

Probability and Statistics 2021 — Assignment 3

1. A maths teacher has set a test with a maximum time of 15π minutes. The random time for a student to complete the test, T , has pdf

$$f(t) = \begin{cases} \frac{1}{30} \sin(t/30), & 0 \leq t \leq 15\pi, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Calculate $E[T]$.
- (b) Calculate the cumulative distribution function of T .
- (c) Calculate the probability that a randomly selected student takes between 15 and 30 minutes to complete the test.
- (d) Given that a randomly selected student takes longer than 15 minutes to complete the test, what is the probability that the student completes the test in less than 30 minutes?

[7 marks]

2. Let Z be a random variable with support on the positive integers $\{1, 2, 3, \dots\}$ with probability mass function $f(z) = c2^{-z}$ for $z = 1, 2, 3, \dots$

- (a) For what value(s) of c is the function f a valid probability mass function?

Hint: use results for a geometric series.

- (b) Show that the moment generating function of Z is $m(t) = \frac{e^t}{2 - e^t}$.
- (c) Using the mgf, calculate the expected value and the variance of Z .

[8 marks]

3. The amount of yeast (in tonnes) used by Coopers brewery can be modelled by a random variable $Y \sim \text{Exp}(1/2)$. The cost of producing the yeast is $C = 4Y + 1$. Find the probability density function for C .

[4 marks]

4. During a storm, the time in minutes between lightening strikes can be modelled as a random variable $T \sim \text{Exp}(1)$.

- (a) If you have been waiting for 2 minutes since the last strike, what is the probability that the next one occurs in the next 1 minute?
- (b) What is the probability of 3 strikes in the next 3 minutes?
- (c) What is the expected number of strikes in the next hour? (assuming the storm continues that long)
- (d) If we start counting strikes at some arbitrary time, what is the expected time until the 5th strike is observed?

[5 marks]