Probability and Statistics 2021 — Assignment 1

For questions 2 and 3, give your answers in terms of binomial coefficients.

1. (a) Let A and B be events with probabilities not equal to 0 or 1. Show that if $P(B|A) = 1$, then $P(A^c|B^c) = 1$.

   (b) Show that if now $P(A) = 1$, then $P(A|B) = 1$ for any B with $P(B) > 0$.

   [7 marks]

2. Four people are playing a card game. A shuffled, 52 card deck is dealt out to the players (so each gets a 13-card hand.)

   (a) How many possibilities are there for the hand the first player will get? (assuming the order of the cards does not matter).

   (b) How many possibilities are there for what hands everyone will get, again assuming that the order of the cards does not matter?

   To get all the marks you should also give a short explanation of your answers.

   [4 marks]

3. Adelaide Casino has a new game that uses 10 identical decks of cards mixed into 1 megadeck. How many 10 card hands can be dealt from this megadeck if the neither the order, nor the suits of the cards matter?

   Hint: think about this as a stars and bars problem.

   [2 marks]

4. SAFM weekdays breakfast show is presented by Bec, Cosi and Lehmo (B, C and L). What is the probability that L is older than B, given that L is older than C?

   [2 marks]

5. A bag contains 9 fair coins and 1 biased coin (where the probability of heads is 0.9). You draw a coin at random from the bag, toss it 7 times and get 7 heads. What is the probability that the coin sampled is the biased one?

   [4 marks]

6. Ringo and John stay at a college in North Adelaide with 50 other students. Let S be the set of other students, let R be the set of people in S that are friends with Ringo and let J be the set of people in S who are friends with John. Assume that each person in the college is friends with Ringo with probability $1/2$ and likewise for John with all friendship pairs being independent.

   (a) Let $A \subseteq S$. What is $P(R = A)$?

   (b) Find $P(R \subseteq J)$.

   (c) Find $P(R \cup J = S)$.

   [7 marks]