

$$\begin{aligned}
\min_{\beta} \|X\beta - y\|^2 &= \min_{\beta} \|QR\beta - QQ^T y + \tilde{Q}\tilde{Q}^T y\|^2 \\
&= \min_{\beta} \|QR\beta - QQ^T y\|^2 + \|\tilde{Q}\tilde{Q}^T y\|^2 \\
&= \min_{\beta} \|R\beta - Q^T y\|^2 + \|\tilde{Q}\tilde{Q}^T y\|^2.
\end{aligned}$$