

①

$$x_3' = b_1 x_3 - b_2 x_4, \quad x_4' = b_1 x_4 + b_2 x_3$$

$$x_1 = 1, \quad x_2 = t, \quad x_3 = e^{b_1 t} \cos b_2 t, \quad x_4 = e^{b_1 t} \sin b_2 t,$$

$$x_5 = e^{b_3 t} \cos b_4 t, \quad x_6 = e^{b_3 t} \sin b_4 t$$

$$x(t) = \sum_{i=1}^6 a_i x_i(t)$$

$$x'(t) = \sum a_i x_i'(t)$$

$$x''(t) = \sum a_i x_i''(t)$$

$$x_3' = b_3 x_5 - b_4 x_6$$

$$x_6' = b_3 x_6 + b_4 x_5$$

$$x(0) = 0: a_1 + a_3 + a_5 = 0$$

$$x(1) = 20: a_1 + a_2 + a_3 e^{b_1} \cos b_2 + a_4 e^{b_1} \sin b_2 + a_5 e^{b_3} \cos b_4 + a_6 e^{b_3} \sin b_4 = 20$$

$$x'(0) = 0: a_2 + a_3 (b_1 \cos b_2 - b_2 \sin b_2) + a_4 (b_1 \sin b_2 + b_2 \cos b_2) + a_5 (b_3 \cos b_4 - b_4 \sin b_4) + a_6 (b_3 \sin b_4 + b_4 \cos b_4) = 0$$

$$x_1'' = 0, \quad x_2'' = 0, \quad x_3'' = (b_1 x_3 - b_2 x_4)' = b_1 x_3' - b_2 x_4' =$$

$$\Rightarrow x_3'' = b_1 (b_3 x_5 - b_4 x_6) - b_2 (b_3 x_6 + b_4 x_5)$$

$$\Rightarrow x_3'' = (b_1^2 - b_2^2) x_3 - 2 b_1 b_2 x_4$$

$$x_4'' = (b_1 x_4 + b_2 x_3)' = b_1 x_4' + b_2 x_3' =$$

$$\Rightarrow x_4'' = b_1 (b_3 x_5 - b_4 x_6) + b_2 (b_3 x_6 + b_4 x_5)$$

$$\Rightarrow x_4'' = 2 b_1 b_2 x_3 + (b_1^2 - b_2^2) x_4$$

$$x_5'' = (b_3 x_5 - b_4 x_6)' = b_3 x_5' - b_4 x_6' =$$

$$x_5'' = b_3 (b_3 x_5 - b_4 x_6) - b_4 (b_3 x_6 + b_4 x_5)$$

$$\Rightarrow x_5'' = (b_3^2 - b_4^2) x_5 - 2 b_3 b_4 x_6$$

②

$$x_6'' = (b_3 x_6 + b_4 x_5)' = b_3 (b_3 x_6 + b_4 x_5) + b_4 (b_3 x_5 - b_4 x_6)$$

$$\Rightarrow x_6'' = 2b_3 b_4 x_5 + (b_3^2 - b_4^2) x_6$$

$$x'(0) \Rightarrow a_2 + a_3 \boxed{x_3'(0)} + a_4 x_4'(0) + a_5 x_5'(0) + a_6 \underline{x_6'(0)} = 0$$

$$0 = a_2 + a_3 (b_1 x_3(0) - b_2 x_4(0)) + a_4 (b_1 x_4(0) + b_2 x_3(0)) +$$

$$+ a_5 (b_3 x_5(0) - b_4 x_6(0)) + a_6 (b_3 x_6(0) + b_4 x_5(0))$$

$$\boxed{0 = a_2 + a_3 \cancel{x_3'(0)} (b_1) + a_4 b_2 + a_5 b_3 + a_6 b_4}$$

$$x'(1) = 0 \Rightarrow a_2 + a_3 \boxed{x_3'(1)} + a_4 x_4'(1) + a_5 x_5'(1) + a_6 x_6'(1) = 0$$

$$\Rightarrow 0 = a_2 + a_3 (b_1 x_3(1) - b_2 x_4(1)) + a_4 (b_1 x_4(1) + b_2 x_3(1))$$

$$+ a_5 (b_3 x_5(1) - b_4 x_6(1)) + a_6 (b_3 x_6(1) + b_4 x_5(1))$$

$$\Rightarrow 0 = a_2 + a_3 (b_1 e^{b_1} \cos b_2 + b_2 e^{b_1} \sin b_2)$$

$$+ a_4 (b_1 e^{b_1} \sin b_2 + b_2 e^{b_1} \cos b_2)$$

$$+ a_5 (b_3 e^{b_3} \cos b_4 - b_4 e^{b_3} \sin b_4)$$

$$+ a_6 (b_3 e^{b_3} \sin b_4 + b_4 e^{b_3} \cos b_4)$$

$$x''(0) = 0 \Rightarrow a_3 \boxed{x_3''(0)} + a_4 x_4''(0) + a_5 x_5''(0) + a_6 x_6''(0) = 0$$

$$\Rightarrow 0 = a_3 [(b_1^2 - b_2^2) x_3(0) - 2b_1 b_2 x_4(0)] +$$

$$+ a_4 [2b_1 b_2 x_3(0) + (b_1^2 - b_2^2) x_4(0)]$$

$$+ a_5 [(b_3^2 - b_4^2) x_5(0) - 2b_3 b_4 x_6(0)] + \Rightarrow$$

(3)

$$\Rightarrow + a_6 [2b_3b_4 x_5(0) + (b_3^2 - b_4^2) x_6(0)]$$

$$\Rightarrow 0 = a_3 (b_1^2 - b_2^2) e^{b_1} \cos b_2 + a_4 2b_1b_2 e^{b_1} \cos b_2 + a_5 (b_3^2 - b_4^2) e^{b_3} \cos b_4 + a_6 2b_3b_4 e^{b_3} \cos b_4$$

$$x''(1) = 0 \Rightarrow a_3 [(b_1^2 - b_2^2) x_3(1) - 2b_1b_2 x_4(1)] + a_4 [2b_1b_2 x_3(1) + (b_1^2 - b_2^2) x_4(1)] + a_5 [(b_3^2 - b_4^2) x_5(1) - 2b_3b_4 x_6(1)] + a_6 [2b_3b_4 x_5(1) + (b_3^2 - b_4^2) x_6(1)] = 0$$

where $x_3(1) = e^{b_1} \cos b_2 = c_1$

$x_4(1) = e^{b_1} \sin b_2 = c_2$

$x_5(1) = e^{b_3} \cos b_4 = c_3$

$x_6(1) = e^{b_3} \sin b_4 = c_4$

\Rightarrow system of six eqns [1] - [6]

[1] $a_1 + a_3 + a_5 = 0$

[2] $a_1 + a_2 + a_3 c_1 + a_4 c_2 + a_5 c_3 + a_6 c_4 = 20$

~~$a_3 (b_1^2 - b_2^2) c_1 + a_4 2b_1b_2 c_1$~~

[3] $a_2 + a_3 b_1 + a_4 b_2 + a_5 b_3 + a_6 b_1 = 0$

[4] $a_2 + a_3 (b_1 c_1 - b_2 c_2) + a_4 (b_1 c_2 - b_2 c_1) + a_5 (b_3 c_3 - b_4 c_4) + a_6 (b_3 c_4 + b_4 c_3) = 0$

[5] $a_3 (b_1^2 - b_2^2) c_1 + a_4 2b_1b_2 c_1 + a_5 (b_3^2 - b_4^2) c_3 + a_6 (2b_3b_4 c_3) = 0$

④

$$\begin{aligned}
 & \boxed{6} \quad a_3 [(b_1^2 - b_2^2) c_1 - 2b_1 b_2 c_2] \\
 & + a_4 [2b_1 b_2 c_1 + (b_1^2 - b_2^2) c_2] \\
 & + a_5 [(b_3^2 - b_4^2) c_3 - 2b_3 b_4 c_4] \\
 & + a_6 [2b_3 b_4 c_3 + (b_3^2 - b_4^2) c_4] = 0
 \end{aligned}$$

$$\boxed{x(t) = a_1 \underbrace{1}_{x_1(t)} + a_2 \underbrace{t}_{x_2(t)} + a_3 x_3(t) + a_4 x_4(t) + a_5 x_5(t) + a_6 x_6(t)}$$

$$\boxed{v(t) = x'(t)} = a_2 + a_3 \underbrace{x_3'(t)}_{x_3'(t)} + a_4 x_4'(t) + a_5 x_5'(t) + a_6 x_6'(t)$$

$$\begin{aligned}
 v(t) = & a_2 + a_3 (b_1 x_3 - b_2 x_4) + a_4 (b_1 x_4 + b_2 x_3) + \\
 & + a_5 (b_3 x_5 - b_4 x_6) + a_6 (b_3 x_6 + b_4 x_5)
 \end{aligned}$$