19-20

1. $P = P(A_1)\cdot\{1-[1-P(B_1)P(B_2)][1-P(C_1)][1-P(C_2)]\}\cdot P(A_2) = 0.798984$

2. 1) $P = P(Z_1=0) + P(Z_2=0, Z_1=1) + P(Z_{21}=0, Z_{22}=0, Z_1=2)$

$= \frac{1}{4} + \frac{1}{2}\cdot\frac{1}{4} + \frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4} = \frac{25}{64}$

2) $P = P(Z_{21}=2, Z_{22}=2, Z_1=2) = \frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4} = \frac{1}{64}$

3. $P(X_1<X_2<X_3) = \cancel{A} \sum_{i=1}^{\infty}(1-P_1)P_1^{i-1} \sum_{j=i+1}^{\infty}(1-P_2)P_2^{j-1} \sum_{k=j+1}^{\infty}(1-P_3)P_3^{k-1}$

$= \sum_{i=1}^{\infty}(1-P_1)P_1^{i-1}\sum_{j=i+1}^{\infty}(1-P_2)P_3(P_2P_3)^{j-1} = \sum_{i=1}^{\infty}(1-P_1)P_1^{i-1}\frac{(1-P_2)P_3(P_2P_3)^i}{1-P_2P_3}$

$= \frac{(1-P_1)(1-P_2)P_2P_3^2}{(1-P_2P_3)(1-P_1P_2P_3)}$

$\int_{-1}^{1}\frac{1}{2}x^4\,dx$

$\frac{1}{5}$

$\frac{1}{5}-\frac{1}{9}$

4. 1) 设 $W_3 = \xi_1+\xi_2+\xi_3$ 则 $\xi_1 = W_1W_3$, $\xi_2 = (W_2-W_1)W_3$, $\xi_3 = W_3 - W_2W_3$

$\frac{\partial(\xi_1,\xi_2,\xi_3)}{\partial(W_1,W_2,W_3)} = W_3^2$

$\therefore P(W_1,W_2,W_3) = e^{-W_1W_3}\cdot e^{-(W_2-W_1)W_3}\cdot e^{-(W_3-W_2W_3)}\cdot W_3^2$

$= e^{-W_3}W_3^2$

故 $P(W_1,W_2) = \int_0^{+\infty} e^{-W_3}W_3^2\,dW_3 = 2 \quad (0<W_1<W_2<1)$

2) $\because P(0<W_2<\frac{1}{2}, \frac{1}{2}<W_1<1) = 0$

而 $P(0<W_2<\frac{1}{2})$, $P(\frac{1}{2}<W_1<1)$ 皆不为0 故不独立

5. $D(U+V) = D(3\xi-3\eta) = 9[Var(\xi)+Var(\eta)-2Cov(\xi,\eta)]$

$\because \xi,\eta$ 独立 $Var(\xi)=Var(\eta)=\frac{1}{3}$ $\quad\therefore D(U+V) = 6$

$D(U^2+V^2) = D(5(\xi^2+\eta^2)) = 25[Var(\xi^2)+Var(\eta^2)] = \frac{40}{9}$

$Var(\xi^2) = E(\xi^4) - E^2(\xi^2) = \frac{1}{5}-\frac{1}{9} = \frac{4}{45}\cdot 2$

6. ?

7. 由泊松分布的可加性 记 $\sum_{i=1}^{n}\lambda_i = m$ 则 $\frac{\xi_1+\cdots\xi_n}{\lambda_1\cdots\lambda_n} \sim \frac{P(m)}{m}$

则 $\frac{P(m)}{m}$ 的特征函数为 $f_{P(m)}(\frac{t}{m}) = e^{m(e^{i\frac{t}{m}}-1)}$ ∵ $m\to+\infty$

∴ $e^{m(e^{i\frac{t}{m}}-1)} = e^{m(i\frac{t}{m}+o(\frac{t}{m}))} \to e^{it}$

而 e^{it} 为1的特征函数 由特征函数的唯一性可知 $\frac{\xi_1+\cdots\xi_n}{\lambda_1+\cdots\lambda_n} \xrightarrow{d} 1$

等价于 $\frac{\xi_1+\cdots\xi_n}{\lambda_1+\cdots\lambda_n} \xrightarrow{P} 1$