1 Geometry revision

1. The points A, B, C, D lie, in this order, on a circle ω, where AD is a diameter of ω. Furthermore, $AB = BC = a$ and $CD = c$ for some relatively prime integers a and c. Show that if the diameter d of ω is also an integer, then either d or $2d$ is a perfect square.

2. Let $ABCDE$ be a convex pentagon such that $AB = BC = CD$, $\angle EAB \angle BCD$, and $\angle EDC = \angle CBA$. Prove that the perpendiular line from E to BC and the line segments AC and BD are concurrent.

3. The heights of triangle ABC for triangle $A_1B_1C_1$. The heights of triangle $A_1B_1C_1$ form triangle $A_2B_2C_2$. Prove that $ABC \sim A_2B_2C_2$

4. Vertex A of square $ABCD$ is symmetric to the midpoint of side CD with respect to line l. Find the ratio of the areas of the two quadrilaterals on formed of square on either side of line l.

5. Acute triangle ABC has circumcircle ω. The tangents of ω at points B and C intesect at P. D and E are the projections of P to lines AB and AC respectively. Prove that the orthocentre of triangle ADE coincides with the midpoint of line BC.

6. The bisector of the $\angle A$ of a triangle ABC intersects BC in a point D and intersects the circumcircle of the triangle ABC in a point E. Let K, L, M and N be the midpoints of the segments AB, BD, CD and AC, respectively. Let P be the circumcenter of the triangle EKL, and Q be the circumcenter of the triangle EMN. Prove that $\angle PEQ = \angle BAC$.