Name: Zhan Yu
E-mail: zyu293@wisc.edu
Dis: 211

Problem 1:

Part a: $A = \{0, 1, 2, 3, 4, 5\}$

i) $R = \{(0,0), (1,1), (2,2), (3,3), (4,4), (5,5), (1,5), (5,1), (2,0)$
$(0,2), (4,1), (1,4), (5,4), (4,5)\}$

ii) equivalence classes: $[1] = \{1,4,5\} = [4] = [5]$
$[2] = \{2,0\} = [0]$ $\{3\} = [3]$

Part b: i) No, S is not a equivalence relation
because $|X| < |Y|$
Then $|Y|$ cannot be $< |X|$
So, S is not symmetric, and not a equivalence relation.

ii) S is an order relation
If $|X| \geq |Y|$ and $|Y| \geq |X|$, the $|X|$ must equals to $|Y|$
so S is antisymmetric
If $|X| \leq |Y|$ and $|Y| \leq |Z|$, then $|X|$ must $\leq |Z|$
so S is transitive

So S is an order relation

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Problem 2:

Part a:

i) if $a \leq b$ and $b \leq a$, b must equal to a

So R is ~~antis~~ antisymmetric

if $a \leq b$ and $b \leq c$, then $a \leq b \leq c$, so $a \leq c$

~~so~~ Thus, R is transitive.

Since R is antisymmetric and transitive, R is an order relation.

ii) suppose R is a total order.

$(\forall a, b \in S)$ $a \neq b$, so $aRb \vee bRa$

let $a = \{A\}$ and $b = \{\Pi\}$

so $a \neq b$, but $a \leq b$ and $b \leq a$ are not true, so we don't have aRb or bRa

So, R is not a total order.

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iii) R is a partial order

$\forall a \in S$, $a \subseteq a$, So R is ~~reflexive~~ reflective, the R is a partial order

iv) R is not strict order

R is ~~reflexive~~ reflective ~~reflexive~~, ~~and~~ so it is not antireflective

Thus, R is not a strict order.

Part b.

i) maximal element: $\{A, B, \Delta\}$, $\{B, \Gamma, \Delta\}$

ii) minimal element: $\{\phi\}$

iii) There is no greatest element

iv) The least element is $\{\phi\}$