559

E: Variance Estimation

Fish 559; Day 2: 15h30-17h00

Variance Estimation

- There are a variety of ways to compute standard errors and confidence intervals for model parameters and model outputs. This lecture considers two of these:
  - asymptotic methods; and
  - likelihood profile.

Asymptotic Methods

Asymptotic Standard Errors and Correlations-I

- TMB automatically calculates asymptotic standard errors and parameter correlations:

$$\Sigma = \left[-\frac{\partial \ell nL}{\partial \theta_i \, \partial \theta_j} \right]^{-1}$$

- The parameter estimates and their asymptotic standard errors can be extracted using the sdreport(model) call.

Asymptotic Standard Errors and Correlations-II

- The default asymptotic variance-covariance calculations relate to the parameters of the model. However, we often want uncertainty estimates for derived quantities.
- The way to do this is via the delta method:

$$Cov[g(\underline{\theta}), h(\underline{\theta})] \approx \sum_i \sum_j \frac{\partial g(\underline{\theta})}{\partial \theta_i} \frac{\partial h(\underline{\theta})}{\partial \theta_j} Cov[\theta_i, \theta_j]$$

Asymptotic Standard Errors and Correlations-III

- To obtain asymptotic variances for derived parameters simply add

ADREPORT(variable)

to the CPP file.

- Note: the results from asymptotic theory are approximations and may be bad approximations.

Asymptotic Standard Errors and Correlations-IV

- Hints to speed things up:
  - Only calculate the derived variables when they are needed:

```
if (Do_sd==1)
  { code; }
```

  - Look at the correlation matrix (which you can get by inverting the Hessian matrix; "hessian=T" in the MakeADFun call). If you see large (>0.80) correlations among the parameters, it may be time to consider reparameterizing the model!

Exercise-I

- Use the age-structured model from the last lecture and the dataset LECTE.DAT (a dataset with error) and compute:
  - the variance-covariance matrix for the parameters;
  - the standard error for the ratio:

$$B_{20} / B_1$$

Exercise-II

- The correlation matrix:

33	Sel50	1.0000					
34	Sel95	***0.9663***	1.0000				
35	logFish	0.6845	0.6354	1.0000			
36	logFish	0.7331	0.6794	0.7865	1.0000		
37	logFish	0.7521	0.6950	0.7468	***0.8133***	1.0000	
38	logFish	0.7597	0.6992	0.7034	0.7712	***0.8081***	1.0000
39	logFish	0.7597	0.6949	0.6708	0.7372	0.7792	***0.8175*** 1.0000
40	logFish	0.7643	0.6962	0.6444	0.7057	0.7470	0.7887 ***0.8250*** 1.0000

"Large" values in bold

- The standard error of B_{20} / B_1:

  0.038787

Likelihood profile

Likelihood profile-I

- A likelihood profile is:

$$\max_{\underline{\theta}}(\ell \mathrm{n} L(\mathbf{D} \mid \underline{\theta} \mid) \mid g(\underline{\theta}) = \alpha)$$

1. Fit the model to find the ML parameter estimates and find the corresponding negative log-likelihood.
2. Select a set of fixed values for the quantity of interest.
3. Minimize the negative log-likelihood fixing the quantity to each value in turn.
4. Plot the difference between the negative log-likelihood from step 1 and those from step 3

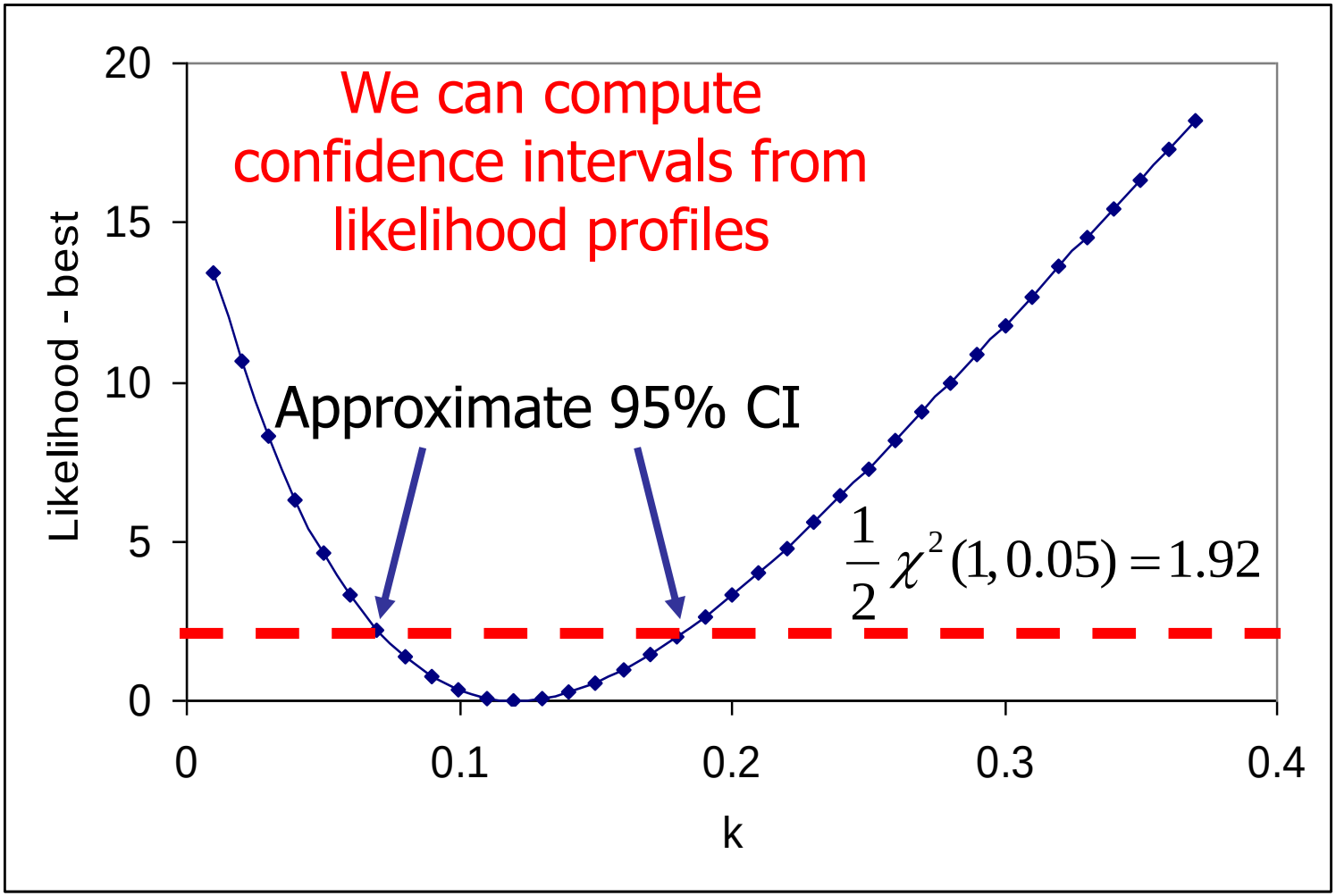
Likelihood profile-II (Confidence intervals)

- An 100-x% confidence interval for p parameters is determined by finding the values for the parameter(s) for which:

$$\ell \mathrm{n} L - \ell \mathrm{n} L_{MLE} = 0.5\, \chi^2(p, x/100)$$

- $\ell \mathrm{n} L_{MLE}$ is the negative log-likelihood corresponding to the maximum likelihood estimates.

Likelihood profile-III



Likelihood Profile-IV
(How to create one)

- To obtain a likelihood profile for any parameter (lets say x). Call the function:

```
prof <- tmbprofile(model,“x")
```

- You can use the profile to determine confidence intervals for a parameter for which a profile has been determined using:

```
print confint(prof,level=0.8)
```

Likelihood Profile-V
(Using profiles)

- Profiles can be used to evaluate the reliability of asymptotic variance estimates (check for non-symmetric profiles).
- Profiles can be used to assess the information content of the data relative to some parameter.
- Profiles can be used to assess data conflicts by plotting a the components of the likelihood.

Likelihood Profile-VI
(Constructing likelihood profiles for derived parameters)

- Add code that involves the computing the square of the difference between the quantity and a (read-in) target value.
- Run the program for many different target values.
- Construct a likelihood profile manually.