Name_______________________________

Student Number_______________________

Computer Engineering 3SK3

DAY CLASS **Dr. D. Zhao**
DURATION OF EXAMINATION: 3 Hours
MCMASTER UNIVERSITY FINAL EXAMINATION **April, 2012**

THIS EXAMINATION PAPER INCLUDES **2** PAGES AND **7** QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

Use of Casio FX-991 calculator only is allowed.
No other aids are allowed.

Answer **all** questions.
Show all your work. Partial credit will be given.

1. Consider the integration of the function $f(x) = 0.5+x^2+2x^3+5x^4$ over the interval $x = -1$ to $x = 1$.

(a) (3 marks) Use the Simpson's 1/3 rule with a step size $h = 0.5$ to approximate the above integral.

(b) (2 marks) Find the truncation error in (a).

(c) (3 marks) If the **relative** error (in absolute value) is to be less than 10^{-5}, what is the minimum number of segments when using the Simpson's 1/3 rule to estimate the integral?

2. Consider the differential equation $dy/dx = 2-3x-4y$ with the initial condition $y(0) = 1$.

(a) (3 marks) Find the analytical solution to the above equation.

(b) (3 marks) Use the Euler's method with a step size of 0.2 to estimate $y(0.4)$.

(c) (3 marks) Find the truncation error in (b) and briefly explain reasons that cause the error.

3. Consider the function $f(x) = 3\cos(3x-2)$.

(a) (3 marks) Use the Newton's method to find the local optimum with relative error less than 1%, starting from x_0=1.

(b) (2 marks) Is the solution in (a) a local minimum or local maximum, and why?

(c) (3 marks) If the golden section search method is used with initial points x_l=0 and x_u=1, estimate the number of iterations required to ensure an **absolute** error less than 0.0001.

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4. Consider fitting the function $y = \alpha \sin(2x)$ to the data (x_i, y_i), $i = 1, 2, \ldots, n$.

(a) (4 marks) Using the Least-Squares criterion, derive an expression to find α from (x_i, y_i)'s.

(b) (3 marks) Given the following data, find the value of α using the expression derived in (a).

i	1	2	3
x_i	0	$\pi/3$	$2\pi/3$
y_i	0.02	2.61	-2.50

(c) (2 marks) Given that y =2.85 when $x = \pi/5$, find the approximation error in the above evaluation.

5. Given the matrix

$$A = \begin{bmatrix} 4 & 0 & 1 \\ 3 & 1 & 3 \\ 0 & 1 & 2 \end{bmatrix}.$$

The singular value decomposition of the matrix can be written as $A = USV^T$, where U, S and V are given by

$$U = \begin{bmatrix} u_{11} & 0.6109 & 0.4520 \\ -0.7283 & -0.3309 & -0.6001 \\ u_{31} & -0.7193 & 0.6600 \end{bmatrix},\quad S = \begin{bmatrix} s_{11} & 0 & 0 \\ 0 & 2.5537 & 0 \\ 0 & 0 & 0.0667 \end{bmatrix},\quad V = \begin{bmatrix} -0.8149 & 0.5682 & 0.1142 \\ -0.1610 & -0.4112 & 0.8972 \\ -0.5568 & -0.7128 & -0.4266 \end{bmatrix},$$

and V^T is the transpose of V.

(a) (3 marks) Find u_{11} and u_{31} in matrix U.

(b) (2 marks) Find s_{11} in matrix S.

6. (5 marks) Use Gauss elimination to find solution to the system $AX=b$ where

$$A = \begin{bmatrix} 0 & 1/2 & 2 \\ 2 & -3 & 3/2 \\ 1/2 & 2 & 1 \end{bmatrix} \text{ and } b = \begin{bmatrix} 1/2 \\ 31/4 \\ -1/2 \end{bmatrix}.$$

7. Consider a 12-bit computer that has 1 sign bit for s, 5 bits for c (c is between 0 and 31, and $c = e + 15$), and 6 bits for f. In terms of s, e, and f, the base 10 numbers are given by $x = (-1)^s 2^e (1+f)$.

(a) (3 marks) What is the machine precision on this computer? Consider both chopping and rounding.

(b) (3 marks) What are the smallest and largest **positive** numbers that can be represented accurately on this computer? Do not consider any special reservations.

End of questions.