STAT 353

Assignment 3

Note: Write up your solution carefully and with sufficient details for each problem. For data analysis problems, you also need to submit your R codes and related R outputs.

Due in class on Friday, November 16

1. Question 4.5: Fit a multiple linear regression model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_5 x_5 + \beta_7 x_7 + \epsilon.$$

Perform residual analysis and answer the following.

(a) Construct a normal probability plot of the residuals. Does there seem to be any problem with the normality assumption?

(b) Construct a plot of the residuals versus the fitted values and comment on the plot.

(c) Are there any outliers?

2. Question 4.6

3. Question 5.5

4. Question 5.15 [4th] or 5.17 [5th] Hint: use the following steps to prove the result.

Define matrix $\mathbf{A} = \mathbf{V}^{-1} - \mathbf{V}^{-1}\mathbf{X}\left(\mathbf{X}^T\mathbf{V}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{V}^{-1}$.

(i) Show that $\mathbf{AX} = 0$.

(ii) Show that $\mathbf{X}^T\mathbf{A} = 0$.

(iii) Show that $\mathbf{y}^T\mathbf{A}\mathbf{y} = \boldsymbol{\varepsilon}^T\mathbf{A}\boldsymbol{\varepsilon}$.

(iv) Show that $E(\boldsymbol{\varepsilon}^T\mathbf{A}\boldsymbol{\varepsilon}) = (n-p)\sigma^2$.