$\boxed{1}$

$$v(t) = a_1 + a_2 e^{b_1 t}\cos b_2 t + a_3 e^{b_1 t}\sin b_2 t + a_4 e^{b_3 t}\cos b_4 t + a_5 e^{b_3 t}\sin b_4 t.$$

boundary conditions:

$$v(0) = v(1) = v'(0) = v'(1) = 0.$$

$$v(0) = 0 \Rightarrow a_1 + a_2 + a_4 = 0. \quad (1) \checkmark$$

$$v(1) = 0 \Rightarrow \checkmark (2) \left\{ \begin{array}{l} a_1 + a_2 e^{b_1}\cos b_2 + a_3 e^{b_1}\sin b_2 \\ + a_4 e^{b_3}\cos b_4 + \boxed{a_5} e^{b_3}\sin b_4 = 0 \end{array} \right.$$

$$\begin{aligned} v'(t) = {} & a_2 (b_1 e^{b_1 t}\cos b_2 t - e^{b_1 t} b_2 \sin b_2 t) \\ & + a_3 (b_1 e^{b_1 t}\sin b_2 t + e^{b_1 t} b_2 \cos b_2 t) \\ & + a_4 (b_3 e^{b_3 t}\cos b_4 t - e^{b_3 t} b_4 \sin b_4 t) \\ & + a_5 (b_3 e^{b_3 t}\sin b_4 t + e^{b_3 t} b_4 \cos b_4 t). \end{aligned}$$

?

$$v'(0) = 0 \Rightarrow a_2 b_1 + a_3 b_2 + a_4 b_3 + \boxed{a_5} b_4 = 0 \quad (3) \checkmark$$

$$\begin{aligned} v'(1) = 0 \Rightarrow {} & a_2 (b_1 e^{b_1}\cos b_2 - e^{b_1} b_2 \sin b_2) + \\ & + a_3 (b_1 e^{b_1}\sin b_2 + e^{b_1} b_2 \cos b_2) + \\ & + a_4 (b_3 e^{b_3}\cos b_4 - e^{b_3} b_4 \sin b_4) + \\ & + a_5 (e^{b_3} b_3 \sin b_4 + e^{b_3} b_4 \cancel{\sin}\,^{\cos} b_4) \end{aligned} \quad (4) \checkmark \quad ?$$

From (2) and (3), we find a_5 in two ways

$$\Rightarrow a_5 = -\frac{(a_2 b_1 + a_3 b_2 + a_4 b_3)}{b_4}$$

and $a_5 = \dfrac{f(a_1, a_2, a_3, b_i)}{-e^{b_3}\sin b_4}$

$$\Rightarrow a_3 = a_3(a_1, a_2, b_i)$$

~~4~~ ~~1~~ $\boxed{2}$

$e^{b_3}\sin b_4$

$-b_4 a_5 = a_2 b_1 + \boxed{a_3} b_2 + a_4 b_3$

$-e^{b_3}\sin b_4\, a_5 = a_1 + a_2 e^{b_1}\cos b_2 + \boxed{a_3} e^{b_1}\sin b_2 + a_4 e^{b_3}\cos b_4$

b_4

$$-b_4(e^{b_3}\sin b_4)a_5 = e^{b_3}\sin b_4\,(a_2 b_1 + \boxed{a_3} b_2 + a_4 b_3) = b_4\,(a_1 + a_2 e^{b_1}\cos b_2 + \boxed{a_3} e^{b_1}\sin b_2 + a_4 e^{b_3}\cos b_4)$$

✓

$\Rightarrow a_3 = a_3(a_1, a_2, b)$

$\Rightarrow \cancel{a_3(e^{b_1}\sin b_2 - b_2 e^{b_3}\sin b_4)} = e^{b_3}\sin b_4\,(\cancel{a_2 b_1 + a_4 b_3})$

$g_3(b)$

$$\Rightarrow ✓\ a_3(b_2 e^{b_3}\sin b_4 - b_4 e^{b_1}\sin b_2) = -e^{b_3}\sin b_4\,(a_2 b_1 + a_4 b_3) + b_4(a_1 + a_2 e^{b_1}\cos b_2 + a_4 e^{b_3}\cos b_4)$$

$$\Rightarrow \boxed{\begin{matrix} a_3 \\ \shortparallel \\ a_3(a_1, a_2) \end{matrix}} = \frac{1}{g_3(b)}\left[-e^{b_3}\sin b_4\,(a_2 b_1 + a_4 b_3) + b_4(a_1 + a_2 e^{b_1}\cos b_2 + a_4 e^{b_3}\cos b_4)\right]$$

✓

$-(a_1 + a_2)$

$\boxed{3}$

$$\Rightarrow a_3 \underbrace{\left(b_2 e^{b_3} \sin b_4 - b_4 e^{b_1} \sin b_2\right)}_{g_3}$$

$$= -e^{b_3} \sin b_4 \,(a_2 b_1 + a_4 b_3) + b_4\left(a_1 + a_2 e^{b_1} \cos b_2 + a_4 e^{b_3} \cos b_4\right)$$

$$\Rightarrow a_3 = a_3(a_1, a_2) = \cdots$$

$$\Rightarrow \underbrace{a_5}_{a_5(a_1,a_2)} = -\frac{a_2 b_1 + a_3 b_2 + a_4 b_3}{b_4}$$

$h_2(b_i)$

From (4) : $v'(1) = 0 \Rightarrow$

$$(4) \Leftrightarrow 0 = a_2 h_2 + a_3 h_3 + a_4 h_4 + a_5 h_5 \quad (**)$$

$$0 = a_2 h_2 + \frac{h_3}{g_3}\left[-e^{b_3} \sin b_4 \,(\boxed{a_2} b_1 + a_4 b_3) + b_4\left(a_1 + a_2 e^{b_1} \cos b_2 + a_4 e^{b_3} \cos b_4\right)\right]$$

$$+ a_4 h_4 + \left(-\frac{h_5}{b_4}\right)(a_2 b_1 + a_3 b_2 + a_4 b_3)$$

$$-\frac{h_3}{g_3} b_1 e^{b_3} \sin b_4$$

$\boxed{2}$

$$\Rightarrow$$

$$0 = a_2\left(h_2 - \frac{h_5 b_1}{b_4}\right) + a_3\left(h_3 - \frac{h_5 b_2}{b_4}\right) + a_4\left(h_4 - \frac{h_5 b_3}{b_4}\right)$$

$$0 = a_2\left(h_2 - \frac{h_5 b_1}{b_4}\right) + \boxed{a_3}\left(h_3 - \frac{h_5 b_2}{b_4}\right) + \underbrace{(-a_1 - a_2)}_{a_4}\left(h_4 - \frac{h_5 b_3}{b_4}\right)$$

$\Rightarrow$ continues

4

$$\boxed{a_3 = \frac{1}{g_3}\left[-e^{b_3}\sin b_4 (a_2 b_1 + a_4 b_3) + b_4 (a_1 + a_2 e^{b_1}\cos b_2 + a_4 e^{b_3}\cos b_4)\right]}$$

✓

continues

$$0 = a_2 \underbrace{\left(h_2 - \frac{h_5 b_1}{b_4}\right)}_{c_2} + \underbrace{\left(\frac{h_3 - \frac{h_5 b_2}{b_4}}{g_3}\right)}_{c_3(b_i)}\left[-e^{b_3}\sin b_4 (a_2 b_1 + a_4 b_3) + b_4 (a_1 + a_2 e^{b_1}\cos b_2 + a_4 e^{b_3}\cos b_4)\right]$$

P. 3

$$+ a_4 \underbrace{\left(h_4 - \frac{h_5 b_3}{b_4}\right)}_{c_4}$$

$$\Rightarrow$$

$$0 = a_2 c_2 + c_3 \left[-e^{b_3}\sin b_4 \left(a_2 (b_1 - b_3) - a_1 b_3\right) + b_4 \left(a_1 + a_2 e^{b_1}\cos b_1 + (-a_1 - a_2) e^{b_3}\cos b_4\right)\right] - (a_1 + a_2) c_4$$

$$\Rightarrow 0 = a_2 \underbrace{\left(c_2 - (b_1 - b_3) c_3 e^{b_3}\sin b_4 + c_3 b_4 e^{b_1}\cos b_2 - c_3 b_4 e^{b_3}\cos b_4 - a_2 c_4\right)}_{d_1}$$

$$+ a_1 \underbrace{\left(c_3 b_3 e^{b_3}\sin b_4 + c_3 b_4 - c_3 b_4 e^{b_3}\cos b_4 - c_4\right)}_{d_2}$$

$\cancel{4}$ $\boxed{5}$

$$\Rightarrow \boxed{a_2 = \frac{d_2}{d_1} a_1}$$

$$\Rightarrow a_3 = \frac{a_1}{g_3 d_1} \overbrace{\left(\frac{d_2 g_2 - (d_1 + d_4) g_4}{b_1} \right)}^{f_4}$$

(**)
P. 3.
$0 = a_2 h_2 + a_3 h_3 + a_4 h_4 + a_5 h_5 -$

$$\boxed{a_3 = a_1 f_4}$$

$$\Rightarrow \cancel{-b_4}\, a_5 = a_2 b_1 + a_3 b_2 + a_4 b_3 =$$

$$= a_1 \underbrace{\left(\frac{\frac{b_1 d_2}{d_1} + \frac{b_2 f_4}{g_3} - \frac{d_1 + d_2}{d_1}}{-b_4} \right)}_{f_5}$$

$$\boxed{a_5 = f_5 a_1}$$

$$-a_3 h_3 = \underbrace{a_2}_{\frac{d_2}{d_1} a_1} h_2 + \underbrace{a_4}_{(-a_1 - a_2) h_4} h_4 + \underbrace{a_5}_{f_5 a_1} h_5 -$$