

MATH/CS 714: PROJECT 1

PAGE LIMIT: 15 pages (single-sided).

NOTE: Please include a cover page. This will not count toward the 15 page limit.

NOTE: Don't forget to include code where appropriate. This *does* count toward the 15 page limit.

PURPOSE: To use a Fortran code to find numerical solutions of PDEs.
(Later, we will also use it to create a *parallel* code.)

PROVIDED FOR YOU:

1. A Fortran code that solves the heat equation.
2. A document on “Getting Started” with Fortran.

WHAT TO DO:

1. Read the document on “Getting Started”.
2. Practice as much Fortran as you need in order to complete the next tasks.
3. Run the provided Fortran code for solving the heat equation.
4. Do a convergence test to demonstrate first-order convergence.
5. Modify the code to solve the 2D advection equation,

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x}(uc) + \frac{\partial}{\partial y}(vc) = 0, \tag{1}$$

using the Lax–Friedrichs method,

$$c_{ij}^{n+1} = \frac{1}{4}(c_{i-1,j}^n + c_{i+1,j}^n + c_{i,j-1}^n + c_{i,j+1}^n) - k \frac{(uc)_{i+1,j}^n - (uc)_{i-1,j}^n}{2h} - k \frac{(vc)_{i,j+1}^n - (vc)_{i,j-1}^n}{2h}. \tag{2}$$

The chemical concentration is $c(x, y, t)$; for simplicity, choose $c = 0$ on the boundary (if chosen that way at $t = 0$, then it will remain true for all times). The velocity field is $\mathbf{u}(x, y) = (u(x, y), v(x, y))$, and a no-flux boundary condition is imposed: $\mathbf{u} \cdot \hat{\mathbf{n}} = 0$, where $\hat{\mathbf{n}}$ is the (outward) unit normal vector on the boundary.

6. Do a convergence test to demonstrate first-order convergence.

Assigned: Tuesday, September 24, 2019
Due (early): Thursday, October 24, 2019
Due (regular): Thursday, November 7, 2019

WHAT TO TURN IN:

1. Plots to illustrate the solution of the *heat* equation.
2. Description of the setup for your convergence tests for the *heat* equation.
3. Plots and/or other data to demonstrate first-order convergence for the *heat* equation.
4. Plots to illustrate the solution of the *advection* equation.
5. Description of the setup for your convergence tests for the *advection* equation.
6. Plots and/or other data to demonstrate first-order convergence for the *advection* equation.
7. Code for solving the *advection* equation.
(Also acceptable to just show the *modifications* that you made to the original heat equation code.)
8. Survey: Complete the survey on Canvas.