In[33]:= `x[t_] = yv Exp[(Ψ + 𝕚 Ω) t] Exp[𝕚 fiv];`
`p[t_] = pu Exp[(Ψ + 𝕚 Ω ) t] - pd Exp[(Ψ + 𝕚 Ω) t] // FullSimplify`

Out[34]= $e^{t (Ψ + 𝕚 Ω)} (-pd + pu)$

In[35]:= `Eq = D[x[t], {t, 2}] + k D[x[t], {t, 1}] + x[t] - p[t] // Simplify`

Out[35]= $e^{t (Ψ + 𝕚 Ω)} \left(pd - pu + e^{𝕚 \, fiv} yv \left(1 + k Ψ + Ψ^2 + 𝕚 k Ω + 2 𝕚 Ψ Ω - Ω^2\right)\right)$

In[36]:= `Eq2 = Eq / Exp[(Ψ + 𝕚 Ω) t] // Simplify`

Out[36]= $pd - pu + e^{𝕚 \, fiv} yv \left(1 + k Ψ + Ψ^2 + 𝕚 k Ω + 2 𝕚 Ψ Ω - Ω^2\right)$

In[37]:= `EqRe = FullSimplify[Re[ComplexExpand[Eq2]],`
`Assumptions → {{yv, k, Ψ, Ω, yv, yd, δ, pd, pu, fiv, k} ∈ Reals, fiv > 0, Ω > 0}]`

Out[37]= $pd - pu + yv \left(1 + Ψ (k + Ψ) - Ω^2\right) Cos[fiv] - yv (k + 2 Ψ) Ω Sin[fiv]$

In[42]:= `EqIm = FullSimplify[Im[ComplexExpand[Eq2]],`
`Assumptions → {{yv, k, Ψ, Ω, yv, yd, δ, pd, pu, fiv, k} ∈ Reals, fiv > 0, Ω > 0}]`

Out[42]= $yv \left((k + 2 Ψ) Ω Cos[fiv] + \left(1 + Ψ (k + Ψ) - Ω^2\right) Sin[fiv]\right)$

In[51]:= `fisol = FullSimplify[Solve[{EqIm == 0}, fiv],`
`Assumptions → {{yv, k, Ψ, Ω, yv, yd, δ, pd, pu, fiv, k} ∈ Reals, fiv > 0, Ω > 0}]`
`yvsol = Solve[EqRe == 0, yv] // FullSimplify`

Out[51]= $\left\{\left\{fiv → ConditionalExpression\left[ArcTan\left[1 + Ψ (k + Ψ) - Ω^2, - (k + 2 Ψ) Ω\right] + 2 π c_1, c_1 ∈ ℤ\right]\right\},\right.$
$\left.\left\{fiv → ConditionalExpression\left[ArcTan\left[-1 - Ψ (k + Ψ) + Ω^2, (k + 2 Ψ) Ω\right] + 2 π c_1, c_1 ∈ ℤ\right]\right\}\right\}$

Out[52]= $\left\{\left\{yv → \dfrac{-pd + pu}{\left(1 + Ψ (k + Ψ) - Ω^2\right) Cos[fiv] - (k + 2 Ψ) Ω Sin[fiv]}\right\}\right\}$