Chapter 1

Functions and Models

1.1 Function Basics

1.1.1 Functions

Definition 1.1 (*Function*)

A function f is the rule that assigns to each element in a set D exactly one element, called $f(x)$, in a set E.

Notes about Definition 1.1

- We usually consider functions for which sets D and E are sets of *real numbers*.
- The set D is called **domain** of function f.
- **Domain Convention**: The domain of the function is the set of *all inputs for which the formula makes sense and gives a real-number output.*
- The number of $f(x)$ is the **value of f at** x and is read f of x.
- The **range** of f is the set of all possible values of $f(x)$ as x varies throughout the domain.
- A symbol that represents an arbitrary number in the domain of a function f is called an **independent variable**.
- A symbol that represents a number in the range of f is called dependent variable.
- 4 ways to represent a function: by an equation, in a table, by a graph, or in words.

Theorem 1.1 (*The Vertical Line Test*)

A curve in the xy-plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.

If each vertical line at $x = a$ intersects a curve only once, then exactly one function value is defined by $f(a) = b$. But if a line $x = a$ intersects the curve twice, at (a, b) and (a, c), then the curve can't represent a function because *a function can't assign 2 different values to a* according to Definition 1.1.

1.1.2 Piecewise Defined Function

Definition 1.2 (*Piecewise Defined Function*)

a function f is called ***piecewise defined function*** *if the function is defined by different formulas in different parts of their domains.*

Example: A function f is defined by

$$f(x) = \begin{cases} 1 - x & \text{if } x \leq 1 \\ x^2 & \text{if } x > 1 \end{cases}$$

Example: Absolute value function:

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

1.1.3 Even and Odd Functions

Definition 1.3 (*Even Function*)

If a function f satisfies $f(-x) = f(x)$ for every number x in its domain, then f is called an ***even function****.*

Example: The function $f(x) = x^2$ is even because

$$f(-x) = (-x)^2 = x^2 = f(x).$$

Definition 1.4 (*Odd Function*)

If a function f satisfies $f(-x) = -f(x)$ for every number x in its domain, then f is called an ***odd function****.*

Example: The function $f(x) = x^3$ is odd because

$$f(-x) = (-x)^3 = -(x^3) = -f(x)$$

1.1.4 Increasing and Decreasing Functions

Definition 1.5 (*Increasing and Decreasing Functions*)

A function f is called ***increasing function*** *on interval I If*

$$f(x_1) < f(x_2) \quad \text{whenever } x_1 < x_2 \text{ for } \forall x_1, x_2 \in I.$$

It's called ***decreasing function*** *If*

$$f(x_1) > f(x_2) \quad \text{whenever } x_1 < x_2 \text{ for } \forall x_1, x_2 \in I.$$

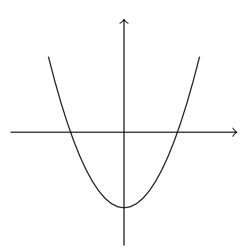

Figure 1.1: The figure of $f(x) = 2x^2 - 1$

Example: For $f(x) = 2x^2 - 1$ which is shown in Fig 1.1, it's decreasing in interval $(-\infty, 0)$ and it's increasing in interval $(0, +\infty)$.