MATH 442/551 Assignment #4

Due Wednesday December 5, in class

1. Consider the system (this is a SIRS disease model with imperfect vaccination, where $V = 1 - S - I - R$ is the fraction of vaccinated population)

$$\begin{aligned}\frac{dS}{dt} &= -rSI - \phi S + \theta(1 - S - I - R) + \rho R \\ \frac{dI}{dt} &= rSI + \sigma r(1 - S - I - R)I - I, \\ \frac{dR}{dt} &= I - \rho R\end{aligned}$$

   (a) Show that

$$(\frac{\theta}{\phi + \theta}, 0, 0)$$

   is always an equilibrium.

   (b) Discuss the bifurcartion of this equilibrium with the parameter r, and draw bifurcation diagram (equilibrium as a function of r) about the bifurcation point for the two cases:

      i. θ is very small,

      ii. θ is very large.