Q3 ...

a) When eliminating D we generate a new factor f_2 as follows:

$$f_2(A,+c,E,F) = \sum_d P(E|d)\,P(F|d)\,f_1(A,+c,d)$$

This leaves us with the factors:

$$P(A),\ P(+c),\ P(G|+c,F),\ f_2(A,+c,E,F)$$

When eliminating G we generate a new factor f_3 as follows:

$$f_3(+c,F) = \sum_g P(g|+c,F)$$

This leaves us with the factors:

$$f_3(+c,F),\ f_2(A,+c,F,E),\ P(A),\ P(+c)$$

When eliminating F we generate a new factor f_4 as follows:

$$f_4(+c,A,E) = \sum_f f_2(+c,A,E,f)\,f_3(+c,f)$$

This leaves us with the factors:

$$P(+c),\ f_4(+c,A,E),\ P(A)$$

b) $$P(A,E|+c) = \frac{P(+c)\,P(A)\,f_4(A,E,+c)}{\sum_{a,e} P(a)\,f_4(+c,a,e)\,P(+c)}$$

c) The largest: f_2 $[2^3 = 8 \text{ entries}]$

d)

Variable eliminated	Factor Generated
B	$f_1(+c,A,D)$
G	$f_2(+c,F)$
F	$f_3(+c,D)$
D	$f_4(+c,A,E)$

Q1

a) b & c   [a has F_1, which is conditionally independent from N given M_1]

b) The Best is b, because it is the simplest from the valid Bayesian networks (b, c), and easy to interpret

c)

M\N	1	2	3
0	$\frac{e(1-f)}{2}+f$	f	f
1	$(1-e)(1-f)$	$\frac{e(1-f)}{2}$	0
2	$\frac{e(1-f)}{2}$	$(1-e)(1-f)$	$\frac{e(1-f)}{2}$
3	0	$\frac{e(1-f)}{2}$	$(1-e)(1-f)$
4	0	0	$\frac{e(1-f)}{2}$

d) N can be 2, 4 OR $N > 5$ (N = integer)

e) $e >> f \Rightarrow N = 2$ is the most likely number of stars, given the observations

$$P(N=2) = \frac{e^2(1-f)^2}{4}$$

$$P(N=4) = \frac{e(1-f)f}{4}$$

$$P(N>5) = f^2$$

$$P(N=2) > P(N=4) > P(N>5)$$

Q4 a) $\frac{5}{8}$ ii) $\frac{2}{3}$

b)

Sample	Weight
-a +b +c -d	5/18
+a +b +c -d	1/6
+a +b -c -d	1/40
-a +b -c -d	1/24

c) $$P(-a \mid +b, -d) = \frac{5/18 + 1/24}{5/18 + 1/24 + 5/30 + 1/40} = \frac{5/18 + 1/24}{5/18 + 1/24 + 1/6 + 1/40} =$$

$$= 0.625$$

d) $P(D|A)$ is better suited for likelihood weighting, Because likelihood weighting conditions on upstream evidence.

Q2 a) $P(B \mid +j, +m) = \alpha P(B) \sum_e P(e) \sum_a P(a \mid b, e) P(m \mid a) P(j \mid a) =$

$= \alpha P(B) \sum_e P(e) \left[0.9 \times 0.7 \times \begin{pmatrix} 0.95 & 0.29 \\ 0.94 & 0.001 \end{pmatrix} + 0.05 \times 0.01 \times \begin{pmatrix} 0.05 & 0.71 \\ 0.06 & 0.999 \end{pmatrix} \right] = \alpha P(B) \sum_e P(e) \times \begin{pmatrix} 0.598525 & 0.183055 \\ 0.59223 & 0.0011295 \end{pmatrix}$

$= \alpha P(B) \left[0.002 \times \begin{pmatrix} 0.598525 \\ 0.183055 \end{pmatrix} + 0.998 \times \begin{pmatrix} 0.59223 \\ 0.0011295 \end{pmatrix} \right] =$

$= \alpha \begin{pmatrix} 0.001 \\ 0.999 \end{pmatrix} \times \begin{pmatrix} 0.59224259 \\ 0.001493351 \end{pmatrix} = \alpha \begin{pmatrix} 0.00059224259 \\ 0.001491857 6 \end{pmatrix} =$

$\approx \langle 0.284, 0.716 \rangle$

b) VE: 7 additions, 16 multiplications, 2 divisions

Enumeration: 7 additions, 18 multiplications, 2 divisions