Formula & Concept Sheet until Quiz 1

Concept

Lecture 1

Intensity, Gray Level, Image Sensor, Pixel, Coordinates, Sampling/Discretization, Quantization, Spatial Resolution, Gray-Level Resolution, Color Space, Color Primaries, Additive Primaries, Subtractive Primaries

Lecture 2

Visible Light, Rods and Cones, Spectral Sensitivity, Luminance, Brightness, Simultaneous Contrast, Weber's Law, Lightness, Chrominance, Hue, Saturation, Chromatic Adaptation, CIE-XYZ, CIE-L*a*b*, Color Difference

Lecture 3

Impulse Sequence, Linear and Shift Invariant System, Principle of Superposition, Impulse Response, Convolution, Fourier Transform, Spatial/Frequency Variables, Basis Functions, Fourier/Power Spectrum, Discrete Fourier Transform, Conjugate Symmetry, Discrete Cosine Transform, Properties of Transform, Periodicity, Separable Property

Lecture4

Bandwidth, Band-Limited Signal Sampling, Function Sampling, Frequency/Rate, Sampling Interval, Dirac Delta Function, Sampling Theorem, Nyquist Frequency/Rate/Interval, Aliasing, Decision Level/Interval, Reconstruction Level, Quantization Error, Mean Square Error (MSE), Llyod-Max Quantizer, Linear Quantizer, Signal-to-Noise Ratio (SNR)

Lecture5

Image Enhancement, Point Processing, Gray-Level Reversal, Low/High Contrast, Contrast Stretching, Thresholding, Dynamic Range, Image Histogram, Histogram Equalization, Histogram Matching, Image Filtering, Lowpass/Highpass Filters, Spatial Filtering, Smoothing/Sharpening Filters, Unsharp Masking, High-Boost Filters, Gradient/Edge Filters

Formula

Lecture 1

- Color space

$$Color\ space:\ C = aP_1 + bP_2 + CP_3 < h3 > Lecture2$$

- Luminance (CIE definition)

$$Y = \int_{\lambda} R(\lambda)V(\lambda)d\lambda,\ where\ R(\lambda) = \rho(\lambda)E(\lambda)$$
$$\rho:\ refelctivity\ or\ transmissivity;\ E:\ energy\ distribution\ at\ \lambda;\ \lambda:\ wavelength$$

- Simultaneous Contrast - *Weber's Law*

$$\frac{|Y_s - Y|}{Y} \approx d(lnY) = \Delta C = constant$$

- Lightness (w.r.t *luminance*)

$$L^* = \begin{cases} 116(\frac{Y}{Y_n})^{\frac{1}{3}} - 16, & if \frac{Y}{Y_n} > 0.008856 \\ 903.3\frac{Y}{Y_n}, & otherwise \end{cases}$$

- RGB - XYZ Conversion

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.4124 & 0.3576 & 0.1804 \\ 0.2127 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9502 \end{bmatrix} \begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix}; \quad \begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix} = \begin{bmatrix} 3.2405 & -1.5372 & -0.4985 \\ -0.9692 & 1.8760 & 0.0416 \\ 0.0556 & -0.2040 & 1.0573 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

- XYZ - Lab Conversion

$$L^* = 116f(\frac{Y}{Y_n}) - 16,$$
$$a^* = 500(f(\frac{X}{X_n}) - f(\frac{Y}{Y_n}),$$
$$b^* = 200(f(\frac{Y}{Y_n}) - f(\frac{Z}{Z_n}),$$
$$where \;\; f(u) = \begin{cases} u^{\frac{1}{3}} \quad if u > 0.008856 \\ 7.787u + \frac{16}{116} \quad otherwise \end{cases}$$
$$Chroma: \;\; C^*_{ab} = \sqrt{(a^*)^2 + (b^*)^2};$$
$$Hue: \;\; h_{ab} = tan^{-1}(\frac{b^*}{a^*});$$
$$Color\ difference: \;\; \Delta E = \sqrt{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2}$$

Lecture 3

- Definition of image as 2D sequence

$$f(x, y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(m, n)\delta(x - m, y - n)$$

- 2D LSI (Linear Shift-Invariant) System

$$g(x, y) = T[f(x, y)] = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(m, n)\; T[\delta(x - m, y - n)]$$
$$= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(m, n)h(x - m, y - n) = f(x, y) * h(x, y)$$

- Fourier Transform

$$F(u, v) = \int_{-\infty}^{\infty} f(x, y)\; e^{-j2\pi(ux+vy)}\; dxdy$$
$$f(x, y) = \int_{-\infty}^{\infty} f(u, v)\; e^{j2\pi(ux+vy)}\; dxdy$$

$$Fourier\ spectrum: |F(u, v)| = \sqrt{R^2(u, v) + I^2(u, v)}$$
$$Phase\ angle: \phi = tan^{-1}[\frac{I(u, v)}{R(u, v)}]$$
$$Power\ spectrum = |F(u, v)|^2 = R^2(u, v) + I^2(u, v), \quad where F(u, v) = R(u, v) + jI(u, v)$$

- Discrete Fourier Transform

$$F(u,v) = \frac{1}{MN}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} f(x,y)e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})}$$

$$f(x,y) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} F(u,v)e^{j2\pi(\frac{ux}{M}+\frac{vy}{N})}$$

- Properties of 2D FT

$$f(x,y)e^{j2\pi(\frac{u_0x}{M})+\frac{v_0y}{N})} \Leftrightarrow F(u-u_0, v-v_0)$$
$$f(x-x_0, y-y_0) \Leftrightarrow F(u,v)e^{-j2\pi(\frac{u_0x}{M})+\frac{v_0y}{N})}$$
$$f(ax, by) \Leftrightarrow \frac{1}{|ab|}f(\frac{u}{a}, \frac{v}{b})$$
$$f*h \Leftrightarrow FH,\ fh \Leftrightarrow F*H$$

- DCT (Discrete Cosine Transform)

$$F(u,v) = \frac{2}{N}C(u)C(v)\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)cos(\frac{(2x+1)u\pi}{2N})cos(\frac{(2y+1)v\pi}{2N}),$$
$$f(x,y) = \frac{2}{N}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} C(u)C(v)F(u,v)cos(\frac{(2x+1)u\pi}{2N})cos(\frac{(2y+1)v\pi}{2N}),$$
$$where\ \ C(u), C(v) = \begin{cases} \frac{1}{\sqrt{2}}\ for\ u,v=0 \\ 1\ for\ u,v = 1,\dots,N-1 \end{cases}$$

Lecture4

- Bandwidth: F(u,v)=0 for any |u|, |v| greater than U_0, V_0
- Image Sampling

$$In\ spatial\ domain:\ f_s(x,y) = F(x,y)x(x,y) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} f(m\Delta x, n\Delta y)\delta(x-m\Delta x, y-n\Delta y)$$
$$In\ spectral\ domain:\ F_s(u,v) = F(u,v)*S(u,v) = \frac{1}{\Delta x\Delta y}\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} F(u,v)*\delta(u-\frac{k}{\Delta x}, v-\frac{l}{\Delta y})$$
$$= \frac{1}{\Delta x\Delta y}\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} F(u-\frac{k}{\Delta x}, v-\frac{l}{\Delta y})$$

- Sampling Theorem - *Nyquist Rate*

$$f_{xs} = \frac{1}{\Delta x} > 2U_0\ \ f_{ys} = \frac{1}{\Delta y} > 2V_0$$

Reconstructing original image from sampled image

$$\tilde{f}(x,y) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} f(m\Delta x, n\Delta y)sinc(xf_{xs}-m)sinc(yf_{ys}-n)$$

- Definition of Quantization

$$\bar{f} = Q(f) = r_k,\ if\ \ t_k \le f \le t_{k+1}\ \ for\ k=1,\dots,L$$

Lloyd-Max Quantizer - based on *probability density function*

$$Quantization\ error:\ \varepsilon = E[(f-\bar{f})^2] = \int_{t_1}^{t_{L+1}} (f-Q(f))^2 P(f)df$$

$$= \sum_{k=1}^{L} \int_{t_k}^{t_{k+1}} (f-Q(f))^2 df$$

$$Let\ \frac{d\varepsilon}{df} = 0,\ we\ get:$$

$$t_k = \frac{r_k + r_{k-1}}{2},\ for\ k = 2, \dots, L$$

$$r_k = \frac{\int_{t_k}^{t_{k+1}} fP_f(f)df}{\int_{t_k}^{t_{k+1}} P_f(f)df},\ for\ k = 1, \dots, L$$

If p(f) is uniform (namely,):

$$p(f) = \frac{1}{t_{k+1} - t_1}$$

$$t_k = t_{k-1} + q,\ r_k = t_k + \frac{q}{2}, where\ q\ is\ defined\ as\ q = \frac{t_{L+1} - t_1}{L}$$

$$In\ this\ case:\ \varepsilon_{linear} = \frac{q^2}{12}, and\ SNR = 10log_{10}\frac{\sigma^2}{\varepsilon} = 10log_{10}2^{2B} \approx 6B\ dB$$

Lecture5

- Image Histogram

$$histogram:\ \sum_{k=0}^{L-1} h_f(k) = MN,\ where\ L\ is\ number\ of gray\ level,\ MN\ is\ pixels\ amount$$

- Histogram equalization - to get a *uniform* histogram

$$probabilities:\ p_f(k) = \frac{h_f(k)}{\sum_{k=0}^{L-1} h_f(k)} for\ k = 0, \dots, L-1$$

$$pdf:\ c_f(f) = \sum_{k=0}^{f} p_f(k), for\ f = 0, \dots, L-1$$

$$Thus,\ equalized\ histogram:\ g = T(f) = Round[(\frac{c_f(f) - c_{min}}{1 - c_{min}})(L-1)]$$

- Filtering - **to be continued**