# CS486 - A3

March 15, 2020

## 1 Part A

### 1.1 (1)

[1]:

```
import numpy as np
import copy
```

[2]:

```
class Factor:
    def __init__(self, vars, probs):
        self.header = np.array(sorted(vars))
        if type(probs) is np.ndarray:
            self.table = probs
        else:
            order = np.argsort(vars)
            probs.sort(key=lambda x: tuple(x[i] for i in order))
            self.table = np.array([p[-1] for p in probs])
            self.table = self.table.reshape((2, ) * len(vars))

    def restrict(self, var, val):
        if var not in self.header:
            return

        index = 1 if val else 0
        axis = np.where(self.header == var)[0][0]
        self.header = np.delete(self.header, axis)
        self.table = np.take(self.table, index, axis=axis)

    def __mul__(self, other):
        new_header = np.array(sorted(np.union1d(self.header, other.header)))
        a = self.table.copy()
        b = other.table.copy()

        for index, var in enumerate(new_header):
            if var not in self.header:
                a = np.expand_dims(a, axis=index)
            if var not in other.header:
                b = np.expand_dims(b, axis=index)
        new_table = a * b
        return Factor(new_header, new_table)

    def sumout(self, var):
        if var not in self.header:
            return

        axis = np.where(self.header == var)[0][0]
        self.header = np.delete(self.header, axis)
        self.table = np.sum(self.table, axis=axis)

    def normalize(self):
        self.table /= np.sum(self.table)

    def possibility(self, vars):
        if len(vars) != len(self.header):
            raise Exception("Invalid query size.")

        index = []
        for var in self.header:
            if var in vars:
                index.append(1)
            elif "not " + var in vars:
                index.append(0)
            else:
                raise Exception("Invalid query.")

        return self.table.item(tuple(index))

    def __copy__(self):
        return Factor(self.header, self.table)

    def __str__(self):
        table_format = '{:<15}' * (len(self.header) + 1)
        result = [table_format.format(*(self.header.tolist() +␣
↪['probability']))]
        for index, value in np.ndenumerate(self.table):
            row = ['True' if boolean > 0 else 'False' for boolean in index]
            result.append(table_format.format(*(row + [value])))

        return '\n'.join(result) + '\n'
```

[3]:
```
def inference(factors, query, hidden_list, evidence_list, print_step=True):
    def step_printer(factor):
        print(factor)
    factor_list = [copy.copy(factor) for factor in factors]
    for factor in factor_list:
        for evidence, value in evidence_list.items():
            factor.restrict(evidence, value)

    product = np.prod(factor_list)

    if print_step:
        print('Before summing out:')
        step_printer(product)

    for hidden in hidden_list:
        product.sumout(hidden)
        print('Eliminate ' + hidden)
        step_printer(product)

    product.normalize()

    if set(query) == set(product.header):
        return product
    else:
        raise Exception("Invalid query size.")
```

### 1.2 (2)

[4]:

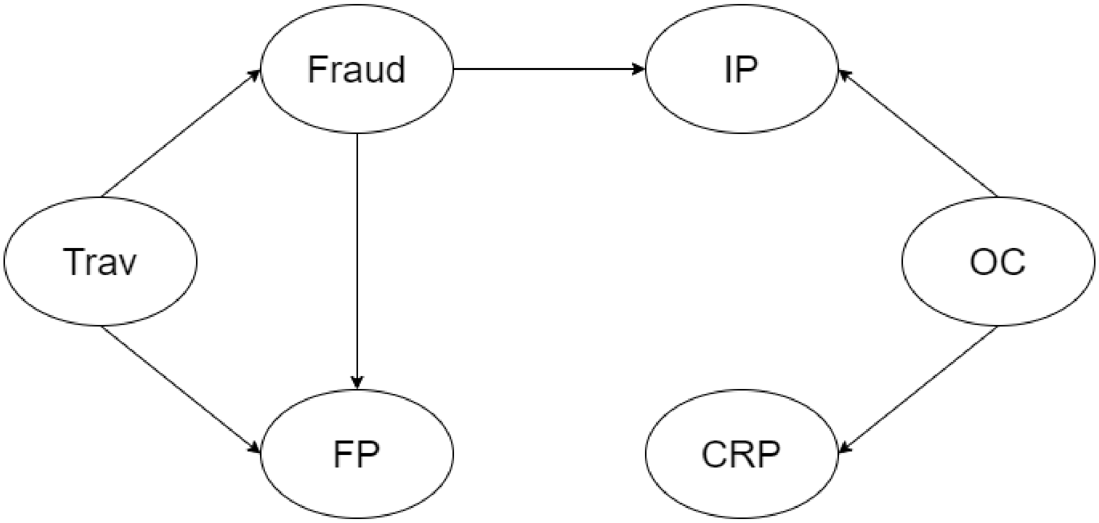
```
# Fraud | Trav
f1 = Factor(['Fraud', 'Trav'],
            [(0, 0, 1 - 0.004),
             (0, 1, 1 - 0.01),
             (1, 0, 0.004),
             (1, 1, 0.01)])

# Trav
f2 = Factor(['Trav'],
            [(0, 1 - 0.05),
             (1, 0.05)])

# FP | Fraud, Trav
f3 = Factor(['FP', 'Fraud', 'Trav'],
            [(0, 0, 0, 1 - 0.01),
             (0, 0, 1, 1 - 0.9),
             (0, 1, 0, 1 - 0.1),
             (0, 1, 1, 1 - 0.9),
             (1, 0, 0, 0.01),
             (1, 0, 1, 0.9),
             (1, 1, 0, 0.1),
             (1, 1, 1, 0.9)])

# OC
f4 = Factor(['OC'],
            [(0, 1 - 0.6),
             (1, 0.6)])

# IP | OC, Fraud
f5 = Factor(['IP', 'OC', 'Fraud'],
            [(0, 0, 0, 1 - 0.001),
             (0, 0, 1, 1 - 0.011),
             (0, 1, 0, 1 - 0.01),
             (0, 1, 1, 1 - 0.02),
             (1, 0, 0, 0.001),
             (1, 0, 1, 0.011),
             (1, 1, 0, 0.01),
             (1, 1, 1, 0.02)])

# CRP | OC
f6 = Factor(['CRP', 'OC'],
            [(0, 0, 1 - 0.001),
             (0, 1, 1 - 0.1),
             (1, 0, 0.001),
             (1, 1, 0.1)])
```

### 1.2.1 a)

[5]:

```
from IPython.display import Image
Image('./A3Q2a.png')
```

[5]:



### 1.2.2 b)

```
[6]: factor_list = [f1, f2, f3, f4, f5, f6]
ans = inference(factor_list, ['Fraud'], ['Trav', 'FP', 'IP', 'OC', 'CRP'], dict())
print("P(Fraud) = " + str(ans.possibility(['Fraud'])))

ans = inference(factor_list, ['Fraud'], ['Trav', 'OC'], {'FP': True, 'IP': False, 'CRP': True})
print("P(Fraud | FP, ~IP, CRP) = " + str(ans.possibility(['Fraud'])))
```

```
Before summing out:
CRP            FP             Fraud          IP             OC             Trav
probability
False          False          False          False          False          False
0.3739461842952
False          False          False          False          False          True
0.00197604198
False          False          False          False          True           False
0.5007801348
False          False          False          False          True           True
0.0026462699999999996
False          False          False          True           False          False
0.0003743205048
False          False          False          True           False          True
1.97802e-06
False          False          False          True           True           False
0.0050583851999999995
False          False          False          True           True           True
2.6729999999999996e-05
False          False          True           False          False          False
0.0013515990480000002
False          False          True           False          False          True
1.9760219999999996e-05
False          False          True           False          True           False
0.001809864
False          False          True           False          True           True
2.645999999999999e-05
False          False          True           True           False          False
1.5032952e-05
False          False          True           True           False          True
2.1977999999999995e-07
False          False          True           True           True           False
3.6936e-05
```

```
False          False          True           True           True           True
5.399999999999999e-07
False          True           False          False          False          False
0.0037772341848
False          True           False          False          False          True
0.017784377820000002
False          True           False          False          True           False
0.0050583851999999995
False          True           False          False          True           True
0.023816430000000003
False          True           False          True           False          False
3.7810152e-06
False          True           False          True           False          True
1.7802180000000002e-05
False          True           False          True           True           False
5.10948e-05
False          True           False          True           True           True
0.00024057000000000004
False          True           True           False          False          False
0.000150177672
False          True           True           False          False          True
0.00017784198000000002
False          True           True           False          True           False
0.00020109600000000003
False          True           True           False          True           True
0.00023814
False          True           True           True           False          False
1.670328e-06
False          True           True           True           False          True
1.97802e-06
False          True           True           True           True           False
4.104000000000001e-06
False          True           True           True           True           True
4.86e-06
True           False          False          False          False          False
0.0003743205048
True           False          False          False          False          True
1.97802e-06
True           False          False          False          True           False
0.0556422372
True           False          False          False          True           True
0.00029403
True           False          False          True           False          False
3.746952e-07
True           False          False          True           False          True
1.98e-09
True           False          False          True           True           False
0.0005620428
```

```
True           False          False          True           True           True
2.97e-06
True           False          True           False          False          False
1.3529520000000002e-06
True           False          True           False          False          True
1.9779999999999996e-08
True           False          True           False          True           False
0.000201096
True           False          True           False          True           True
2.939999999999999e-06
True           False          True           True           False          False
1.5048e-08
True           False          True           True           False          True
2.1999999999999996e-10
True           False          True           True           True           False
4.104e-06
True           False          True           True           True           True
6e-08
True           True           False          False          False          False
3.7810152e-06
True           True           False          False          False          True
1.7802180000000002e-05
True           True           False          False          True           False
0.0005620428
True           True           False          False          True           True
0.0026462700000000005
True           True           False          True           False          False
3.7848e-09
True           True           False          True           False          True
1.7820000000000003e-08
True           True           False          True           True           False
5.6772000000000005e-06
True           True           False          True           True           True
2.6730000000000007e-05
True           True           True           False          False          False
1.50328e-07
True           True           True           False          False          True
1.7802000000000003e-07
True           True           True           False          True           False
2.2344e-05
True           True           True           False          True           True
2.646e-05
True           True           True           True           False          False
1.672e-09
True           True           True           True           False          True
1.98e-09
True           True           True           True           True           False
4.5600000000000006e-07
```

```
True           True           True           True           True           True
5.4e-07

Eliminate Trav
CRP            FP             Fraud          IP             OC
probability
False          False          False          False          False
0.3759222262752
False          False          False          False          True
0.5034264048
False          False          False          True           False
0.0003762985248
False          False          False          True           True
0.0050851151999999995
False          False          True           False          False
0.0013713592680000002
False          False          True           False          True
0.001836324
False          False          True           True           False
1.5252732e-05
False          False          True           True           True
3.7476e-05
False          True           False          False          False
0.0215616120048
False          True           False          False          True
0.0288748152
False          True           False          True           False
2.1583195200000003e-05
False          True           False          True           True
0.0002916648
False          True           True           False          False
0.00032801965200000003
False          True           True           False          True
0.000439236
False          True           True           True           False
3.648348e-06
False          True           True           True           True
8.964000000000001e-06
True           False          False          False          False
0.0003762985248
True           False          False          False          True
0.0559362672
True           False          False          True           False
3.766752e-07
True           False          False          True           True
0.0005650128
True           False          True           False          False
1.3727320000000003e-06
```

```
True           False          True           False          True
0.000204036
True           False          True           True           False
1.5268e-08
True           False          True           True           True
4.164e-06
True           True           False          False          False
2.1583195200000003e-05
True           True           False          False          True
0.0032083128000000003
True           True           False          True           False
2.1604800000000003e-08
True           True           False          True           True
3.2407200000000004e-05
True           True           True           False          False
3.2834800000000005e-07
True           True           True           False          True
4.8804e-05
True           True           True           True           False
3.6520000000000002e-09
True           True           True           True           True
9.96e-07

Eliminate FP
CRP            Fraud          IP             OC             probability
False          False          False          False          0.39748383828
False          False          False          True           0.53230122
False          False          True           False
0.00039788172000000004
False          False          True           True           0.005376779999999999
False          True           False          False
0.0016993789200000003
False          True           False          True
0.0022755600000000003
False          True           True           False          1.890108e-05
False          True           True           True
4.6439999999999996e-05
True           False          False          False
0.00039788172000000004
True           False          False          True           0.05914458
True           False          True           False
3.9828000000000004e-07
True           False          True           True           0.00059742
True           True           False          False
1.7010800000000004e-06
True           True           False          True           0.00025284
True           True           True           False          1.892e-08
True           True           True           True
```

```
5.1600000000000006e-06

Eliminate IP
CRP            Fraud          OC             probability
False          False          False          0.39788172
False          False          True           0.537678
False          True           False          0.0017182800000000004
False          True           True           0.0023220000000000003
True           False          False          0.00039828000000000004
True           False          True           0.059742
True           True           False          1.7200000000000005e-06
True           True           True           0.000258

Eliminate OC
CRP            Fraud          probability
False          False          0.93555972
False          True           0.00404028
True           False          0.060140280000000004
True           True           0.00025971999999999996

Eliminate CRP
Fraud          probability
False          0.9957
True           0.0043

P(Fraud) = 0.0043
Before summing out:
Fraud          OC             Trav           probability
False          False          False          3.7810152e-06
False          False          True           1.7802180000000002e-05
False          True           False          0.0005620428
False          True           True           0.0026462700000000005
True           False          False          1.50328e-07
True           False          True           1.7802000000000003e-07
True           True           False          2.2344e-05
True           True           True           2.646e-05

Eliminate Trav
Fraud          OC             probability
False          False          2.1583195200000003e-05
False          True           0.0032083128000000003
True           False          3.2834800000000005e-07
True           True           4.8804e-05

Eliminate OC
Fraud          probability
False          0.0032298959952000005
True           4.9132348e-05
```

```
P(Fraud | FP, ~IP, CRP) = 0.014983813147541077
```

### 1.2.3 c)

```
[7]: ans = inference(factor_list, ['Fraud'], ['OC'], {'FP': True, 'IP': False, 'CRP':
      ↪ True, 'Trav': True})
     print("P(Fraud | FP, ~IP, CRP, Trav) = " + str(ans.possibility(['Fraud'])))
```

```
Before summing out:
Fraud          OC             probability
False          False          1.7802180000000002e-05
False          True           0.0026462700000000005
True           False          1.7802000000000003e-07
True           True           2.646e-05

Eliminate OC
Fraud          probability
False          0.0026640721800000005
True           2.663802e-05

P(Fraud | FP, ~IP, CRP, Trav) = 0.009899995919292979
```

### 1.2.4 d)

```
[8]: ans = inference(factor_list, ['Fraud'], ['Trav', 'FP', 'OC', 'CRP'], {'IP':␣
      ↪True})
     print("P(Fraud | IP) = " + str(ans.possibility(['Fraud'])))

     cond_vars = ['FP', 'CRP']
     for enum in range(2 ** 2):
         bin = "{0:02b}".format(enum)
         cond = {var: dig == '1' for var, dig in zip(cond_vars, bin)}
         cond['IP'] = True
         ans = inference(factor_list, ['Fraud'], ['Trav', 'OC'], cond)

         cond_to_print = [var if dig == '1' else '~' + var for var, dig in␣
      ↪zip(cond_vars, bin)]
         print('P(Fraud | ' + ', '.join(cond_to_print) + ') = ' + str(ans.
      ↪possibility(['Fraud'])))
```

```
Before summing out:
CRP            FP             Fraud          OC             Trav
probability
False          False          False          False          False
```

```
0.0003743205048
False          False          False          False          True
1.97802e-06
False          False          False          True           False
0.0050583851999999995
False          False          False          True           True
2.6729999999999996e-05
False          False          True           False          False
1.5032952e-05
False          False          True           False          True
2.1977999999999995e-07
False          False          True           True           False
3.6936e-05
False          False          True           True           True
5.399999999999999e-07
False          True           False          False          False
3.7810152e-06
False          True           False          False          True
1.7802180000000002e-05
False          True           False          True           False
5.10948e-05
False          True           False          True           True
0.00024057000000000004
False          True           True           False          False
1.670328e-06
False          True           True           False          True
1.97802e-06
False          True           True           True           False
4.104000000000001e-06
False          True           True           True           True
4.86e-06
True           False          False          False          False
3.746952e-07
True           False          False          False          True
1.98e-09
True           False          False          True           False
0.0005620428
True           False          False          True           True
2.97e-06
True           False          True           False          False
1.5048e-08
True           False          True           False          True
2.1999999999999996e-10
True           False          True           True           False
4.104e-06
True           False          True           True           True           6e-08
True           True           False          False          False
3.7848e-09
```

```
True           True           False          False          True
1.7820000000000003e-08
True           True           False          True           False
5.6772000000000005e-06
True           True           False          True           True
2.6730000000000007e-05
True           True           True           False          False
1.672e-09
True           True           True           False          True
1.98e-09
True           True           True           True           False
4.5600000000000006e-07
True           True           True           True           True
5.4e-07

Eliminate Trav
CRP            FP             Fraud          OC             probability
False          False          False          False          0.0003762985248
False          False          False          True
0.0050851151999999995
False          False          True           False          1.5252732e-05
False          False          True           True           3.7476e-05
False          True           False          False
2.1583195200000003e-05
False          True           False          True           0.0002916648
False          True           True           False          3.648348e-06
False          True           True           True
8.964000000000001e-06
True           False          False          False          3.766752e-07
True           False          False          True           0.0005650128
True           False          True           False          1.5268e-08
True           False          True           True           4.164e-06
True           True           False          False
2.1604800000000003e-08
True           True           False          True
3.2407200000000004e-05
True           True           True           False
3.6520000000000002e-09
True           True           True           True           9.96e-07

Eliminate FP
CRP            Fraud          OC             probability
False          False          False          0.00039788172000000004
False          False          True           0.005376779999999999
False          True           False          1.890108e-05
False          True           True           4.6439999999999996e-05
True           False          False          3.9828000000000004e-07
True           False          True           0.00059742
True           True           False          1.892e-08
True           True           True           5.1600000000000006e-06

Eliminate OC
CRP            Fraud          probability
False          False          0.005774661719999999
False          True           6.534107999999999e-05
True           False          0.00059781828
True           True           5.17892e-06

Eliminate CRP
Fraud          probability
False          0.006372479999999999
True           7.052e-05

P(Fraud | IP) = 0.010945211857830204
Before summing out:
Fraud          OC             Trav           probability
False          False          False          0.0003743205048
False          False          True           1.97802e-06
False          True           False          0.0050583851999999995
False          True           True           2.6729999999999996e-05
True           False          False          1.5032952e-05
True           False          True           2.1977999999999995e-07
True           True           False          3.6936e-05
True           True           True           5.399999999999999e-07

Eliminate Trav
Fraud          OC             probability
False          False          0.0003762985248
False          True           0.0050851151999999995
True           False          1.5252732e-05
True           True           3.7476e-05

Eliminate OC
Fraud          probability
False          0.0054614137247999996
True           5.2728731999999996e-05

P(Fraud | ~FP, ~CRP) = 0.009562453711179572
Before summing out:
Fraud          OC             Trav           probability
False          False          False          3.746952e-07
False          False          True           1.98e-09
False          True           False          0.0005620428
False          True           True           2.97e-06
True           False          False          1.5048e-08
True           False          True           2.1999999999999996e-10
True           True           False          4.104e-06
True           True           True           6e-08

Eliminate Trav
Fraud          OC             probability
False          False          3.766752e-07
False          True           0.0005650128
True           False          1.5268e-08
True           True           4.164e-06

Eliminate OC
Fraud          probability
False          0.0005653894752
True           4.179268e-06

P(Fraud | ~FP, CRP) = 0.007337600684545429
Before summing out:
Fraud          OC             Trav           probability
False          False          False          3.7810152e-06
False          False          True           1.7802180000000002e-05
False          True           False          5.10948e-05
False          True           True           0.00024057000000000004
True           False          False          1.670328e-06
True           False          True           1.97802e-06
True           True           False          4.104000000000001e-06
True           True           True           4.86e-06

Eliminate Trav
Fraud          OC             probability
False          False          2.1583195200000003e-05
False          True           0.0002916648
True           False          3.648348e-06
True           True           8.964000000000001e-06

Eliminate OC
Fraud          probability
False          0.0003132479952
True           1.2612348e-05

P(Fraud | FP, ~CRP) = 0.03870476498043534
Before summing out:
Fraud          OC             Trav           probability
False          False          False          3.7848e-09
False          False          True           1.7820000000000003e-08
False          True           False          5.6772000000000005e-06
False          True           True           2.6730000000000007e-05
True           False          False          1.672e-09
True           False          True           1.98e-09
True           True           False          4.5600000000000006e-07
True           True           True           5.4e-07

Eliminate Trav
Fraud          OC             probability
False          False          2.1604800000000003e-08
False          True           3.2407200000000004e-05
True           False          3.6520000000000002e-09
True           True           9.96e-07

Eliminate OC
Fraud          probability
False          3.24288048e-05
True           9.99652e-07

P(Fraud | FP, CRP) = 0.02990422220148673
```

As is illustrated above, we are supposed to satisfy conditions ~FP, CRP to get the minimal probability. Thus, I should use domestic currency for the payment after making a computer related purchase. This will help reduce the probability from 1.094% to 0.734%, which is approximately 0.36%.

## 2 Part B

[9]:

```python
import math
```

[10]:

```python
train_data_path = './dataset/trainData.txt'
train_label_path = './dataset/trainLabel.txt'
test_data_path = './dataset/testData.txt'
test_label_path = './dataset/testLabel.txt'
words_path = './dataset/words.txt'

# Labels must start from 1 since we will minus 1 when storing
actual_labels = [1, 2]

labels = [label - 1 for label in actual_labels]
```

[11]:

```python
class Document:
    def __init__(self, label, doc_id=0):
        self.id = doc_id
        self.label = label - 1
        # Set labels to start from 0, it makes no difference
        self.word_list = set()

    def __contains__(self, word):
        return word in self.word_list
    def add_word(self, word):
        self.word_list.add(word - 1)
        # We store word - 1 here so that we only need to consider
        # the index issue when mapping word id to its word


def load_train_data():
    return load_data(train_data_path, train_label_path)


def load_test_data():
    return load_data(test_data_path, test_label_path)


def load_data(data_path, label_path):
    docs = []
    with open(label_path, 'r', encoding='utf-8') as file:
        for index, line in enumerate(file):
            doc_id = index + 1
            label = int(line.strip())
            docs.append(Document(label, doc_id))

    with open(data_path, 'r', encoding='utf-8') as file:
        for line in file.readlines():
            [doc_id, word_id] = list(map(int, line.strip().split(' ')))
            docs[doc_id - 1].add_word(word_id)

    return docs


def load_words(filename=words_path):
    word_map = dict()

    with open(filename, 'r', encoding='utf-8') as file:
        for index, word in enumerate(file):
            # Similar to above, we minus 1 for the word id
            word_map[index] = word.strip()

    return word_map
```

[12]:

```
class NaiveBayes:
    def __init__(self, word_map, labels):
        self.word_set = set(word_map)
        self.poss_labels = labels

    def fit(self, docs):
        num_words = len(self.word_set)
        num_label = len(self.poss_labels)
        stat = np.zeros((num_words, num_label))

        for document in docs:
            for word in document.word_list:
                stat[word][document.label] += 1

        label_sum = np.sum(stat, axis=1, keepdims=True)
        word_sum = np.sum(stat, axis=0, keepdims=True)
        # prob_given_label[word][label] stores the value of Pr(word | label)
        self.prob_given_label = (stat + 1) / (label_sum + num_words)
        # prob_given_word[label][word] stores the value of Pr(label | word)
        self.prob_given_word = np.transpose((stat + 1) / (word_sum + num_label))

    def predict(self, word_list):
        p = dict()
        for label in self.poss_labels:
            p[label] = 1
            for word in self.word_set:
                if word in word_list:
                    p[label] *= self.prob_given_label[word][label]
                else:
                    p[label] *= 1 - self.prob_given_label[word][label]

        return max(p, key=p.get)

    def __diff(self, word, label1, label2):
        return abs(math.log(self.prob_given_label[word][label1]) -
                      math.log(self.prob_given_label[word][label2]))

    def discriminative(self, n, label1=0, label2=1):
        rank = {word: self.__diff(word, label1, label2) for word in self.
↪word_set}
        return sorted(rank, key=rank.get, reverse=True)[: n]
```

[13]:

```
def test(model, data):
    correct, incorrect = 0, 0
    for doc in data:
        label = doc.label
        pred = model.predict(doc)
        if pred == label:
            correct += 1
        else:
            incorrect += 1

    return correct / (correct + incorrect)
```

```python
train_data = load_train_data()
test_data = load_test_data()
word_map = load_words()

nb = NaiveBayes(word_map, labels)
nb.fit(train_data)
disc_words = nb.discriminative(10)
print("Top 10 discriminative words:")
print([word_map[word] for word in disc_words])

print("Training accuracy: " + str(test(nb, train_data)))
print("Testing accuracy: " + str(test(nb, test_data)))
```

```
Top 10 discriminative words:
['christian', 'religion', 'atheism', 'books', 'christians', 'library',
'religious', 'libraries', 'novel', 'beliefs']
Training accuracy: 0.908
Testing accuracy: 0.7286666666666667
```

The assumption that all word features are independent is not reasonable. One example is shown above in the most discriminative 10 words that the same words may appear in an article in different forms, say `christian' and `christians'.

To take into account dependencies between words, we would better apply a stemming algorithm first, which will reduce all words to their roots so that different forms of the same words will be counted as only one.