Combinatorial geometry

1. There are 5 points on the plane, no three of them lie on the same line. Prove that there exists 4 of them which form a convex quadrilateral

2. A finite number of points is chosen on the plane, no three of them lie on the same line. It is known that there exists a non-convex polygons with its vertices at given points. Prove that there exists a non-convex quadrilateral with its vertices at given points.

3. Let $n \geq 3$ be an integer. Find the largest number of angles which can be greater than $180°$ in an n-gon whose sides are all equal.

4. Every point on the sides of an equilateral triangle is coloured either red or blue. Is it always possible to find a right angle triangle with all its vertices having the same colour.

5. Prove that there are more than 30000 points with integral coordinates which lie within a circle of radius 100.

6. There are n points on the plane. Starting from one of those points, in each step we move to the second closest point. After n steps we have visited all the points and returned to the original point. Find all possible values for n.

7. Let k be a positive integer. Find all positive integers n for which it is possible to choose n points on the sides of a triangle (different from its vertices) and connect some of them with a line such that

   (a) There is at least 1 point on each side

   (b) For each pair of points X and Y which are on different sides of the triangle, there exists exactly k points on the third side which are all connected to both X and Y, and exactly k points which are all connected to neither of X or Y.