Each multiple choice question has **EXACTLY ONE** correct answer.

1. Fill in the blanks to complete the following pumping lemma proof that the language $L = \{0^n 1^n \mid n \geq 0\}$ is not regular:

   Given $p > 0$, choose $w =$_____

   Given x, y, z such that $w = xyz$, $|xy| \geq p$, and $y \neq \epsilon$, choose $i =$_____

   Proof that $xy^i z \notin L$: since $|xy| \geq p$, it must be the case that xy has the form _____.
   But then, since $y \neq \epsilon$, it must be the case that y has the form _____. This means that
   $xy^i z$ must have the form _____, and $xy^i z \notin L$ because _____.

2. Which one of the following is an unambiguous context-free grammar for

   $$\{w \in \{(,)\}^* \mid |w| \geq 0 \text{ and } w \text{ is balanced}\}?$$

   (Note: the empty string is balanced)

   (a) $S \to SS \mid (S)$

   (b) $S \to S(S) \mid \epsilon$

   (c) $S \to SS \mid ()$

   (d) $S \to S(S) \mid ()$

   (e) $S \to SS \mid (S) \mid \epsilon$

3. Which one of the following is a CNF grammar that is equivalent to $S \to 1S0S \mid 0S1S \mid \epsilon$

   (a) $S_0 \to S \mid \epsilon$
       $S \to UR \mid ZT \mid SS \mid ZU \mid UZ$
       $R \to SZ$
       $T \to SU$
       $U \to 1$
       $Z \to 0$

   (b) $S_0 \to USZ \mid ZT \mid SS \mid ZU \mid UZ \mid \epsilon$
       $S \to USZ \mid ZT \mid SS \mid ZU \mid UZ$
       $T \to SU$
       $U \to 1$
       $Z \to 0$

   (c) $S_0 \to UR \mid ZT \mid SS \mid 01 \mid 10 \mid \epsilon$
       $S \to UR \mid ZT \mid SS \mid 01 \mid 10$
       $R \to S0$
       $T \to S1$

   (d) $S_0 \to UR \mid ZT \mid SS \mid ZU \mid UZ \mid \epsilon$
       $S \to UR \mid ZT \mid SS \mid ZU \mid UZ$
       $R \to SZ$
       $T \to SU$
       $U \to 1$
       $Z \to 0$

   (e) None of the above.

4. Consider the following CNF grammar

   $$\begin{aligned}
   S &\to SS \mid LX \mid LR \\
   X &\to SR \\
   L &\to (\\
   R &\to)
   \end{aligned}$$

   Suppose we run the CYK algorithm on this grammar, and the input ()(()), producing the following table (with missing entries):

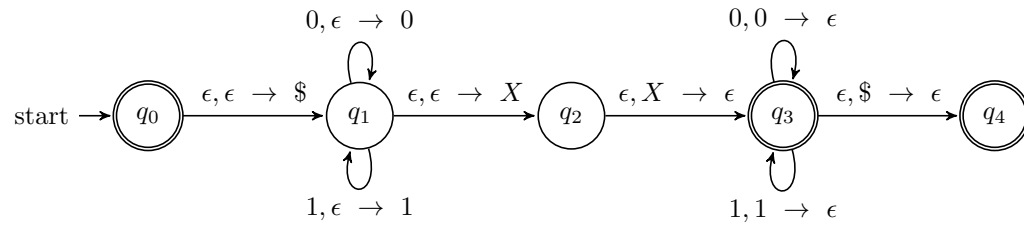 | 1 | 2 | 3 | 4 | 5 | 6 | |
   |---|---|---|---|---|---|---|
   | L | (i) | ∅ | ∅ | ∅ | (vi) | 1 |
   | | R | ∅ | ∅ | ∅ | ∅ | 2 |
   | | | L | ∅ | ∅ | S | 3 |
   | | | | (ii) | (iii) | (v) | 4 |
   | | | | | (iv) | ∅ | 5 |
   | | | | | | R | 6 |

   What are the correct values for the missing entries?

(a) (i)=S, (ii)=R, (iii)=S, (iv)=L, (v)=X, (vi)=S

(b) (i)=S, (ii)=R, (iii)=S, (iv)=R, (v)=X, (vi)=S

(c) (i)=S, (ii)=L, (iii)=S, (iv)=S, (v)=X, (vi)=S

(d) (i)=S, (ii)=S, (iii)=S, (iv)=R, (v)=X, (vi)=S

(e) None of the above.

5. Which one of the following grammars (with start symbol S) generates the language recognized by the following DFA?
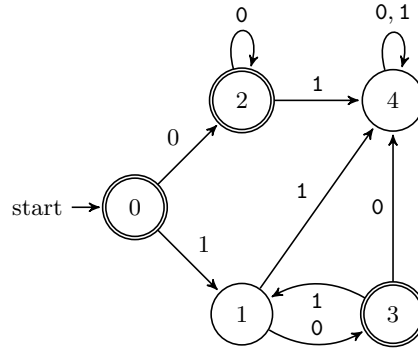
(a)

$$S \rightarrow 0B \mid 1A \mid \epsilon$$
$$A \rightarrow 0D \mid 1C$$
$$B \rightarrow 0B \mid 1D \mid \epsilon$$
$$C \rightarrow 0D \mid 1A \mid \epsilon$$
$$D \rightarrow 0D \mid 1D$$

(b)

$$S \rightarrow 0B \mid 1A \mid \epsilon$$
$$A \rightarrow 0C \mid 1D \mid \epsilon$$
$$B \rightarrow 0B \mid 1D$$
$$C \rightarrow 0D \mid 1A \mid \epsilon$$
$$D \rightarrow 0D \mid 1D$$

(c)

$$S \rightarrow 0B \mid 1A \mid \epsilon$$
$$A \rightarrow 0C \mid 1D$$
$$B \rightarrow 0B \mid 1D \mid \epsilon$$
$$C \rightarrow 0D \mid 1A \mid \epsilon$$
$$D \rightarrow 0D \mid 1D$$

(d)

$$S \rightarrow 0B \mid 1A \mid \epsilon$$
$$A \rightarrow 0D \mid 1C \mid \epsilon$$
$$B \rightarrow 0B \mid 1D$$
$$C \rightarrow 0D \mid 1A \mid \epsilon$$
$$D \rightarrow 0D \mid 1D$$

(e) None of the preceding.

6. Which one of the following grammars generates the language recognized by the following PDA?

$$0, \epsilon \rightarrow 0 \qquad\qquad 0, 0 \rightarrow \epsilon$$

$$\text{start} \rightarrow \boxed{q_0} \xrightarrow{\epsilon, \epsilon \rightarrow \$} \boxed{q_1} \xrightarrow{\epsilon, \epsilon \rightarrow X} \boxed{q_2} \xrightarrow{\epsilon, X \rightarrow \epsilon} \boxed{q_3} \xrightarrow{\epsilon, \$ \rightarrow \epsilon} \boxed{q_4}$$

$$1, \epsilon \rightarrow 1 \qquad\qquad 1, 1 \rightarrow \epsilon$$

(a) $S \rightarrow \epsilon \mid 1S1 \mid 0S0$

(b) $S \rightarrow \epsilon \mid 0 \mid 1 \mid 1S1 \mid 0S0$

(c) $S \rightarrow \epsilon \mid 1S1 \mid 0S0 \mid SS$

(d) $S \rightarrow \epsilon \mid 0 \mid 1 \mid 1S1 \mid 0S0 \mid SS$

(e) None of the preceding