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Problem 1

$f(n) = (n^2+1)(\log n + 1) + (n^2\log n + 1)^2$

$= n^2\log n + n^2 + \log n + 1 + n^4(\log n)^2 + 2n^2\log n + 1$

$= 3n^2\log n + n^2 + \log n + n^4(\log n)^2 + 2$

$$\lim_{n\to\infty} \frac{3n^2\log n + n^2 + \log n + n^4(\log n)^2 + 2}{n^5}$$

$$= \lim_{n\to\infty} \frac{3\log n}{n^3} + \frac{1}{n^3} + \frac{\log n}{n^5} + \frac{(\log n)^2}{n} + \frac{2}{n^5}$$

~~$= 0 + 0 + 0 +$~~ since when $n \geq 1$, $\frac{\log n}{n^3}, \frac{\log n}{n^5}, \frac{(\log n)^2}{n^5}$ are all less than 1

and $\lim_{n\to\infty} \frac{1}{n^3} + \frac{2}{n^5} = 0$

So $\lim_{n\to\infty} \frac{f(n)}{n^5} = \frac{3\log n}{n^3} + \frac{1}{n^3} + \frac{\log n}{n^5} + \frac{(\log n)^2}{n} + \frac{2}{n^5} < 3+1+1 = 5$

Thus, there is an upper bound that is $\in \mathbb{R}_{>0}$

So $t = 5$, $f(n) = O(n^5)$

$$\lim_{n\to\infty} \frac{f(n)}{n^4} = \frac{3\log n}{n^2} + \frac{1}{n^2} + \frac{\log n}{n^4} + (\log n)^2 + \frac{2}{n^4}$$

since $(\log n)^2 \doteq \infty$

$\lim_{n\to\infty} \frac{f(n)}{n^4} = \infty$ since there is no upper bound

t should be 5 and not be 4.

Problem 2

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If $f(n)=n^3$, $g(n)=2n^3$, ~~which fulfill that $f(n)=\Omega(g(n))$~~

$$2^{f(n)} = 2^{n^3}$$

$$2^{g(n)} = 2^{2n^3}$$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{n^2}{2n^2} = \frac{1}{2} \geq \frac{1}{2}$$, so $f(n)=\Omega(g(n))$

$$\lim_{n\to\infty} \frac{2^{f(n)}}{2^{g(n)}} = \lim_{n\to\infty} 2^{f(n)-g(n)} = \lim_{n\to\infty} 2^{-n^3} = 0$$

So $2^{f(n)} \neq \Omega(2^{g(n)})$

Problem 3

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since we have $f(n) = O(g(n))$

$$g(n) = O(f(n))$$

and given $f: N \to R_{>0}$ and $g: N \to R_{>0}$, we have

$$(\exists d \in R_{>0})(\exists N \in N)(\forall n \geq N)\ \frac{f(n)}{g(n)} \leq d$$

and

$$(\exists c \in R_{>0})(\exists N \in N)(\forall n \geq N)\ \frac{g(n)}{f(n)} \leq c$$

so we get
$$f(n) \leq d \cdot g(n)$$
$$g(n) \leq c \cdot f(n)$$

so

$$(\exists c, d \in R_{>0})(\exists N \in N)(\forall n \geq N)\ \frac{c \cdot f(n)}{f(n)} \leq \frac{f(n)}{g(n)} \leq \frac{d \cdot g(n)}{g(n)}$$

which is $c \leq \frac{f(n)}{g(n)} \leq d$

So, by definition $f(n) = \Theta(g(n))$