

Diophantine equations

1. Diophantine equations are polynomial equations, usually in two or more unknowns, such that only integer solutions are sought.
2. A Diophantine equation equates two or more monomials, each of degree 1 in one of the variables, to a constant.
3. An exponential Diophantine equation is one in which exponents on terms can be unknowns.

Problems

1. Given integers a, b, c , find the conditions on a, b, c such that there is an integer solution x, y to $ax + by = c$.
2. Prove that for every positive integer m there is a positive integer n such that $m + n + 1$ is a perfect square and $mn + 1$ is a perfect cube.
3. Show that if a, b are integers such that $\frac{a^2+b^2}{1+ab}$ is also an integer, then $\frac{a^2+b^2}{1+ab}$ is a perfect square.
4. Solve in integers

$$x^3 + x^2 + x + 1 = y^2$$

5. Prove that the equation $y^2 = x^5 - 4$ has no integer solutions.
6. Prove that there are no integer solutions to

$$x^3 + y^4 = 19^{19}$$

7. Find all solutions to the equation $x^5 = y^2 + 4$ in positive integers.
8. Find the positive integer solutions to the equation $3^x + 4^y = 5^z$
9. Prove that if there exists a triple of positive integers (x, y, z) such that

$$x^2 + y^2 + 1 = xyz$$

then $z = 3$ and find all such triplets.

10. Given an odd prime p , find all pairs of nonnegative integers x, y that solve the equation

$$p^x - y^p = 1$$