MATH 442/551 Assignment #2

Due Thursday October 4, in class

1. Consider the following system

$$\frac{dx}{dt} = \begin{bmatrix} e^{-t} & \frac{t^2+1}{t^2} \\ (2-a)\frac{1-t}{t} & a\frac{1-t}{t} \end{bmatrix} x,$$

where $x \in \mathbb{R}^3$. Determine the range of a that makes the origin unstable (ignore that case where eigenvalues has zero real parts).

2. Consider the following linear system,

$$\frac{dx}{dt} = Ax + B(t)x,$$

where $x \in \mathbb{R}^n$, A is an $n \times n$ matrix which eigenvalues all have negative real parts. $B(t)$ is a continuous $n \times n$ matrix. Show that, there exists a constant $b > 0$, such that the origin is asymptotically stable as long as $\|B(t)\| < b$ for all $t \geq 0$.

3. Consider the system

$$\frac{dx}{dt} = 1 + y - x^2 - y^2,$$
$$\frac{dy}{dx} = 1 - x - x^2 - y^2.$$

   (a) Find the equilibria and classify them (a saddle, or an unstable spiral node, etc).

   (b) Show that there is a periodic solution (using polar coordinates).

   (c) Linearize about the periodic solution, and determine the characteristic exponents of the linearized system.