Assignment 3 Written Solution

Problem 1 | **Problem 2**

Problem 1:

First we note that $S = \{ x \in \mathbb{Z} : (x > 10) \wedge (4 \mid x) \} = \{12, 16, 20, 24, 28, 32, ...\}$

To prove an infinite set S is countable, we need to prove that there is an enumeration consisting exactly of all elements of the set. Every position in the enumeration should correspond to a different element of the set and every element of the set should appear in it. Thus, we need to find a 1-1 correspondence between S and the set $\mathbb{N}^+ = \{1, 2, 3, 4, 5, ...\}$.

Part a: Consider the following enumeration of S: 12, 16, 20, 24, 28, 32, ...

This gives us a 1-1 correspondence between S and $\mathbb{N}^+$ (and the following two steps show that this it is a 1-1 correspondence):

Part b: Suppose $i \in \mathbb{N}^+$. Then the corresponding element of S that appears at position i is $4 \cdot (i + 2)$.

> Note that since $i \in \mathbb{N}^+$, $4 \cdot (i + 2)$ will be a natural number greater than 10. Moreover, 4 divides $4 \cdot (i + 2)$, thus $4 \cdot (i + 2) \in S$.

Part c: Suppose $k \in S$. Then the position in the enumeration corresponding to k is $(k/4) - 2$.

> Note that since $k \in S$, k is an integer greater than 10 and divisible by 4. This means $k = 4 \cdot n$ for some integer n and since $k > 10$, $n > 2$. Thus, $(k/4) - 2$ simplifies to an element of $\mathbb{N}^+$.

Since we have given an enumeration of S and shown that it is a 1-1 correspondence between S and $\mathbb{N}^+$, S is countable. ■

Problem 2:

Proof that $A \cap (B - C) = (A \cap B) - (A \cap C)$:

We will start with the right side.

	Justification
$(A \cap B) - (A \cap C)$	right side of equation
$= \{x : x \in (A \cap B) - (A \cap C)\}$	convert to set builder notation

$= \{x :\ x \in (A \cap B) \wedge x \notin (A \cap C)\}$	definition of set difference
$= \{x :\ x \in (A \cap B) \wedge \neg(x \in (A \cap C))\}$	definition of $\notin$
$= \{x :\ (x \in A \wedge x \in B) \wedge \neg(x \in A \wedge x \in C))\}$	definition of intersection ($\cap$)
$= \{x :\ (x \in A \wedge x \in B) \wedge (\neg(x \in A) \vee \neg(x \in C))\}$	DeMorgan's law
$= \{x :\ ((x \in A \wedge x \in B) \wedge \neg(x \in A)) \vee ((x \in A \wedge x \in B) \wedge \neg(x \in C))\}$	distributive property
$= \{x :\ ((x \in B \wedge x \in A) \wedge \neg(x \in A)) \vee ((x \in A \wedge x \in B) \wedge \neg(x \in C))\}$	commutative property
$= \{x :\ (x \in B \wedge (x \in A \wedge \neg(x \in A))) \vee ((x \in A \wedge x \in B) \wedge \neg(x \in C))\}$	associative property
$= \{x :\ (x \in B \wedge FALSE) \vee ((x \in A \wedge x \in B) \wedge \neg(x \in C))\}$	complement property
$= \{x :\ FALSE \vee ((x \in A \wedge x \in B) \wedge \neg(x \in C))\}$	domination property
$= \{x :\ (x \in A \wedge x \in B) \wedge \neg(x \in C)\}$	identity property
$= \{x :\ x \in A \wedge (x \in B \wedge \neg(x \in C))\}$	associative property
$= \{x :\ x \in A \wedge (x \in B \wedge x \notin C)\}$	definition of $\notin$
$= \{x :\ x \in A \wedge x \in (B - C)\}$	definition of set difference
$= \{x :\ x \in A \cap (B - C)\}$	definition of intersection ($\cap$)
$= A \cap (B - C)$	convert from set builder notation

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