Zhan Yu
zyu293@wisc.edu

Problem 1:

reflective: a vertex always has a path to itself.

transitive: if a and b ($a, b \in V$) are mutually reachable and b and c ($c \in V$) are mutually reachable,

through the path $a \to b \to c$, a can reach c

through the path $c \to b \to a$, c can reach a

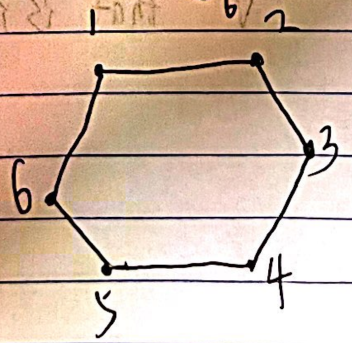
so a and c are mutually reachable and it's transitive

symmetric: since two vertices are mutually reachable, it's symmetric

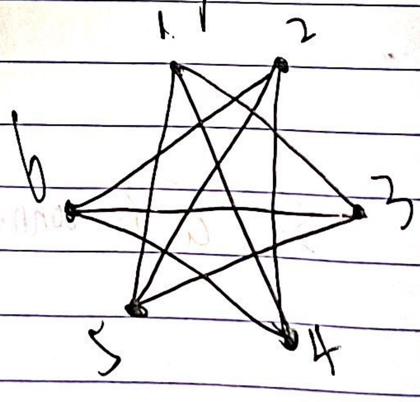
Problem 2:

a.

i. C_6



complement



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ii. $K_{m,n}$ is the graph with m vertices in one set A and n vertices in the other set B.

Also, all the vertices of set A are adjacent to all vertices of set B.

The complement of $K_{m,n}$ is the set of edges not in $K_{m,n}$

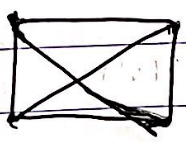
i.e. Both subset of $K_{m,n}$ is a complete graph and there's no edge between two vertices from different subsets

iii. K_n is the complete graph and every vertices is adjacent to every other vertex.

Complement of K_n is the edge not in K_n

$|E|$ of $\overline{K_n}$ is 0

eg. K_4 K_4 complement



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b. ~~Proof~~

Suppose $G=(V,E)$ is a simple graph on n vertices with no self-loops with two connected components, proof that G is connected.

Assumn u, v are two vertices in G

G is a simple graph with two components A and B, so there is a vertex in A and at least a vertex in B. Hence $n \geq 2$.

Case 1: u, v are not in the same connected component.

For G, there is no edges to form a path between u and v, so $\{u,v\}$ is not in G

For $\bar{G}$, by definition $\{u,v\}$ is an edge in $\bar{G}$, so there's a path between u and v in $\bar{G}$

Case 2: u, v are in the same connected component of G, which assumed to be A

then u, v are connected in G.

So, there is no path between u and the vertices in set B.

On $\bar{G}$, there is a path from u to every vertices in set B.

Also, there is a path from every vertices in B to u.

Thus, u and v are connected in $\bar{G}$

erefore, From both case, there exits a path for every vertices u and v that is in $\bar{G}$

So $\bar{G}$ is connected.