FISH 559: Example Application IV (Length-structured model)

Many of the world's fish and (particularly) invertebrate stocks are assessed using size-structured population dynamics models because either there are no data on the age-composition of the population or growth is indeterminate. This example has two steps: (a) fit a (simple; 6 stage) size-structured population dynamics model and (b) project into the future under various levels of fishing intensity to implement a rebuilding strategy.

Step A: Population assessment

A.1 Population dynamics model

The basic dynamics of the population are:

$$\underline{N}_{y+1} = \mathbf{X}\mathbf{S}_y \underline{N}_y + \underline{R}_{y+1} \tag{1}$$

where $\underline{N}_y$ is the vector of numbers-at-stage at the start of year y, $\mathbf{X}$ is the growth transition matrix (assumed to be lower triangular), $\mathbf{S}_y$ is the survival matrix for year y, and $\underline{R}_y$ is the vector of recruits for year y (with zeros on all elements except the first). The matrix $\mathbf{S}_y$ is diagonal with elements:

$$S_{y,i,i} = e^{-M - \tilde{S}_i F_y} \tag{2}$$

where M is the instantaneous rate of natural mortality (assumed to be time-and size-invariant), $\tilde{S}_i$ is selectivity on animals in size-class i, and F_y is the fully-selected fishing mortality rate during year.

Recruitment occurs to the first size-class only and is modelled as $R_{y,1} = \bar{R}e^{\varepsilon_y}$

The catch in numbers and by weight are given by:

$$C_{y,i} = \frac{\tilde{S}_i F_y}{M + \tilde{S}_i F_y} N_{y,i} (1 - e^{-M - \tilde{S}_i F_y}) \tag{3a}$$

$$\tilde{C}_y = \sum_i W_i C_{y,i} \tag{3b}$$

where W_i is the weight of animals in size-class i.

A.2 Parameterization

The parameters of model are the numbers-at-size at the start of the first year (in log-space and 6 parameters), the parameters that define selectivity and growth, the fully-selected fishing mortalities (one for each year), median recruitment, $\bar{R}$ (in log-space), and the deviations in recruitment about mean recruitment, ε_y. Fishery selectivity as a function of length is modelled as a logistic function of length with lengths at 50% and 95% selectivity of 40 and 70cm while survey selectivity is modelled as a logistic function with lengths at 50% and 95% selectivity of 20 and 60cm. The growth transition matrix is pre-specified (see EX4.DAT). The only parameters that are included in the TMB non-linear search are the numbers-at-size at the start of the first year, the fully-selected fishing mortalities and the parameters that define annual recruitment (the value for the catchability coefficient is estimated analytically).

The data available for assessment purposes are the catches in weight, the size-composition of the catches and an index of survey-selected biomass. The contributions of each of the data sources to the negative of the logarithm of the likelihood function are:

$$L_1 = \frac{1}{2\sigma_C^2} \sum_y \left(\ell n \tilde{C}_y - \ell n C_y^{\text{obs}} \right)^2 \tag{4}$$

where C_y^{obs} is the observed catch-in-weight for year y, σ_c is the standard deviation of the logs of the catches in weight (assumed to be 0.05).

$$L_2 = \frac{1}{2\sigma_I^2} \sum_y \left(\ell n I_y - \ell n(qB_y) \right)^2 \tag{5}$$

where I_y is the index of abundance for year y, σ_I is the pre-specified standard deviation of the logarithm of I_y (0.2), B_y is the biomass corresponding to I_y:

$$B_y = \sum_i \breve{S}_i W_i N_{y,i} \tag{6}$$

$\breve{S}_i$ is survey selectivity for animals in size-class i, q is a constant of proportionality.

$$L_3 = -\sum_y \sum_i N^{eff} \rho_{y,i}^{\text{obs}} \ell n(\hat{\rho}_{y,i} / \rho_{y,i}^{\text{obs}}) \tag{7}$$

where N^{eff} is the effective sample size (set to 100), $\rho_{y,\text{i}}^{\text{obs}}$ is the observed proportion which animals in size-class i constitute of the catch in numbers during year y, and $\hat{\rho}_{y,i}$ is the model estimate corresponding to $\rho_{y,\text{i}}^{\text{obs}}$:

$$\hat{\rho}_{y,i} = C_{y,i} / \sum_j C_{y,j} \tag{8}$$

A penalty is placed on the deviations in recruitment about mean recruitment:

$$P_1 = \frac{1}{2\sigma_R^2} \sum_y \varepsilon_y^2 \tag{9}$$

where σ_R is the standard error of the ε_y (assumed to be 0.6).

A.3 Problem statement

Update the provided code (EX4Class.R and EX4Class.cpp) and estimate the values for the parameters of the model. Use the provided MCMC module construct a posterior distribution. Implement the MCMC algorithm by running 1,000,000 cycles, within a burn-in of 10,000, saving every 1,000$^{\text{th}}$ vector (900 parameter vectors in total). Based on the samples from the posterior construct a posterior distribution for the time-trajectory for B_y. Hints:

- Treat the logarithms of average recruitment, annual fishing mortality and initial numbers as parameters to ensure they stay positive.
- Set the value for q to the maximum likelihood estimate. The maximum likelihood estimate is:

$$\hat{q} = \exp\left(\tfrac{1}{n} \sum \ell n(I_y / B_y) \right)$$

- You need to include B_y as a REPORT() variable in the cpp code.
- You can get a variance-covariance matrix by setting "hessian=T" in the MakeADFun and inverting the resultant hessian matrix, i.e. "VarCo <- solve(model$he())". You can check you have things correct by comparing the standard errors of the parameters from TMB with the square roots of the diagonal of the variance-covariance matrix.
- The object "post1a" contains the vectors of parameters.

Step B: Risk Analysis

The aim of this step is to find the constant level of F_y, which if implemented over the next 20 years will result in B_y rebuilding to 1000t with 0.5 probability by year 45, i.e. $P(B_{45} > 1000t) = 0.5$.. For "bonus points" plot the future F as a function of $P(B_{45} > 1000t)$.

Hints:

- Given step B, I included an extra "Nproj" recruitment deviations and projected the model Nyear+Nproj years ahead with the fully-selected fishing mortality F_y included as data.
- Create code in R that solve for the F you wish to project the model forward for (or just do it manually).