ASSIGNMENT 1
CSC 425, FALL 2018

Due Thursday Nov 15 at 11:55 PM online. Helpful readings: CLR chapter on Probabiliistic Analysis and Randomized Algorithms, Approximation Algorithms, Maxflow (Maximum bipartite graph) (chapters 5, 26, 35 in edition 2).

Homework policy: See homework 1.

(1) Consider the algorithm for mincut. Suppose the input to the graph is given as an array of the nodes 1...,n and for each node there is a linked list of neighbors. Our goal is to implement the algorithm with the $O(n^2)$ repetitions in $O(mn^3)$ time.

a) How can you perform the repeated merges quickly? Analyze the running time over the course of the algorithm for this.

b) How can you select an edge uniformly at random efficiently, assuming you have access to a random number generator which can randomly generate a number between 0 and 1? Analyze the cost of picking a random edge (and maintaining the data structure to store the edges in between picks.

(2) As a variant of the above LRU algorithm, the 1-bit LRU algorithm (also known as the marking algorithm) associates each page in the cache with 1 bit.' If a cache page is recently used, the corresponding bit value is 1 (marked); otherwise, the bit value is 0 (unmarked). The algorithm works as follows:

Initially, all cache pages are unmarked; - Whenever a page is requested:

```
If the page is already in the cache, mark the page;
    Else:
       If there is at least one unmarked page in the cache,
              evict an arbitrary unmarked page,
             bring the requested page in, and mark it;
      Else unmark all the pages and start a new phase.
```

Show that the above deterministic 1-bit LRU algorithm has a competitive ratio of k.

(3) Bin packing approximation: Approx Problem 35-1 parts b,c,d,e,f only from CLR (see attachment)

(4) Bipartite matching Problem 26-7– Hopcroft-Karp Problem from CLR (see attachment)