FISH 559: Numerical Computing for the Natural Resources
Homework 3 : Root Finding (Out of 110 points)

Fisheries management decisions are often based on abundance relative to target and limit reference points. The most common reference point is the population size at which *MSY* is achieved. The fully-selected fishing mortality corresponding to *MSY*, F_{MSY}, is defined as the instantaneous rate of fishing mortality at which yield is maximized, i.e.:

$$\left.\frac{dY(F)}{dF}\right|_{F_{MSY}} = 0 \tag{1}$$

where $Y(F)$ is yield as a function of fully-selected fishing mortality, i.e.:

$$Y(F) = \tilde{Y}(F)\, R(F) \tag{2}$$

$\tilde{Y}(F)$ is yield-per-recruit as a function of *F*, and
$R(F)$ is recruitment as a function of *F*.

Yield-per-recruit is defined according to the formula:

$$\tilde{Y}(F) = \sum_s \sum_{a=0}^{x} w_a^s \frac{S_a^s F}{Z_a^s(F)} N_a^s(F)(1 - e^{-Z_a^s(F)}) \tag{3}$$

where w_a^s is the weight of an animal of sex *s* and age *a*,
S_a^s is the selectivity for animals of sex *s* and age *a*,
$Z_a^s(F)$ is the total mortality on fish of sex *s* and age *a*,

$$Z_a^s(F) = M + S_a^s F \tag{4}$$

$N_a^s(F)$ is the number of fish of sex *s* and age *a* relative to the number of animals of age 0 (both sexes combined):

$$N_a^s(F) = \begin{cases} 0.5 & \text{if } a = 0 \\ N_{a-1}^s(F) e^{-Z_{a-1}^s(F)} & \text{if } 0 < a < x \\ N_{x-1}^s(F) e^{-Z_{x-1}^s(F)} / (1 - e^{-Z_x^s(F)}) & \text{if } a = x \end{cases} \tag{5}$$

x is the maximum age-class.

The recruitment as a function of *F* depends on the assumed form of the stock-recruitment relationship, e.g.:

$$R(F) = \frac{S(F)}{\alpha + \beta S(F)} \tag{6}$$

where $S(F)$ is spawner biomass as a function of *F*:

$$S(F) = \tilde{S}(F)\, R(F) \tag{7}$$

$\tilde{S}(F)$ is spawner biomass-per-recruit as a function of *F*:

$$\tilde{S}(F) = \sum_{a=1}^{x} f_a\, N_a^{\text{fem}}(F) \tag{8}$$

f_a is fecundity as a function of age.

Substituting Equation (7) into Equation (6) and solving for $R(F)$ leads to:

$$R(F) = \frac{\tilde{S}(F) - \alpha}{\beta \tilde{S}(F)} \tag{9}$$

Question 1 (10 points)
Find the relationship between recruitment and spawner biomass-per-recruit (see Equation 9) for the Ricker and Pella-Tomlinson stock-recruitment relationships:

$$R(F) = \alpha S(F) e^{-\beta S(F)} \tag{10a}$$
$$R(F) = \alpha\, S(F)(1+\beta(1-[S(F)/S(0)]^{\gamma}) \tag{10b}$$

<u>Ricker</u>

$R(F) = \alpha\, S(F)\, e^{-\beta S(F)}$

$\cancel{R(F)} = \alpha\, \tilde{S}(F) \cancel{R(F)}\, e^{-\beta R(F) \tilde{S}(F)}$

$\ln\left(1 = \alpha \tilde{S}(F)\, e^{-\beta R(F)\tilde{S}(F)}\right)$

$0 = \ln \alpha + \ln \tilde{S}(F) - \beta R(F) \tilde{S}(F)$

$\frac{\cancel{\beta} R(F) \cancel{\tilde{S}(F)}}{\cancel{\beta \tilde{S}(F)}} = \frac{\ln \alpha + \ln \tilde{S}(F)}{\beta \tilde{S}(F)}$

$R(F) = \frac{\ln(\alpha \tilde{S}(F))}{\beta \tilde{S}(F)}$

Pella - Tomlinson

$$R(F) = \alpha S(F)\left(1+\beta\left(1-\left(\frac{S(F)}{S(0)}\right)^{\gamma}\right)\right)$$

$$R(F) = \alpha \tilde{S}(F) R(F)\left(1+\beta\left(1-\left(\frac{\tilde{S}(F) R(F)}{S(0)}\right)^{\gamma}\right)\right)$$

$$\frac{1}{\alpha \tilde{S}(F)} = 1+\beta\left(1-\left(\frac{\tilde{S}(F) R(F)}{S(0)}\right)^{\gamma}\right)$$

$$\frac{\frac{1}{\alpha \tilde{S}(F)}-1}{\beta} = 1-\left(\frac{\tilde{S}(F) R(F)}{S(0)}\right)^{\gamma}$$

$$\frac{\left(\frac{1}{\alpha \tilde{S}(F)}-1\right)}{\beta}-1 = -\left(\frac{\tilde{S}(F) R(F)}{S(0)}\right)^{\gamma}$$

$$\left(\frac{\tilde{S}(F) R(F)}{S(0)}\right)^{\gamma} = 1-\left(\frac{1-\alpha \tilde{S}(F)}{\beta \alpha \tilde{S}(F)}\right)$$

$$\boxed{R(F) = \frac{S(0)}{\tilde{S}(F)}\left[1-\left(\frac{1-\alpha \tilde{S}(F)}{\beta \alpha \tilde{S}(F)}\right)\right]^{\frac{1}{\gamma}}}$$

Question 2 (10 points)

Reparameterize the Beverton-Holt, Ricker and Pella-Tomlinson models in terms of R_0 (the number of age-0 animals in the absence of exploitation), spawner biomass-per-recruit in the absence of exploitation, and steepness (the fraction of R_0 to be expected when the spawner biomass is reduced to 20% of that in the absence of exploitation). When reparameterizing the Pella-Tomlinson model, assume that γ remains one of the parameters.

Beverton - Holt

Eqn 6: $R(F) = \dfrac{S(F)}{\alpha + \beta S(F)}$ and Eqn 7: $S(F) = \tilde{S}(F)R(F)$

$\therefore$ $R(0) = \dfrac{S(0)}{\alpha + \beta S(0)}$ $\therefore$ $S(0) = \tilde{S}(0)R(0)$

and $R(0) = \dfrac{S(0)}{\tilde{S}(0)}$.

Putting these together:

$$\frac{S(0)}{\tilde{S}(0)} = \frac{S(0)}{\alpha + \beta S(0)}$$

$$\alpha = \tilde{S}(0) - \beta S(0)$$

Sub for α

$$R(0) = \frac{S(0)}{\tilde{S}(0) - \beta S(0) + \beta S(0)}$$

If h = the fraction of R(0) when S is reduced to 20% of that in the absence of exploitation:

$$hR(0) = \frac{\frac{1}{5}S(0)}{\tilde{S}(0) - \beta S(0) + \frac{1}{5}\beta S(0)} = \frac{\frac{1}{5}S(0)}{\tilde{S}(0) - \frac{4}{5}\beta S(0)}$$

$$hR(0)\tilde{S}(0) - \frac{4}{5}hR(0)\beta S(0) = \frac{1}{5}S(0)$$ ← Sub $R(0)\tilde{S}(0)$ for $S(0)$ and cancel

$$\left(h - \frac{4}{5}hR(0)\beta = \frac{1}{5}\right)5$$

$$5h - 4hR(0)\beta = 1$$

$$\beta = \frac{5h-1}{4hR(0)}$$

$$\alpha = \tilde{S}(0) - \left(\frac{5h-1}{4hR(0)}\right)S(0)$$ ← sub $\dfrac{S(0)}{R(0)}$ for $\tilde{S}(0)$

$$= \tilde{S}(0)\left(1 - \frac{5h-1}{4h}\right)$$

$$\alpha = \tilde{S}(0)\left(\frac{1-h}{4h}\right)$$

Ricker

Eqn 10a : $R(F) = \alpha S(F) e^{-\beta S(F)}$

$R(0) = \alpha S(0) e^{-\beta S(0)}$

$\frac{S(0)}{\tilde{S}(0)} = \frac{\alpha S(0)}{e^{\beta S(0)}}$

$\frac{1}{\tilde{S}(0)} = \frac{\alpha}{e^{\beta S(0)}}$

$\frac{1}{\alpha} = \frac{\tilde{S}(0)}{e^{\beta S(0)}}$

$\alpha = \frac{e^{\beta S(0)}}{\tilde{S}(0)}$

$h \cancel{R(0)} = \left(\frac{e^{\beta S(0)}}{\tilde{S}(0)}\right) \frac{S(0)}{5 e^{\beta \frac{1}{5} S(0)}} = \frac{\cancel{S(0)}}{5 \cancel{\tilde{S}(0)}} \frac{e^{\beta \frac{4}{5} S(0)}}{5}$

$\ln \left(5h = e^{\beta \frac{4}{5} S(0)}\right)$

$\ln(5h) = \beta \frac{4}{5} S(0) =$

$\beta = \frac{5 \ln(5h)}{4 \tilde{S}(0) R(0)}$

$\alpha = e^{\left(\frac{5 \ln(5h) \cancel{S(0)}}{4 \cancel{S(0)}}\right)} \cdot \frac{1}{\tilde{S}(0)}$

$\alpha = \frac{1}{\tilde{S}(0)} e^{\left(\frac{5 \ln(5h)}{4}\right)}$

Pella - Tomlinson

Eqn 10b $R(F) = \alpha S(F)(1 + \beta(1 - [S(F)/S(0)]^{\gamma}])$

$R(0) = \alpha S(0)\,(1 + \beta(1 - [S(0)/S(0)]^{\gamma}))$

$R(0) = \alpha S(0)$

$\alpha = \frac{1}{\tilde{S}(0)}$

$h R(0) = \frac{S(0)}{5\tilde{S}(0)}\,(1 + \beta(1 - (\frac{1}{5})^{\gamma}))$

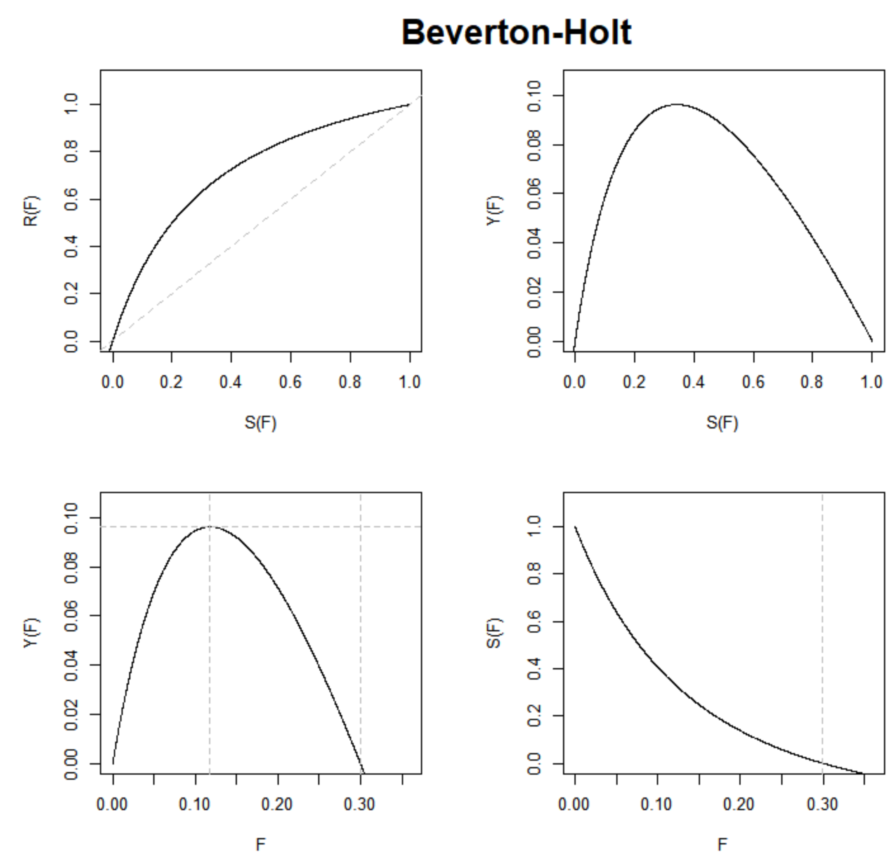
$5h = 1 + \beta(1 - (\frac{1}{5})^{\gamma})$

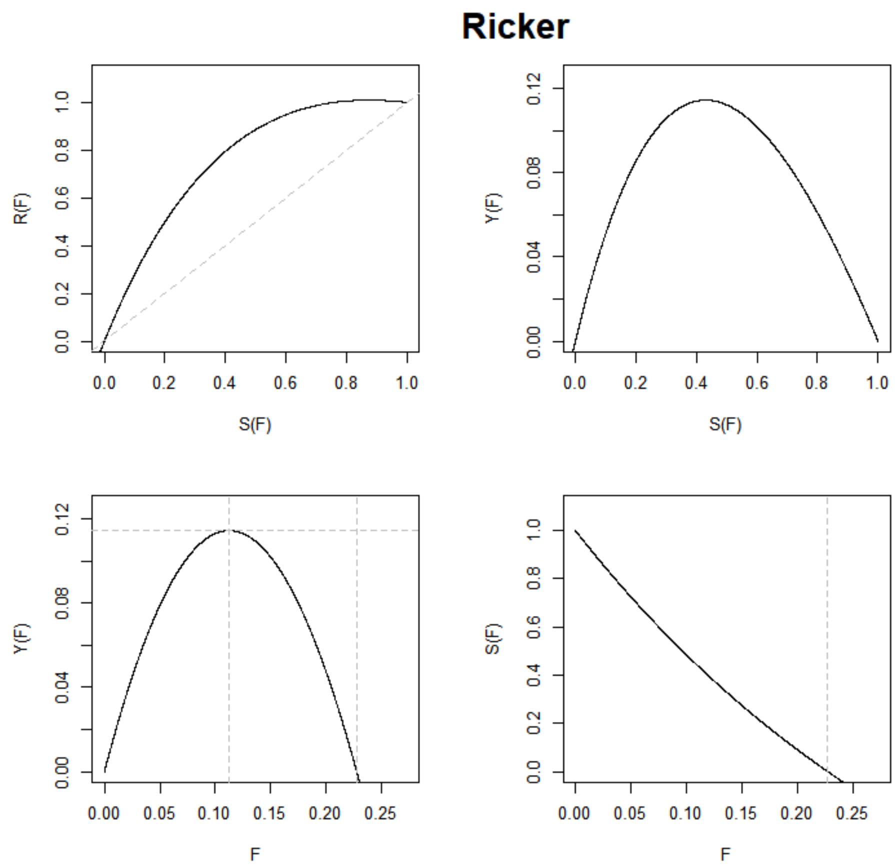
$\beta = (5h - 1)/(1 - (\frac{1}{5})^{\gamma})$

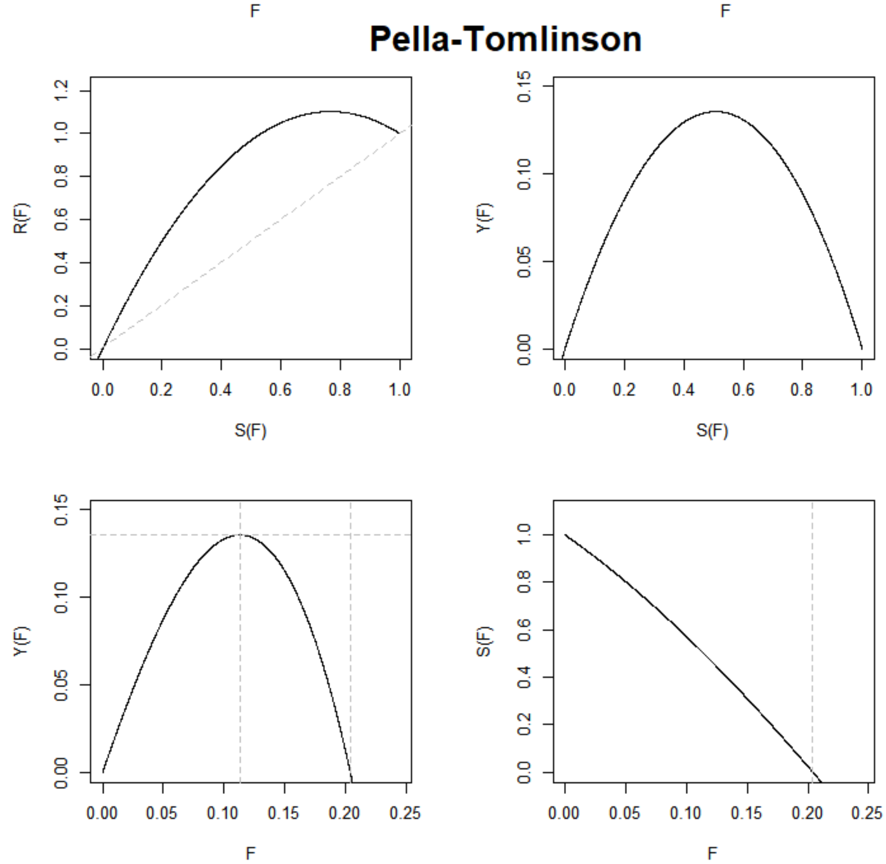
Question 3 (75 points)

Write R functions that take information on natural mortality, fecundity-at-age, weight-at-age, selectivity, R_0, steepness, the form of the stock-recruitment relationship and γ and produce:

a) a plot of the relationship between recruitment and spawning biomass (show the replacement line on this plot), spawning biomass and yield, fishing mortality and yield, and fishing mortality and spawner biomass (four panels on one figure); and




Ricker
R(F)
S(F)
Y(F)
S(F)
Y(F)
F
S(F)
F


Pella-Tomlinson
R(F)
S(F)
Y(F)
S(F)
Y(F)
F
S(F)
F

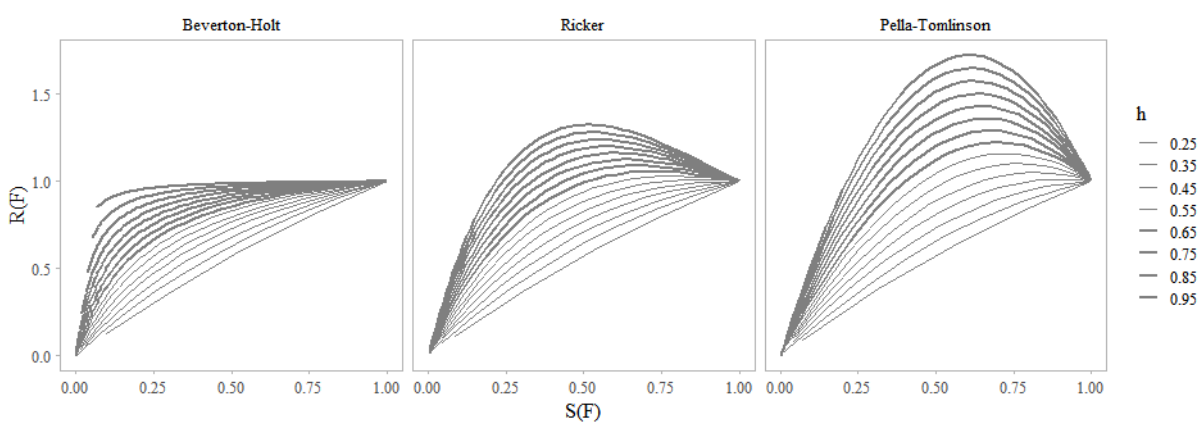
b) estimates of F_{MSY}, MSY, and the lowest fishing mortality corresponding to extinction, (F_τ or F_{crash}).

Model	Fmsy	MSY	Fcrash
Beverton-Holt	0.1179	0.0962	0.2996
Ricker	0.1125	0.1145	0.2275
Pella-Tomlinson	0.1129	0.1355	0.2039

Question 4 (15 points)

Compare the ratio $S(F_{MSY})/S(0)$ across different values for steepness for the Ricker, Beverton-Holt and Pella-Tomlinson forms. The target depletion for many west coast groundfish resources, including widow rockfish, is 0.4*S*(0). Comment on the implications of your results for how west coast groundfish species are managed. Hint: you may wish to review some recent groundfish stock assessments.

The figures below show the stock-recruitment relationship over varying steepness levels (thicker curves indicate higher levels of *h*). They demonstrate that changes in steepness affect the Beverton-Holt relationship the least and the Pella-Tomlinson relationship the most.



The table below shows the ratio of S(Fmsy)/S(0) for each stock-recruitment model over a range of steepness values.

	S(Fmsy)/S(0)		
h	**Beverton-Holt**	**Ricker**	**Pella-Tomlinson**
0.25	0.4659	0.4839	0.5017
0.30	0.4365	0.4703	0.5031
0.35	0.4102	0.4584	0.5042
0.40	0.3861	0.4478	0.5051
0.45	0.3636	0.4383	0.5058
0.50	0.3422	0.4297	0.5064
0.55	0.3216	0.4218	0.5070
0.60	0.3013	0.4145	0.5075
0.65	0.2811	0.4078	0.5079
0.70	0.2606	0.4016	0.5083
0.75	0.2394	0.3958	0.5086
0.80	0.2170	0.3903	0.5089
0.85	0.1923	0.3851	0.5092
0.90	0.1637	0.3803	0.5095
0.95	0.1267	0.3757	0.5097

Fmsy is widely accepted as a reference point over which overfishing occurs. In the table above, every time the ratio S(Fmsy)/S(0) exceeds 0.4, the target depletion exceeded Fmsy.

The table below shows the percentage of times over the range of *h* from 0.25 to 0.95, each stock recruitment model was on or below target or results in overfishing:

Model	On or below target	Overfishing
Beverton-Holt	80%	20%
Ricker	33.3%	66.7%
Pella-Tomlinson	0%	100%

These results show that if recruitment follows a Beverton-Holt relationship then managers are on or below the depletion target 80% of the time over this range of steepness values. However, if recruitment for this stock follows a Ricker or Pella-Tomlinson relationship, they are on target 33% or 0% of the time, respectively.

According to a recent SAFE for Widow rockfish (Hicks and Wetzel 2015), the assessment authors assume a Beverton-Holt relationship and fix steepness at 0.798. If these assumptions are valid, they are not at risk of overfishing. However, according to their report, a likelihood profile of *h* suggests that, if estimated, *h* would be much lower for this stock. This evidence, along with the potential for mis-specifying the stock-recruit relationship, suggests that 0.4*S(0) is too high a target, because it frequently results in overfishing.

References

Hicks, A.C. and Wetzel, C. R. The Status of Widow Rockfish (*Sebastes entomelas*) Along the U.S. West Coast in 2015. SAFE. http://www.pcouncil.org/wp-content/uploads/2016/04/WidowAssessment2015.pdf

Hints:

- Develop separate functions to calculate the numbers-at-age matrix, yield-per-recruit and spawner biomass-per-recruit before writing code to implement Equation 9.
- Use defaults in your function definition / pass a code for the stock-recruitment relationship to each function.
- Show your results for Question 3 by the results when steepness is 0.5 (Figures for each form for the stock-recruitment relationship; Tables of estimates of F_{MSY}, *MSY*, and F_{crash}).
- Show your results for Question 4 by a plot with three panels (one for each stock-recruitment relationship) and a table of $S(F_{MSY})/S(0)$ against steepness values from 0.25 to 0.95 in steps of 0.05.
- Think about ways of testing your functions when you are coding them.
- Base your tests on the data for widow rockfish off the U.S. west coast (Table 1). Assume that R_0 =1 for your calculations.

Table 1. Biological parameters for widow rockfish (Alec MacCall, NWFS, pers. commn). *M* for widow rockfish is assumed to 0.15yr^{-1} for all ages.

Age	**Fecundity**	**Weight (f)**	**Weight (m)**	**Selec (f)**	**Selec (m)**
0	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000

3	0.0001	0.2606	0.3034	0.0006	0.0006
4	0.0002	0.3824	0.4101	0.0057	0.0056
5	0.0150	0.5161	0.5157	0.0548	0.0542
6	0.0642	0.6546	0.6153	0.3626	0.3598
7	0.1612	0.7922	0.7062	0.8465	0.8449
8	0.2766	0.9249	0.7871	0.9981	0.9982
9	0.3687	1.0499	0.8580	0.9636	0.9628
10	0.4412	1.1659	0.9192	0.9050	0.9029
11	0.5085	1.2719	0.9717	0.8477	0.8445
12	0.5666	1.3678	1.0162	0.7939	0.7897
13	0.6188	1.4538	1.0540	0.7433	0.7382
14	0.6652	1.5305	1.0857	0.6953	0.6894
15	0.7064	1.5985	1.1123	0.6491	0.6427
16	0.7428	1.6585	1.1346	0.6041	0.5972
17	0.7747	1.7114	1.1532	0.5593	0.5522
18	0.8028	1.7577	1.1687	0.5139	0.5068
19	0.8274	1.7983	1.1816	0.4670	0.4601
20	0.8839	1.8916	1.2089	0.4180	0.4117