

## Diophantine equations

1. Diophantine equations are polynomial equations, usually in two or more unknowns, such that only integer solutions are sought.
2. A Diophantine equation equates two or more monomials, each of degree 1 in one of the variables, to a constant.
3. An exponential Diophantine equation is one in which exponents on terms can be unknowns.

## Problems

1. Given integers  $a, b, c$ , find the conditions on  $a, b, c$  such that there is an integer solution  $x, y$  to  $ax + by = c$ .
2. Prove that for every positive integer  $m$  there is a positive integer  $n$  such that  $m + n + 1$  is a perfect square and  $mn + 1$  is a perfect cube.
3. Show that if  $a, b$  are integers such that  $\frac{a^2+b^2}{1+ab}$  is also an integer, then  $\frac{a^2+b^2}{1+ab}$  is a perfect square.
4. Solve in integers

$$x^3 + x^2 + x + 1 = y^2$$

5. Prove that the equation  $y^2 = x^5 - 4$  has no integer solutions.
6. Prove that there are no integer solutions to

$$x^3 + y^4 = 19^{19}$$

7. Find all solutions to the equation  $x^5 = y^2 + 4$  in positive integers.
8. Find the positive integer solutions to the equation  $3^x + 4^y = 5^z$
9. Prove that if there exists a triple of positive integers  $(x, y, z)$  such that

$$x^2 + y^2 + 1 = xyz$$

then  $z = 3$  and find all such triplets.

10. Given an odd prime  $p$ , find all pairs of nonnegative integers  $x, y$  that solve the equation

$$p^x - y^p = 1$$