CSC320 Summer 2018 Midterm 4 Sample Anwers

1. Suppose that A is TM-decidable and $A \leq_m B$. Which one of the following statements could be false?

   (a) $\overline{A}$ is TM-decidable.

   (b) A is TM-recognizable.

   (c) $\overline{B}$ is TM-decidable.

   (d) $\overline{A}$ is TM-recognizable.

   (e) None of the above

   The answer is (c). E.g., consider the case where A is $\{0,1\}^*$ and B is A_{TM}. Then A is decidable. Also, we cah show that $A \leq_m B$ as follows: Let M_e be a TM that includes the transition $\delta(q_0, \sqcup) = (q_{accept}, \sqcup, L)$. Then M accepts ϵ. Define f as follows: for any $w \in \{0,1\}^*$, $f(w) = \langle M_e, \epsilon \rangle$. Then for any w, $f(w) \in A_{TM}$, so $w \in A$ iff $f(w) \in A_{TM}$, so f is a mapping reduction, and $A \leq_m B$. But $B = A_{TM}$ is not decidable. So (e) could be false.

2. Which one of the following statements could be false?

   (a) There is a languge L such that neither L nor $\overline{L}$ is decidable.

   (b) There is a languge L such that neither L nor $\overline{L}$ is recognizable.

   (c) If L is undecidable and $\overline{L}$ is not recognizable, then L must be recognizable.

   (d) If L is recognizable and $\overline{L}$ is recognizable, then L must be decidable.

   (e) None of the above.

   The answer is (c). First of all, note that if $\overline{L}$ is unrecognizable, then $\overline{L}$ is undeciable, which implies L is undecidable, so we could just simplify (c) to: If $\overline{L}$ is unrecognizable, then L must be recognizable. So only one of (b) and (c) cam be true. We saw in class that EQ_{TM} is niether recognizable no co-recognizable, so (b) is true.

3. Which one of the following is a valid DIMACS encoding of the formula $\phi = (x_1 \wedge (x_2 \vee x_3)) \rightarrow x_4$, assuming x_i is encoded as i?

   (a)
   ```
   p cnf 4 2
   -1 -2 -4 0
   -1 -3 -4 0
   ```
   (b)
   ```
   p cnf 4 3
   -1 -2  0
   -1 -3 0
   -4 0
   ```
   (c)
   ```
   p cnf 4 3
   -1 -2  0
   -1 -3 0
   4 0
   ```
   (d)
   ```
   p cnf 4 2
   -1 -2 4 0
   -1 -3 4 0
   ```
   (e) None of the preceding.

We need to turn this Boolean formula into a CNF first (NOTE: I remember mentioning in class that you don't have to know how to do this tranlation... so for our upcoming midterm I would have to start with ϕ already in CNF if we ahve a question like this. Anyway, we have

$$\begin{aligned}
\phi &\equiv \overline{(x_1 \wedge (x_2 \vee x_3))} \vee x_4 \\
&\equiv (\overline{x}_1 \vee (\overline{x}_2 \wedge \overline{x}_3)) \vee x_4 \\
&\equiv (\overline{x}_1 \vee x_4) \vee (\overline{x}_2 \wedge \overline{x}_3) \\
&\equiv (\overline{x}_1 \vee \overline{x}_2 \vee x_4) \wedge (\overline{x}_1 \vee \overline{x}_3 \vee x_4) \\
&\equiv \{\{\overline{x}_1, \overline{x}_2, x_4\}, \{\overline{x}_1, \overline{x}_3, x_4\}\}
\end{aligned}$$

In DIMACS format, this gives us (d)

4. Let $EHALT_{TM}$ be the language defined as follows:

$$EHALT_{TM} = \{\langle M \rangle \mid M \text{ halts on all even-length inputs}\}$$

Use the undecidability of the $HALT_{TM}$ to prove that $EHALT_{TM}$ is undecidable.

(a) Define the reduction f, where $f(\langle M, w \rangle) = \langle M' \rangle$, and M' is defined as follows:

On input x, M' does the following:

if $|x|$ is odd **then** accept **else** Run M on w and accept if M accepts

Prove that f is a reduction from $HALT_{TM}$ to $EHALT_{TM}$:

If $\langle M, w \rangle \in HALT_{TM}$, then M' halts on every input, so it alls on all even length inputs, so $f(\langle M, w \rangle) \in EHALT_{TM}$.

If $\langle M, w \rangle \notin HALT_{TM}$, then M' halts only on odd-length inputs, so $f(\langle M, w \rangle) \notin EHALT_{TM}$.
Note that we can define M' without making any modifications to $\langle M, w \rangle$ – M' just contains the code to check that $|x|$ is odd (always the same) and then can just pass $\langle M, w \rangle$ to the Universal TM to run M on w. Hence f is easily seen to be computable.