

Formula & Concept Sheet until Quiz 1

Concept

Lecture 1

Intensity, Gray Level, Image Sensor, Pixel, Coordinates, Sampling/Discretization, Quantization, Spatial Resolution, Gray-Level Resolution, Color Space, Color Primaries, Additive Primaries, Subtractive Primaries

Lecture 2

Visible Light, Rods and Cones, Spectral Sensitivity, Luminance, Brightness, Simultaneous Contrast, Weber's Law, Lightness, Chrominance, Hue, Saturation, Chromatic Adaptation, CIE-XYZ, CIE-L*a*b*, Color Difference

Lecture 3

Impulse Sequence, Linear and Shift Invariant System, Principle of Superposition, Impulse Response, Convolution, Fourier Transform, Spatial/Frequency Variables, Basis Functions, Fourier/Power Spectrum, Discrete Fourier Transform, Conjugate Symmetry, Discrete Cosine Transform, Properties of Transform, Periodicity, Separable Property

Lecture4

Bandwidth, Band-Limited Signal Sampling, Function Sampling, Frequency/Rate, Sampling Interval, Dirac Delta Function, Sampling Theorem, Nyquist Frequency/Rate/Interval, Aliasing, Decision Level/Interval, Reconstruction Level, Quantization Error, Mean Square Error (MSE), Lloyd-Max Quantizer, Linear Quantizer, Signal-to-Noise Ratio (SNR)

Lecture5

Image Enhancement, Point Processing, Gray-Level Reversal, Low/High Contrast, Contrast Stretching, Thresholding, Dynamic Range, Image Histogram, Histogram Equalization, Histogram Matching, Image Filtering, Lowpass/Highpass Filters, Spatial Filtering, Smoothing/Sharpening Filters, Unsharp Masking, High-Boost Filters, Gradient/Edge Filters

Formula

Lecture 1

- Color space

$$\text{Color space : } C = aP_1 + bP_2 + cP_3 \quad \text{Lecture2}$$

- Luminance (CIE definition)

$$Y = \int_{\lambda} R(\lambda)V(\lambda)d\lambda, \text{ where } R(\lambda) = \rho(\lambda)E(\lambda)$$

ρ : reflectivity or transmissivity; E : energy distribution at λ ; λ : wavelength

- Simultaneous Contrast - Weber's Law

$$\frac{|Y_s - Y|}{Y} \approx d(\ln Y) = \Delta C = \text{constant}$$

- Lightness (w.r.t *luminance*)

$$L^* = \begin{cases} 116(\frac{Y}{Y_n})^{\frac{1}{3}} - 16, & \text{if } \frac{Y}{Y_n} > 0.008856 \\ 903.3 \frac{Y}{Y_n}, & \text{otherwise} \end{cases}$$

- RGB - XYZ Conversion

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.4124 & 0.3576 & 0.1804 \\ 0.2127 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9502 \end{bmatrix} \begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix}; \quad \begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix} = \begin{bmatrix} 3.2405 & 1.5372 & 0.4985 \\ -0.9692 & 1.8760 & 0.0416 \\ 0.0556 & 0.2040 & 1.0573 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

- XYZ - Lab Conversion

$$L^* = 116f(\frac{Y}{Y_n}) - 16,$$

$$a^* = 500(f(\frac{X}{X_n}) - f(\frac{Y}{Y_n})),$$

$$b^* = 500(f(\frac{Y}{Y_n}) - f(\frac{Z}{Z_n})),$$

$$\text{where } f(u) = \begin{cases} 116u^{\frac{1}{3}} & \text{if } u > 0.008856 \\ 7.787u + \frac{16}{116} & \text{otherwise} \end{cases}$$

$$\text{Chroma : } C_{ab}^* = \sqrt{(a^*)^2 + (b^*)^2};$$

$$\text{Hue : } h_{ab} = \tan^{-1}(\frac{b^*}{a^*});$$

$$\text{Color difference : } \Delta E = \sqrt{(L^*)^2 + (a^*)^2 + (b^*)^2}$$

Lecture 3

- Definition of image as 2D sequence

$$f(x, y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(x, y) \delta(x - m, y - n)$$

- 2D LSI (Linear Shift-Invariant) System

$$g(x, y) = T[f(x, y)] = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(x, y) T[\delta(x - m, y - n)]$$

$$= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(x, y) h(x - m, y - n) = f(x, y) * h(x, y)$$

- Fourier Transform

$$F(u, v) = \int_{-\infty}^{\infty} f(x, y) e^{-j2\pi(ux+vy)} dx dy$$

$$f(x, y) = \int_{-\infty}^{\infty} f(x, y) e^{j2\pi(ux+vy)} dx dy$$

$$\text{Fourier spectrum : } |F(u, v)| = \sqrt{R^2(u, v) + I^2(u, v)}$$

$$\text{Phase angle : } \phi = \tan^{-1}[\frac{I(u, v)}{R(u, v)}]$$

$$\text{Power spectrum} = |F(u, v)|^2 = R^2(u, v) + I^2(u, v), \quad \text{where } F(u, v) = R(u, v) + jI(u, v)$$

- Discrete Fourier Transform

$$F(u, v) = \frac{1}{MN} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} f(x, y) e^{-j2\pi(\frac{ux}{M} + \frac{vy}{N})}$$

$$F(x, y) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} f(x, y) e^{j2\pi(\frac{ux}{M} + \frac{vy}{N})}$$

- Properties of 2D FT

$$f(x, y) e^{j2\pi(\frac{u_0 x}{M} + \frac{v_0 y}{N})} \Leftrightarrow F(u - u_0, v - v_0)$$

$$f(x - x_0, y - y_0) \Leftrightarrow F(u, v) e^{-j2\pi(\frac{u_0 x}{M} + \frac{v_0 y}{N})}$$

$$f(ax, by) \Leftrightarrow \frac{1}{|ab|} f\left(\frac{u}{a}, \frac{v}{b}\right)$$

$$f * h \Leftrightarrow FH, fh \Leftrightarrow F * H$$

- DCT (Discrete Cosine Transform)

$$F(u, v) = \frac{2}{N} C(u) C(v) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x, y) \cos\left(\frac{(2x+1)u\pi}{2N}\right) \cos\left(\frac{(2y+1)v\pi}{2N}\right),$$

$$\text{where } C(u), C(v) = \begin{cases} \frac{1}{\sqrt{2}} & \text{for } u, v = 0 \\ 1 & \text{for } u, v = 1, \dots, N-1 \end{cases}$$

$$f(x, y) = \frac{2}{N} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} F(u, v) \cos\left(\frac{(2x+1)u\pi}{2N}\right) \cos\left(\frac{(2y+1)v\pi}{2N}\right)$$

Lecture4

- Bandwidth: $F(u, v) = 0$ for any $|u|, |v|$ greater than U_0, V_0
- Image Sampling

$$\text{In spatial domain : } f_s(x, y) = F(x, y) x(x, y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(m\Delta x, n\Delta y) \delta(x - m\Delta x, y - n\Delta y)$$

$$\text{In spectral domain : } F_s(u, v) = F(u, v) * S(u, v) = \frac{1}{\Delta x \Delta y} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} F(u, v) * \delta\left(u - \frac{k}{\Delta x}, v - \frac{l}{\Delta y}\right)$$

$$= \frac{1}{\Delta x \Delta y} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} F\left(u - \frac{k}{\Delta x}, v - \frac{l}{\Delta y}\right)$$

- Sampling Theorem - Nyquist Rate

$$f_{xs} = \frac{1}{\Delta x} \leq 2U_0 \quad f_{ys} = \frac{1}{\Delta y} \leq 2V_0$$

Reconstructing original image from sampled image

$$\tilde{f}(x, y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(m\Delta x, n\Delta y) \text{sinc}(xf_{xs} - m) \text{sinc}(yf_{ys} - n)$$

- Definition of Quantization

$$\bar{f} = Q(f) = r_k, \text{ if } t_k \leq f \leq t_{k+1} \text{ for } k = 1, \dots, L$$

Lloyd-Max Quantizer - based on probability density function

$$\text{Quantization error : } \varepsilon = E[(f - \bar{f})^2] = \int_{-\infty}^{\infty} (f - \bar{f})^2 P(f) df$$

$$\text{Let } \frac{d\varepsilon}{df} = 0, \text{ we get :}$$

$$t_k = \frac{r_k + r_{k-1}}{2}, \text{ for } k = 2, \dots, L$$

$$r_k = \frac{\int_{-\infty}^{\infty} f P(f) df}{\int_{-\infty}^{\infty} P(f) df}, \text{ for } k = 1, \dots, L$$

If $p(f)$ is uniform (namely,):

$$p(f) = \frac{1}{t_{k+1} - t_1}$$

$$t_k = t_{k-1} + q, \quad r_k = t_k + \frac{q}{2}, \text{ where } q \text{ is defined as } q = \frac{t_{L+1} - t_1}{L}$$

$$\text{In this case : } \varepsilon_{\text{linear}} = \frac{q^2}{12}, \text{ and } SNR = 10 \log_{10} \frac{\sigma^2}{\varepsilon} = 10 \log_{10} 2^{2B} \approx 6B \text{ dB}$$

Lecture5

- Image Histogram

$$\text{histogram : } \sum_{k=0}^{L-1} h_f(k) = MN, \text{ where } L \text{ is number of gray level, } MN \text{ is pixels amount}$$

- Histogram equalization - to get a *uniform* histogram

$$\text{probabilities : } p_f(k) = \frac{h_f(k)}{\sum_{k=0}^{L-1} h_f(k)} \text{ for } k = 0, \dots, L-1$$

$$\text{pdf : } c_f(f) = \sum_{k=0}^f p_f(k), \text{ for } f = 0, \dots, L-1$$

$$\text{Thus, equalized histogram : } g = T(f) = \text{Round}\left[\left(\frac{c_f(f) - c_{\min}}{1 - c_{\min}}\right)(L-1)\right]$$

- Filtering - **to be continued**