A Controller Designed for A Buck Converter

*I have not received, nor have I given, any help and assistance on this project

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***Abstract*— Buck converter is a commonly used electrical device in almost every industry. The basic operation of the buck converter has the current in an inductor controlled by two switches (usually a transistor and a diode). In the idealized converter, all the components are considered ideal. Specifically, the switch and the diode have zero voltage drop when on and zero current flow when off, and the inductor has zero series resistance. In the practice, the output voltage of buck converter could have some oscillations, so that a controller is necessary to be designed to control the current and the voltage. By designing a buck converter circuit, a PI controller and simulation, the output voltage can be stable at the desired value. After adding current loop, the current of inductor could be stable at an expected value with a ripple that is limited within 1%.**

***Index Terms*—Buck Converter, Controller Design, Double-Loop Controller**

I. Introduction

Switch mode buck (step-down) DC/DC converters originated with the development of pulse width modulated (PWM) buck converters. The conceptual model of the buck converter is best understood in terms of the relation between current and voltage of the inductor. Beginning with the switch open (off-state), the current in the circuit is zero [1]. When the switch is first closed (on-state), the current will begin to increase, and the inductor will produce an opposing voltage across its terminals in response to the changing current. This voltage drop counteracts the voltage of the source and therefore reduces the net voltage across the load. Over time, the rate of change of current decreases, and the voltage across the inductor also then decreases, increasing the voltage at the load. During this time, the inductor stores energy in the form of a magnetic field. If the switch is opened while the current is still changing, then there will always be a voltage drop across the inductor, so the net voltage at the load will always be less than the input voltage source. When the switch is opened again (off-state), the voltage source will be removed from the circuit, and the current will decrease. The decreasing current will produce a voltage drop across the inductor (opposite to the drop at on-state), and now the inductor becomes a Current Source. The stored energy in the inductor's magnetic field supports the current flow through the load. This current, flowing while the input voltage source is disconnected, when appended to the current flowing during on-state, totals to current greater than the average input current (being zero during off-state). The "increase" in average current makes up for the reduction in voltage, and ideally preserves the power provided to the load. During the off - state, the inductor is discharging its stored energy into the rest of the circuit. If the switch is closed again before the inductor fully discharges (on-state), the voltage at the load will always be greater than zero.

A proportional-integral-derivative controller (PID controller) is a control loop mechanism employing feedback that is widely used in industrial control system and a variety of other applications requiring continuously modulated control [2].

In this study, a buck converter circuit is designed in Sec. II; The state space model of this circuit is built up in Sec. III; In Sec. IV, we designs the controller for only voltage control that a P controller is designed using root locus method, a PI controller is designed using frequency domain method, the performance of PI controller is evaluated by comparison of transfer function form and circuit form; In Sec. V, we designs a double-loop controller that could not only control the voltage but also the current that a P controller is for current control and a PI controller is for voltage control; Sec. VI is our conclusion of this study.

II. Circuit Design

A. Design parameters

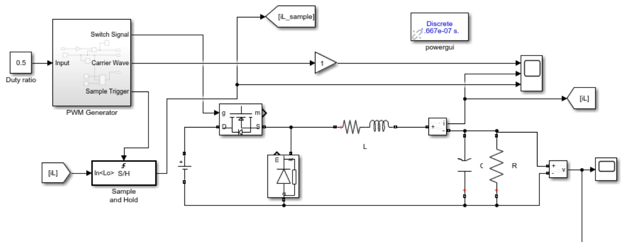
The design parameters of the buck converter were shown at below in Table I.

TABLE I. PARAMETERS OF THE BUCK CONVERTER

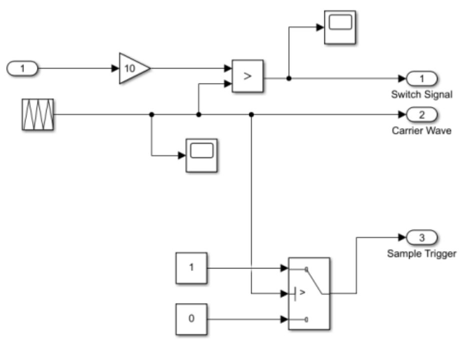
Converter Parameters	Value
Input Voltage, *Vi*	30 V
Output Voltage, *Vo*	15 V
Switching Frequency, fs	60 kHz
PWM Sawtooth Peak Voltage, Vs	10 V
Converter Inductor, L	250 uH
Inductor Resistance, RL	0.2 Ohm
Converter Capacitor, C	30 mF
Load Resistance, R	10 Ohm

B. Open-Loop Circuit Model

To evaluate the performance of the buck converter circuit, firstly build a typical buck converter circuit model in the Simulink as the fig. 1 shows.



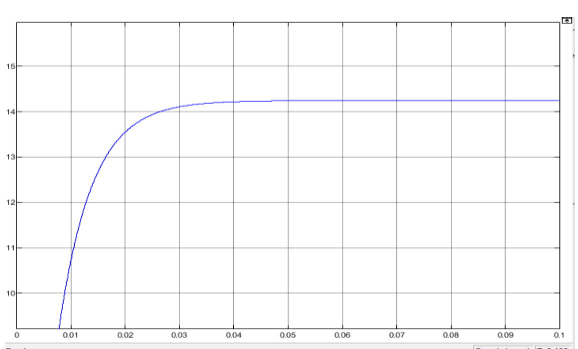
(a) Open-loop circuit



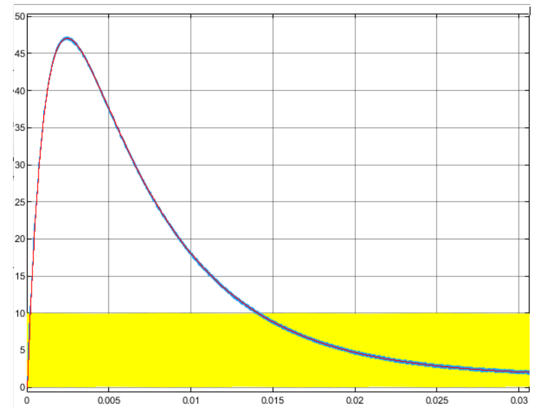
(b) PWM generator

Fig 1. Open-loop buck converter circuit

This circuit model contains PWM generator to generate sawtooth wave, the input terminal of the PMW generator is our duty ration and inside the generator, the duty ration is multiplied by a gain to get the reference voltage that could be compared with the sawtooth wave. This circuit includes an IL sampler, a sample-and-hold block to sample the average IL. Input reference voltage of 5V to the PWM block that could ideally generate a duty ratio around 0.5, run the simulation. The results can be obtained from the following pictures.



(a) Output Voltage



(b) Inductor Current, yellow line represents sawtooth wave, blue line represents IL, red line represents average IL.

Fig 2. Open-loop buck converter simulation

From the results of the simulation of the open-loop system, we can get that the output voltage is around 14.3 volt and the current have a response curve going up and down.

III. MODELLING

This section builds up the dynamic modelling of designed buck converter to attain the transfer function of open loop system on which this study based to evaluate the stability of closed loop system.

A. State Space Averaging Modeling

This subsection derives the generalized equation for the continuous mode of the basic power converters. The continuous mode operation is simpler that it has only two states. The analysis stats from the state-space equations during the switch's on and off states and uses an averaging method to linearize them. When the switching device is turned on, it conducts for a ration D of a period.

To build the model, let

$$\begin{cases} x = \begin{bmatrix} i_L \\ V_C \end{bmatrix} \\ u = [V_i] \\ y = [V_o] \end{cases} \tag{1}$$

- When switch is on:

$$\dot{x} = A_1 x + B_1 x \qquad y = C_1 x \tag{2}$$

$$\begin{cases} \dfrac{dI_L}{dt} = -\dfrac{I_L R_L}{L} - \dfrac{V_C}{L} + \dfrac{V_i}{L} \\ \dfrac{dV_c}{dt} = -\dfrac{1}{RC} V_C + \dfrac{1}{C} I_L \\ V_o = V_C \end{cases} \tag{3}$$

where R is R_{load}. From (1) (2) (3), could derive

$$A_1 = \begin{bmatrix} -\frac{R_L}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \quad B_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \quad C_1 = [0 \ \ 1] \tag{4}$$

- When switch is off

$$\dot{x} = A_2 x + B_2 x \qquad y = C_2 x \tag{5}$$

$$\begin{cases} \frac{dI_L}{dt} = -\frac{I_L R_L}{L} - \frac{V_C}{L} \\ \frac{dV_c}{dt} = -\frac{1}{RC} V_C + \frac{1}{C} I_L \\ V_o = V_C \end{cases} \tag{6}$$

From (4) (5) (6), could get

$$A_2 = \begin{bmatrix} -\frac{R_L}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad C_2 = [0 \ \ 1] \tag{7}$$

$$A_1 = A_2 = \begin{bmatrix} -\frac{R_L}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} = \begin{bmatrix} -800 & -4000 \\ 33.33333 & -3.33333 \end{bmatrix} \tag{8}$$

$$B_1 = \begin{bmatrix} 4000 \\ 0 \end{bmatrix} \ B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{9}$$

$$C_1 = C_2 = [0 \ \ 1] \tag{10}$$

- Average modelling

$$\dot{x} = \{D\, A_1 + (1-D)A_2\}x + \{DB_1 + (1-D)B_2\}u \tag{11}$$

$$y = \{DC_1 + (1-D)\,C_2\}x \tag{12}$$

B. *Calculate Equilibrium Points*

To get the value of duty ratio d, we need to calculate the equilibrium point and do the local linearization around the equilibrium point.

$$x_0 = -\{D\, A_1 + (1-D)A_2\}^{-1}\{DB_1 + (1-D)B_2\}V_i \tag{13}$$

From questions (7) (8) (9)(12) (13),

$$x_0 = \begin{bmatrix} 2.94D \\ 29.41D \end{bmatrix} \tag{14}$$

$$V_o = C_1 x_0 = 29.41D \tag{15}$$

$$d = 0.51 \tag{16}$$

C. *Open-loop System Transfer Function*

The transfer function from the duty cycle to the output voltage could be written as:

$$P(s) = \frac{\hat{y}(s)}{\hat{d}(s)} = \frac{\hat{V_o}(s)}{\hat{d}(s)} = C(sI - A)^{-1}B \tag{17}$$

$$\begin{cases} A = DA_1 + (1-D)A_2 = \begin{bmatrix} -\frac{R_L}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} = \\ \qquad \begin{bmatrix} -800 & -4000 \\ 33.33333 & -3.33333 \end{bmatrix} \\ B = DB_1 + (1-D)B_2 = \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix} = \begin{bmatrix} 2040 \\ 0 \end{bmatrix} \\ C = DC_1 + (1-D)C_2 = [0 \ \ 1] \end{cases} \tag{18}$$

From equation (16) and (17), we can get the open-loop system transfer function,

$$P(s) = \frac{\hat{y}(s)}{\hat{d}(s)} = \frac{\hat{V_o}(s)}{\hat{d}(s)} = \frac{68000}{s^2 + 803.33s + 136000} \tag{19}$$

IV. Voltage Controller Design

The voltage control logic of designed circuit could be presented as the following picture [3].

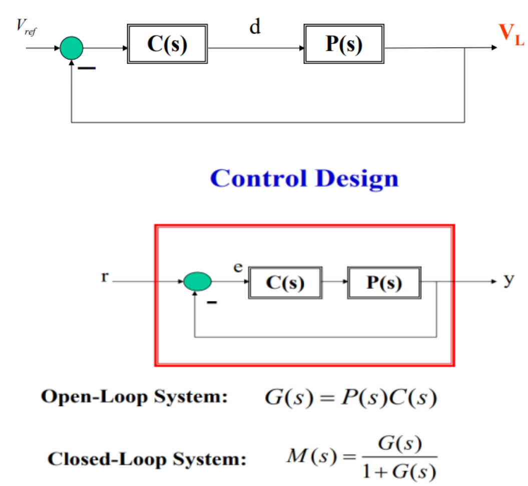


Fig 3. Voltage control logic

From the picture, the P(s) represents the plant model, which have been obtained in last section. C(s) is the controller model which needs to be designed.

A. *Design P Controller Using Root Locus Method*

Chose a Kp that makes the closed-loop system could have a damping ration bigger than 0.55. As the picture below shows, this study selects Kp as 2.9 and the damping ration of the closed-loop system is 0.551 when Gain equals 1.

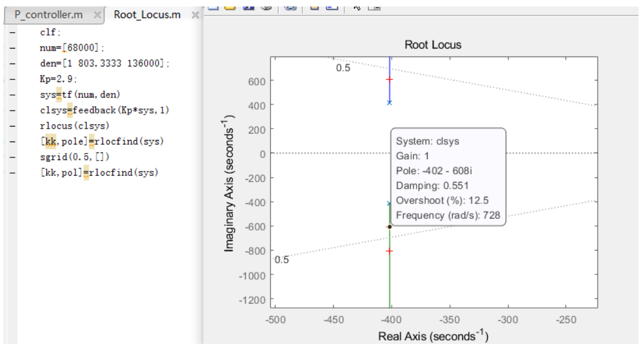


Fig 4. Root locus plot

To get the impact of changing Kp, this study plots the step response of kp from 1 to 5 shown as Fig. 5.

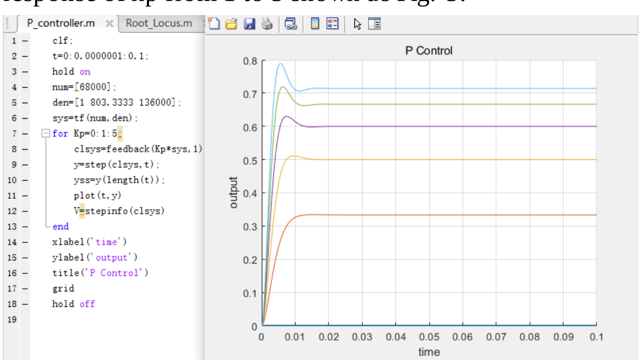


Fig 5. Step response

From the picture, we can get the trade-off of selecting Kp for the closed-loop system. When Kp increases: the peak time decreases; the overshoot is bigger; the settling time is smaller; the steady-state error is smaller. In this case, this study selects Kp = 2.9, so that the C(s) = Kp =2.9.

B. *Design PI Controller Using Frequency Domain Method*

The transfer function of the PI controller could be written as:

$$C(s) = K_P + \frac{K_I}{s} = K_P * \left(\frac{s + \frac{K_I}{K_P}}{s} \right) \qquad (20)$$

To simplify the design process and make it easier to be calculated, we design KI/Kp as a whole. Firstly, we need to draw a Bode plot of open-loop system to meet the requirement that the system has at least a phase margin of 75 degrees. Firstly, I select KI/Kp = 5, plot the Bode plot using different Kp from 0 to 8.

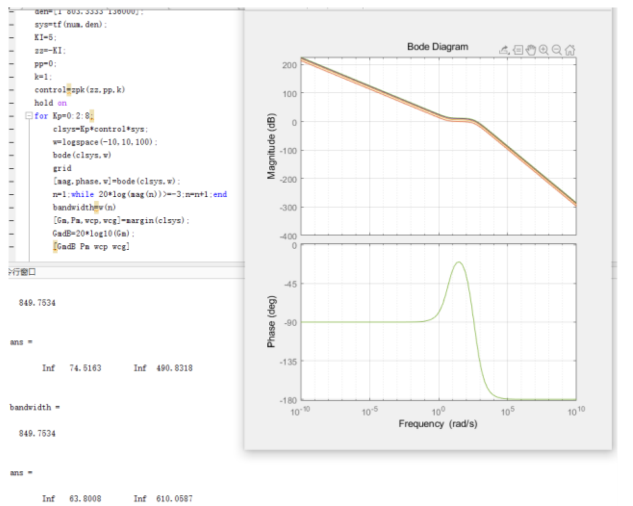


Fig 6. Bode plot

From the simulation I found that increasing Kp will increase the bandwidth from 0 to 849.7534 and maintain this value. Additionally, increasing Kp will decrease the phase margin. To have at least a phase margin of 75 degree, Kp need to be smaller than 5.8. After changing KI/Kp to 10 and 30 with Kp = 5.8, I find that increasing KI could decrease phase margin, but the impact is very small. Increasing KI/Kp to 30 just decrease the phase margin for about 3 degree.

RiseTime:	0.2481	RiseTime:	0.0368
SettlingTime:	0.6794	SettlingTime:	0.1073
SettlingMin:	0.9000	SettlingMin:	0.9004
SettlingMax:	0.9977	SettlingMax:	0.9974
Overshoot:	0	Overshoot:	0
Undershoot:	0	Undershoot:	0
Peak:	0.9977	Peak:	0.9974
PeakTime:	1.2576	PeakTime:	0.1957

Fig 7. Comparison of step responses of closed-loop system with Ki /Kp =5 (left) and Ki /Kp =30 (right)

Then run the step response of closed-loop system with

Ki/Kp =5 and 30. The differences could be obtained from Fig.7. Rising Ki could decrease the rising time, settling time and peak time. So, for a faster response, this study selects Kp = 5.8 and Ki = 30*5.8=174. And the step response curve is shown as Fig. 8.

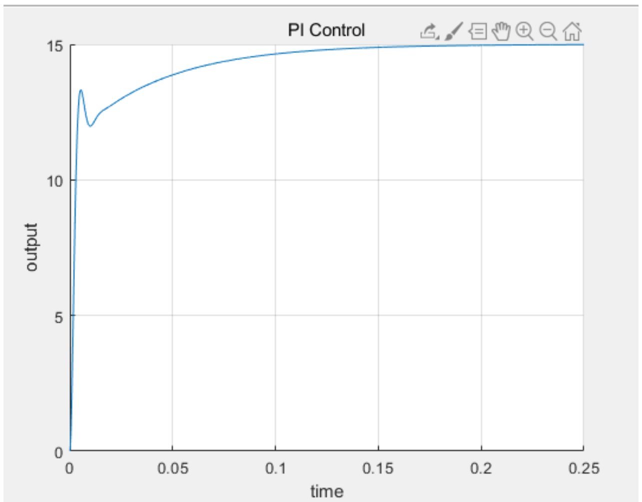


Fig 8. Step response of closed-loop system using PI controller

As a result, the transfer function of this PI controller could be written as:

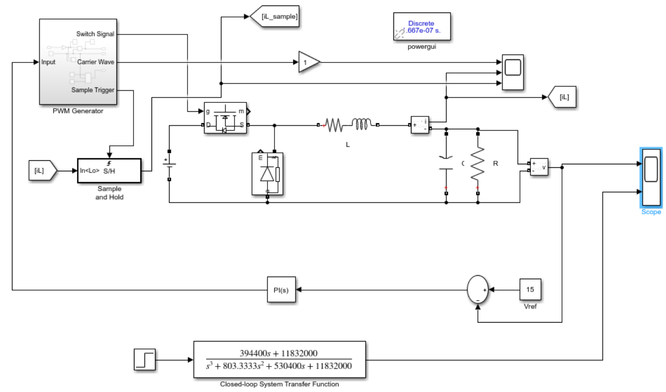
$$C(s) = K_P + \frac{K_I}{s} = 5.8 * \left(\frac{s+30}{s}\right) \tag{21}$$

C. Simulate the Overall Closed-loop System and Performance Analysis

The closed-loop transfer function could be written as:

$$M(s) = \frac{C(s) * P(s)}{1 + C(s) * P(s)} = \frac{394400s + 11832000}{s^3 + 803.33s^2 + 530400s + 11832000} \tag{22}$$

Which represents the transfer function from the reference voltage to the output voltage. Now we simulate the closed-loop system both in transfer function form and in circuit form and compare the simulates results to verify the correctness of this study.



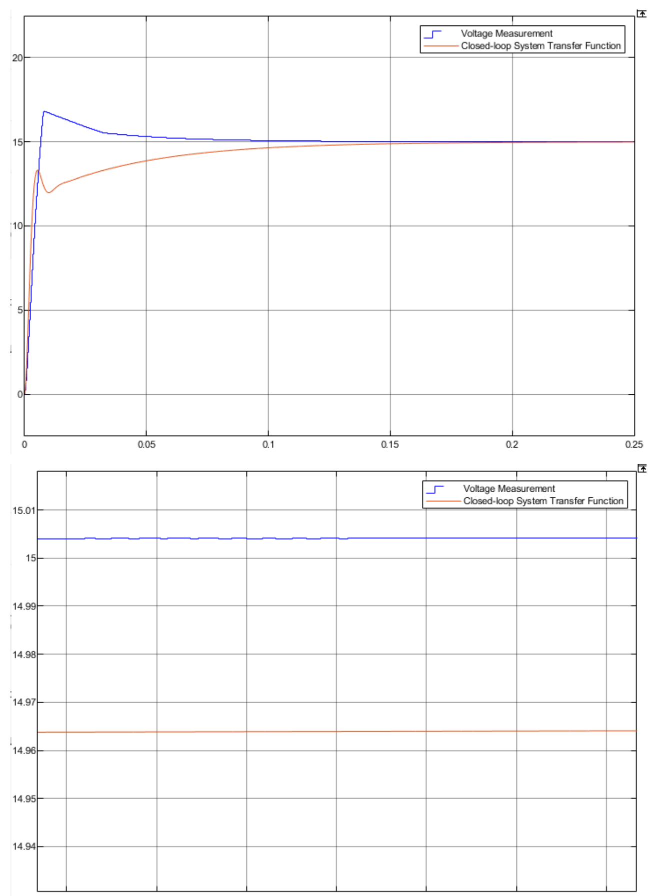
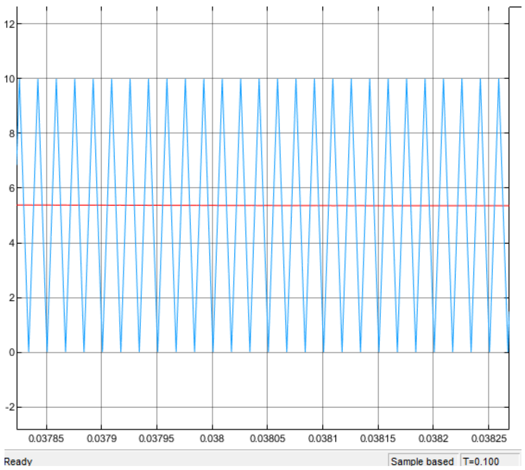


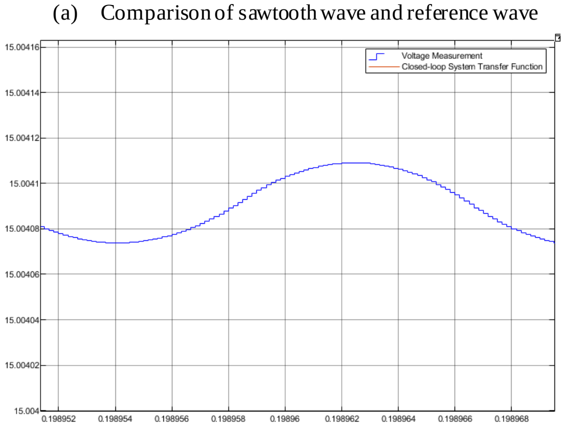
Fig 9. Comparison of two forms

From Fig. 9. We can obtain the results of both transfer function form and circuit form. Due to the MOSFET switch, the circuit model could have differences in results with the transfer function model, but the final value could be steady at the reference value without oscillations. The output voltage of circuit form is finally steady at 15.004V, the output of the transfer function form is finally steady at 14.964. The peak time of the circuit form is 0.007 second, transfer function form takes longer time shown as the following table.

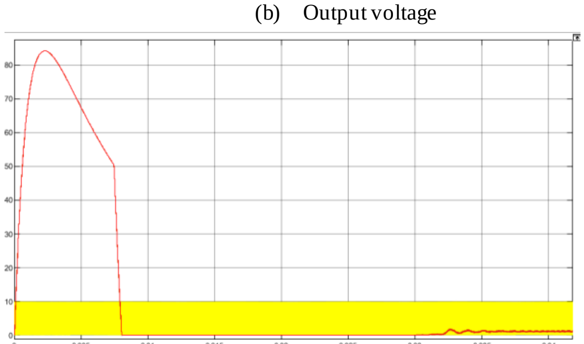
	Transfer Function	Circuit
Peak time	0.1957	0.007
Settling time	0.1073	0.0277
Steady-state error	0.036	0.004
Overshoot	0%	12.3%



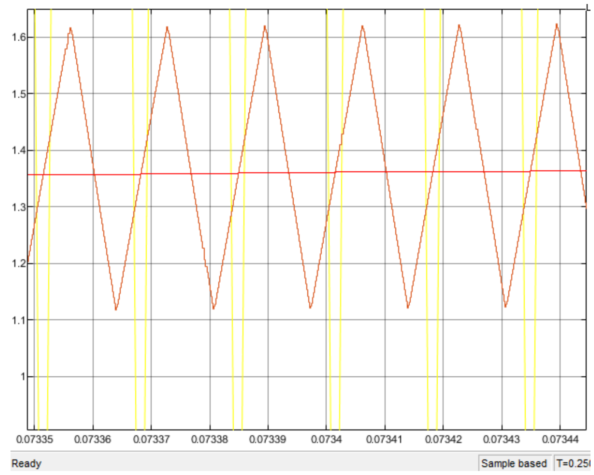
(a) Comparison of sawtooth wave and reference wave



(b) Output voltage



(c) Inductor current



(d) Average Inductor current

Fig 10. Comparison of sawtooth wave and reference wave.

From the Fig.10, we can get that the reference wave generated by PI controller is finally stable at around 5.28V, compared to the peak value of sawtooth 10V, the ratio is around 0.528, which is close to our desired duty ratio 0.51. And the output voltage ripple is smaller than 0.001%. The inductor current firstly increases to 83A and then decrease to a steady state which has a ripple of 36% and its average value could be stable around 1.36 A.

In conclusion, here are the pros and cons of this design based on the simulation results of circuit form. The pros of this design are:

1. The closed-loop system could be stable at steady state with a ripple smaller than 0.001%.
2. The peak time is small as 0.007 second.

The pros of this design are:

1. Still has 12.3% overshoot.
2. The overshoot of the current is too big and could do harm to the circuit.
3. The current ripple is about 36%, bigger than 20%.

V. Current-loop and Voltage-loop CONTROLLER DESIGN

The double-loop system can not only control the output voltage but also current voltage, of which the configuration can be presented as the following picture.

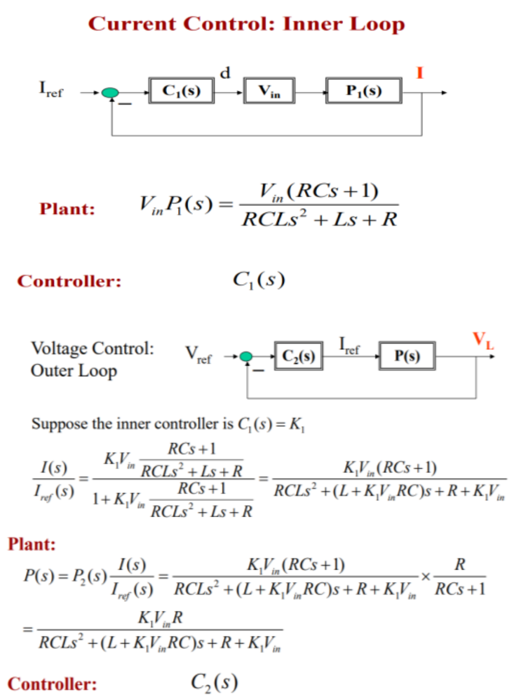


Fig 11. Configuration of the current loop and the voltage loop.

From Fig. 11, we can get the transfer function of current loop,

$$V_{in}P_1(s) = \frac{9s+30}{75us^2+250us+10} \tag{23}$$

After simulation of running step responses, this study designs a P controller with Kp = 100, then we can get the plant of the voltage loop,

$$P(s) = \frac{30000}{75us^2+900us+3000} \tag{24}$$

Then run step responses of obtained plant, design a PI controller with Kp=1000 and Ki = 1000. The whole system could be shown as follows.

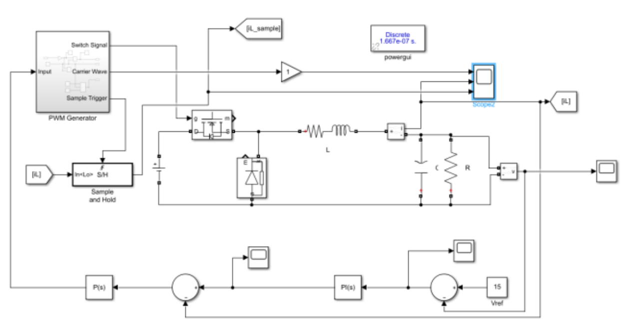


Fig 11. Circuit of the current loop and the voltage loop.

Then do a simulation of this circuit.

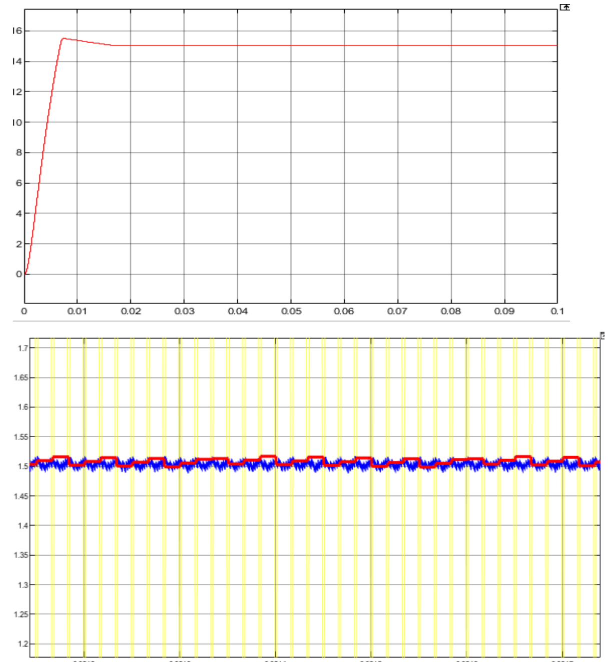


Fig 12. Simulation result.

From the Fig. 12, we can get that the voltage could be stable at around 15V. and the current could be stable at our desired value 1.5A, and the current ripple could be limited in 1% that is much smaller than the circuit with only voltage control.

VI. Conclusion

In this study, a buck converter circuit is designed, a P controller is designed using root locus method, a PI controller is designed using frequency domain method, the performance of PI controller is evaluated by comparison of transfer function form and circuit form. The voltage could be stable at desired value. Then this study designs a double-loop controller, a P controller for current loop and a PI controller for voltage loop. By using the double-loop design, the current could be stable at a desired value 1.5A with a ripple less than 1%, which is much smaller than the circuit with only voltage control.

VII. Reference

1. https://en.wikipedia.org/wiki/Buck_converter
2. https://en.wikipedia.org/wiki/PID_controller
3. Lecture Notes of EVE 5430