

1 Geometry revision

1. The points A, B, C, D lie, in this order, on a circle ω , where AD is a diameter of ω . Furthermore, $AB = BC = a$ and $CD = c$ for some relatively prime integers a and c . Show that if the diameter d of ω is also an integer, then either d or $2d$ is a perfect square.
2. The bisector of the $\angle A$ of a triangle ABC intersects BC in a point D and intersects the circumcircle of the triangle ABC in a point E . Let K, L, M and N be the midpoints of the segments AB, BD, CD and AC , respectively. Let P be the circumcenter of the triangle EKL , and Q be the circumcenter of the triangle EMN . Prove that $\angle PEQ = \angle BAC$.
3. The heights of triangle ABC form triangle $A_1B_1C_1$. The heights of triangle $A_1B_1C_1$ form triangle $A_2B_2C_2$. Prove that $ABC \sim A_2B_2C_2$.
4. Vertex A of square $ABCD$ is symmetric to the midpoint of side CD with respect to line l . Find the ratio of the areas of the two quadrilaterals formed of square on either side of line l .
5. Acute triangle ABC has circumcircle ω . The tangents of ω at points B and C intersect at P . D and E are the projections of P to lines AB and AC respectively. Prove that the orthocentre of triangle ADE coincides with the midpoint of line BC .