1. (a)

X	1	-1
$P(x)$	p	$1-p$

$$EX = \sum x P(x) = 1 \times p + (-1) \times (1-p) = 2p-1$$

Y	1	-1
$P(y)$	q	$1-q$

$$EY = \sum y P(y) = 1 \times q + (-1) \times (1-q)$$
$$= 2q-1$$

(b) $$EX^2 = \sum x^2 P(x) = 1^2 \times p + (-1)^2 \times (1-p) = 1$$

$$Var(X) = EX^2 - (EX)^2 = 1-(2p-1)^2 = (1+2p-1)(1-2p+1)$$
$$= 2p(2-2p) = 4p(1-p)$$

$$EY^2 = \sum y^2 P(y) = 1^2 \times q + (-1)^2 \times (1-q) = 1$$
$$Var(Y) = EY^2 - (EY)^2 = 1-(2q-1)^2 = (1+2q-1)(1-2q+1)$$
$$= 2q(2-2q) = 4q(1-q)$$

(c) $$EXY = \sum xy P(xy)$$
$$= 1 \times 1 \times (pq+\alpha) + (-1) \times (-1)[(1-p)(1-q)+\alpha] + 1 \times (-1) \times [p(1-q)-\alpha] + (-1) \times 1 \times [(1-p)q-\alpha]$$
$$= pq+\alpha+1-q-p+pq+\alpha-p+pq+\alpha-q+pq+\alpha$$
$$= 4\alpha+4pq-2q-2p+1$$
$$= 4\alpha+(2p-1)(2q-1)$$
$$Cov(X,Y) = EXY - EXEY = 4\alpha+(2p-1)(2q-1)-(2p-1)(2q-1)$$
$$= 4\alpha$$

So the covariance between X and Y only depends on the value of α.

(d) when $\alpha=0$ X and Y are independent

$$P(X=+1, Y=+1)=pq+\alpha=P(X=+1)P(Y=+1)=p\cdot q \quad \therefore \alpha=0$$

$$P(X=-1, Y=+1)=(1-p)q=P(X=-1)P(Y=+1)=(1-p)q$$

$$P(X=+1, Y=-1)=p(1-q)=P(X=+1)P(Y=-1)=p(1-q)$$

$$P(X=-1, Y=-1)=(1-p)(1-q)=P(X=-1)P(Y=-1)=p(1-q)$$

(f) $$P(Y=1 \mid X=1)=\frac{P(X=1, Y=1)}{P(X=1)}=\frac{pq+\alpha}{p}=q+\frac{\alpha}{p}$$

$$P(Y=-1 \mid X=-1)=\frac{P(X=-1, Y=-1)}{P(X=-1)}=\frac{(1-p)(1-q)+\alpha}{1-p}=1-q+\frac{\alpha}{1-p}$$

(g) $T=aX+bY$

$$ET=E(aX+bY)=aEX+bEY=a(2p-1)+b(2q-1)$$
$$=2ap+2bq-a-b$$

$$Var(T)=Var(aX+bY)=Var(aX)+Var(bY)+2\,cov(aX,bY)$$
$$=a^2Var(X)+b^2Var(Y)+2ab\,cov(X,Y)$$
$$=a^2 4p(1-p)+b^2 4q(1-q)+2ab\cdot 4\alpha$$
$$=4a^2p(1-p)+4b^2q(1-q)+8ab\alpha$$

2. (a) $\int_0^1 dx \int_0^1 f(x,y)dy = \int_0^1 dx \int_0^1 (ax^2+bxy+ay^2)dy = 1$

$= \int_0^1 dx\,(ax^2y + \frac{b}{2}xy^2 + \frac{a}{3}y^3)\Big|_0^1 = \int_0^1 (ax^2 + \frac{b}{2}x + \frac{a}{3})dx$

$= (\frac{a}{3}x^3 + \frac{b}{4}x^2 + \frac{a}{3}x)\Big|_0^1 = \frac{2}{3}a + \frac{b}{4} = 1$

$\therefore 8a + 3b = 12$

(b) $b=0$ so $a=\frac{3}{2}$ $f(x,y) = ax^2 + ay^2 = \frac{3}{2}(x^2+y^2)$

$D = \{(x,y) \in R^2 \mid 0<x\le 1, 0<y\le 1\}$

$Eg(x,y) = \iint_D g(x,y) f(x,y)\,dxdy = \iint_D \frac{1}{x^2+y^2} \frac{3}{2}(x^2+y^2)dxdy$

$= \frac{3}{2}\iint_D 1\,dxdy = \frac{3}{2} \times 1 = \frac{3}{2}$

(c) $a=0$ $\frac{2}{3}a + \frac{b}{4} = 1$ so $b=4$ $f(x,y) = 4xy$ $0\le x\le 1, 0\le y\le 1$

① $f_X(x) = \int_0^1 f(x,y)dy = \int_0^1 4xy\,dy = 2xy^2\Big|_0^1 = 2x$ $0\le x\le 1$

$f_Y(y) = \int_0^1 f(x,y)dx = \int_0^1 4xy\,dx = 2x^2y\Big|_0^1 = 2y$ $0\le y\le 1$

$f(x,y) = 4xy = f_X(x)f_Y(y) = 2x\cdot 2y$

so x, y are independent

② $x<0$ or $y<0$ $F(x,y)=0$

$0\leq x<1$, $0\leq y<1$ $F(x,y)=P(X\leq x, Y\leq y)=\int_0^x du\int_0^y f(u,v)dv$

$=\int_0^x du\int_0^y 4uv\,dv=\int_0^x 2uy^2 du$

$=u^2y^2\Big|_0^x=x^2y^2$

$0\leq x<1$ $y\geq 1$ $F(x,y)=F(x,1)=x^2$

$x\geq 1$, $0\leq y<1$ $F(x,y)=F(1,y)=y^2$

$x\geq 1$ and $y\geq 1$ $F(x,y)=F(1,1)=1$

so $F(x,y)=\begin{cases}0 & x<0 \text{ or } y<0\\ x^2y^2 & 0\leq x<1,\ 0\leq y<1\\ x^2 & 0\leq x<1,\ y\geq 1\\ y^2 & 1\leq x,\ 0\leq y<1\\ 1 & x\geq 1,\ y\geq 1\end{cases}$

③ $P(X\leq\frac{1}{2}, Y\leq\frac{1}{2})=F(\frac{1}{2},\frac{1}{2})=(\frac{1}{2})^2(\frac{1}{2})^2=\frac{1}{16}$

(d) $P(x^2+y^2\le 1)=\iint_{\substack{x^2+y^2=1\\ 1\ge x\ge 0\\ 1\ge y\ge 0}} a(x^2+y^2)+bxy\ dxdy = \cancel{\int_0^1 rdr\int_0^{\frac{\pi}{2}}(ar^2+br\cos\theta r\sin\theta)d\theta}$

$$=\int_0^1 rdr\int_0^{\frac{\pi}{2}}(ar^2+br\cos\theta\, r\sin\theta)d\theta$$

$$=\int_0^1 rdr\int_0^{\frac{\pi}{2}}(ar^2+\frac{br}{2}\sin 2\theta)d\theta=\int_0^1 rdr\left(ar^2\theta-\frac{br}{4}\cos 2\theta\right)\Big|_0^{\frac{\pi}{2}}$$

$$=\int_0^1\left(\frac{br^2}{2}+\frac{a\pi}{2}r^3\right)dr$$

$$=\left(\frac{b}{6}r^3+\frac{a\pi}{8}r^4\right)\Big|_0^1=\frac{b}{6}+\frac{a\pi}{8}$$

3. yellow rose $=1.3$ Pink rose $=3.3$ red rose $=4.6$

(a) minimum $=1.1\times 12=13.2$

maximum $=4.6\times 12=55.2$

(b)

X	P(X)	XP(X)	X^2P(X)
1.1	0.333	0.367	0.403
3.3	0.333	1.1	3.63
4.6	0.333	1.533	7.053

	P(X)	X·P(X)	X^2·P(X)
total sum	1	3	11.09

mean $=EX=\sum x\cdot P(X)=3$

Expected cost $=3\times 12=36$

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(c) $E[X^2] = \sum x^2 \cdot P(X) = 11.0867$

Variance $= E[X^2] - (E[X]^2) = 2.0867$

(d) $\mu = 36$

$\sigma = 5.003998401$

$P(30 < X < 45)$

$= P((30-36)/5.0039984012787 < (X-\mu)/\sigma < (45-36)/5.003998401)$

$P(-1.199 < Z < 1.799)$

$= P(Z < 1.799) - P(Z < -1.199) = 0.9640 - 0.1153 = 0.8487$

4.

(a) T

(b) F

(c) F

(d) F

(e) T

(f) F ($\rho = 0$, the X, Y are linear uncorrelated)

(g) T

(h) T

(i) F because $f_X(x) = \int_0^1 x y^{-\frac{1}{2}} dy = 2x\sqrt{y}\Big|_0^1 = 2x \quad 0 \le x \le 1$

$f_Y(y) = \int_0^1 x y^{-\frac{1}{2}} dx = \frac{1}{2} x^2 y^{-\frac{1}{2}}\Big|_0^1 = \frac{1}{2} y^{-\frac{1}{2}} \quad 0 < y \le 1$

(j) T