- Question 1:

(a) $a = -1$, $b = 1$, $h = 0.5$, $n = (b-a)/h = 4$.
$x_0 = a = -1$, $x_1 = -0.5$, $x_2 = 0$, $x_3 = 0.5$, $x_4 = 1$.
$f(x_0) = 4.5$, $f(x_1) = 0.8125$, $f(x_2) = 0.5$, $f(x_3) = 1.3125$, $f(x_4) = 8.5$.
Use $I \approx \frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)] + \frac{h}{3}[f(x_2) + 4f(x_3) + f(x_4)]$ or
$I \approx (b-a)\frac{f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+f(x_4)}{6}$
we get $I \approx 3.75$.

(b) Method 1:
$\int_{-1}^{1} f(x)dx = 90.5x + \frac{x^3}{3} + \frac{1}{2}x^4 + x^5)|_{-1}^{1} = 3.67$
Truncation error is $3.67 - 3.75 = -0.08$ (or 0.08).

Method 2:
$n = 4$, 2-applications of Simpson's 1/3 rule. Total truncation error is $E_t = -2 \times \frac{1}{90}h^5 f^{(4)}(\xi)$
$f^{'}(x) = 2x + 6x^2 + 20x^3$, $f^{''}(x) = 2 + 12x + 60x^2$, $f^{(3)}(x) = 12 + 120x$, and $f^{(4)}(x) = 120$.
Truncation error $= -2 \times \frac{1}{90}0.5^5 \times 120 = -0.08$.

(c) Let n (even) be the total number of segments. Then $n/2$ be the number of Simpson's 1/3 rules used, and $h = (b-a)/n = 2/n$. Then the truncation error is $E_t = -\frac{n}{2} \times \frac{1}{90}h^5 f^{(4)}(\xi)$.
Since $n = 2/h$ and $f^{(4)}(\xi) = 120$, $E_t = -\frac{120}{90}h^4$, and relative error is $\frac{4h^4}{3\times 3.67} < 10^{-5}$, then $h < 0.0724$, and $n > 27.6$. Therefore, $n = 28$.

- Question 2:

(a) $y_p = Ax + B$, $y_p^{'} = A$. From $y_p^{'} = 2 - 3x - 4y$, we have $A = 2 - 3x - 4Ax - 4B$. Then $A = 2 - 4B$, and $0 = -3 - 4A$. Then $A = -0.75$, and $B = 0.6875$
$y_h = Ce^{-4x}$, and $y = y_p + y_h = -0.75x + 0.6875 + Ce^{-4x}$. When $x = 0$, $1 = 0.6875 + C$. Then $C = 0.3125$.

(b) $x_0 = 0$, $y_0 = 1$, and $h = 0.2$.
$x_1 = 0.2$, $y_1 = y_0 + f(x_0, y_0)h = 1 + (2-4) \times 0.2 = 0.6$
$x_2 = 0.4$, $y_2 = y_2 + f(x_1, y_1)h = 0.6 + (2 - 3 \times 0.2 - 4 \times 0.6) \times 0.2 = 0.4$

(c) Based on part (a), when $x = 0.4$, $y = 0.4506$. Therefore, truncation error is .0505 (or 11.2%)

- Question 3:

(a) $f(x) = 3\cos(3x-2)$, $f^{'}(x) = -9\sin(3x-2)$, and $f^{''}(x) = -27\cos(3x-2)$.
$x_{k+1} = x_k - \frac{-9\sin(3x_k-2)}{-27\cos(3x_k-2)} = x_k - \frac{\sin(3x_k-2)}{3\cos(3x_k-2)}$.
$x_0 = 1$, $x_1 = 0.4809$, $e_a = 108\%$; $x_2 = 0.6886$, $e_a = 30.16\%$; $x_3 = 0.6666$, $e_a = 3\% x_4 = 0.6667$, $e_a = 0.01\%$.

(b) $f''(x) = -27\cos(3x-2) = -27\cos(3 \times 0.6667 - 2) = -27 < 0$, local maximum.

(c) $0.618^n \times (1-0) \leq 0.0001$, $n \geq 20$.

- Question 4:

(a) $\min\ S_r = \min\ \sum_{i=1}^{n} e_i^2 = \min \sum_{i=1}^{n} (\alpha \sin(2x_i) - y_i)^2$.
$\frac{dS_r}{d\alpha} = 2\sum_{i=1}^{n} (\alpha \sin(2x_i) - y_i)\sin(2x_i) = 0$
$\alpha = \frac{\sum_{i=1}^{n} \sin(2x_i) y_i}{\sum_{i=1}^{n} (\sin(2x_i))^2}$

(b) $\sum_{i=1}^{3} (\sin(2x_i))^2 = 1.50$, $\sum_{i=1}^{3} \sin(2x_i) y_i = 4.4254$, and $\alpha = 2.9503$.

(c) From (b), when $x = \pi/5$, $y = 2.8059$, $e_a = |(2.8059 - 2.85)/2.85| = 1.55\%$.

- Question 5:

(a) $u_{11}u_{21} + u_{12}u_{22} + u_{13}u_{23} = 0$, then $u_{11} = -0.6500$.
$u_{31}u_{21} + u_{32}u_{22} + u_{33}u_{23} = 0$, then $u_{31} = -0.2171$

- Question 6:

Do pivoting, row 1: $[2\ \ -3\ \ 3/2 \mid 31/4]$; row 2: $[0\ \ 1/2\ \ 2 \mid 1/2]$, and row 3: $[1/2\ \ 2\ \ 1 \mid\ -1/2]$.

Row 3 - row 1 $\times 1/4$: $[0\ \ 11/4\ \ 5/8 \mid -39/16]$.

Row 3 - row 2 $\times 11/2$: $[0\ \ 0\ \ -83/8 \mid -83/16]$.

From row 3: $x_3 = 1/2$; from row 2: $x_2 = -1$, and from row 3, $x_1 = 2$.

- Question 7:

(a): For chopping, $(0.000001)_2 = 2^{-6}$. For rounding, 2^{-7}.

(b): Minimum number: $s = -1$, $c = 0$, and $e = -15$, and $f = 0$. Then $x_{\min} = 2^{-15}$.

Maximum number: $s = 1$, $c = 31$, and $e = 16$, and $f = (0.111111)_2$. Then $x_{\max} = 2^{16} \times \sum_{i=0}^{6} 2^{-i}$