1 Stereometry

1. Find the volume of an icosahedron with sidelength a.
2. Find the volume of a dodecahedron with sidelength a.
3. Find the density of close packed spheres.
4. Points $P_1, \dots, P_n$ are in space such that no three are collinear. Any triangle $P_iP_jP_k$ contains a side which is shorter than a. Prove that there exist spheres S_1 and S_2 with radii, such that all of the n points are in at least one of them.
5. P_1 is a convex polyhedron with vertices $A_1, A_2, \dots, A_9$. Polyhedra $P_2, \dots, P_9$ are formed by shifting P_1 such that A_1 is shifted to $A_2, \dots, A_9$ respectively. Prove that at least two of the polyhedra $P_1, \dots, P_9$ are intersecting.
6. Find all integers n for which there exists a convex polyhedron and which satisfies all the following:
   (a) All faces of the polyhedron are regular polygons
   (b) Among the faces of the polyhedron there are at most two polygons with different number of sides
   (c) There exist two faces which share an edge and are both n-gons.
7. A convex polyhedron has faces $S_1, \dots, S_n$ with areas $A_1, \dots, A_n$ respectively. For each side S_i, let us define a vector $\vec{v_i}$ with length A_i and normal to S_1. Prove that $\sum_{i=1}^{n} \vec{v_i} = \vec{0}$