1 Geometry revision

1. The points A, B, C, D lie, in this order, on a circle ω, where AD is a diameter of ω. Furthermore, $AB = BC = a$ and $CD = c$ for some relatively prime integers a and c. Show that if the diameter d of ω is also an integer, then either d or $2d$ is a perfect square.

2. Let $ABCDE$ be a convex pentagon such that $AB = BC = CD$, $\angle EAB = \angle BCD$, and $\angle EDC = \angle CBA$. Prove that the perpendiular line from E to BC and the line segments AC and BD are concurrent.

3. The heights of triangle ABC for triangle $A_1B_1C_1$. The heights of triangle $A_1B_1C_1$ form triangle $A_2B_2C_2$. Prove that $ABC \sim A_2B_2C_2$

4. Vertex A of square $ABCD$ is symmetric to the midpoint of side CD with respect to line l. Find the ratio of the areas of the two quadrilaterals on either side of line l which make up the square.

5. Acute triangle ABC has circumcircle ω. The tangents of ω at points B and C intesect at P. D and E are the projections of P to lines AB and AC respectively. Prove that the orthocentre of triangle ADE coincides with the midpoint of line BC.

6. The bisector of the $\angle A$ of a triangle ABC intersects BC in a point D and intersects the circumcircle of the triangle ABC in a point E. Let K, L, M and N be the midpoints of the segments AB, BD, CD and AC, respectively. Let P be the circumcenter of the triangle EKL, and Q be the circumcenter of the triangle EMN. Prove that $\angle PEQ = \angle BAC$.

2 Auxilliary constructions

7. Vertices A, B, C, D of parallelogram are connected to the midpoints of sides BC, CD, DA, AB respectively. The segments intersect at points K, L, M, N. Find the ratio of the areas of $KLMN$ and $ABCD$.

8. In triangle ABC $\angle A = 90°$. Let M be the midpoint of AB. Perpendicular line to CM through A intersects AB at P. Prove that $\angle AMC = \angle BMP$.

9. Let ABC be an acute triangle and D the projection of A to side BC. Point E lies on segment AD and satisfies
$$\frac{AE}{ED} = \frac{CD}{DB}$$
Let F be the projection of point D to BE. Prove that $\angle AFC = 90°$.

10. Points E and F are chosen on the sides CD and BC of square $ABCD$, such that $\angle AEB = \angle AEF$. Find $\angle EAF$.

11. The angle bisectors from points A and B of triangle ABC intersect sides BC and AC at points D and E respectively. Find $\angle C$, if $AE + BD = AB$

12. Let us have a parallelogram $ABCD$. A circle which goes through A intersects segments AB, AC and AD at points M, K and N. Prove that $|AB| \cdot |AM| + |AD| \cdot |AN| = |AK| \cdot |AC|$