FISH 559: Example Application V (Hierarchical Modelling)

The following nonlinear mixed effects method uses biomass and recruitment data from the application of a stock assessment method to multiple stocks to estimate a distribution that represents the uncertainty of the steepness of the stock-recruitment relationship among stocks. Given the estimates of spawning biomass and recruitment from an assessment method, the Beverton-Holt model is defined as in equation (1) for each stock *i* and observation *t* in the time series.

$$R_{it} = \frac{4R_{0i}h_i B_{it}}{\phi_{0i}R_{0i}(1-h_i)+(5h_i-1)B_{it}} e^{\varepsilon_{it}-\sigma_i^2/2} = F(B_{it})e^{\varepsilon_{it}-\sigma_i^2/2};\ \varepsilon_{it} \sim N(0;\sigma_i^2) \qquad (1)$$

where h_i is the value of steepness for stock *i*, R_{0i} is the value of R_0 for stock *i*, ϕ_{0i}, is the spawner biomass-per-recruit in the absence of exploitation for stock *i* (i.e. $R_{0,i} = B_{0,i} / \phi_{0,i}$), B_{it} is the spawning biomass for year *t* and stock *i*, and R_{it} is the recruitment (at age 0) for year *t* and stock *i*.

This method assumes that the stock-specific values for ϕ_{0i} are known, and estimates the stock-specific unfished recruitment, R_{0i}, and the extent of variation about the stock-recruitment relationship, σ_i, for each stock as fixed effects. Given values for R_{it} and B_{it} from a stock assessment, the steepness parameter after logit transformation is assumed to be a random effect and normally distributed, *i.e.*:

$$\beta = \log\left(\frac{h-0.2}{1-h}\right); \qquad \beta \sim N(\mu,\ \tau^2) \qquad (2)$$

where μ and τ are respectively the mean and standard deviation of the distribution of logit-transformed steepness. The process errors (the ε_{it} in Equation 1) are assumed to be temporally independent, and independent among species.

The likelihood function for this problem assumes that the deterministic component of Equation (1) represents the mean of the distribution, i.e.:

$$-\ell\text{n}L = \sum_i \sum_t \left(\left(\ell\text{n}R_{it} - \ell\text{n}F(B_{it}) + \sigma_i^2/2 \right)^2 / (2\sigma_i^2) + \ell\text{n}\sigma_i \right) \qquad (3)$$

In addition to Equation 3, the objective function also contains contributions from the probability of the random effects given μ and .

Develop a nonlinear mixed effects model fit using maximum likelihood, to estimate the parameters of the distribution for the steepness parameter.

Hints:

- Model β for each stock as $\beta_i = \mu + \tau\eta_i$.
- The estimable parameters for should be μ, $\ell\text{n}\tau$ (bounded between -7 and 4), B_0 (each bounded between 0.1 and 1000), $\ell\text{n}\sigma_i$ (each bounded between -7 and 4), and η_i (no explicit bounds).
- Don't forget to add a "random("X")" statement to the MakeADFun call.
- Use ADREPORT to obtain estimates of *h*, R_0, and τ, along with their asymptotic standard errors.