MATH 442/551 Assignment #4

Due Monday November 19, in class

1. Consider a Hamiltonian system

$$\frac{dp}{dt} = -\frac{\partial H(p,q)}{\partial q},$$
$$\frac{dq}{dt} = \frac{\partial H(p,q)}{\partial p}.$$

   (a) Show that, if the point (p^*, q^*) is a local minimum of the Hamiltonian $H(p,q)$, then it is a stable equilibrium of the system.

   (b) Show that the pendulum equation

$$\dot{x} = y$$
$$\dot{y} = -\omega^2 \sin x$$

   is a Hamiltonian system, (i.e., identify the Hamiltonian $H(y,x)$), and show that the origin is locally stable. Is it locally asymptotically stable?

2. Consider the system

$$\frac{dx}{dt} = -ax + y,$$
$$\frac{dy}{dt} = 1 + x^2 - y,$$

   where the parameter $a > 0$.

   (a) Find the bifurcation point (equilibrium and the corresponding parameter value).

(b) Approximate the extended center manifold (including a) with a polynomial up to the second order. (First shift the equilibrium at the bifurcation point to the origin, and shift the parameter value to zero).

(c) Show that a saddle node bifurcation occurs at the bifurcation point.