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Problem 1

a. given $A_n = A_{n-1} + 4$ and $n > 0$

n	M(n)
1	$A_1 = A_{1-1} + 4 = A_0 + 4 = 2+4 = 6 = 4\times1+2$
2	$A_2 = A_{2-1} + 4 = A_1 + 4 = 6+4 = 10 = 4\times2+2$
3	$A_3 = A_{3-1} + 4 = A_2 + 4 = 10+4 = 14 = 4\times3+2$
4	$A_4 = A_{4-1} + 4 = A_3 + 4 = 18 = 4\times4+2$
5	$A_5 = A_{5-1} + 4 = A_4 + 4 = 22 = 4\times5+2$
n	$4n+2$

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b. proposed solution:

$$A(n) = 4n+2 \ (\forall n > 0)$$

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c. Prove the solution works using induction

Let $P(n)$: $A(n) = 4n+2$ show $P(n)$ holds $\forall n > 0$

Base case: show $A(1)$ holds

(LHS) $A(1) = A_{1-1}+4 = A_0 + 4 = 2+4 = 6$ (by definition of recurrence relation)

(RHS) $4\times1+2=6$

so $A(1) = 4\times1+2$ and thus $P(1)$ holds

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Inductive step: show $P(k) \Rightarrow P(k+1)$

Assume $P(k)$ holds, i.e, $A(k) = 4k+2$ (IH)

$A(k+1) = A_{k+1-1}+4$ using recurrence relation

$= A_k + 4$

$= 4k+2+4$

$= 4(k+1)+2$ so $P(k+1)$ holds

$\therefore$ by induction $A(n) = 4n+2 \quad \forall n > 0$ $\blacksquare$

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Problem 2

1 inch : f, i, j, t
2 inch : a, b, c, e, g, h, k, n, o, p, s, u

a.

Let $P(n)$ be the number of ways to create a string that has length n inches

String length	Number of ways
0	1 (No string has length 0) (do nothing)
1	1+1+1+1 = 4 (four 1 inch wide letters)
2	case 1: consist of two 1 inch letters. since every letter can have the other 1 inch letter behind it, so 4×4 = 16 (ways). Case 2: consist of 1 two inch letter, there are 12 two inch letters. so 16+12×1 = 28
3	Case 1: start with 1 inch letter: 4×($P_{(2)}$) (four 1 inch letter with banner that's 2 inches) = 4×28. Case 2: start with 2 inch letter: 12 × $P_{(1)}$ (twelve 2 inches letter with 1 inch banner) = 12×4. 4×28+12×4 = 160

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Case 1: start with 1 inch

4×($P_{(3)}$) four 1 inch letter with 3 inches banner

= 4×160

Case 2: start with 2 inch

12×($P_{(2)}$) twelve 1 inch letters with 2 inches banner

= 12×28

= 336

4×160 + 336 = 976

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b. $P(0) = 1 \quad P(1) = 4$

$P(n) = 4 \times P(n-1) + 12 \times P(n-2)$ for $n \geq 2$

since we have four 1inch letters and twelve 2inches letters,

we can use the four 1inch letters to pair with banner that we get from last time, which has length-1

Also, we can use the twelve 2inches letters to pair with length-2 banner

~~For pair~~ pairing with ~~metas~~ means we make product of them,

So, this is how I we get the recurrence relation