Homework 4–CSC 320 Summer 2018

Due by conneX submission, Sunday July 22, 11:55pm

1. Let u, v be strings. We will write $u \prec v$ if u (strictly) precedes v in the standard string ordering: $\epsilon \prec 0 \prec 1 \prec 00 \prec 01 \ldots$. An enumerator E *respects* $\prec$ if for any strings u and v that it enumerates, if it outputs u before it outputs v then it must be the case that $u \prec v$. Prove the following: a language L is Turing-decidable *if and only if* it is enumerated by an enumerator that respects $\prec$.

2. Prove that the language

   $L = \{\langle M \rangle \mid M$ when started on the blank tape, eventually writes a \$ somewhere on the tape$\}$

   is undecidable. Use the undecidability of A_{TM} to do this. I.e., give a computable reductiion f such that $f(\langle M, w \rangle) = \langle M_1 \rangle$. Prove that f is computable and that $\langle M, w \rangle \in A_{TM}$ iff $\langle M_1 \rangle \in L$.

3. Give a reduction from

   $$HALT_{TM} = \{\langle M, w \rangle \mid M \text{ is a TM which halts on } w\}$$

   to

   $$L = \{\langle M, j \rangle \mid M \text{ halts on all inputs with less than } j \text{ 1's}\}.$$

   I.e., give a computable reductiion f such that $f(\langle M, w \rangle) = \langle M_1, j_1 \rangle$ (for an appropriately chosen j_1.) Prove that f is computable and that $\langle M, w \rangle \in A_{TM}$ iff $\langle M_1, j_1 \rangle \in L$.

4. Show that $\overline{E_{TM}}$ is recognizable by giving a high level description of a nondeterministic TM which recognizes it.

5. Give a reduction to show that $\{\langle M_1, M_2 \rangle \mid L(M_1) \cap L(M_2) = \emptyset\}$ is not recognizable. Use E_{TM}.

6. Say a TM M is *reversible* if for every string w, M accepts w iff M accepts w^R. (Recall that if $w = w_1 w_2 \ldots w_k$ then $w^R = w_k w_{k-1} \ldots w_1$.) Prove that the language

   $$T = \{\langle M \rangle \mid M \text{ is a reversible } \mathsf{TM}\}$$

   is undecidable. I.e., give a computable reductiion f such that $f(\langle M, w \rangle) = \langle M_1 \rangle$. Prove that f is computable and that $\langle M, w \rangle \in A_{TM}$ iff $L(M_1)$ is reversible. (HINT: Σ^* is reversible.)