CS 240 Exam 1 Written Answer Sheet

Print/type name (First Last):	Zhan Yu
Print/type NetID:	zyu293@wisc.edu
Print/type student ID:	9081553720

Instructions:

Overview:

1. Get a copy of this sheet (you will put your answers to the "written" questions on your copy)
2. The actual questions you will answer will be available to you in the Exam 1 Canvas quiz once you start taking it. Put your answers on your copy of your sheet.
3. Upload your answer sheet as your answer to the last question of the Exam 1 Canvas quiz. You must upload a single PDF file.

More details:

<u>Before starting the exam:</u>

From the **File** tab in the upper-left corner, select **Make a copy.** You will not be able to edit this version.

Open up your answer sheet in your Google Drive (if it does not automatically open).

Fill out the name, NetID, and student ID information on the first page. Do not remove any of the text provided in this answer sheet. This will ensure that your exam is able to be uploaded and graded in GradeScope.

<u>Filling in the answer sheet with your answers:</u>

You may do any of the following:

- print the answer sheet and hand-write your answers in the appropriate locations
- write your answers on your own paper and copy-paste a picture of your work into the appropriate locations **Note:** if you use this option, make sure to delete the provided structure within each answer box (to avoid extra pages)
- use a tablet & stylus to hand-write your solutions electronically in the appropriate locations

Answers to free response questions **must** be hand-written.

<u>To turn in your answer sheet:</u>

Create a single pdf from your answer sheet and upload your file as your answer to the last question in the Exam 1 Canvas quiz.

Note: to create a pdf from within Google Docs,
From the **File** tab in the upper-left corner, select **Download**.
Select **PDF Document (.pdf)** and save your file to your computer.

If time expires before you have a chance to upload your file, upload your pdf file to the *Exam 1 late* assignment in Canvas. If your pdf is uploaded within 10 minutes of the ending time of your quiz, it will be accepted for grading.

Question 14

Write your answer for Question 14 in the box below:

Part a:

P: Pat rides the bus tomorrow.

Q: Quinn rides the bus tomorrow.

R: Riley rides the bus tomorrow.

Part b:

1. $P \Rightarrow Q$
2. $(Q \vee R) \wedge \neg(Q \wedge R)$
3. $R \Rightarrow P$
4. $(P \vee R) \vee (P \wedge R) = P \vee R$

Part c:

Since all of the 4 statements in part b are true,

P, R should be TT, TF, FT by given $(P \vee R)$ from 4

Since $R \Rightarrow P$ is true in 3, P, R can only be TT or TF by giving 3 and 4.

Case 1: if $P = T$, $R = T$

Since $P \Rightarrow Q$ is true from 1, P is true, so Q must be true.

However, for 2, it will be false if P, Q are both true, so P, R cannot be both true.

Case 2: if $P = T$ $R = F$

since $P \Rightarrow Q$ is true from 1, P is true, so Q must be true.

For 2, if Q is true and R is false, this statement will be true.

Therefore, all 4 statements are true if $P = T$, $R = F$, $Q = T$, Pat and Quinn rides the bus tomorrow

Question 15

Write your answer for Question 15 below:

Statement	Justification
$(A \cap \bar{B}) \cup (A \cap \bar{C})$	right side of the equation
$= \{x : x \in (A \cap \bar{B}) \cup (A \cap \bar{C})\}$	convert to set builder notation
$= \{x : (x \in (A \cap \bar{B})) \vee (x \in (A \cap \bar{C}))\}$	definition of uion ($\cup$)
$= \{x : (x \in A \wedge x \in \bar{B}) \vee (x \in A \wedge x \in \bar{C})\}$	definition of intersection ($\cap$)
$= \{x : (x \in A \wedge x \notin B) \vee (x \in A \wedge x \notin C)\}$	definition of set complement
$= \{x : (x \in A) \wedge (x \notin B \vee x \notin C)\}$	distributive property
$= \{x : (x \in A) \wedge (\neg(x \in B) \vee \neg(x \in C)\}$	~~DeMorgan's law~~ definition of $\notin$
$\cancel{= \{x : (x \in A) \wedge \neg(\neg\neg(x \in B) \vee \neg\neg(x \in C))\}}$	~~DeMorg~~
$= \{x : (x \in A) \wedge \neg(\neg\neg(x \in B) \wedge \neg\neg(x \in C))\}$	DeMorgan's law
$= \{x : (x \in A) \wedge \neg(x \in B \wedge x \in C)\}$	Double negtion
$= \{x : (x \in A) \wedge \neg(x \in (B \cap C))\}$	definition of intersection ($\cap$)
$= \{x : (x \in A) \wedge x \in \overline{(B \cap C)}\}$	definition of set complement
$= \{x : x \in A \cap \overline{(B \cap C)}\}$	definition of interaction ($\cap$)
$= A \cap \overline{(B \cap C)}$	convert from set builder notation

Question 16

Write your answer for Question 16 in the box below:

Part a:

~~Proof:~~

we want to prove that if n^2+1 is ~~even~~ old, then n is even.

Proof: we prove the contrapositive: if n is odd, then n^2+1 is even

Suppose n is odd. Then $n=2K+1$ by definition for some integer k.

~~Then n^2+1~~

Then $n^2+1 = (2K+1)^2+1$ (by plugging $n=2K+1$)

$= 4K^2+4K+1+1$

$= 4K^2+4K+2$

$= 2(2K^2+2K+1)$ (by algebra)

Since K is an integer, so is $2K^2+2K+1$ (by closure).

Thus, $n^2+1 = 2m$ for integer $m = 2K^2+2K+1$ so, by definition, n^2+1 = even

~~Since~~ So, since the contrapositive n is odd implies n^2+1 is even is true, n^2+1 is old implies n is even is also true ∎

Part b:

I use proof by contrapositive

Question 17

Write your answer for Question 17 in the box below:

Part a: Proof by induction: Some structure of a proof by induction is provided for you.

What is P(*n*)? $P(n) = (\forall n \in \mathbb{N})\ (3 \mid 4^n - 1)$

Base case: We want to prove $P(0)$ holds

Proof:

To show $3 \mid 4^0 - 1$

$4^0 - 1 = 1 - 1 = 0$

since $3 \mid 0$, $P(0)$ holds

Inductive step: We want to prove $P(n+1) = (\forall n \in \mathbb{N})\ (3 \mid 4^{n+1} - 1)$ holds

(*inductive hypothesis*) Assume $P(n) = (\forall n \in \mathbb{N})\ (3 \mid 4^n - 1)$ is true

Proof:

by ~~Inductive H~~ IH,

$4^n - 1 = 3k$, where $k \in \mathbb{Z}$

Thus $4^n = 3k + 1$ ~~and $4^{n+1} = 3$~~

$4^{n+1} = (3k+1) \cdot 4 = 12k + 4$

$4^{n+1} - 1 = 12k + 3 = 3(4k+1)$

since $k \in \mathbb{Z}$, $4k+1$ is also an integer (by closure)

So, $3 \mid 4^{n+1} - 1 = 3 \mid 3(4k+1)$

$P(n+1)$ holds

∎

Conclusion: Therefore, by induction, P(*n*) holds for all positive natural numbers n

Part b: I use regular induction