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Problem 1

$A = \{0, 1, 2, 3, 4, 5\}$

Part a

i) $R = \{(0,0), (1,1), (2,2), (3,3), (4,4), (5,5), (1,5), (5,1), (2,0), (0,2), (4,1), (1,4), (5,4), (4,5)\}$ where $A = \{0,1,2,3,4,5\}$

ii) equivalence class

$[1] = \{1, 4, 5\} = [4] = [5]$

$[2] = \{2, 0\} = [0]$ $\{3\} = [3]$

Part b

i)

$S = \{(X, Y) : |X| \leq |Y|\}$

No, S is not a equivalence relation

Let $X = (1)$ $Y = (1,2)$ so $|X| \leq |Y|$

but 2 is not less or equal to 1 so $|Y| \nleq |X|$

So S is not symmetric.

Therefore, S is not a equivalence relation

ii) Yes, S is a order relation

if $(|X| \geq |Y|) \wedge (|Y| \geq |X|)$ then must $|X| = |Y|$

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Problem 1 CONT

So S is antisymmetric

if $(|x| \geq |y|) \wedge (|y| \geq |z|)$

then there must be $|x| \geq |y| \geq |z|$

so $|x| \geq |z|$

So S is transitive.

Therefore, since S is both antisymmetric and transitive, so S is a order relation.

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Problem 2.

Part a:

i) if $a \subseteq b$ and $b \subseteq a$
then b must equal to a
So R is antisymmetric
if $a \subseteq b$ and $b \subseteq c$
then there must be $a \subseteq b \subseteq c$
So $a \subseteq c$
so R is transitive
Since R is both antisymmetric and transitive, so R is a order relation.

ii) prove by contradiction
suppose it is a total order:
$((\forall a, b \in S)\ a \neq b \Rightarrow (aRb \vee bRa)$
then let $a = \{A\}$ $b = \{\Gamma\}$
we have $a \neq b$, so we must have aRb or bRa
but $a \nsubseteq b$ and $b \nsubseteq a$
so the assumption is wrong
R is not a total order

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Problem 2. CONT

iii) R is a partial order because for all $a \in S$, $a \leq a$
so R is reflexive
Therefore R is a partial order

iv) R is not strict order
because R is reflexive, so R is not antireflexive
Therefore R is not a strict order.

Part b:

i) maximal element: $\{A, B, \Delta\}$
$\{B, \Gamma, \Delta\}$

ii) minimal element: $\{\emptyset\}$

iii) There is no greatest element

iv) The least element is $\{\emptyset\}$.