STAT 353 Assignment 1

Due in class on Wednesday, September 26

Note: Write up your solution carefully and with sufficient details for each problem. For data analysis problems, you also need to submit your R codes and related R outputs. For hypothesis tests, you must specify the null and alternative hypotheses, find the p-values and state your conclusions (use $\alpha = 0.05$ unless specified otherwise). All data sets in the textbook can be downloaded from the following link:

`ftp://ftp.wiley.com/public/sci_tech_med/introduction_linear_regression/`

1. Suppose $(X_1, X_2, X_3)^\top \sim N(\mu, \Sigma)$ with $\mu^\top = (0, 5, -3)$ and

$$\Sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 10 & -1 \\ 0 & -1 & 5 \end{pmatrix}.$$

(a) Find $E(X_3 - 2X_1 + 1)$ and $Var(X_3 - 2X_1 + 1)$.

(b) Find $E(3X_1 + 2X_2 + X_3 + 1)$ and $Var(3X_1 + 2X_2 + X_3 + 1)$.

(c) Find $Cov(2X_1, X_3 + 1)$ and $Cov(X_1 - X_2, X_1 - X_3)$.

2. Suppose $Z_1 \sim \chi^2_5$, $Z_2 \sim \chi^2_3$ and $Y = (Y_1, Y_2, Y_3)^\top \sim N(\mu, \Sigma)$ where

$$\mu^\top = (0, 2, 5) \quad \text{and} \quad \Sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 5 & -1 \\ 0 & -1 & 5 \end{pmatrix}.$$

Also, Z_1, Z_2 and Y are independent. For the random variable X defined by each of the following equations, find its distribution if it is one of the following four distributions: $N(\mu, \sigma^2)$, t_k, χ^2_k and $F(m, n)$. If not, answer "distribution unknown". If your answer is one of the four, you must show values of the parameters or degrees of freedom.

(a) $X = Y_1 + 2Y_2 + 3Y_3$,

(b) $X + Z_2 = Z_1$,

(c) $X = Z_1 - Z_2$,

(d) $X = \frac{Y_3 - 5}{\sqrt{Z_1}}$,

(e) $X = Y_1^2 + \frac{1}{5}(Y_2 - 2)^2 + Z_1$,

(f) $X = \frac{(Y_1 + Y_2 - 2)^2}{2Z_2}$.

3. Question 2.6 in the textbook: Notice that there are 24 observations in the data set. Make changes to parts **d** and **e** and add part **f** as follows.

d. Find a 95% CI on β_1. What is the interpretation of β_1?

e. Find a 95% CI on the mean selling price of a house for which the current taxes are \$7500.

f. Plot the selling price versus the current taxes and add the LS line. Label x-axis and y-axis and give a title to the plot.