**(1)**

**Code:**

function [Q,S]=ArnoldiMethod(A,n)

m=length(A);

b=rand(m,1);

S=zeros(n+1,n);

Q(:,1)=b/norm(b);

for i=2:n+1

Q(:,i)=A\*Q(:,i-1);

for j=1:i-1

S(j,i-1)=Q(:,i)'\*Q(:,j);

end

Q(:,i)=Q(:,i)-Q(:,1:i-1)\*S(1:i-1,i-1);

S(i,i-1)=norm(Q(:,i));

Q(:,i)=Q(:,i)/S(i,i-1);

end

S=S(1:n,:);

Q=Q(:,1:n);

end

A=rand(100);

sigma1=max(abs(eig(A)));

e=zeros(10,1);

for n=1:10

[Q,S]=ArnoldiMethod(A,n);

sigma2=max(abs(eig(S)));

e(n)=abs(sigma1-sigma2);

end

figure

plot(abs(eig(A)),'.');

title('The distribution of absolute values of the eigenvalues')

figure

plot(1:10,log(e));

xlabel('n = # of iterations')

ylabel('log(error)')

title('log(errors) with respect to # of iterations')

**(2)**

**Code:**

function x=GMRES(A,b,n)

S=zeros(n+1,n);

t=zeros(n,1);

Q(:,1)=b/norm(b);

for i=2:n+1

Q(:,i)=A\*Q(:,i-1);

for j=1:i-1

S(j,i-1)=Q(:,i)'\*Q(:,j);

end

Q(:,i)=Q(:,i)-Q(:,1:i-1)\*S(1:i-1,i-1);

S(i,i-1)=norm(Q(:,i));

Q(:,i)=Q(:,i)/S(i,i-1);

end

Q=Q(:,1:n);

b=[norm(b) zeros(1,n)]';

t=inv(S'\*S)\*S'\*b;

x=Q\*t;

end

format long

v=zeros(100,1);

e=zeros(4,1);

N=[4 5 10];

e\_=zeros(20,5);

for i=1:100

r=randperm(3);

v(i)=N(r(1));

end

v\_=v;

v=rand(100,1)+v;

A=diag(v);

P=rand(100);

A=P\*A\*P^-1;

b=rand(100,1);

x0=A\b;

temp=[5 10 20 90];

for i=1:4

n=temp(i);

x=GMRES(A,b,n);

e(i)=norm(x-x0);

end

for j=1:5

v=v\_+rand(100,1)\*10^(j-5);

A=diag(v);

A=P\*A\*P^-1;

x0=A\b;

for n=1:22

x=GMRES(A,b,n);

e\_(n,j)=norm(x-x0);

end

end

figure

for i=1:5

plot(1:22,log(e\_(:,i)))

hold on

end

% e(1) denotes the error of solution for n=5

% e(2) denotes the error of solution for n=10

% e(3) denotes the error of solution for n=20

% e(4) denotes the error of solution for n=90

xlabel('# of iterations')

ylabel('log(error)')

legend('ε=10^-4','ε=10^-3','ε=10^-2','ε=10^-1','ε=10^0','location','northeast')

>> u=[10^-4 10^-3 10^-2 10^-1 10^0];

>> v=[8 9 11 15 18];

>> plot(log(u),v);

>> xlabel('log(ε)');

>> ylabel('# of iterations for the convergence');

>> title('choice of ε affects the speed of convergence')