatrix} -\mathbf{I}\Delta t_k & \mathbf{0} & \frac{1}{2}\mathbf{R}_{b_k c_0}\begin{bmatrix}\mathbf{b}_1^T \\ \mathbf{b}_2^T\end{bmatrix}\Delta t_k^2 & \mathbf{R}_{b_k c_0}(\bar{\mathbf{p}}_{c_0 b_{k+1}} - \bar{\mathbf{p}}_{c_0 b_k}) \\ -\mathbf{I} & \mathbf{R}_{b_k c_0}\mathbf{R}_{c_0 b_{k+1}} & \mathbf{R}_{b_k c_0}\begin{bmatrix}\mathbf{b}_1^T \\ \mathbf{b}_2^T\end{bmatrix}\Delta t_k & 0 \end{bmatrix}$$

观测方程变为

$$\hat{\mathbf{z}}_{b_{k+1}}^{b_k} = \begin{bmatrix} \hat{\boldsymbol{\alpha}}_{b_k b_{k+1}} - \mathbf{p}_{bc} + \mathbf{R}_{b_k c_0}\mathbf{R}_{c_0 b_{k+1}}\mathbf{p}_{bc} - \frac{1}{2}\mathbf{R}_{b_k c_0}\tilde{\mathbf{g}}^{c_0}\Delta t_k^2 \\ \hat{\boldsymbol{\beta}}_{b_k b_{k+1}} - \mathbf{R}_{b_k c_0}\tilde{\mathbf{g}}^{c_0}\Delta t_k \end{bmatrix}$$
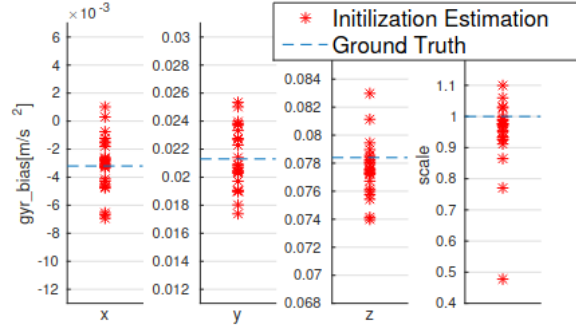
利用最小二乘对 $\mathcal{X}_I^k$ 进一步优化

视觉惯性对齐

根据旋转的性质和李代数的指数映射，我们可以构建从 c_0 系到 w 系的旋转矩阵 $\mathbf{R}_{wc_0}$

$$\mathbf{R}_{wc_0} = \exp\left[\arctan\left(\frac{||\hat{\mathbf{g}}^{c_0} \times \hat{\mathbf{g}}^w||}{\hat{\mathbf{g}}^{c_0} \cdot \hat{\mathbf{g}}^w}\right) \cdot \frac{\hat{\mathbf{g}}^{c_0} \times \hat{\mathbf{g}}^w}{||\hat{\mathbf{g}}^{c_0} \times \hat{\mathbf{g}}^w||}\right]$$

接着为所有 c_0 系为坐标系的向量左乘 $\mathbf{R}_{wc_0}$，同时将非米制的 $\bar{\mathbf{p}}$ 通过尺度因子 s 恢复为 $\mathbf{p}$

未估计的量

作者通过实验指出二者加速度计偏置 $\mathbf{b}^a$ 和相机与IMU间的平移向量 $\mathbf{p}_{bc}$ 对系统精度影响极小，可以不在初始化中显式优化

Visual Front End

## Performance evaluation of visual SLAM using several feature extractors

J. Klippenstein and H. Zhang, "Performance evaluation of visual SLAM using several feature extractors," in *2009 IEEE/RSJ International Conference on Intelligent Robots and Systems*, St. Louis, MO, USA: IEEE, Oct. 2009, pp. 1574–1581. doi: 10.1109/IROS.2009.5354001.
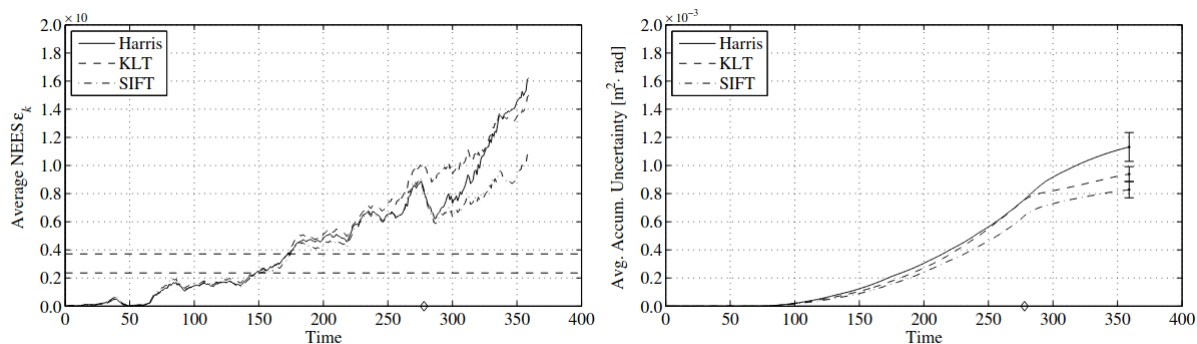
在平均归一化误差和累计不确定性上对 Harris, KLT 和 SIFT 进行测试

$$\bar{\epsilon}_k = \frac{1}{N}\sum_{i=1}^{N}||\mathbf{r}_k^{(i)} - \hat{\mathbf{r}}_k^{(i)}||_{\Sigma_{r,k}^{-1}}^2$$

$$\overline{\mathrm{AU}} = \frac{1}{N}\sum_{i=1}^{N}\sum_{k=1}^{N_s}\frac{4}{3}\pi\sqrt{\det(\Sigma_{k,r})}$$

实验表明，在大部分室内情况下三种特征提取器的累计不确定性没有显著差异，平均归一化误差也遵循相同的趋势，表现几乎相似。在累计不确定性上，SIFT的表现稍稍更优秀。
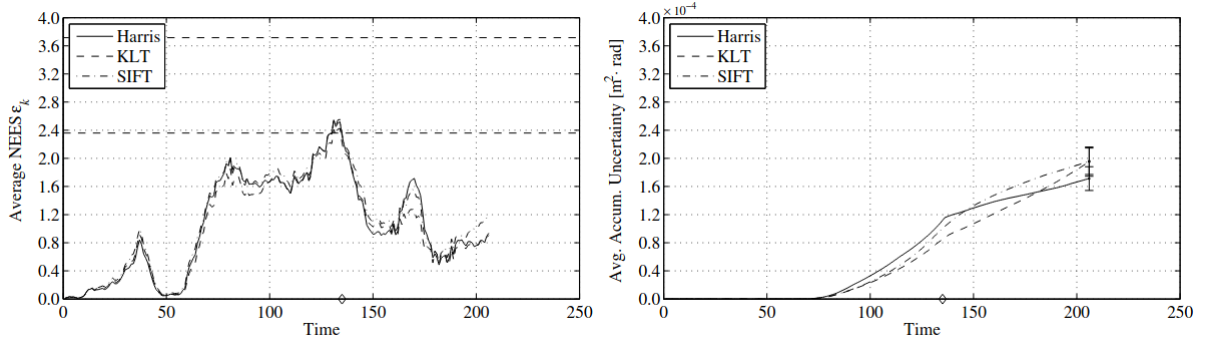
A comparison of feature descriptors for visual SLAM

J. Hartmann, J. H. Klussendorff, and E. Maehle, "A comparison of feature descriptors for visual SLAM," in *2013 European Conference on Mobile Robots*, Barcelona, Catalonia, Spain: IEEE, Sep. 2013, pp. 56–61. doi: 10.1109/ECMR.2013.6698820.

特征提取器希望找到特征点亮度、尺度、平移不变的描述，使系统能够完成运动结构恢复和回环检测的任务。

Stereo Camera

VINS-Multi: A Robust Asynchronous Multi-camera-IMU State Estimator

L. Wang, Y. Xu, and S. Shen, "VINS-Multi: A Robust Asynchronous Multi-camera-IMU State Estimator," May 23, 2024, *arXiv*: arXiv:2405.14539. doi: 10.48550/arXiv.2405.14539.

针对不同类型相机在异步时间戳上的融合问题，设计了一套平行视觉前端和关键帧选择器的算法。视觉前端平行地为每一个相机提取并追踪特征，加入一个联合筛选器负责挑选放入后端优化的特征和关键帧。特征按照最后一帧追踪到的特征点的数量占比来分配。关键帧筛选则负责将最后帧特征点占比最高的帧以及帧更新间隔最长的帧发送到后端进行优化。