# CS486 - A3

March 14, 2020

## 1 Part A

### 1.1 (1)

[1]:
```python
import numpy as np
import copy
```

[2]:
```python
class Factor:
    def __init__(self, vars, probs):
        self.header = np.array(sorted(vars))
        if type(probs) is np.ndarray:
            self.table = probs
        else:
            order = np.argsort(vars)
            probs.sort(key=lambda x: tuple(x[i] for i in order))
            self.table = np.array([p[-1] for p in probs])
            self.table = self.table.reshape((2, ) * len(vars))

    def restrict(self, var, val):
        if var not in self.header:
            return

        index = 1 if val else 0
        axis = np.where(self.header == var)[0][0]
        self.header = np.delete(self.header, axis)
        self.table = np.take(self.table, index, axis=axis)

    def __mul__(self, other):
        new_header = np.array(sorted(np.union1d(self.header, other.header)))
        a = self.table.copy()
        b = other.table.copy()

        for index, var in enumerate(new_header):
            if var not in self.header:
                a = np.expand_dims(a, axis=index)
            if var not in other.header:
                b = np.expand_dims(b, axis=index)
```

```
        new_table = a * b
        return Factor(new_header, new_table)

    def sumout(self, var):
        if var not in self.header:
            return

        axis = np.where(self.header == var)[0][0]
        self.header = np.delete(self.header, axis)
        self.table = np.sum(self.table, axis=axis)

    def normalize(self):
        self.table /= np.sum(self.table)

    def possibility(self, vars):
        if len(vars) != len(self.header):
            raise Exception("Invalid query size.")

        index = []
        for var in self.header:
            if var in vars:
                index.append(1)
            elif "not " + var in vars:
                index.append(0)
            else:
                raise Exception("Invalid query.")

        return self.table.item(tuple(index))

    def __copy__(self):
        return Factor(self.header, self.table)

    def __str__(self):
        table_format = '{:<15}' * (len(self.header) + 1)
        result = [table_format.format(*(self.header.tolist() +␣
↪['probability']))]
        for index, value in np.ndenumerate(self.table):
            row = ['True' if boolean > 0 else 'False' for boolean in index]
            result.append(table_format.format(*(row + [value])))

        return '\n'.join(result) + '\n'
```

[3]:
```
def inference(factors, query, hidden_list, evidence_list, print_step=True):
    def step_printer(factor_list):
        for factor in factor_list:
            print(factor)
```

```python
    factor_list = [copy.copy(factor) for factor in factors]

    if print_step:
        print('Initialized factors:')
        step_printer(factor_list)

    for factor in factor_list:
        for evidence, value in evidence_list.items():
            factor.restrict(evidence, value)
    if print_step:
        print('After restriction:')
        step_printer(factor_list)

    product = np.prod(factor_list)

    if print_step:
        print('After production:')
        step_printer([product])

    for hidden in hidden_list:
        product.sumout(hidden)

    if print_step:
        print('After summing out:')
        step_printer([product])

    product.normalize()

    if print_step:
        print('After normalization:')
        step_printer([product])

    if set(query) == set(product.header):
        return product
    else:
        raise Exception("Invalid query size.")
```

### 1.2 (2)

[4]:

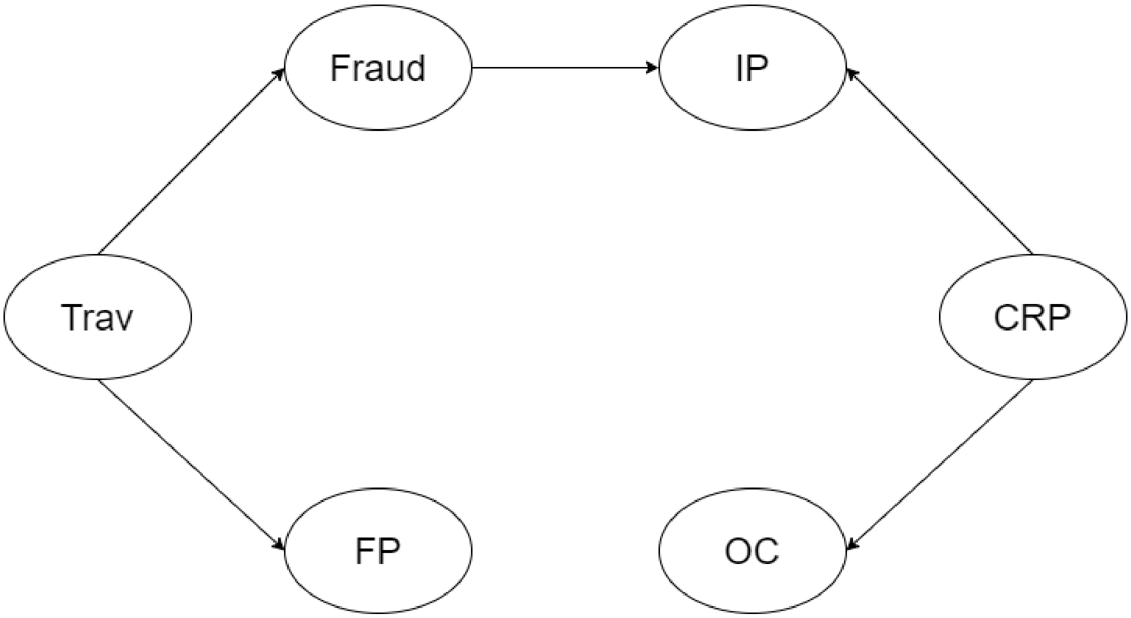
```python
# Fraud | Trav
f1 = Factor(['Fraud', 'Trav'],
            [(0, 0, 1 - 0.004),
             (0, 1, 1 - 0.01),
             (1, 0, 0.004),
             (1, 1, 0.01)])
```

```
# Trav
f2 = Factor(['Trav'],
            [(0, 1 - 0.05),
             (1, 0.05)])

# FP | Fraud, Trav
f3 = Factor(['FP', 'Fraud', 'Trav'],
            [(0, 0, 0, 1 - 0.01),
             (0, 0, 1, 1 - 0.9),
             (0, 1, 0, 1 - 0.1),
             (0, 1, 1, 1 - 0.9),
             (1, 0, 0, 0.01),
             (1, 0, 1, 0.9),
             (1, 1, 0, 0.1),
             (1, 1, 1, 0.9)])

# OC
f4 = Factor(['OC'],
            [(0, 1 - 0.6),
             (1, 0.6)])

# IP | OC, Fraud
f5 = Factor(['IP', 'OC', 'Fraud'],
            [(0, 0, 0, 1 - 0.001),
             (0, 0, 1, 1 - 0.011),
             (0, 1, 0, 1 - 0.01),
             (0, 1, 1, 1 - 0.02),
             (1, 0, 0, 0.001),
             (1, 0, 1, 0.011),
             (1, 1, 0, 0.01),
             (1, 1, 1, 0.02)])

# CRP | OC
f6 = Factor(['CRP', 'OC'],
            [(0, 0, 1 - 0.001),
             (0, 1, 1 - 0.1),
             (1, 0, 0.001),
             (1, 1, 0.1)])
```

### 1.2.1 a)

[5]:
```
from IPython.display import Image
Image('./A3Q2a.png')
```

[5]:




#### 1.2.2 b)

[6]:
```
factor_list = [f1, f2, f3, f4, f5, f6]
ans = inference(factor_list, ['Fraud'], ['Trav', 'FP', 'IP', 'OC', 'CRP'],␣
 ↪dict())
print("P(Fraud) = " + str(ans.possibility(['Fraud'])))

ans = inference(factor_list, ['Fraud'], ['Trav', 'OC'], {'FP': True, 'IP':␣
 ↪False, 'CRP': True})
print("P(Fraud | FP, ~IP, CRP) = " + str(ans.possibility(['Fraud'])))
```

```
Initialized factors:
Fraud           Trav            probability
False           False           0.996
False           True            0.99
True            False           0.004
True            True            0.01

Trav            probability
False           0.95
True            0.05

FP              Fraud           Trav            probability
False           False           False           0.99
False           False           True            0.09999999999999998
False           True            False           0.9
```

| | | | |
|---|---|---|---|
| False | True | True | 0.09999999999999998 |
| True | False | False | 0.01 |
| True | False | True | 0.9 |
| True | True | False | 0.1 |
| True | True | True | 0.9 |

| OC | probability |
|---|---|
| False | 0.4 |
| True | 0.6 |

| Fraud | IP | OC | probability |
|---|---|---|---|
| False | False | False | 0.999 |
| False | False | True | 0.99 |
| False | True | False | 0.001 |
| False | True | True | 0.01 |
| True | False | False | 0.989 |
| True | False | True | 0.98 |
| True | True | False | 0.011 |
| True | True | True | 0.02 |

| CRP | OC | probability |
|---|---|---|
| False | False | 0.999 |
| False | True | 0.9 |
| True | False | 0.001 |
| True | True | 0.1 |

After restriction:

| Fraud | Trav | probability |
|---|---|---|
| False | False | 0.996 |
| False | True | 0.99 |
| True | False | 0.004 |
| True | True | 0.01 |

| Trav | probability |
|---|---|
| False | 0.95 |
| True | 0.05 |

| FP | Fraud | Trav | probability |
|---|---|---|---|
| False | False | False | 0.99 |
| False | False | True | 0.09999999999999998 |
| False | True | False | 0.9 |
| False | True | True | 0.09999999999999998 |
| True | False | False | 0.01 |
| True | False | True | 0.9 |
| True | True | False | 0.1 |
| True | True | True | 0.9 |

| OC | probability |
|---|---|

| | |
|---|---|
| False | 0.4 |
| True | 0.6 |

| Fraud | IP | OC | probability |
|---|---|---|---|
| False | False | False | 0.999 |
| False | False | True | 0.99 |
| False | True | False | 0.001 |
| False | True | True | 0.01 |
| True | False | False | 0.989 |
| True | False | True | 0.98 |
| True | True | False | 0.011 |
| True | True | True | 0.02 |

| CRP | OC | probability |
|---|---|---|
| False | False | 0.999 |
| False | True | 0.9 |
| True | False | 0.001 |
| True | True | 0.1 |

After production:

| CRP | FP | Fraud | IP | OC | Trav | probability |
|---|---|---|---|---|---|---|
| False | False | False | False | False | False | 0.3739461842952 |
| False | False | False | False | False | True | 0.00197604198 |
| False | False | False | False | True | False | 0.5007801348 |
| False | False | False | False | True | True | 0.0026462699999999996 |
| False | False | False | True | False | False | 0.0003743205048 |
| False | False | False | True | False | True | 1.97802e-06 |
| False | False | False | True | True | False | 0.0050583851999999995 |
| False | False | False | True | True | True | 2.6729999999999996e-05 |
| False | False | True | False | False | False | 0.0013515990480000002 |
| False | False | True | False | False | True | 1.9760219999999996e-05 |
| False | False | True | False | True | False | 0.001809864 |
| False | False | True | False | True | True | 2.645999999999999e-05 |
| False | False | True | True | False | False | 1.5032952e-05 |

```
False           False           True            True            False           True
2.1977999999999995e-07
False           False           True            True            True            False
3.6936e-05
False           False           True            True            True            True
5.399999999999999e-07
False           True            False           False           False           False
0.0037772341848
False           True            False           False           False           True
0.017784377820000002
False           True            False           False           True            False
0.0050583851999999995
False           True            False           False           True            True
0.023816430000000003
False           True            False           True            False           False
3.7810152e-06
False           True            False           True            False           True
1.7802180000000002e-05
False           True            False           True            True            False
5.10948e-05
False           True            False           True            True            True
0.00024057000000000004
False           True            True            False           False           False
0.000150177672
False           True            True            False           False           True
0.00017784198000000002
False           True            True            False           True            False
0.00020109600000000003
False           True            True            False           True            True
0.00023814
False           True            True            True            False           False
1.670328e-06
False           True            True            True            False           True
1.97802e-06
False           True            True            True            True            False
4.104000000000001e-06
False           True            True            True            True            True
4.86e-06
True            False           False           False           False           False
0.0003743205048
True            False           False           False           False           True
1.97802e-06
True            False           False           False           True            False
0.0556422372
True            False           False           False           True            True
0.00029403
True            False           False           True            False           False
3.746952e-07
```

```
True            False           False           True            False           True
1.98e-09
True            False           False           True            True            False
0.0005620428
True            False           False           True            True            True
2.97e-06
True            False           True            False           False           False
1.3529520000000002e-06
True            False           True            False           False           True
1.9779999999999996e-08
True            False           True            False           True            False
0.000201096
True            False           True            False           True            True
2.939999999999999e-06
True            False           True            True            False           False
1.5048e-08
True            False           True            True            False           True
2.1999999999999996e-10
True            False           True            True            True            False
4.104e-06
True            False           True            True            True            True
6e-08
True            True            False           False           False           False
3.7810152e-06
True            True            False           False           False           True
1.7802180000000002e-05
True            True            False           False           True            False
0.0005620428
True            True            False           False           True            True
0.0026462700000000005
True            True            False           True            False           False
3.7848e-09
True            True            False           True            False           True
1.7820000000000003e-08
True            True            False           True            True            False
5.6772000000000005e-06
True            True            False           True            True            True
2.6730000000000007e-05
True            True            True            False           False           False
1.50328e-07
True            True            True            False           False           True
1.7802000000000003e-07
True            True            True            False           True            False
2.2344e-05
True            True            True            False           True            True
2.646e-05
True            True            True            True            False           False
1.672e-09
```

```
True            True            True            True            False           True
1.98e-09
True            True            True            True            True            False
4.5600000000000006e-07
True            True            True            True            True            True
5.4e-07

After summing out:
Fraud           probability
False           0.9957
True            0.0043

After normalization:
Fraud           probability
False           0.9957
True            0.0043

P(Fraud) = 0.0043
Initialized factors:
Fraud           Trav            probability
False           False           0.996
False           True            0.99
True            False           0.004
True            True            0.01

Trav            probability
False           0.95
True            0.05

FP              Fraud           Trav            probability
False           False           False           0.99
False           False           True            0.09999999999999998
False           True            False           0.9
False           True            True            0.09999999999999998
True            False           False           0.01
True            False           True            0.9
True            True            False           0.1
True            True            True            0.9

OC              probability
False           0.4
True            0.6

Fraud           IP              OC              probability
False           False           False           0.999
False           False           True            0.99
False           True            False           0.001
False           True            True            0.01
```

| | | | |
|---|---|---|---|
| True | False | False | 0.989 |
| True | False | True | 0.98 |
| True | True | False | 0.011 |
| True | True | True | 0.02 |

| CRP | OC | probability |
|---|---|---|
| False | False | 0.999 |
| False | True | 0.9 |
| True | False | 0.001 |
| True | True | 0.1 |

After restriction:

| Fraud | Trav | probability |
|---|---|---|
| False | False | 0.996 |
| False | True | 0.99 |
| True | False | 0.004 |
| True | True | 0.01 |

| Trav | probability |
|---|---|
| False | 0.95 |
| True | 0.05 |

| Fraud | Trav | probability |
|---|---|---|
| False | False | 0.01 |
| False | True | 0.9 |
| True | False | 0.1 |
| True | True | 0.9 |

| OC | probability |
|---|---|
| False | 0.4 |
| True | 0.6 |

| Fraud | OC | probability |
|---|---|---|
| False | False | 0.999 |
| False | True | 0.99 |
| True | False | 0.989 |
| True | True | 0.98 |

| OC | probability |
|---|---|
| False | 0.001 |
| True | 0.1 |

After production:

| Fraud | OC | Trav | probability |
|---|---|---|---|
| False | False | False | 3.7810152e-06 |
| False | False | True | 1.7802180000000002e-05 |
| False | True | False | 0.0005620428 |
| False | True | True | 0.0026462700000000005 |

```
True            False           False           1.50328e-07
True            False           True            1.7802000000000003e-07
True            True            False           2.2344e-05
True            True            True            2.646e-05

After summing out:
Fraud           probability
False           0.0032298959952000005
True            4.9132348e-05

After normalization:
Fraud           probability
False           0.985016186852459
True            0.014983813147541077

P(Fraud | FP, ~IP, CRP) = 0.014983813147541077
```

### 1.2.3 c)

[7]:

```python
ans = inference(factor_list, ['Fraud'], ['OC'], {'FP': True, 'IP': False, 'CRP':
 ↪ True, 'Trav': True})
print("P(Fraud | FP, ~IP, CRP, Trav) = " + str(ans.possibility(['Fraud'])))
```

```
Initialized factors:
Fraud           Trav            probability
False           False           0.996
False           True            0.99
True            False           0.004
True            True            0.01

Trav            probability
False           0.95
True            0.05

FP              Fraud           Trav            probability
False           False           False           0.99
False           False           True            0.09999999999999998
False           True            False           0.9
False           True            True            0.09999999999999998
True            False           False           0.01
True            False           True            0.9
True            True            False           0.1
True            True            True            0.9

OC              probability
False           0.4
True            0.6
```

| Fraud | IP | OC | probability |
|---|---|---|---|
| False | False | False | 0.999 |
| False | False | True | 0.99 |
| False | True | False | 0.001 |
| False | True | True | 0.01 |
| True | False | False | 0.989 |
| True | False | True | 0.98 |
| True | True | False | 0.011 |
| True | True | True | 0.02 |

| CRP | OC | probability |
|---|---|---|
| False | False | 0.999 |
| False | True | 0.9 |
| True | False | 0.001 |
| True | True | 0.1 |

After restriction:

| Fraud | probability |
|---|---|
| False | 0.99 |
| True | 0.01 |

| probability |
|---|
| 0.05 |

| Fraud | probability |
|---|---|
| False | 0.9 |
| True | 0.9 |

| OC | probability |
|---|---|
| False | 0.4 |
| True | 0.6 |

| Fraud | OC | probability |
|---|---|---|
| False | False | 0.999 |
| False | True | 0.99 |
| True | False | 0.989 |
| True | True | 0.98 |

| OC | probability |
|---|---|
| False | 0.001 |
| True | 0.1 |

After production:

| Fraud | OC | probability |
|---|---|---|
| False | False | 1.7802180000000002e-05 |
| False | True | 0.0026462700000000005 |
| True | False | 1.7802000000000003e-07 |

```
True            True            2.646e-05

After summing out:
Fraud           probability
False           0.0026640721800000005
True            2.663802e-05

After normalization:
Fraud           probability
False           0.990100004080707
True            0.009899995919292979

P(Fraud | FP, ~IP, CRP, Trav) = 0.009899995919292979
```

### 1.2.4 d)

[8]:
```python
ans = inference(factor_list, ['Fraud'], ['Trav', 'FP', 'OC', 'CRP'], {'IP':␣
 ↪True})
print("P(Fraud | IP) = " + str(ans.possibility(['Fraud'])))

cond_vars = ['FP', 'CRP']
for enum in range(2 ** 2):
    bin = "{0:02b}".format(enum)
    cond = {var: dig == '1' for var, dig in zip(cond_vars, bin)}
    cond['IP'] = True
    ans = inference(factor_list, ['Fraud'], ['Trav', 'OC'], cond)

    cond_to_print = [var if dig == '1' else '~' + var for var, dig in␣
 ↪zip(cond_vars, bin)]
    print('P(Fraud | ' + ', '.join(cond_to_print) + ') = ' + str(ans.
 ↪possibility(['Fraud'])))
```

```
Initialized factors:
Fraud           Trav            probability
False           False           0.996
False           True            0.99
True            False           0.004
True            True            0.01

Trav            probability
False           0.95
True            0.05

FP              Fraud           Trav            probability
False           False           False           0.99
False           False           True            0.09999999999999998
```

| | | | |
|---|---|---|---|
| False | True | False | 0.9 |
| False | True | True | 0.09999999999999998 |
| True | False | False | 0.01 |
| True | False | True | 0.9 |
| True | True | False | 0.1 |
| True | True | True | 0.9 |

| OC | probability |
|---|---|
| False | 0.4 |
| True | 0.6 |

| Fraud | IP | OC | probability |
|---|---|---|---|
| False | False | False | 0.999 |
| False | False | True | 0.99 |
| False | True | False | 0.001 |
| False | True | True | 0.01 |
| True | False | False | 0.989 |
| True | False | True | 0.98 |
| True | True | False | 0.011 |
| True | True | True | 0.02 |

| CRP | OC | probability |
|---|---|---|
| False | False | 0.999 |
| False | True | 0.9 |
| True | False | 0.001 |
| True | True | 0.1 |

After restriction:

| Fraud | Trav | probability |
|---|---|---|
| False | False | 0.996 |
| False | True | 0.99 |
| True | False | 0.004 |
| True | True | 0.01 |

| Trav | probability |
|---|---|
| False | 0.95 |
| True | 0.05 |

| FP | Fraud | Trav | probability |
|---|---|---|---|
| False | False | False | 0.99 |
| False | False | True | 0.09999999999999998 |
| False | True | False | 0.9 |
| False | True | True | 0.09999999999999998 |
| True | False | False | 0.01 |
| True | False | True | 0.9 |
| True | True | False | 0.1 |
| True | True | True | 0.9 |

| OC | probability |
|---|---|
| False | 0.4 |
| True | 0.6 |

| Fraud | OC | probability |
|---|---|---|
| False | False | 0.001 |
| False | True | 0.01 |
| True | False | 0.011 |
| True | True | 0.02 |

| CRP | OC | probability |
|---|---|---|
| False | False | 0.999 |
| False | True | 0.9 |
| True | False | 0.001 |
| True | True | 0.1 |

After production:

| CRP | FP | Fraud | OC | Trav | probability |
|---|---|---|---|---|---|
| False | False | False | False | False | 0.0003743205048 |
| False | False | False | False | True | 1.97802e-06 |
| False | False | False | True | False | 0.0050583851999999995 |
| False | False | False | True | True | 2.6729999999999996e-05 |
| False | False | True | False | False | 1.5032952e-05 |
| False | False | True | False | True | 2.1977999999999995e-07 |
| False | False | True | True | False | 3.6936e-05 |
| False | False | True | True | True | 5.399999999999999e-07 |
| False | True | False | False | False | 3.7810152e-06 |
| False | True | False | False | True | 1.7802180000000002e-05 |
| False | True | False | True | False | 5.10948e-05 |
| False | True | False | True | True | 0.00024057000000000004 |
| False | True | True | False | False | 1.670328e-06 |
| False | True | True | False | True | 1.97802e-06 |
| False | True | True | True | False | |

```
4.104000000000001e-06
False          True           True           True           True
4.86e-06
True           False          False          False          False
3.746952e-07
True           False          False          False          True
1.98e-09
True           False          False          True           False
0.0005620428
True           False          False          True           True
2.97e-06
True           False          True           False          False
1.5048e-08
True           False          True           False          True
2.1999999999999996e-10
True           False          True           True           False
4.104e-06
True           False          True           True           True           6e-08
True           True           False          False          False
3.7848e-09
True           True           False          False          True
1.7820000000000003e-08
True           True           False          True           False
5.6772000000000005e-06
True           True           False          True           True
2.6730000000000007e-05
True           True           True           False          False
1.672e-09
True           True           True           False          True
1.98e-09
True           True           True           True           False
4.5600000000000006e-07
True           True           True           True           True
5.4e-07

After summing out:
Fraud          probability
False          0.006372479999999999
True           7.052e-05

After normalization:
Fraud          probability
False          0.9890547881421697
True           0.010945211857830204

P(Fraud | IP) = 0.010945211857830204
Initialized factors:
Fraud          Trav           probability
```

```
False           False           0.996
False           True            0.99
True            False           0.004
True            True            0.01

Trav            probability
False           0.95
True            0.05

FP              Fraud           Trav            probability
False           False           False           0.99
False           False           True            0.09999999999999998
False           True            False           0.9
False           True            True            0.09999999999999998
True            False           False           0.01
True            False           True            0.9
True            True            False           0.1
True            True            True            0.9

OC              probability
False           0.4
True            0.6

Fraud           IP              OC              probability
False           False           False           0.999
False           False           True            0.99
False           True            False           0.001
False           True            True            0.01
True            False           False           0.989
True            False           True            0.98
True            True            False           0.011
True            True            True            0.02

CRP             OC              probability
False           False           0.999
False           True            0.9
True            False           0.001
True            True            0.1

After restriction:
Fraud           Trav            probability
False           False           0.996
False           True            0.99
True            False           0.004
True            True            0.01

Trav            probability
False           0.95
```

```
True            0.05

Fraud           Trav            probability
False           False           0.99
False           True            0.09999999999999998
True            False           0.9
True            True            0.09999999999999998

OC              probability
False           0.4
True            0.6

Fraud           OC              probability
False           False           0.001
False           True            0.01
True            False           0.011
True            True            0.02

OC              probability
False           0.999
True            0.9

After production:
Fraud           OC              Trav            probability
False           False           False           0.0003743205048
False           False           True            1.97802e-06
False           True            False           0.005058385199999995
False           True            True            2.6729999999999996e-05
True            False           False           1.5032952e-05
True            False           True            2.1977999999999995e-07
True            True            False           3.6936e-05
True            True            True            5.399999999999999e-07

After summing out:
Fraud           probability
False           0.005461413724799996
True            5.272873199999996e-05

After normalization:
Fraud           probability
False           0.9904375462888204
True            0.009562453711179572

P(Fraud | ~FP, ~CRP) = 0.009562453711179572
Initialized factors:
Fraud           Trav            probability
False           False           0.996
False           True            0.99
```

```
True            False           0.004
True            True            0.01

Trav            probability
False           0.95
True            0.05

FP              Fraud           Trav            probability
False           False           False           0.99
False           False           True            0.09999999999999998
False           True            False           0.9
False           True            True            0.09999999999999998
True            False           False           0.01
True            False           True            0.9
True            True            False           0.1
True            True            True            0.9

OC              probability
False           0.4
True            0.6

Fraud           IP              OC              probability
False           False           False           0.999
False           False           True            0.99
False           True            False           0.001
False           True            True            0.01
True            False           False           0.989
True            False           True            0.98
True            True            False           0.011
True            True            True            0.02

CRP             OC              probability
False           False           0.999
False           True            0.9
True            False           0.001
True            True            0.1

After restriction:
Fraud           Trav            probability
False           False           0.996
False           True            0.99
True            False           0.004
True            True            0.01

Trav            probability
False           0.95
True            0.05
```

```
Fraud           Trav            probability
False           False           0.99
False           True            0.09999999999999998
True            False           0.9
True            True            0.09999999999999998

OC              probability
False           0.4
True            0.6

Fraud           OC              probability
False           False           0.001
False           True            0.01
True            False           0.011
True            True            0.02

OC              probability
False           0.001
True            0.1

After production:
Fraud           OC              Trav            probability
False           False           False           3.746952e-07
False           False           True            1.98e-09
False           True            False           0.0005620428
False           True            True            2.97e-06
True            False           False           1.5048e-08
True            False           True            2.1999999999999996e-10
True            True            False           4.104e-06
True            True            True            6e-08

After summing out:
Fraud           probability
False           0.0005653894752
True            4.179268e-06

After normalization:
Fraud           probability
False           0.9926623993154545
True            0.007337600684545429

P(Fraud | ~FP, CRP) = 0.007337600684545429
Initialized factors:
Fraud           Trav            probability
False           False           0.996
False           True            0.99
True            False           0.004
True            True            0.01
```

```
Trav          probability
False         0.95
True          0.05

FP            Fraud         Trav          probability
False         False         False         0.99
False         False         True          0.09999999999999998
False         True          False         0.9
False         True          True          0.09999999999999998
True          False         False         0.01
True          False         True          0.9
True          True          False         0.1
True          True          True          0.9

OC            probability
False         0.4
True          0.6

Fraud         IP            OC            probability
False         False         False         0.999
False         False         True          0.99
False         True          False         0.001
False         True          True          0.01
True          False         False         0.989
True          False         True          0.98
True          True          False         0.011
True          True          True          0.02

CRP           OC            probability
False         False         0.999
False         True          0.9
True          False         0.001
True          True          0.1

After restriction:
Fraud         Trav          probability
False         False         0.996
False         True          0.99
True          False         0.004
True          True          0.01

Trav          probability
False         0.95
True          0.05

Fraud         Trav          probability
False         False         0.01
```

```
False          True           0.9
True           False          0.1
True           True           0.9

OC             probability
False          0.4
True           0.6

Fraud          OC             probability
False          False          0.001
False          True           0.01
True           False          0.011
True           True           0.02

OC             probability
False          0.999
True           0.9

After production:
Fraud          OC             Trav           probability
False          False          False          3.7810152e-06
False          False          True           1.7802180000000002e-05
False          True           False          5.10948e-05
False          True           True           0.00024057000000000004
True           False          False          1.670328e-06
True           False          True           1.97802e-06
True           True           False          4.104000000000001e-06
True           True           True           4.86e-06

After summing out:
Fraud          probability
False          0.0003132479952
True           1.2612348e-05

After normalization:
Fraud          probability
False          0.9612952350195646
True           0.03870476498043534

P(Fraud | FP, ~CRP) = 0.03870476498043534
Initialized factors:
Fraud          Trav           probability
False          False          0.996
False          True           0.99
True           False          0.004
True           True           0.01

Trav           probability
```

```
False          0.95
True           0.05

FP             Fraud          Trav           probability
False          False          False          0.99
False          False          True           0.09999999999999998
False          True           False          0.9
False          True           True           0.09999999999999998
True           False          False          0.01
True           False          True           0.9
True           True           False          0.1
True           True           True           0.9

OC             probability
False          0.4
True           0.6

Fraud          IP             OC             probability
False          False          False          0.999
False          False          True           0.99
False          True           False          0.001
False          True           True           0.01
True           False          False          0.989
True           False          True           0.98
True           True           False          0.011
True           True           True           0.02

CRP            OC             probability
False          False          0.999
False          True           0.9
True           False          0.001
True           True           0.1

After restriction:
Fraud          Trav           probability
False          False          0.996
False          True           0.99
True           False          0.004
True           True           0.01

Trav           probability
False          0.95
True           0.05

Fraud          Trav           probability
False          False          0.01
False          True           0.9
True           False          0.1
```

```
True            True            0.9

OC              probability
False           0.4
True            0.6

Fraud           OC              probability
False           False           0.001
False           True            0.01
True            False           0.011
True            True            0.02

OC              probability
False           0.001
True            0.1

After production:
Fraud           OC              Trav            probability
False           False           False           3.7848e-09
False           False           True            1.7820000000000003e-08
False           True            False           5.6772000000000005e-06
False           True            True            2.6730000000000007e-05
True            False           False           1.672e-09
True            False           True            1.98e-09
True            True            False           4.5600000000000006e-07
True            True            True            5.4e-07

After summing out:
Fraud           probability
False           3.24288048e-05
True            9.99652e-07

After normalization:
Fraud           probability
False           0.9700957777985133
True            0.02990422220148673

P(Fraud | FP, CRP) = 0.02990422220148673
```

As is illustrated above, we are supposed to satisfy conditions ~FP, CRP to get the minimal probability. Thus, I should use domestic currency for the payment after making a computer related purchase. This will help reduce the probability from 1.094% to 0.734%, which is approximately 0.36%.

## 2 Part B

[9]:
```
import math
```

[10]:
```
train_data_path = './dataset/trainData.txt'
train_label_path = './dataset/trainLabel.txt'
test_data_path = './dataset/testData.txt'
test_label_path = './dataset/testLabel.txt'
words_path = './dataset/words.txt'

# Labels must start from 1 since we will minus 1 when storing
actual_labels = [1, 2]

labels = [label - 1 for label in actual_labels]
```

[11]:
```
class Document:
    def __init__(self, label, doc_id=0):
        self.id = doc_id
        self.label = label - 1
        # Set labels to start from 0, it makes no difference
        self.word_list = set()

    def __contains__(self, word):
        return word in self.word_list

    def add_word(self, word):
        self.word_list.add(word - 1)
        # We store word - 1 here so that we only need to consider
        # the index issue when mapping word id to its word


def load_train_data():
    return load_data(train_data_path, train_label_path)


def load_test_data():
    return load_data(test_data_path, test_label_path)


def load_data(data_path, label_path):
    docs = []
    with open(label_path, 'r', encoding='utf-8') as file:
        for index, line in enumerate(file):
            doc_id = index + 1
            label = int(line.strip())
            docs.append(Document(label, doc_id))
```

```python
    with open(data_path, 'r', encoding='utf-8') as file:
        for line in file.readlines():
            [doc_id, word_id] = list(map(int, line.strip().split(' ')))
            docs[doc_id - 1].add_word(word_id)

    return docs


def load_words(filename=words_path):
    word_map = dict()

    with open(filename, 'r', encoding='utf-8') as file:
        for index, word in enumerate(file):
            # Similar to above, we minus 1 for the word id
            word_map[index] = word.strip()

    return word_map
```

[12]:

```python
class NaiveBayes:
    def __init__(self, word_map, labels):
        self.word_set = set(word_map)
        self.poss_labels = labels

    def fit(self, docs):
        num_words = len(self.word_set)
        num_label = len(self.poss_labels)
        stat = np.zeros((num_words, num_label))

        for document in docs:
            for word in document.word_list:
                stat[word][document.label] += 1

        label_sum = np.sum(stat, axis=1, keepdims=True)
        word_sum = np.sum(stat, axis=0, keepdims=True)
        # prob_given_label[word][label] stores the value of Pr(word | label)
        self.prob_given_label = (stat + 1) / (label_sum + num_words)
        # prob_given_word[label][word] stores the value of Pr(label | word)
        self.prob_given_word = np.transpose((stat + 1) / (word_sum + num_label))

    def predict(self, word_list):
        p = dict()
        for label in self.poss_labels:
            p[label] = 1
            for word in self.word_set:
                if word in word_list:
                    p[label] *= self.prob_given_label[word][label]
                else:
```

```
                    p[label] *= 1 - self.prob_given_label[word][label]

        return max(p, key=p.get)

    def __diff(self, word, label1, label2):
        return abs(math.log(self.prob_given_label[word][label1]) -
                   math.log(self.prob_given_label[word][label2]))

    def discriminative(self, n, label1=0, label2=1):
        rank = {word: self.__diff(word, label1, label2) for word in self.
↪word_set}
        return sorted(rank, key=rank.get, reverse=True)[: n]
```

[ ]:
```
def test(model, data):
    correct, incorrect = 0, 0
    for doc in data:
        label = doc.label
        pred = model.predict(doc)
        if pred == label:
            correct += 1
        else:
            incorrect += 1

    return correct / (correct + incorrect)


train_data = load_train_data()
test_data = load_test_data()
word_map = load_words()

nb = NaiveBayes(word_map, labels)
nb.fit(train_data)
disc_words = nb.discriminative(10)
print("Top 10 discriminative words:")
print([word_map[word] for word in disc_words])

print("Training accuracy: " + str(test(nb, train_data)))
print("Testing accuracy: " + str(test(nb, test_data)))
```

```
Top 10 discriminative words:
['christian', 'religion', 'atheism', 'books', 'christians', 'library',
'religious', 'libraries', 'novel', 'beliefs']
Training accuracy: 0.908
```

The assumption that all word features are independent is not reasonable. One example is shown above in the most discriminative 10 words that the same words may appear in an article in different forms, say `christian' and `christians'.

To take into account dependencies between words, we would better apply a stemming algorithm first, which will reduce all words to their roots so that different forms of the same words will be counted as only one.