MATH 442/551 Assignment #3

Due Thursday October 25, in class

1. Show that, if an orbit Γ is not a closed orbit, and its ω limit set $\omega(\Gamma)$ is a closed orbit, then, there exists a positively invariant set bounded by the orbit Γ and $\omega(\Gamma)$, such that $\omega(\Gamma)$ is the omega limit set of all orbits starting in this positively invariant set. (Hint, firstly, show that there exists a neighborhood of a closed orbit that contains no equilibrium; secondly, consider a cross section ℓ of $\omega(\Gamma)$).

2. Consider the following system

$$\begin{aligned} \frac{dx}{dt} &= x(3-2x-y), \\ \frac{dy}{dt} &= y(3-x-2y). \end{aligned}$$

   (a) Show that all orbit starting in the first quadrant are positively bounded. (Hint, show that the square $[0, M] \times [0, M]$ is positively invariant for large enough M).

   (b) Find the equilibria and classify them.

   (c) Show that all orbits with a positive initial condition must approach the positive equilibrium. (Hint, we have shown in class that there is no closed orbit in the first quadrant).