

Comp Eng 3SK3 Assignment 2

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Topic 1:

1.1)

Task 1.1.1

Derive the matrix

$$z = Ax^2 + By^2 + Cxy + Dx + Ey + F$$

$$Ax^2 + By^2 + Cxy + Dx + Ey + F - z = 0$$

$$\sum_{i=1}^N (Ax^2 + By^2 + Cxy + Dx + Ey + F - z)^2 = L$$

$$0 = \frac{\partial L}{\partial A} = 2 \sum_{i=1}^N (Ax^2 + By^2 + Cxy + Dx + Ey + F - z)x^2$$

$$\sum_{i=1}^N z x^2 = \sum_{i=1}^N A x^4 + \sum_{i=1}^N B y^2 x^2 + \sum_{i=1}^N C x^3 y + \sum_{i=1}^N D x^3 + \sum_{i=1}^N E y x^2 + \sum_{i=1}^N F x^2$$

$$\frac{\partial L}{\partial B} = 2 \sum_{i=1}^N (Ax^2 + By^2 + Cxy + Dx + Ey + F - z)y^2$$

$$\sum_{i=1}^N z y^2 = \sum_{i=1}^N A x^2 y^2 + \sum_{i=1}^N B y^4 + \sum_{i=1}^N C x y^3 + \sum_{i=1}^N D x y^2 + \sum_{i=1}^N E y^3 + \sum_{i=1}^N F y^2$$

$$\frac{\partial L}{\partial C} = 2 \sum_{i=1}^N (Ax^2 + By^2 + Cxy + Dx + Ey + F - z)xy$$

$$\sum_{i=1}^N z xy = \sum_{i=1}^N (A x^3 y + B x y^3 + C x^2 y^2 + D x^2 y + E x y^2 + F x y)$$

$$\frac{\partial L}{\partial D} = 2 \sum_{i=1}^n (Ax_i^2 + By_i^2 + Cx_iy_i + \underline{Dx_i} + Ey_i + F - z_i) \underline{x_i}$$

$$\sum_{i=1}^n z_i x_i = \sum_{i=1}^n (Ax_i^3 + By_i^2 x_i + Cx_i^2 y_i + Dx_i^2 + Ey_i x_i + Fx_i)$$

$$\frac{\partial L}{\partial E} = 2 \sum_{i=1}^n (Ax_i^2 + By_i^2 + Cx_iy_i + Dx_i + \underline{Ey_i} + F - z_i) y_i$$

$$\sum_{i=1}^n z_i y_i = \sum_{i=1}^n (Ax_i^2 y_i + By_i^3 + Cx_i y_i^2 + Dx_i y_i + Ey_i^2 + Fy_i)$$

$$\frac{\partial L}{\partial F} = 2 \sum_{i=1}^n (Ax_i^2 + By_i^2 + Cx_iy_i + Dx_i + Ey_i + \underline{F} - z_i)$$

$$\sum_{i=1}^n z_i = \sum_{i=1}^n (Ax_i^2 + By_i^2 + Cx_iy_i + Dx_i + Ey_i + \underbrace{F}_{\substack{\uparrow \\ n \cdot F}} - z_i)$$

$$\begin{bmatrix} \sum x_i^4 & \sum x_i^2 y_i^2 & \sum x_i^3 y_i & \sum x_i^3 & \sum x_i^2 y & \sum x_i^2 \\ \sum x_i^2 y_i^2 & \sum y_i^4 & \sum x_i y_i^3 & \sum x_i y_i^2 & \sum y_i^3 & \sum y_i^2 \\ \sum x_i^3 y_i & \sum x_i y_i^3 & \sum x_i^2 y_i^2 & \sum x_i^2 y_i & \sum x_i y_i^2 & \sum x_i y_i \\ \sum x_i^3 & \sum x_i y_i^2 & \sum x_i^2 y_i & \sum x_i^2 & \sum x_i y_i & \sum x_i \\ \sum x_i^2 y_i & \sum y_i^3 & \sum x_i y_i^2 & \sum x_i y_i & \sum y_i^2 & \sum y_i \\ \sum x_i^2 & \sum y_i^2 & \sum x_i y_i & \sum x_i & \sum y_i & n \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \\ E \\ F \end{bmatrix} = \begin{bmatrix} \sum x_i^2 z_i \\ \sum y_i^2 z_i \\ \sum x_i y_i z_i \\ \sum z_i x_i \\ \sum z_i y_i \\ \sum z_i \end{bmatrix}$$

The above is the derived matrix

Task 1.1.2

After solving equation through Matlab, I got the Estimated Parameters:

A = 1.0018
B = 1.9955
C = 3.0001
D = -0.0299
E = -0.0236
F = 1.0042

6x1 double		
	1	
1	1.0018	
2	1.9955	
3	3.0001	
4	-0.0299	
5	-0.0236	
6	1.0042	
7		

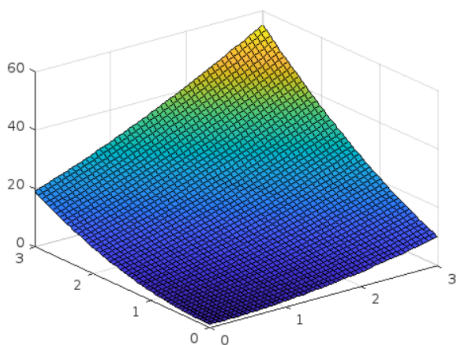
Task 1.1.3

From the dataset, I got the Xmin, Xmax, Ymin, Ymax:

xmin = 0.0007;
xmax = 3.0096;
ymin = 0.0001;
ymax = 3.0093;

Volume is 151.82

Surface looks like:



1.2)

Task 1.2.1

Cubic Surface

$$1) \quad z = Ax^3 + By^3 + Cx^2y + Dxy^2 + Ex^2 + Fy^2 + Gxy + Hx + Iy + J$$

$$0 = Ax^3 + By^3 + Cx^2y + Dxy^2 + Ex^2 + Fy^2 + Gxy + Hx + Iy + J - z$$

$$\frac{\partial \mathcal{L}}{\partial A} = 0$$

$$0 = \sum_{i=0}^n (Ax^3 + By^3 + Cx^2y + Dxy^2 + Ex^2 + Fy^2 + Gxy + Hx + Iy + J - z) x^3$$

$$0 = \sum_{i=0}^n (Ax^6 + By^3x^3 + Cx^5y + Dx^4y^2 + Ex^5 + Fx^3y^2 + Gx^4y + Hx^4 + Ix^3y + Jx^3 - zx^3)$$

$$\sum_{i=0}^n zx^3 = \sum_{i=0}^n (Ax^6 + Bx^3y^3 + Cx^5y + Dx^4y^2 + Ex^5 + Fx^3y^2 + Gx^4y + Hx^4 + Ix^3y + Jx^3)$$

$$\frac{\partial \mathcal{L}}{\partial B} = 0$$

$$0 = \sum_{i=0}^n (Ax^3 + By^3 + Cx^2y + Dxy^2 + Ex^2 + Fy^2 + Gxy + Hx + Iy + J - z) y^3$$

$$\sum_{i=0}^n zy^3 = \sum_{i=0}^n (Ax^3y^3 + By^6 + Cx^2y^4 + Dxy^5 + Ex^3y^3 + Fy^5 + Gxy^4 + Hxy^3 + Iy^4 + Jy^3)$$

$$\frac{\partial \mathcal{L}}{\partial C} = 0$$

$$0 = \sum_{i=0}^n (Ax^3 + By^3 + Cx^2y + Dxy^2 + Ex^2 + Fy^2 + Gxy + Hx + Iy + J - z) x^2y$$

$$\sum_{i=0}^n zx^2y = \sum_{i=0}^n (Ax^5y + Bx^2y^4 + Cx^4y^2 + Dx^3y^3 + Ex^4y + Fx^2y^3 + Gx^3y^2 + Hx^3y + Ix^2y^2 + Jx^2y)$$

$$\frac{\partial \mathcal{L}}{\partial D} = 0$$

$$0 = \sum_{i=0}^n (Ax^3 + By^3 + Cx^2y + Dxy^2 + Ex^2 + Fy^2 + Gxy + Hx + Iy + J - z) xy^2$$

$$\sum_{i=0}^n zxy^2 = \sum_{i=0}^n (Ax^4y^2 + Bxy^5 + Cx^3y^3 + Dx^2y^4 + Ex^3y^2 + Fxy^4 + Gx^2y^3 + Hx^2y^2 + Ixy^3 + Jxy^2)$$

$$\frac{\partial \mathcal{L}}{\partial E} = 0$$

$$0 = \sum_{i=0}^n 2(Ax^3 + By^3 + Cx^2y + Dxy^2 + Ex^2 + Fy^2 + Gxy + Hx + Iy + J - \epsilon) x^2$$

$$\sum_{i=0}^n \epsilon x^2 = \sum_{i=0}^n (Ax^5 + Bx^2y^3 + Cx^4y + Dx^3y^2 + Ex^4 + Fx^2y^2 + Gx^3y + Hx^3 + Ix^2y + Jx^2)$$

$$\frac{\partial \mathcal{L}}{\partial F} = 0$$

$$0 = \sum_{i=0}^n 2(Ax^3 + By^3 + Cx^2y + Dxy^2 + Ex^2 + Fy^2 + Gxy + Hx + Iy + J - \epsilon) y^2$$

$$\epsilon y^2 = \sum_{i=0}^n (Ax^3y^2 + By^5 + Cx^2y^3 + Dxy^4 + Ex^2y^2 + Fy^4 + Gxy^3 + Hxy^2 + Iy^3 + Jy^2)$$

$$\frac{\partial \mathcal{L}}{\partial G} = 0$$

$$0 = \sum_{i=0}^n 2(Ax^3 + By^3 + Cx^2y + Dxy^2 + Ex^2 + Fy^2 + Gxy + Hx + Iy + J - \epsilon) xy$$

$$\epsilon xy = \sum_{i=0}^n (Ax^4y + Bxy^4 + Cx^3y^2 + Dx^2y^3 + Ex^3y + Fxy^3 + Gx^2y^2 + Hx^2y + Ixy^2 + Jxy)$$

$$\frac{\partial \mathcal{L}}{\partial H} = 0$$

$$0 = \sum_{i=0}^n 2(Ax^3 + By^3 + Cx^2y + Dxy^2 + Ex^2 + Fy^2 + Gxy + Hx + Iy + J - \epsilon) x$$

$$\epsilon x = \sum_{i=0}^n (Ax^4 + Bxy^3 + Cx^3y + Dx^2y^2 + Ex^3 + Fxy^2 + Gx^2y + Hx^2 + Ixy + Jx)$$

$$\frac{\partial f}{\partial I} = 0$$

$$0 = \sum_{i=0}^n (Ax^3 + By^3 + Cx^2y + Dxy^2 + Ex^2 + Fy^2 + Gxy + Hx + Iy + J - z) y$$

$$zy = \sum_{i=0}^n (Ax^3y + By^4 + Cx^2y^2 + Dxy^3 + Ex^2y + Fy^3 + Gxy^2 + Hxy + Iy^2 + Jy)$$

$$\frac{\partial f}{\partial J} = 0$$

$$0 = \sum_{i=0}^n (Ax^3 + By^3 + Cx^2y + Dxy^2 + Ex^2 + Fy^2 + Gxy + Hx + Iy + J - z)$$

$$\sum_{i=0}^n z = \sum_{i=0}^n (Ax^3 + By^3 + Cx^2y + Dxy^2 + Ex^2 + Fy^2 + Gxy + Hx + Iy + J)$$

$$\begin{bmatrix} z x^6 & z x^3 y^3 & z x^5 y & z x^4 y^2 & z x^5 & z x^3 y^2 & z x^4 y & z x^4 & z x^3 y & z x^3 \\ z x^3 y^3 & z y^6 & z x^1 y^4 & z x y^5 & z x^2 y^3 & z y^5 & z x y^4 & z x y^3 & z y^4 & z y^3 \\ z x^5 y & z x^2 y^4 & z x^4 y^2 & z x^3 y^3 & z x^4 y & z x^3 y^3 & z x^3 y^2 & z x^3 y & z x^3 y^3 & z x^2 y^2 \\ z x^4 y^2 & z x y^5 & z x^3 y^3 & z x^2 y^4 & z x^3 y^2 & z x y^4 & z x^2 y^3 & z x^2 y^2 & z x y^3 & z x y^2 \\ z x^5 & z x^2 y^3 & z x^4 y & z x^3 y^2 & z x^4 & z x^4 y^2 & z x^3 y & z x^3 & z x^2 y & z x^2 \\ z x^3 y^2 & z y^5 & z x^2 y^3 & z x y^4 & z x^2 y^2 & z y^4 & z x y^3 & z x y^2 & z y^3 & z y^2 \\ z x^4 y & z x y^4 & z x^3 y^2 & z x^2 y^3 & z x^3 y & z x y^3 & z x^2 y^2 & z x^2 y & z x y^2 & z x y \\ z x^4 & z x y^3 & z x^3 y & z x^2 y^2 & z x^3 & z x y^2 & z x^4 y & z x^2 & z x y & z x \\ z x^3 y & z y^4 & z x^2 y^2 & z x y^3 & z x^3 y & z y^3 & z x y^2 & z x y & z y^2 & z y \\ z x^3 & z y^3 & z x^4 y & z x y^2 & z x^2 & z y^2 & z x y & z x & z y & N \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \\ I \\ J \end{bmatrix} = \begin{bmatrix} z x^3 z \\ z y^3 z \\ z x^2 y z \\ z x y^2 z \\ z x^2 z \\ z y^2 z \\ z x y z \\ z x z \\ z y z \\ z z \end{bmatrix}$$

1.2.2) After solving equation through Matlab, I got the Estimated Parameters:

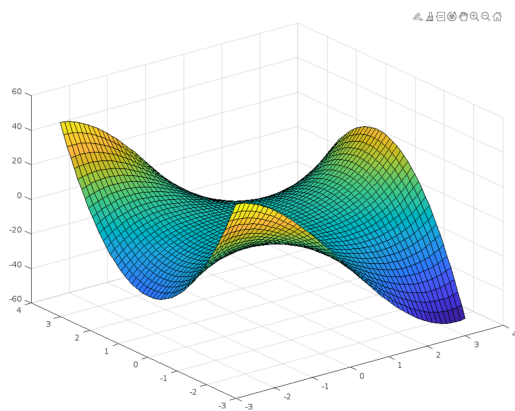
A = 1.0001;
B = -0.00002829;
C = -0.0001198;
D = -3.0007;
E = 0.4852;
F = -0.4848;
G = 0.02888;
H = 0.9949;
I = 0.005009;
J = 0.9994;

1	1.0001		
2	-0.0000		
3	-0.0001		
4	-3.0007		
5	0.4852		
6	-0.4848		
7	0.0289		
8	0.9949		
9	0.0050		
10	0.9994		

Task 1.1.3

From the dataset, I got the Xmin, Xmax, Ymin, Ymax:

xmin = -2.9999;
xmax = 3.01;
ymin = -2.9996;
ymax = 3.0098;
Volume is 36.4054



Topic 2:
Task 2.1

Derive ten linear equations matrix

$$\frac{\partial \mathcal{L}}{\partial A} : \mathcal{L} = \frac{1}{N} \sum_{i=1}^N (Ax_i^2 + By_i^2 + Cz_i^2 + Dx_iy_i + Ey_iz_i + Fx_iz_i + Gx_i + Hy_i + Iz_i + J - 0)^2 \cdot x^2$$

$$0 = \frac{\partial \mathcal{L}}{\partial A} = 2 \sum (Ax^4 + Bx^2y^2 + Cx^2z^2 + Dx^3y + Ex^2yz + Fx^3z + Gx^3 + Hx^2y + Jx^2z + Jx^2)$$

$$\frac{\partial \mathcal{L}}{\partial B} : \mathcal{L} = \frac{1}{N} \sum_{i=1}^N (Ax_i^2 + By_i^2 + Cz_i^2 + Dx_iy_i + Ey_iz_i + Fx_iz_i + Gx_i + Hy_i + Iz_i + J - 0)^2 \cdot y^2$$

$$0 = 2 \sum (Ax^2y^2 + By^4 + Cy^2z^2 + Dxy^3 + Ey^3z + Fxy^2z + Gxy^2 + Hy^3 + Jy^2z + Jy^2)$$

$$\frac{\partial \mathcal{L}}{\partial C} : \mathcal{L} = \frac{1}{N} \sum_{i=1}^N (Ax_i^2 + By_i^2 + Cz_i^2 + Dx_iy_i + Ey_iz_i + Fx_iz_i + Gx_i + Hy_i + Iz_i + J - 0)^2 \cdot z^2$$

$$0 = 2 \sum (Ax^2z^2 + By^2z^2 + Cz^4 + Dxyz^2 + Eyz^3 + Fxz^3 + Gxz^2 + Hyz^2 + Jz^3 + Jz^2)$$

$$\frac{\partial \mathcal{L}}{\partial D} : \mathcal{L} = \frac{1}{N} \sum_{i=1}^N (Ax_i^2 + By_i^2 + Cz_i^2 + Dx_iy_i + Ey_iz_i + Fx_iz_i + Gx_i + Hy_i + Iz_i + J - 0)^2 \cdot xy$$

$$0 = 2 \sum (Ax^3y + Bxy^3 + Cxyz^2 + Dx^2y^2 + Exyz^2 + Fx^2yz + Gx^2y + Hxy^2 + Ixyz + Jxy)$$

$$\frac{\partial \mathcal{L}}{\partial \epsilon} = \frac{1}{N} \sum_{i=1}^N (Ax_i^2 + By_i^2 + Cz_i^2 + Dx_iy_i + Ey_iz_i + Fx_iz_i + Gx_i + Hy_i + Iz_i + J - 0)^2 \cdot yz$$

$$0 = 2 \sum (Ax^2yz + By^3z + Cyz^3 + Dxy^2z + Ey^2z^2 + Fxyz^2 + Gxyz + Hy^2z + Iz^2y + Jyz)$$

$$\frac{\partial \mathcal{L}}{\partial f} = \frac{1}{N} \sum_{i=1}^N (Ax_i^2 + By_i^2 + Cz_i^2 + Dx_iy_i + Ey_iz_i + Fx_iz_i + Gx_i + Hy_i + Iz_i + J - 0)^2 \cdot xz$$

$$0 = 2 \sum (Ax^3z + Bxy^2z + Cxz^3 + Dx^2yz + Exyz^2 + Fx^2z^2 + Gx^2z + Hxyz + Ixz^2 + Jxz)$$

$$\frac{\partial \mathcal{L}}{\partial g} = \frac{1}{N} \sum_{i=1}^N (Ax_i^2 + By_i^2 + Cz_i^2 + Dx_iy_i + Ey_iz_i + Fx_iz_i + Gx_i + Hy_i + Iz_i + J - 0)^2 \cdot x$$

$$0 = 2 \sum (Ax^3 + Bxy^2 + Cxz^2 + Dx^2y + Exyz + Fx^2z + Gx^2 + Hxy + Ixz + Jx)$$

$$\frac{\partial \mathcal{L}}{\partial h} = \frac{1}{N} \sum_{i=1}^N (Ax_i^2 + By_i^2 + Cz_i^2 + Dx_iy_i + Ey_iz_i + Fx_iz_i + Gx_i + Hy_i + Iz_i + J - 0)^2 \cdot y$$

$$0 = 2 \sum (Ax^2y + By^3 + Cz^2y + Dxy^2 + Ey^2z + Fxyz + Gxy + Hy^2 + Iz y + Jy)$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{z}} = \frac{1}{N} \sum_{i=1}^N (Ax_i^2 + By_i^2 + Cz_i^2 + Dx_iy_i + Ey_iz_i + Fx_iz_i + Gx_i + Hy_i + Iz_i + J - 0)^2 \cdot \mathbf{z}$$

$$0 = 2 \sum (Ax^2z + By^2z + Cz^3 + Dxy z + Eyz^2 + Fxz^2 + Gxz + Hyz + Iz^2 + Jz)$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{z}} = \frac{1}{N} \sum_{i=1}^N (Ax_i^2 + By_i^2 + Cz_i^2 + Dx_iy_i + Ey_iz_i + Fx_iz_i + Gx_i + Hy_i + Iz_i + J - 0)^2 \cdot \mathbf{z}$$

$$0 = 2 \sum (Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fxz + Gx + Hy + Iz) \cdot \mathbf{z}$$

$$\begin{bmatrix} z^4 & z^2y^2 & z^2z^2 & z^3y & z^2yz & z^3z & z^3 & z^2y & z^2z & z^3 \\ z^2y^2 & z^4 & z^2z^2 & zxy^3 & zy^3z & zxy^2z & zxy^2 & zy^3 & zyz^2 & zy^2 \\ z^2z^2 & zy^2z^2 & z^4 & zxy^2z & zyz^3 & zxz^3 & zxz^2 & zyz^2 & z^3 & z^2 \\ z^3y & zxy^3 & zxy^2z & z^2y^3 & zxy^2z & z^2yz & z^2y & zxy^2 & zxy^2 & zxy \\ z^2yz & zy^3z & zyz^3 & zxy^2z & zy^3z^2 & zxy^2z^2 & zxy^2z & zy^3z & zyz^2 & zyz \\ z^2z^2 & zxy^2z & zxz^3 & z^2yz & zxy^2z^2 & z^2z^2 & z^2z & zxy^2z & zxz^2 & zxz \\ z^3 & zxy^2 & zxz^2 & z^2y & zxy^2z & z^2z & z^2 & zxy & zxz & zx \\ z^2y & zy^3 & zy^2z & zxy^2 & zyz^2 & zxy^2z & zxy & zy^2 & z^2y & zy \\ z^2z & zyz^2 & z^3 & zxy^2z & zyz^2 & zxz^2 & zxz & zyz & z^2z & z^2 \\ z^2 & zy^2 & z^2z & zxy & zyz & zxz & zx & zy & z^2 & N \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \\ I \\ J \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Above derive the matrix

Task 2.2.1

Based on implicit_1.mat, I found the smallest eigenvalue vectors

A = -0.2220

B = -0.3938

C = -0.0984

D = -0.0004

E = -0.0004

F = 0.0001

G = 0.0010

H = 0.0002

I = -0.0002

J = 0.8865

1	-0.2220	
2	-0.3938	
3	-0.0984	
4	-0.0004	
5	-0.0004	
6	0.0001	
7	0.0010	
8	0.0002	
9	-0.0002	
10	0.8865	

From solving all the parameters, The parameters A, B, and C, which represent the squared semi-principal axes of an ellipsoid, have been found to possess the same sign. This uniformity in sign is consistent with the typical representation of an ellipsoid, and the rest of D/E/F/G/H/I their value is very small, Given their minimal magnitude, the impact of these parameters on the ellipsoidal shape is negligible and J is around 1, The parameter J, which appears to act as a scaling or normalization factor in the ellipsoidal equation, is approximately equal to 1. The analyzed parameters strongly suggest that the geometric form in question is an ellipsoid, nearly aligned with the principal axes and centered near the origin.

Task 2.2.2

Based on implicit_2.mat, here is the parameters result

$$A = 0.0864$$

$$B = 0.1821$$

$$C = -0.1345$$

$$D = -0.000024677944464$$

$$E = -0.000001943480139$$

$$F = 0.000002518532185$$

$$G = -0.0008$$

$$H = -0.0016$$

$$I = 0.0013$$

$$J = 0.9702$$

10x1 double		
	1	
1	0.0864	
2	0.1821	
3	-0.1345	
4	-0.0000	
5	-0.0000	
6	0.0000	
7	-0.0008	
8	-0.0016	
9	0.0013	
10	0.9702	

Based on the analyzed parameters, A and B are positive while C is negative, suggesting a distinct geometric configuration. The negligible magnitudes of parameters D/E/F/G/H/I indicate minimal deviation from the principal axes, simplifying the model. The negative value of J further supports this, identifying the shape as a hyperboloid of two sheets. This conclusion is drawn from the sign and relative sizes of these parameters, defining the nature and orientation of the geometric figure in question.