Each question has **EXACTLY ONE** correct answer.

1. Which one of the following statements is true for every $L, L' \subseteq \Sigma^*$?

   (a) If L is not regular and $L' \subseteq L$ then L' is not regular.
   (b) If L is regular and $L' \subseteq L$ then L' is regular.
   (c) If L is regular and $L \cup L'$ is not regular, then L' is not regular.
   (d) If L is regular and $L \cup L'$ is regular, then L' is regular.
   (e) None of the above.

   (a) False: take $L' = \emptyset$
   (b) False: take $L = \{0,1\}^*$ and $L' = \{0^n 1^n \mid n \geq 0\}$
   (c) True since regular languages are closed under union
   (d) False: take $L = \{0,1\}^*$ and $L' = \{0^n 1^n \mid n \geq 0\}$

2. Which one of the following languages is regular?

   (a) $\{ww \mid w \in \{0,1\}^+\}$
   (b) $\{ww^R \mid w \in \{0,1\}^*\}$
   (c) $\{wcw^R \mid w \in \{0,1\}^*\}$, $0, 1 \neq c \in \Sigma$
   (d) $\{wtw \mid w, t \in \{0,1\}^+\}$
   (e) $\{wtw^R \mid w, t \in \{0,1\}^*\}$

   (a)-(c) are all non-regular just using variations of proofs we have seen. For (d), let $L' = \{wtw \mid w, t \in \{0,1\}^+\} \cap 0^*10^*$. Then $L' = \{0^n 1 0^n \mid n \geq 0\}$, for which we can again use a pumping lemma proof to prove non-regularity. For (e), note that any string in $\{0,1\}^*$ is in this language (just take $w = \epsilon$.) So this language is just $\{0,1\}^*$, which is regular. (This is probably a bit too tricky for a test question.)

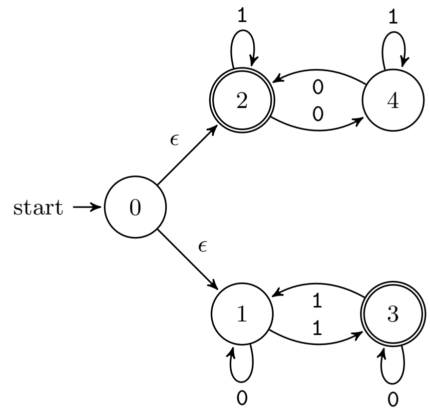
3. Which of the following is a regular expression for the language

   $$\{w \in \{0,1\}^* \mid w \text{ contains the same number of occurrences of 01 and 10}\}?$$

   (a) The language is not regular
   (b) $(010)^*$
   (c) $0(0+1)^*0 + 1(0+1)^*1 + \epsilon$
   (d) $(01+10)^*$
   (e) $(0+1)(0+1)^+(0+1) + \epsilon$

   This question is a bit too tricky too. Suppose the string starts with 0. As long as we have 0's, there are the same number of 01's and 10's. As soon as we have a 1, we eventually have to have a 0 (otherwise there will be more 01's than 10's.) At this point we have the same number of 01's and 10's. This just continues on, but it means the last symbol has to be a 0. Similary, if the string starts with 1 it has to end with one. Between the beginning and end anything can happen. So the answer is (c).

For questions 4-5 refer to the following NFA $M = (Q, \Sigma, \delta, q_0, F)$:

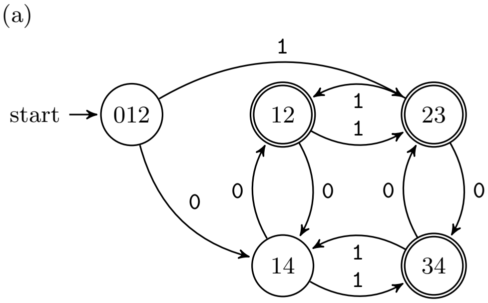


4. Which of the following is true? (Remember: 0 is an even number.)
   (a) $L(M) = \{w \in \{0,1\}^* \mid w$ contains more 0's than 1's$\}$
   (b) $L(M) = \{w \in \{0,1\}^* \mid w$ contains an odd number of 0's or an even number of 1's$\}$
   (c) $L(M) = \{w \in \{0,1\}^* \mid w$ contains an even number of 0's or an odd number of 1's$\}$
   (d) $L(M) = \{0,1\}^*$
   (e) None of the above.

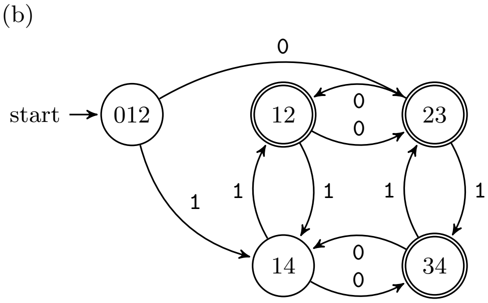
   The top branch accepts strings with an even number of 0's and the bottom accepts strings with an odd number of 1's. So the answer if (c)

5. Which of the following DFAs results when the lazy algorithm presented in class is applied to M?

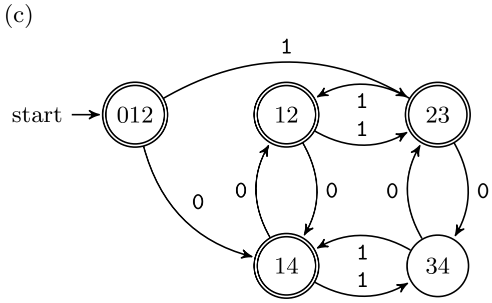
(a)



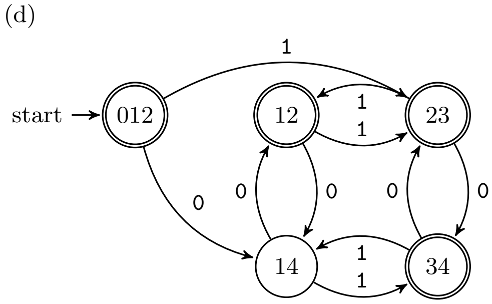
(b)



(c)



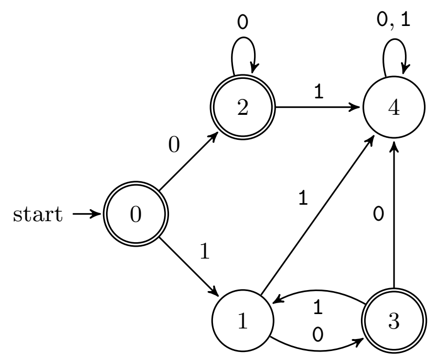
(d)



(e) None of the above.

(a) and (b) are out because 012 should be an accepting state. (c) is out because 14 should not be an accepting state. Using the lazy algorithm will lead to (d).

Consider the following DFA $M = (Q, \Sigma, \delta, q_0, F)$.



6. For which one of the following regular expressions R do we have $L(R) = L(M)$?
   (a) $(0 \cup 1)(0 \cup 01)^*$ (b) $0^* \cup (01)^*$ (c) $00^* \cup 1(01)^*$ (d) $(0 \cup 01)^*$ (e) None of the preceding.

The answer is (e). (a) and (c) are out because they do not include ϵ. (b) and (d) are out because they do not include 10, which is accepted by M (going from state 0 to state 1 to state 3.)