Assignment 2

Question 1: Y_1 = finger dexterity of music students who study piano

Y_2 = finger dexterity of music students who study singing.

$n_1 = 137$, $s_1 = 4.34$, $\bar{y}_1 = 37.25$

$n_2 = 137$, $s_2 = 5.19$, $\bar{y}_2 = 35.91$

a) $H_0: u_1 - u_2 = 0$ $\quad H_a: u_1 - u_2 \neq 0$

$$\text{test statistic} = z = \frac{\bar{y}_1 - \bar{y}_2 - \Delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \qquad u_1 - u_2 = 0$$

$$= \frac{37.25 - 35.91 - 0}{\sqrt{\frac{(4.34)^2}{137} + \frac{(5.19)^2}{137}}}$$

$$= 2.3183$$

critical region : $|z| \geq z_{\frac{\alpha}{2}} \approx z_{\frac{0.05}{2}} = \text{qnorm}\left(1 - \frac{0.05}{2}\right) \approx 1.96$ # (using R)

$z \leq -1.96$ or $z \geq 1.96$

Conclusion: Since, the observed test statistic z falls into the critical region of $|z_{\frac{\alpha}{2}}|$. We have sufficient evidence to reject the H_0.

b) power $= 1 - \beta = P(\text{reject } H_0 \mid H_0 \text{ is false})$

$= P(|z^*| \geq 1.96 \mid u_1 - u_2 = 3)$

$= P(z^* \leq -1.96 \mid u_1 - u_2 = 3) + P(z^* \geq 1.96 \mid u_1 - u_3 = 3)$

$$= \Phi\left(-1.96, \frac{u_1 - u_2 - \Delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}, 1\right) + 1 - \Phi\left(1.96, \frac{u_1 - u_2 - \Delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}, 1\right)$$

$$= \Phi\left(-1.96, \frac{3-0}{\sqrt{\frac{(4.34)^2}{137} + \frac{(5.19)^2}{137}}}, 1\right) + 1 - \Phi\left(1.96, \frac{3-0}{\sqrt{\frac{(4.34)^2}{137} + \frac{(5.19)^2}{137}}}, 1\right)$$

using R → $=$ pnorm(-1.96, 5.1902, 1) + 1 - pnorm(1.96, 5.1902, 1)

$= 4.3326 \times 10^{-13} + 0.9993815$

$= 0.9993815$ #

every

c) $\alpha = 0.05$

power $= 0.95$, therefore $\beta = 1 - \text{power}$

$= 1 - 0.95$

$= 0.05$

$u_1 - u_2 = 3$

$$\sigma^2 = S_p^2 = \frac{(137-1)(4.34)^2 + (137-1)(5.19)^2}{137+137-2}$$

$= 22.8859$ #

The required sample size n is:

$$n = \frac{2\sigma^2 (z_{\frac{\alpha}{2}} + z_\beta)^2}{\delta = (u_1 - u_2)}$$

$$= \frac{2 \times (22.8859)(1.96 + 1.6449)^2}{3^2}$$

$= 66.09 \approx 67$ samples (rounded up)

The sample size needed for the test is 67 samples.

Question 2

a) The sampling distribution of $\frac{(n_1-1)s_1^2}{\sigma_1^2}$ is a chi-square distribution with n-1 degree of freedom.

$$\frac{(n_1-1)s_1^2}{\sigma_1^2} \sim \chi^2_{n-1}$$

b) The symmetric 95% confidence interval for σ_1^2 is defined as

$$\left(\frac{(n-1)s_1^2}{c_2}, \frac{(n-1)s_1^2}{c_1}\right).$$

To find the c_1 and c_2 =

$P(x < c_1) = \frac{\alpha}{2}$ and $P(x > c_2) = \frac{\alpha}{2}$ where $x \sim \chi^2_{136}$

$= \frac{0.05}{2}$ $\quad\quad = \frac{0.05}{2}$

$= 0.025$ $\quad\quad = 0.025$

using R qchisq(0.025, 136) = 105.61 $\quad\quad$ qchisq (0.025, 136, lower.tail= FALSE) = 170.18

$c_1 = 105.61$ $\quad\quad$ $c_2 = 170.18$

$$\left(\frac{(137-1)(4.34)^2}{170.18}, \frac{(137-1)(4.34)^2}{105.61}\right)$$

$$= \left(\frac{136 \times 4.34^2}{170.18}, \frac{136 \times 4.34^2}{105.61}\right)$$

$$= (15.05, 24.36)_{\#}$$

Therefore, the symmetric 95% confidence interval for σ_1^2 is between 15.05 and 24.36.

c) 1) Test statistic is $F = \frac{\frac{s_1^2}{\sigma_1^2}}{\frac{s_2^2}{\sigma_2^2}}$ but under the Ho, $F = \frac{s_1^2}{s_2^2} \sim F_{136, 136}$ where the

F is an F-distribution with $n_1 - 1 = 136$ and $n_2 - 1 = 136$ degrees of freedom.

2) $F = \frac{(4.34)^2}{(5.19)^2} = 0.6993$ ~ the observed test statistic F is 0.6993.

3) critical region is $F < F_{136, 136, 1-\frac{0.05}{2}}$ ≈ using R qf($\frac{0.05}{2}$, 136, 136) = 0.7135

$F > F_{136, 136, \frac{0.05}{2}}$ ≈ using R qf(1- $\frac{0.05}{2}$, 136, 136) = 1.4015

4) Conclusion: Since the test statistic F is in the critical region, there is sufficient evidence to reject the Ho at significance level α of 0.05.

or

[we can check the P-value = 2 × pf (0.6993, 136, 136)
= 0.03787 which is greater than $\alpha = 0.05$.]

Therefore, we have enough evidence to reject the Ho.