15

SPATIAL NETWORKS

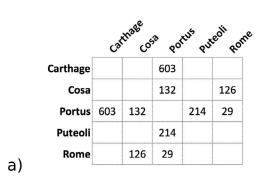
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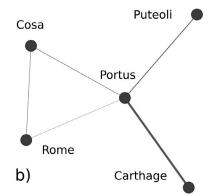
Introduction

What are spatial networks?

A network is a formal representation of the structure of relations among a set of entities of interest. In many cases, networks are analysed as mathematical graphs where the entities are defined as nodes with the connections among pairs of nodes defined as edges representing a formal dyadic relationship (edges are also sometimes called arcs, ties, or links). Nodes and edges can be used to represent any features and relationships of interest, the only requirements being that they can be formally described and that their boundaries can be unambiguously defined (at least for analytical purposes). Networks can be described and visualized in a variety of formats (see Figure 15.1) which can provide information on the presence/ absence or weights of edges or the direction of flows across a network, as well as attributes of nodes and edges defined without direct reference to the network itself (e.g. node age, size, population estimates, edge length, etc.). The methods and models used to collect, manage, analyse, present, and interpret network data are diverse, but generally connected by the notion that the properties of nodes, edges, attributes, and global structures of a network (or any combination thereof) depend on one another in ways that can provide us with unique insights and testable ideas about the drivers of a range of social processes (Brandes, Robins, McCranie, & Wasserman, 2013, pp. 9–11).

Here we focus on a specific class of networks that have received considerable attention in archaeology: spatial networks. Spatial networks refer to any set of formally defined nodes and edges where these network features are located in geometric space, and where network topology (the structural arrangement of network elements) is at least partly constrained by the spatial relationships among them (Barthelemy, 2011). Common examples include road networks, power grids, or even the internet as a spatialized set of connections among computers and routers. In archaeology, spatial networks have been used to investigate a range of phenomena including transportation or flows across roads, rivers, currents, or other cost-paths; line-of-sight networks for exploring intervisibility; space-syntax graphs for exploring the accessibility of features, settlements, or broader landscapes (see Thaler, this volume); and material culture networks of exchange, interaction, or similarity constrained by the geographic loci of production and consumption of those materials. We discuss these various applications in detail in this chapter.





c)

Edge ID	Source	Target	Length	Transport
	node	node	(km)	type
1	Carthage	Portus	603	Sea
2	Cosa	Portus	132	Sea
3	Cosa	Rome	126	Road
4	Portus	Puteoli	214	Sea
5	Portus	Rome	29	River

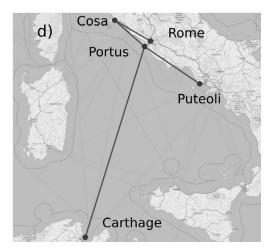


FIGURE 15.1 Four different network data representations of the same hypothetical Mediterranean transport network. (a) adjacency matrix with edge length (in km) in cells corresponding to a connection; (b) nodelink-diagram where edge width represents length (in km). Please refer to the colour plate for a breakdown by transport type where red lines = sea, green = river, grey = road); (c) edge list; (d) geographical layout. Once again, please refer to the colour plate for a breakdown of transport type. A colour version of this figure can be found in the plates section.

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Spatial network data allow us to directly explore the systematic spatial relationships among nodes, edges and attributes that would otherwise be difficult to characterize. The abstract transport network shown in Figure 15.1 provides an instructive example. The different roles played in the Roman transport system by Cosa and Portus cannot be understood only with reference to their spatial locations and proximity to other towns, but also by the opportunities afforded by their relationships with all other towns by way of connections across roads, rivers, and seas. From Portus all other towns can be reached directly in one step over the transport network, whereas from Cosa two steps are needed to reach either Puteoli or Carthage. Moreover, the maritime route between Cosa and Portus could come into use or become popular as a result of the slower alternative route via Rome. When such dependencies are of interest, spatial network methods, often coupled with GIS analytical tools, can offer extremely valuable approaches.

Before we proceed with the archaeological application of spatial networks, we want to briefly consider the interchangeable use of the words network and graph. The word graph is more commonly used in the fields of mathematics, computer science and computational geometry. Indeed, graph theory is a long-established subdiscipline of mathematics and one of the fundamentals of computer science (Harary, 1969). In many disciplines where graph theory is applied to real-world phenomena the term network is used, and this is the case for the two disciplines with the most active traditions of network research: Social Network Analysis (SNA) and statistical physics. However, in practice the terms graph and network are commonly used interchangeably and we will here consistently use the term network.

An overview of archaeological network research: introduction

Spatial networks have a long history in archaeology and many of the earliest applications of network methods drew upon tools for creating and analysing geographically explicit networks to explore settlement patterns and exchange systems in particular (see Stjernquist, 1966; Doran & Hodson, 1975, pp. 12-15; Hodder & Orton, 1976, pp. 68-73 for some early examples of network visualizations). Building on these early calls, formal spatial network analytical approaches have been sporadically applied by archaeologists to a host of issues since the 1970s (Terrell (1977) is often cited as the first formal example) but perhaps surprisingly given the frequent use of spatial data in archaeology, network methods in general and spatial networks in particular have only recently seen a dramatic increase in popularity (see Brughmans & Peeples, 2017; Collar, Coward, Brughmans, & Mills, 2015). In this section we briefly discuss four of the most common applications of spatial network data in archaeology. Some of these applications concern network representations of observed relationships such as roads connecting places, whereas others concern network representations of relationships derived from archaeological data through an intermediary method, such as the use of similarity measures to represent material culture similarity networks. This overview is by no means exhaustive, but our discussion highlights the most common ways that archaeological data are abstracted and formally represented as network data.

Roads, rivers, oceans, traversal and transportation

Perhaps the most direct method for representing a network based on archaeological spatial data involves the assessment of transportation and flows at various scales based on formal features like roads, trails, or rivers or simply the likely paths across various landscapes or waterways. In such networks, nodes are typically defined as discrete features at a site or on the landscape (rooms, sites, settlements, etc.) and edges are defined by the features or paths that connect them. In some cases, edges represent easily identifiable formal features like roads and trails (Isaksen, 2007, 2008; Jenkins, 2001; Pailes, 2014; Menze & Ur, 2012) or riverine paths (Peregrine, 1991) where the edges themselves have clear spatial information. In other cases the connections between pairs of nodes may be defined using models of the costs of traversal or proximity across the physiographic environment (Bevan & Wilson, 2013; Hill, Peeples, Huntley, & Carmack, 2015; Mackie, 2001; Verhagen, Brughmans, Nuninger, & Bertoncello, 2013; White & Barber, 2012), or seas/ oceans (Broodbank, 2000; Evans, 2016; Hage & Harary, 1991, 1996; Irwin, 1978; Knappett, Evans, & Rivers, 2008; Terrell, 1977) that are derived from analyses using GIS, spatially explicit models or related tools. Networks based on either formal features or models of traversal have been used to explore a broad range of social processes from the relationship between node position and prominence to the rise of expansive trade systems, pilgrimages, and settlement hierarchies.

Visibility networks

Another common topic in archaeological spatial network research is the study of visibility, usually represented as lines-of-sight: the ability for an observer to observe an object of interest within a natural or built-up environment or to be observed (see Brughmans & Brandes, 2017, for a recent overview). Visibility networks are typically defined based on line-of-sight data, often derived through GIS analyses (see Gillings & Wheatley, this volume). In line-of-sight networks the set of nodes represents the observation locations and the edges represent lines-of-sight. A pair of nodes is connected by an edge if a line-of-sight starting at the eye level of an observer at one observation point can reach the second observation point, i.e. if the line-of-sight is not blocked by a natural or cultural feature. In some studies, this point-to-point model of visibility is expanded to landscape scale assessments of viewsheds where the total cumulative area viewable from a given viewpoint is defined and networks are created based on areas with overlapping viewsheds or when certain key features are mutually viewable (see O'Sullivan & Turner, 2001; Brughmans & Brandes, 2017; Bernardini & Peeples, 2015). The method is most commonly used to study hypothesised visual signalling networks, communities sharing visual landmarks and to explore processes of site positioning and the possible expression of power relationships through visual control (Bernardini & Peeples, 2015; Brughmans, Keay, & Earle, 2014, 2015, Brughmans, de Waal, Hofman, and Brandes, 2017; Brughmans & Brandes, 2017; De Montis & Caschili, 2012; Earley-Spadoni, 2015; Fraser, 1980, 1983; Ruestes Bitrià, 2008; Shemming & Briggs, 2014; Swanson, 2003; Tilley, 1994, pp. 156-166). Analyses of visibility network data frequently involve assessments of the relative importance of different nodes for sending or receiving information or resources across that network, or to evaluate the likelihood that a given configuration suggests a concern for signaling, defense, or other factors among the people who built those features.

Access analyses

A somewhat different use for network methods in spatial data draws on a body of work referred to as space syntax (Hillier & Hanson, 1984; Hillier, 1996; for a detailed discussion see Thaler, this volume). The access analysis approach in space syntax is particularly popular in archaeological research. It uses network graphs and related visualizations to explore the nature of physical or sometimes visible access within features, buildings, or larger landscapes. The basic idea behind the approach is that we can think of discrete spaces being "reachable" from one another through tree-like networks that let us both examine the overall structure of mutual reachability among spaces and also assess the relative depth (the number of edges crossed) from one space to another. In this way individual spaces (however they are defined) are characterized as nodes, and edges are drawn between pairs of nodes that are reachable (i.e. that share a doorway or are mutually visible). A number of studies have employed space syntax graphs to argue that tracking or comparing the cultural logics of spatial organization can provide insights into a range of issues including social organization, public versus private spaces, the distribution of urban services, and social stratification (see Branting, 2007; Brusasco, 2004; Cutting, 2003; Fairclough, 1992; Ferguson, 1996; Foster, 1989; Grahame, 1997; Wernke, 2012). Analyses of space syntax graphs are often limited to qualitative assessments, due in part to concerns over incomplete data in archaeological contexts (see Cutting, 2003) but archaeologists are also starting to take advantage of quantitative tools for assessing the topology of access networks (e.g. Wernke, 2012; Wernke, Kohut, & Traslaviña, 2017).

Spatial material culture networks

The final common approach to spatial networks in archaeology involves analyses of network data generated through material cultural data which are assessed in relation to the spatial arrangement of nodes and

edges. The methods used to abstract networks from archaeological material cultural data are quite diverse but often involve the use of geochemically sourced materials or regions (e.g. Golitko, Meierhoff, Feinman, & Williams, 2012), or the shared presence or similarities in material cultural assemblages to define edges among settlements or regions (e.g. Mills et al., 2013). Although the presence and/or weights of edges in such material cultural networks are typically defined using aspatial data (such as artefact type frequencies) the samples from which these data are drawn are often associated with spatial locations that allow for a consideration of the propinquity of social and spatial relations. In many cases, geographic proximity or other spatial information is used to generate a null model of geographic connections expected under certain constraints which is then compared to the network based on material cultural data. For example, Mills and colleagues (2013) created a two-mode network of obsidian distribution in the late Prehispanic Southwest and compared the obsidian network to geographic expectations based on the costs of travel across the landscape, to identify times and places where the material networks deviated from the geographic expectation. Most material cultural networks explored using archaeological data have a spatial component and such direct comparisons between material and geographic distance are becoming increasingly common (e.g. Gjesfjeld, 2015; Gjesfjeld & Phillips, 2013; Hill et al., 2015).

Method

In this section we will introduce some key concepts in spatial network research, commonly applied analytical techniques and a range of spatial network models.

Building spatial networks

The range of archaeological applications of spatial networks reviewed above reveals that spatial network data can either be generated through models, such as those introduced below, or derived from observations. Regardless of their source, at least three things are needed to build a spatial network dataset: a set of nodes, a set of edges connecting these nodes, and information about their spatial embeddedness. The latter could take the form of spatial coordinates of nodes' point locations or of edges' starting and ending locations. Such information is commonly included in attributes attached to the nodes and edges, along with other additional information about nodes and edges. The most common network data formats are shown in Figure 15.1, and network data represented in these formats can be imported into most network science software. The adjacency matrix (Figure 15.1(a)) represents the set of nodes as the column and row headers and includes a value in the cell referring to a pair of nodes that have an edge. The node-linkdiagram (Figure 15.1(b)) represents nodes as points and edges as lines between them, and is a particularly appropriate data format to emphasise the presence of edges unlike the adjacency matrix which is a more powerful representation of the absence of edges. The edge list (Figure 15.1(c)) consists of three columns listing the pair of nodes that are connected by an edge and the value of their connection.

Planar and non-planar networks

A planar network is a network where the edges do not cross but instead always end in nodes (Figure 15.2). A key feature of many spatial networks is planarity, which is often enforced precisely because nodes and sometimes edges are spatially embedded. Planar spatial networks have traditionally received more attention in network science than non-planar spatial networks, and many network analysis methods and models have been purposely developed to study planar networks, some of which are introduced below (Barthelemy, 2011).

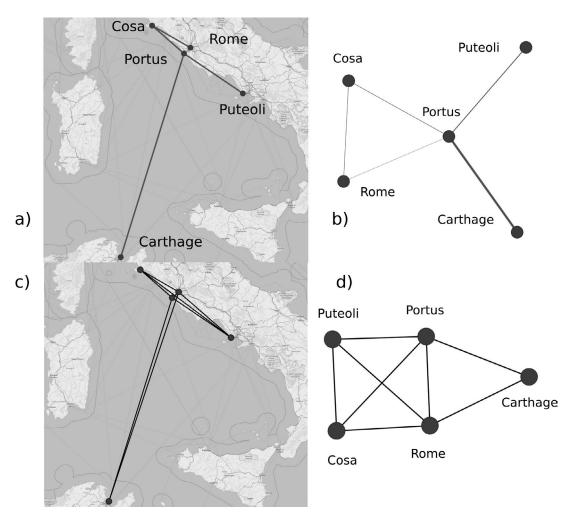


FIGURE 15.2 A planar network representing transport routes plotted geographically (a) and topologically (b). A non-planar social network representing social contacts between communities plotted geographically (c) and topologically (d). Note the crossing edges in the non-planar network. A colour version of this figure can be found in the plates section.

Source: Background © Openstreetmap

Local and global spatial network analysis measures

A range of statistical network methods can be used to explore the structure of spatial networks. Spatial network measures usually take the form of aspatial network science techniques modified to include a physical distance variable reflecting edge distance. Many of these network science measures, when applied to spatial networks, reveal particular properties of spatial networks such as the generally limited density of planar networks (see further in chapter). In this chapter we will limit ourselves to listing the most common network science analytical measures with spatial variants, some of which will be applied in the case-study below, but see Barthelemy (2011) for an exhaustive overview of spatial network analysis measures and properties.

Network analysis measures are commonly divided into local measures that reveal structural properties of nodes or small sets of nodes, and global measures that reveal structural properties of the network as a whole. The most common procedure for creating spatial variants of all these measures is to consider the physical distance of edges, or any other spatially derived attributes of edges such as transport time or effort of moving between two places, as a repelling "weight" in the algorithm: the higher the physical distance between two nodes, the lower the score of the measure.

Local measures include degree, paths, centralities, and a node clustering coefficient. A node's degree refers to the number of edges it has, and spatial degree refers to the number of edges weighted by their summed distance. A path is a sequence of connected node pairs from one node to another in the network. The shortest path from any one node i to any other node j is the minimum number of connected nodes between i and j that need to be traversed in order to reach j from i. A spatial variant of the shortest path

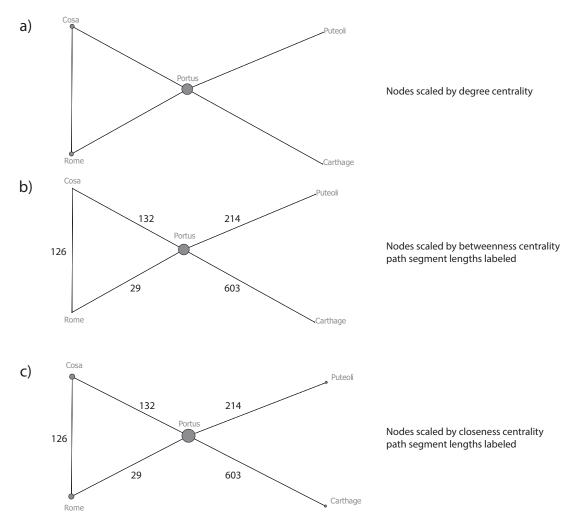


FIGURE 15.3 Examples of three different node centrality measures: (a) nodes scaled by degree centrality, (b) nodes scaled by betweenness centrality with path segment lengths shown, (c) nodes scaled by closeness centrality with path segment lengths shown.

includes the summed distance of all edges on the path as a weight. Centrality refers to a very large number of network measures that each reflect a node's importance in the network according to different structural features, the most popular of which are degree, closeness and betweenness. A node's closeness centrality refers to the network or spatial distance from this node over the set of shortest paths to each other node. A node's betweenness centrality refers to the number of all shortest paths between all node pairs in the network that this node is positioned on. A node's clustering coefficient is the existing proportion of all edges that could exist between its direct network neighbours, i.e. the density in the direct neighbourhood of the network (see O'Sullivan & Turner (2001) for a spatial variant applied to total viewsheds.

Global measures include average degree, degree distribution, density, average shortest path length, diameter, and network clustering coefficient. The network's average degree is the average of all nodes' degree scores. A network's node degree scores are most commonly explored as a distribution (see the case study in this chapter for examples). The density is the existing proportion of all edges that could exist in a network. Spatial networks where the edges are spatially embedded such as transport systems tend to have very low densities, whereas spatial networks where only the nodes are explicitly embedded such as artefact similarity networks typically have much higher densities. The average shortest path length is the average of all shortest path lengths between all node pairs in the network. The network diameter is the longest shortest path between any pair of nodes in the network. The network clustering coefficient is the average of all nodes' clustering coefficient scores.

Spatial network models

A body of techniques has been developed, mainly in computational geometry, to represent core structures and patterns of a set of spatially embedded nodes. These models are used in archaeological research as representations of archaeological theories of the interactions or interaction opportunities between the entities under study. Only a set of nodes and their spatial location are required to apply them, and the fundamental patterning they derive from this information can be compared to observed network patterning to understand how far removed the empirical network structure is from ideal theorized network structures. Here we will limit ourselves to introducing some of the most fundamental spatial network models, but additional and more elaborate models can be found in computational geometry and physics handbooks and reviews (Chorley & Haggett, 1967; Barthelemy, 2011). More complex models that have received a lot of attention in archaeology but fall outside the scope of the current overview are models that evaluate the cost of interaction between node pairs to propose interaction probabilities and derive hierarchical relationships between nodes. These include gravity models and their modification by Rihll and Wilson (1987) for the study of the emergence of Greek city-states (see Bevan & Wilson (2013) for a further applied example), as well as the ARIADNE model which has been used for the study of interactions between island communities in the Middle Bronze Age Aegean (Evans & Rivers, 2017; Knappett et al., 2008).

Relative neighbourhood networks, beta skeletons and Gabriel graphs

A pair of nodes are relative neighbours and are connected by an edge if they are at least as close to each other as they are to any other point (Toussaint, 1980). It can be derived for a pair of nodes N_i and N_j in a set of nodes N by considering a circle around each with a radius equal to the distance between N_i and N_j . If the almond-shaped intersection of the two circles does not include any other points then N_i and N_j are relative neighbours (Figure 15.4(a–b)). The relative neighbourhood network is a subset of the Delaunay triangulation and contains the minimum spanning tree (introduced below). A Gabriel graph is derived

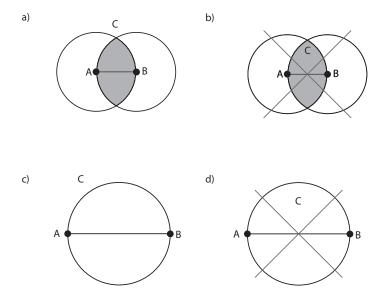


FIGURE 15.4 Examples showing relative and Gabriel graph neighborhood definitions: (a) A is a relative neighbor of B because there are no nodes in the shaded overlap between the circles around A and B, (b) A and B are not relative neighbors because C falls within the shaded overlap. (c) A and B are Gabriel neighbors because there are no nodes within the circle with a diameter AB, (d) A and B are not Gabriel neighbors because C falls within the circle with a diameter AB.

when the same principle is applied to a circular (rather than almond-shaped) region between every pair of nodes: if no other nodes lie within the circular region with diameter d(i,j) between N_i and N_i then N_i and N_i are connected in the Gabriel graph (Figure 15.4(c-d)). The concept of relative proximity can be controlled and varied in an interesting way using the concept of beta skeletons (Kirkpatrick & Radke, 1985). Rather than fixing the diameter of the circle as in the Gabriel graph, the diameter can be varied using a parameter β . Varying the value of β leads to interesting alternative network structures that are denser with lower values of β , sparser with higher values of β , and the beta skeleton equals the Gabriel graph when $\beta = 1$ (i.e. when the diameter of the circles equlls d(i,j)). These models create planar networks and have been applied in archaeology to study site and artefact distributions as well as to represent the theoretical flow of ceramics between settlements (Brughmans, 2010; Jiménez-Badillo, 2012).

Minimum spanning tree

In a set of nodes in the Euclidean plane, edges are created between pairs of nodes to form a tree where each node can be reached by each other node, such that the sum of the Euclidean edge lengths is less than the sum for any other spanning tree. The model has been applied by Per Hage and Frank Harary (1996) to study a diverse range of phenomena in Pacific archaeology: kinship networks and descent, the evolution and devolution of social and linguistic networks, and classification systems. They also used a model dynamically generating a minimum spanning tree edge by edge as a theoretical representation of the growth of a past social network. Herzog (2013) also uses minimum spanning trees as one representation of least-cost paths between places.

Delaunay triangulation

A triangulation network aims to create as many triangles as possible without allowing for any crossing edges and therefore creates planar networks. The Delaunay triangulation specifically is derived from the Voronoi diagram or Thiessen polygons: a pair of nodes are connected by an edge if and only if their corresponding tiles in a Voronoi diagram (or Thiessen polygons) share a side. The model has seen widespread application for representing archaeological theories, but mainly for the study of transport systems. To name just a few, Fulminante (2012) used Delaunay triangulation as a theoretical model for a road and river transport system between Iron Age towns in Central Italy (Latium Vetus), and Herzog (2013) used it as a representation of least-cost path networks. Evans and Rivers (2017) apply Delaunay triangulation for exploring the rise of Greek city-states.

K-nearest neighbours and maximum distance

In the previously discussed models nodes were connected to their nearest neighbours relative to the location of all other nodes. However, a simpler way of creating nearest neighbour networks is to connect a node to the closest other nodes regardless of the location of all other nodes. This is the approach taken in K-nearest neighbour networks, where each node is connected to the K other nodes closest to it. The method is sometimes called Proximal Point Analysis (Terrell, 1977). Another alternative to relative neighbourhood networks is offered by maximum distance networks: a node pair N_i and N_i is connected if the distance from each other d(i,j) is lower or equal than a threshold distance value d_{max} . In archaeological applications of these two models the edges are usually considered to represent the most likely channels for the flow of material or immaterial resources between individuals, settlements or island communities (Broodbank, 2000; Collar, 2013; Terrell, 1977). An applied example of these two models is given in the case study.

Case study

We will illustrate some of the network measures and models introduced in this chapter through an exploration of the structure of the Roman transport system. By applying a wide range of spatial network models and methods we will illustrate how interesting insights can be gained by taking a topological as well as spatial look at a past phenomenon. The following research questions will guide our exploration of the transport system:

- In what regions is the transport system particularly dense and in what regions is it particularly sparse?
- How important is each urban settlement as an intermediary in the flow of information or goods between all other settlements?
- How did the Roman transport system structure flows of supplies to the capital of Rome, and which regions and supplying towns were better positioned in the system to supply Rome?
- Does the Roman transport system reveal a particular spatial structure: nearest-neighbour, relativeneighbour or maximum distance?

An abstract representation of the Roman transport system will be used here: the Orbis geospatial network model of the Roman world (Scheidel, 2015; Meeks, Scheidel, Weiland, & Arcenas, 2014). Orbis offers a static and hypothetical representation of the Roman transport system with limited detail. Therefore, our present analysis merely aims to explore our research questions within the context of the coarse-grained structure of the Roman transport system in the second century AD as hypothesised by the Orbis team.

Data

We decided to use the Orbis dataset because it is well-studied and well-known among Roman archaeology scholars, it is open access and reusable for research purposes (Meeks et al., 2014), and it provides the only functional network dataset covering the entire Roman Empire at its largest extent. However, a key limitation of Orbis is that it is not as detailed as our current knowledge of Roman settlements and routes allows, precisely because it aims to represent the broad Empire-wide structure of the Roman transport system in a comparable way. Moreover, the selection of nodes and edges, as well as the distance assigned to edges, reflect decisions by its creators and should be submitted to sensitivity analyses (which is not within the scope of this chapter). Finally, Orbis represents a static picture of what the Roman transport system might have looked like in the second century AD, and does not offer the ability to explore how this system changed through time. The longitude and latitude of all nodes was cross-checked with the Pleiades gazetteer of ancient placenames (Bagnall et al., 2018) and corrected where necessary. The resulting network dataset includes a set of 678 nodes, 570 of which represent urban settlements and the remainder cultural features such as crossroads or natural features such as capes. The node attributes include the settlement name and latitude longitude coordinates. These nodes are connected by a set of 2208 directed links representing the ability to travel between a node pair in a particular direction. Edge attributes include the type of transport link (road, river, sea) and the distance in kilometres.

Spatial network visualisation

An initial visual exploration of this network can be performed to identify key structural features, using both geographical and topological layout algorithms. A geographical visualisation places the nodes in their correct geographical positions, which allows for an intuitive and recognisable exploration of the regional differences in node and edge distribution (Figure 15.5(a)). For example, we can easily identify the difference between maritime and terrestrial routes, the geographical extent of the Roman Empire, the Rhine and Danube Rivers making up the edges of the system at the northern borders of the empire, and the strong difference in node and edge density between Italy and the rest of the system. However, this figure has a high degree of node and edge overlap making the structure of the network particularly difficult to interpret. The topological visualisation shown in Figure 15.5(b) aims to avoid such overlap, revealing at a glance a number of interesting structural features that allow us to provide an informal answer to our first research question: the Aegean region is particularly dense; another dense cluster at the centre of the network consists of present-day Italy, France and Spain; the river Nile creates a tree-like pattern at the periphery of the network; provinces along the border of the empire have sparser transport networks.

Distance weighted betweenness centrality

Betweenness centrality allows us to answer our second research question because it measures how important a node is as an intermediary in the flow of information or goods between all other nodes, and it is therefore a particularly appropriate measure to study transport systems. It is calculated by counting how often each node is positioned on the shortest paths between all node pairs. Applying this measure to the Orbis network gives the results shown in Table 15.1 and Figure 15.5(c, d). The topological visualisation (Figure 15.5(d)) reveals that nodes at the centre of the network and in particular those crossing dense clusters score very high whilst nodes at the periphery score very low. The geographical visualisation (Figure 15.5(c)) further reveals that these highly scoring nodes are a chain of port sites connecting Egypt with Britain circling around the Iberian Peninsula.

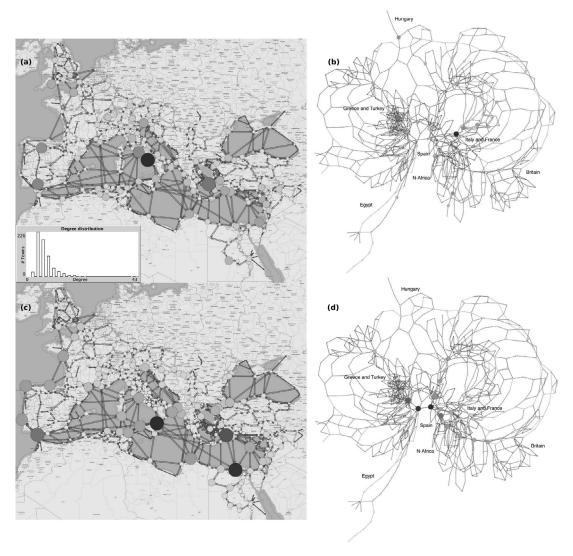


FIGURE 15.5 Network representation of the Orbis network: geographical layout (a, c) and topological layout (b, d). Node size and colour represent betweenness centrality weighted by physical distance in (a) and (b), and they represent unweighted betweenness centrality in (c) and (d): the bigger and darker blue the node, the more important it is as an intermediary for the flow of resources in the network. By comparing (a, b) with (c, d), note the strong differences in which settlement is considered a central one depending on whether physical distance is taken into account (a, b) or not (c, d). Edge colours represent edge type: red = sea, green = river, grey = road. A colour version of this figure can be found in the plates section.

Source: Background © Openstreetmap

However, this unweighted betweenness centrality measure completely ignores physical distance and considers the traversal of each edge equally: all that is considered is the number of hops over the network to get from one node to the other. To make this network analysis more representative of the physical reality of the system we can weigh the edges according to their physical distance, where a shortest path is now defined as the path between a pair of nodes with the lowest summed distance. Results of the distance

TABLE 15.1 Top 20 highest ranking towns according to the topological betweenness centrality measure and the distance weighted betweenness centrality measure. Towns highly ranked according to both measures are highlighted.

Rank	Betweenness	Distance weighted betweenness
1	Messana	Puteoli
2	Alexandria	Delos
3	Rhodos	Hispalis
4	Gades	Roma
5	Apollonia-Sozousa	Palantia
6	Olisipo	Pisae
7	Sallentinum Pr.	Ascalon
8	Flavium Brigantium	Aquileia
9	Acroceraunia Pr.	Rhodos
10	Lilybaeum	Isca
11	Civitas Namnetum	Apollonia-Sozousa
12	Portus Blendium	Lydda
13	Paphos	Iuliobona
14	Ostia/Portus	Placentia
15	Carthago	Constantinopolis
16	Corcyra	Histria
17	Aquileia	Ephesus
18	Caralis	Mothis
19	Sigeion	Patara
20	Constantinopolis	Lancia

weighted betweenness centrality measure are shown in Table 15.1 and Figure 15.5(a, b). Note how different the top scoring towns are (Table 15.1), only four towns occur in both measures' top 20 list. The high scoring towns are still mostly ports but are now more equally spread throughout the system, often with one or a few high scoring towns per province (Figure 15.5a). These high scoring towns can be interpreted as the most important intermediaries for the flow of goods and information through this abstract representation of the Roman transport system if we assume that the shortest possible path between towns was always preferred. The same method can of course be applied to represent other assumptions such as the shortest path in terms of time or financial cost.

Distance from Rome

We now turn to our third research question centred on Rome: the capital of the Roman Empire and a mega city with more than one million inhabitants. The city needed a constant supply of all types of goods and was the largest market for staple goods. Indeed, much of the Roman economy was structured by the need to supply the huge population of the city of Rome. One approach to understanding this structuring is to explore how the Roman transport system could have structured flows of supplies to Rome, and which regions and supplying towns were better positioned on this network to supply Rome. We already know that "All roads lead to Rome", but from some towns the roads take you there much faster than

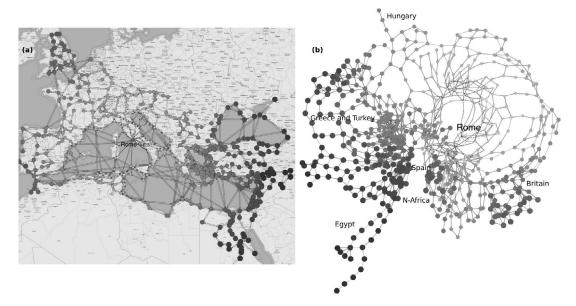


FIGURE 15.6 Geographical network representation of the Orbis network: geographical layout (a) and topological layout (b). Node size and colour represent increasing physical distance over the network away from Rome: the larger and darker the node, the further away this settlement is from Rome following the routes of the transport system. Note the fall-off of the results with distance away from Rome structured by the transport routes rather than as-the-crow-flies distance. Edge colours represent edge type: red = sea, green = river, grey = road. A colour version of this figure can be found in the plates section.

Source: Background © Openstreetmap

from other towns. These differences can be identified using spatial network methods, by calculating the shortest paths from all towns to Rome according to the sum of their physical distance.

The results of this analysis (Figure 15.6) reveal of course a fall-off with distance away from Rome. But note that this does not merely represent a fall-off of towns' scores with as-the-crow-flies distance from Rome, as could be easily calculated in GIS, but rather with their distance to Rome over the shortest path of the network. It offers a representation of physical distance morphed and structured by the Roman transport system. We can observe differences between the outlying regions, like Britain being closer than much of Syria and Egypt. But a more interesting result is the proximity of areas that became the earliest overseas provinces: the proximity of Tunisian towns around Carthage, Sardinia, as well as the relatively short distances to towns in Southern France and Western Spain as compared to much of Greece, for example. These results also offer an appropriate visualisation of what we know about the well-documented large-scale and possibly partly state-organised supplies of foodstuffs to Rome from Tunisia especially from the second century AD onwards, and it highlights the huge organisational efforts that must have gone into the long distance and equally well-documented transport of foodstuffs from Southern Spain and, in particular, Egypt.

Network models

The network models discussed earlier in this chapter can be applied to the Orbis settlement distribution pattern to answer our fourth research question. What spatial structuring does the settlement distribution included in Orbis reveal? To what extent does the Orbis network align with or deviate from this structuring? Does the Roman transport system reveal a nearest-neighbour, relative-neighbour or maximum distance structure? We will use global network measures to compare how similar the structure of the simulated network models are to that of the Orbis network. The models presented in this section were implemented in NetLogo, a very accessible programming language with an intuitive user-interface and comprehensive network science and GIS libraries (Wilensky, 1999).

K-nearest-neighbour networks

This model is very sensitive to the proximity of sets of nodes, and reveals clusters of densely settled areas in the Orbis set of towns (Figure 15.7; Table 15.2). The nearest-neighbour networks with K equals 1 and 2 are very disconnected, although for K equals 2 the global network measures are very similar to the Orbis network but more clustered (Table 15.2). The network becomes connected with 4-nearest-neighbours and the 10-nearest-neighbours network emphasises the clusters in areas where the settlement pattern is densest, but both these networks are much denser and more clustered than the Orbis network (Table 15.2). The degree distributions for these K-nearest-neighbour networks shows very little variance. The lower limit always equals K, and just a few towns have a higher degree than most other towns, a difference that increases as K increases. In contrast, the degree distribution of the real Orbis network is very skewed (Figure 15.5): the large majority of towns are connected to less than eight other towns, whereas very few towns have a much higher degree. The towns with the highest degree are important port towns or large population centres: Delos, Rhodos, Carthago, Ostia/Portus, Lilybaeum, Paphos, Messana, Rome (the first two in this list have the highest degree, but this is partly caused by the very high density of nodes in the Aegean area). The K-nearest-neighbour networks clearly do not capture this feature of the Orbis network. The maritime routes in the Orbis dataset which cross long distances through the Atlantic Ocean and the Mediterranean and Black Sea, are also not recreated by this model. However, aspects of the structure of the terrestrial roads and the dense connections between Aegean islands, as well as the coastal and riverine connections, are better captured by this model where K equals 4.

Maximum distance networks

The maximum distance networks have very different network patterns and degree distributions compared to the previously discussed models (Figure 15.8; Table 15.2). At a maximum distance up to 165km only the densest settled areas in the Orbis dataset in Central Italy, the Aegean and Phoenicia reveal dense clusters. Only at a maximum distance of 220km does the outline of the Orbis transport network start to appear and around a maximum distance of 440km the network becomes connected. However, the 220km and 440km networks are much denser than the Orbis network. The 82.5km and 99km maximum distance networks show a density, number of edges and average degree that is more similar to the Orbis network, but like all other maximum distance networks the degree of clustering is much too high (Table 15.2). Like the other models, this model does not succeed in capturing the long distance maritime routes of the Orbis network but it does slightly better at representing the terrestrial, coastal and riverine connections. The degree distribution is very different from both Orbis and the other models: the higher the maximum distance, the higher the maximum degree; the degree distribution is only very slightly skewed towards the lower degrees but tends to be very spread out.

Gabriel graph and relative neighbourhood network

Aside from the long distance overseas routes, the relative neighbourhood network captures the shape of the transport system rather well (Figure 15.9; Table 15.2). It offers an outline of the Orbis network including most coastal routes, includes some of the maritime connections between the African and

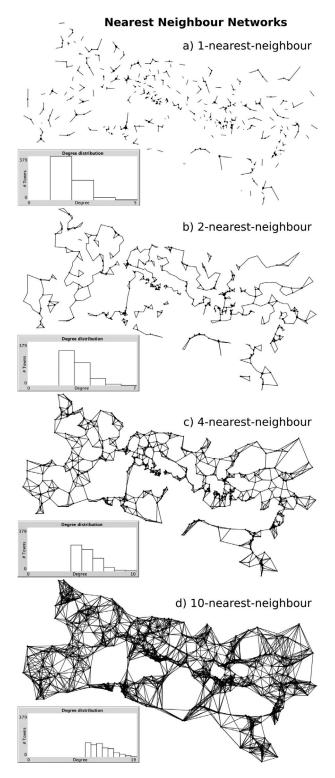


FIGURE 15.7 Nearest neighbour network results of the Orbis set of nodes. Node size represents degree. Insets show degree distributions. Note how the network only becomes connected into a single component when assuming 4-nearest-neighbours.

TABLE 15.2 Results of global network measures for all tested models and the undirected Orbis network (in bold).
Highlighted results show some similarity in global network measures with the Orbis network.

	Edges	Average Degree	Density	Average Clustering Coefficient
Orbis (undirected)	805	2.825	0.005	0.235
1-nearest-neighbour	391	1.372	0.002	0.665
2-nearest-neighbour	743	2.607	0.005	0.447
4-nearest-neighbour	1416	4.968	0.009	0.551
10-nearest-neighbour	3488	12.239	0.022	0.614
82.5km-maximum-distance	684	2.4	0.004	0.818
99km-maximum-distance	981	3.442	0.006	0.771
220km-maximum-distance	3631	12.74	0.022	0.668
440km-maximum-distance	11321	39.723	0.07	0.697
Relative-neighbourhood	663	2.326	0.004	0.079
Gabriel-graph	1040	3.649	0.006	0.239

Eurasian continents and shows some similarities in the density and structure of the terrestrial routes. However, the degree distribution is normally distributed and there is very little variance in nodes' degrees. The Gabriel graph similarly shows little variance in its normally distributed degree distribution, but its triangular structure does succeed in recreating some of the long distance maritime connections. Moreover, it is the only model used here that has an average clustering coefficient close to that of the Orbis network.

Conclusions of network modelling results

This comparison of models suggests that the density, average degree and number of edges can be approximated by a number of models: 2-nearest-neighbour, 82.5km and 99km maximum-distance, relative-neighbourhood-network, and Gabriel graph. However, only the latter two show similarities in the shape of the Orbis network, and only the Gabriel graph succeeds in capturing the degree of clustering. None of the models succeed in reproducing the very skewed degree distribution, suggesting alternative models should be tested that include preferential attachment effects giving rise to a few very highly connected nodes. These modelling results suggest that theories about the structure of the Roman transport system, as hypothesised in the static, coarse resolution Orbis network, should: include a tendency for settlements to be connected to a limited number of their nearest neighbours (e.g. 2-3); mostly avoid the creation of very long distance routes (e.g. > 100km); crucially take into account the position of pairs of nodes relative to all other nearby nodes by avoiding connections between settlement pairs which have other settlements located in the circular neighbourhood described by the diameter between them (i.e. the Gabriel graph). The results further suggest that these models should include an effect to allow for high degree nodes to reproduce the skewed degree distribution (e.g. preferential attachment), a pattern that is rarely reproduced in the explicitly spatial relative or nearest neighbourhood network models presented here.

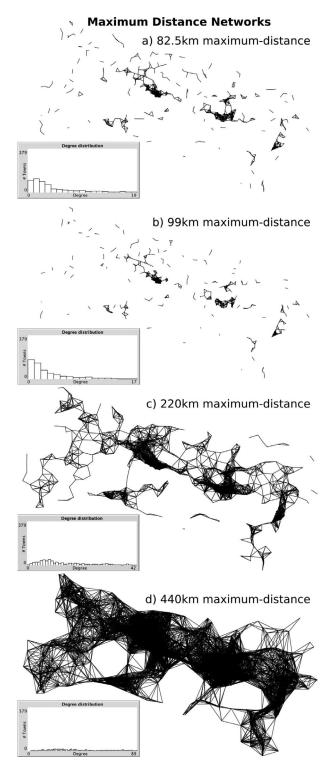


FIGURE 15.8 Maximum distance network results of the Orbis set of nodes. Node size represents degree. Insets show degree distributions. Note how the network only becomes connected into a single component when assuming 440 km as the maximum distance.

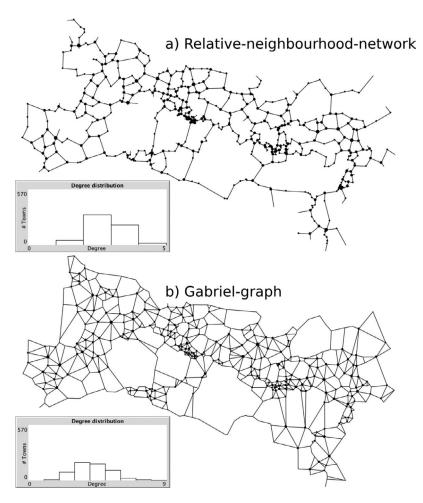


FIGURE 15.9 Results of the Orbis set of nodes; (a) relative neighbourhood network and (b) Gabriel graph. Node size represents degree. Insets show degree distributions. Note how the networks, as compared to the results shown in Figures 15.7 and 15.8, better succeed in representing the shape of the Orbis transport network and the long-distance maritime routes crossing the Mediterranean.

Conclusion

In this chapter we have introduced spatial networks as consisting of sets of spatially embedded nodes and edges whose topology is partly restricted by physical space. A strong research tradition in the archaeological application of spatial networks has focused on a few key themes: transport networks, visibility networks, space syntax and material culture networks. The most commonly applied local and global network measures have been introduced, along with a range of fundamental spatial network models.

Many of the methods and models introduced in this chapter were illustrated through a case study which aimed at exploring the structure of the Roman transport system, as hypothesised by the Orbis network. Geographical and topological visualisations of the Orbis network revealed complementary insights into regional differences in transport network density. The use of a distance weighted

betweenness centrality measure identified settlements that are particularly crucial as intermediaries for the flow of information, people and goods in this system. Calculating the summed distance of the shortest paths from all settlements to Rome highlighted regional differences in the proximity to Rome following the transport network, which has implications for their ability to supply foodstuffs to the capital. Finally, spatial network modelling results suggest that theories about the structure of the Roman transport system should include nearest-neighbourhood, relative-neighbourhood and maximum-distance effects, and a preferential attachment effect is hypothesised to be a further key explanatory factor.

Spatial network applications have a long history in archaeological research, but they have only recently received more attention in the research traditions at the core of network science: social network analysis and physics. We believe the strong archaeological research tradition in spatial networks reveals an important opportunity for archaeologists to contribute to the future development of spatial network methods and models and to their multi-disciplinary application. More intense interaction with the broader network science community will in turn lead to a richer toolbox of spatial network methods and models for archaeologists to let loose on their research topics.

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