

4. **TRUE / FALSE.** Each of these five (unrelated) general statements is either true or false. If the former, formally **prove** it using definitions, etc.; if the latter, find a concrete **counterexample** that disproves it. (5 pts ea)

**Note:** A single example that verifies a given statement is NOT a formal proof that it is true *in general*!

- (a) “**Conditional**” **Complement Rule.** For any two events  $A$  and  $B$ ,

$$\boxed{P(A | B) + P(A^c | B) = 1}.$$

- (b) “**Conditional**” **Addition Rule.** For any three events  $A, B$ , and  $C$ ,

$$\boxed{P(A \cup B | C) = P(A | C) + P(B | C) - P(A \cap B | C)}.$$

- (c) Let  $A$  and  $B$  be two *statistically independent* events, i.e.,  $P(A \cap B) = P(A)P(B)$ . Suppose  $C$  is any other event (with nonzero probability). Then  $A$  and  $B$  must be *conditionally independent* with respect to  $C$ . That is,  $P(A \cap B | C) = P(A | C)P(B | C)$ .

- (d) Two *statistically independent* events  $A$  and  $B$  with nonzero probabilities cannot be disjoint.

- (e) For any two events  $A$  and  $B$ , if  $\boxed{P(A | B) = P(A | B^c)}$ , then  $A$  and  $B$  are **statistically independent**.