Reply to the referee

" This manuscript would fill a hole in current literature in $0\nu\beta\beta$ given its comprehensive approach to a multi-step problem and will be beneficial to a diverse community of physicists, from model builders to nuclear structure physicists, to obtain the rate of $0\nu\beta\beta$ assuming a/multiple high-scale scenario(s) of LNV. Once further clarification is provided on the points mentioned above, I would like to recommend this manuscript for publication in JHEP."

We thank the referee for her/his kind comments and are happy to see that she/he thinks the manuscript is suitable for publication in JHEP.

Referee's comment

"That being said, only two aspects of the discussions seem to require further clarifications:

i Authors mention few discrepancies with previous literature. It appears that these discrepancies can be cast into two categories:

(a) The discrepancy in the number of the higher-dimension short distance operators and their renormalization group flow equations from above to below the electroweak symmetry breaking scale. It is not obvious why the previous work came out with different results and what the source of discrepancy is. particular, the authors not only need to clarify the source of disagreement, but also to confidently state that their result must be considered correct compared to existing results.

(b) The discrepancy in the treatment of matching to hadronic scale, where non-purtarbative features of nuclear interactions seem to have been dismissed in previous work, and is being identified and corrected by the authors. This point has been made clear through the manuscript and this referee is satisfied with provided level of clarification."

Reply

(a) As the referee mentions we find several discrepancies with the previous literature. Starting with the number of short-distance operators, we do not include operators of the form, $(\bar{u}_L\sigma^{\mu\nu}d_R)\times$ $(\bar{u}_R\sigma_{\mu\nu}d_L)$, which were included in Ref. [39] but can be shown to vanish identically. Furthermore, we find additional $SU(3)_c\times U(1)_{\rm em}$-invariant terms that can be induced by $SU(3)_c \times SU(2) \times U(1)_Y$-invariant operators compared to Ref. [24]. This is due to the fact that only the $SU(2)$-invariant operators consisting of two lepton fields and four quark fields were considered in that work, while the additional terms are induced by operators involving Higgs and/or gauge fields.

In addition, we keep the effects of O_3, O_5 and O_7, which is equivalent to considering color-octet interactions which were neglected in Ref. [21,109]. Although the matrix elements of these operators were argued to vanish in Ref. [21], one would expect them to be of similar size as those of their color-singlet cousins by symmetry considerations. This expectation is borne out by the lattice QCD results for the $\pi\pi$ LECs in Table 1.

To clarify these points we have extended our discussion in Appendix F, where we added the following paragraph:

Compared to Ref. [39] our basis does not include tensor operators of the form $(\bar{u}_L\sigma^{\mu\nu}d_R)\times(\bar{u}_R\sigma_{\mu\nu}d_L)$*, which vanish due to the identity* $[\sigma_{\mu\nu}(1\pm\gamma_5)]_{ij}\,[\sigma^{\mu\nu}(1\mp\gamma_5)]_{kl}=0$*. In contrast, we do include the operators* O_3*,* O_5*, and* O_7*, which are related to color-octet interactions of the form* $\sim(\bar{u}\Gamma t^a d)\,(\bar{u}\Gamma t^a d)$*. Such color-octet terms were neglected in Refs. [21,109], however, the matrix elements of these operators are expected to be of the same size as their color-singlet cousins,* O_2*,* O_4*, and* O_6*. This is borne out by the lattice QCD results for the* $\pi\pi$ *LECs in Table 1. Furthermore, compared to Ref. [24] we find additional* $SU(3)_c\times U(1)_{\rm em}$*-invariant operators that can be induced by operators of* $\overline{\textit{dim-9}}$*. These operators are not induced by the* $SU(2)_L$*-invariant two-lepton four-quark operators considered in Ref. [24], but by additional* $SU(3)_c\times SU(2)_L\times U(1)_Y$*-invariant operators involving gauge- and Higgs-bosons. Finally, we note that although we expect the* $SU(3)_c\times U(1)_{\rm em}$*-invariant operators included here to capture the dominant effects in most LNV scenarios, we have not included all* $\overline{\textit{dim-9}}$ *operators as a complete basis is currently unavailable.*

A second difference with the literature appears in the renormalization group equations. In this case it is hard to pinpoint the source of the discrepancy. However, some of the discrepancies mentioned in the previous version of the manuscript have recently been resolved in an erratum that appeared for Ref. [48]. In particular, the results for the RGEs of $C_{2,3}$ were corrected in this erratum, which are now in agreement with the expressions derived here and in Refs. [46,47]. Nevertheless, our results for the RGEs of the vector operators in Eq. 16 still disagree with Ref. [48]. We therefore removed the statement claiming our results for the RGEs of $C_{2,3}$ differ from Ref. [48] in Appendix F. In addition, to clarify that we believe our results to be correct, we added the following sentence below Eq. 16:

The RGEs for $C^{(9)}_{6,7,8,9}$ *correct expressions that have previously appeared in the* $0\nu\beta\beta$ *literature [48].*

(b) We are happy to hear that the presentation of the matching to the hadronic scale was satisfactory to the referee.

Referee's comment

ii *Since the manuscript's main advantage over previous work is considered by the authors to be applying EFT to all steps of the problem, it must be noted that the many-body nuclear problem may involve other scales not captured by the scales relevant in the few-nucleon system, and the convergence of chiral potentials in realm of heavy nuclei has not yet been fully tested and verified. Discussing possible EFT approaches to a nuclear structure calculation in heavy isotopes is beyond the scope of this work and is still not fully considered and developed by nuclear physics community, and it is not expected that this manuscript go beyond what has already been presented. However, the limitations of chiral nuclear potentials must be clearly alluded to in the manuscript to set a realistic tone when it comes to EFT- based nuclear ab initio many-body calculations. In particular, Weinberg's power-counting that is being selectively used in parts of author's framework is known to have convergence issues even in the two-nucleon sector.*

Reply

ii We agree with the referee that one would preferably use an EFT approach in every step in the calculation of $0\nu\beta\beta$. Part of such a program would be the combination of the transition operators derived in this work with many-body calculations that employ Chiral potentials. The strong and weak sectors can then be renormalized in the same way, which would allow one to test of the power counting employed here in a consistent calculation. As the referee mentions, however, EFT approaches to nuclear structure calculations are currently not able to reach the heavy isotopes of experimental interest. As a result, we are currently unable to derive the $0\nu\beta\beta$ decay rate in a fully consistent manner based on Chiral EFT, and the limits we obtain in Section 5 and 6 (using the nuclear matrix elements obtained from non-chiral methods) should be interpreted with some care. To address these issues we have added a paragraph in the introduction:

Although the power counting employed here is consistent with (nonperturbative) renormalization in the two-body sector, it has not been tested in the large nuclei of experimental interest. To do so in a fully consistent manner, one would have to combine the transition operators derived in this work with many-body wavefunctions obtained from chiral potentials that are consistently renormalized. This has not been achieved for systems with more than a few nucleons. An intermediate approach is to use chiral wave functions from ab initio calculations [28-30] and the insertion of the neutrino transition operators derived in this work [31]. Still, it is not guaranteed that after renormalizing the strong interactions the neutrino transition matrix elements are correctly renormalized [27]. Even this approach is limited to relatively light nuclei, but there is a large ongoing effort to increase the reach to larger and denser systems which would allow for better tests of the chiral power counting. As the ab initio methods fall short for nuclei of experimental interests, we will use results obtained from several non-chiral many-body methods [32-35] to estimate limits on the LNV operators.

Regarding the last sentence of the referee's point: We agree with the referee that Weinberg's power counting breaks down already in the two-body sector. Nevertheless, we have made sure to include all operators in the Chiral Lagrangian that are necessary to (non-perturbatively) renormalize the leading-order amplitude. Although the resulting Lagrangians coincide with those one would obtain from Weinberg's power counting in some cases, they do not rely on it.

To obtain limits we did set several unknown low-energy constants to zero in sections 5 and 6, thereby effectively assuming Weinberg's power counting. The main reason for doing so is the fact that these LECs are currently unknown. As the contributions that also appear in Weinberg's counting should be of the same order, we used these terms to estimate of the limits. Thus, although expressions for the transition operators do not rely on Weinberg's power counting scheme, the limits will certainly be affected once more is known about the low-energy constants.

Referee's comment

Few other minor points that require authors' attention are:

1. v (the Higgs vev) is not introduced when it is first used.
2. Fig. 2 must explicitly reference the work by Prezeau, Ramsey-Musolf and Vogel where the nucleonic EFT was introduced in this problem to classify contributions in Weinberg's power counting.
3. All mentions of lattice or lattice data need to be made more accurate: lattice data $\rightarrow$ lattice QCD data, etc.
4. The matrix element each symbol in Table 2 denotes must be clearly defined in the manuscript (some seem to be listed in the Appendix but not all).

Reply

We thank the referee for pointing out these details to us.

1. We added the following in the introduction

   "*... where $v \simeq 246$ GeV is the Higgs vacuum expectation value,* "

2. The first sentence of the caption of Fig. 2 now reads

   The different contributions of dim-9 LNV operators to the $0\nu\beta\beta$ potential, first discussed in Refs.[54-56,21].

3. We have modified the phrases *“lattice”* and *“lattice data”* where they appeared.

4. The caption of Table 2 now refers to Eq. 72 of Appendix A.2, where we believe all the necessary matrix elements are defined. The added sentence reads

   The NMEs are defined in Eq. (72) of Appendix A.2.