

# MATH 442/551 Assignment #2

Key content

Due Thursday October 4, in class

1. Consider the following system

$$\frac{dx}{dt} = \begin{bmatrix} e^{-t} & \frac{t^2+1}{t^2} \\ (2-a)\frac{1-t}{t} & a\frac{1-t}{t} \end{bmatrix} x,$$

where  $x \in \mathbb{R}^3$ . Determine the range of  $a$  that makes the origin unstable (ignore that case where eigenvalues has zero real parts).

2. Consider the following linear system,

$$\frac{dx}{dt} = Ax + B(t)x,$$

where  $x \in \mathbb{R}^n$ ,  $A$  is an  $n \times n$  matrix which eigenvalues all have negative real parts.  $B(t)$  is a continuous  $n \times n$  matrix. Show that, there exists a constant  $b > 0$ , such that the origin is asymptotically stable as long as  $\|B(t)\| < b$  for all  $t \geq 0$ .

3. Consider the system

$$\begin{aligned} \frac{dx}{dt} &= 1 + y - x^2 - y^2, \\ \frac{dy}{dx} &= 1 - x - x^2 - y^2. \end{aligned}$$

- (a) Find the equilibria and classify them (a saddle, or an unstable spiral node, etc).
- (b) Show that there is a periodic solution (using polar coordinates).
- (c) Linearize about the periodic solution, and determine the characteristic exponents of the linearized system.