Name: Zhan Yu
E-mail: zyu293@wisc.edu

Problem 1

Zhan Yu
zyu293@wisc.edu

(a) $S = \{12, 16, 20, 24 \ldots\}$

enumeration: 12, 16, 20, 24 ...

(b) Proof: $(i+2) \times 4$ is the element of S appears at position i in the enumeration, suppose $i \in N^+$

$(i+2) \times 4$ we can map: when $i=1 \to 12$, $i=2 \to 16$, $i=3 \to 20$ ...

For any $i \in N^+$, $4i+8 \geq 10$ and $4i+8 = 4(i+2)$ is divisible by 4

So $4(i+2) \in S$

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(c) Proof: suppose $k \in S$, element k appears at $\frac{k-8}{4}$ in the enumeration.

From (a) and (b), we get $k = 4i+8$

So $\frac{k-8}{4} = i \Leftarrow$ in this position

Since k and 8 are divisible by 4,

$\frac{k-8}{4}$ is also divisible by 4

and $\frac{k-8}{4}$ is an integer

Also, since $k \geq 12$, $\frac{k-8}{4} \geq 1 \Rightarrow \frac{k-8}{4} \in N^+$

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From (b) and (c), we can conclude that the enumeration is a one-to-one correspondence between S and N^+

So, S is countable

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Problem 2

Prove that $A \cap (B-C) = (A \cap B) - (A \cap C)$ for arbitrary sets A, B and C using propositional logic and the definitions of set operators

Proof: we start with right side of what we want to prove and use the definitions of what set operators are and the properties of propositional operators

	Justification
$(A \cap B) - (A \cap C)$	right side of equation
$= \{x : x \in ((A \cap B) - (A \cap C))\}$	convert to set builder notation
$= \{x : (x \in A \land x \in B) - (x \in (A \cap C))\}$	definition of $\cap$
$= \{x : (x \in A \land x \in B) \land \neg(x \in (A \cap C))\}$	definition of difference (-)
$= \{x : (x \in A \land x \in B) \land \neg(\cancel{x \notin A} \; x \in A \land x \in C)\}$	definition of $\cap$
$= \{x : (x \in A \land x \in B) \land (\underset{\neg(x \in A)}{\cancel{x \notin A}} \lor \underset{\neg(x \in C)}{\cancel{x \notin C}})\}$	DeMorgan's Law
$= \{x : (x \in A \land x \in B \land \neg(x \in A)) \lor (x \in A \land x \in B \land \neg(x \in C))\}$	distributive property
$= \{x : \text{FALSE} \lor (x \in A \land x \in B \land \neg(x \in C))\}$	complement property
$\{x : \text{FALSE} \lor x \in (A \cap B \cap \neg C)\}$	definition of $\cap$
$= \{x : x \in (A \cap B \cap \neg C)\}$	identity property
$= \{x : x \in (A \cap (B \cap \neg C))\}$	associative property
$= \{x : x \in (A \cap (B - C))\}$	definition of difference (-)
$= A \cap (B - C)$	convert from set builder notation to get left side of equation

The Justification show each step preserves the equality

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