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ABSTRACT

This paper develops a multi-firm equilibrium model of information acquisition based on differences in firms' characteristics. The model shows that heightened economic uncertainty amplifies stock price reactions to earnings announcements via increased investor attention, which varies by firm characteristics. Firms with higher systematic risk or more informative announcements attract more attention and exhibit stronger reactions to earnings announcements. Moreover, **heightened investor attention caused by high economic uncertainty leads to a steeper CAPM relation** and higher betas for announcing firms. Empirical analyses using firm-level attention measures and CAPM tests on high- versus low-attention days support the model's predictions.

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1. Introduction

We explore the impact of economic uncertainty and investor attention on asset prices in a multi-firm equilibrium model of information acquisition. The motivation for this inquiry starts with a large body of theoretical and empirical research that studies the tradeoffs imposed by the limited attention theory (Sims, 2003; Hirshleifer and Teoh, 2003; Peng, 2005). Limited attention models can explain a wide array of phenomena, such as the home bias puzzle (Van Nieuwerburgh and Veldkamp, 2009), investment and attention allocation behavior (Van Nieuwerburgh and Veldkamp, 2010), the attention allocation of mutual fund managers (Kacperczyk et al., 2016), or the comovement of asset returns (Peng and Xiong, 2006; Veldkamp, 2006). Despite these findings, important questions remain unanswered. Specifically, how are investors' attention choices priced in financial markets, particularly when investors can selectively choose which firms and corporate information to pay attention to? Additionally, how does systematic and cross-sectional variation in investor attention impact risk

exposures as reflected in the capital asset pricing model (CAPM)? Given the first-order effect that attention has on financial markets (Da et al., 2011; Andrei and Hasler, 2015), answering the latter question is particularly important. To this end, we develop an equilibrium model of information acquisition.

Our model is a multi-firm variant of Grossman and Stiglitz (1980), in which firms make earnings announcements and investors tailor their attention to any combination of firms' announcements. We focus on earnings announcements because they are salient information releases by firms that convey firm-specific and, potentially, macroeconomic/systematic information. We allow for investors' attention decisions to depend on the economic uncertainty investors face. This facilitates predictions about how aggregate uncertainty affects information acquisition, investor demand for shares, and the intertwined CAPM pricing of both corporate announcements and macroeconomic risk.

The model predicts that increased uncertainty attracts more investor attention to firm-level information, amplifying stock price reactions to the earnings announcements, hereafter referred to as earnings response coefficients (ERCs). The effect of attention on ERCs varies predictably with firm-specific factors: ERCs increase incrementally more for firms that have (i) a stronger exposure to systematic risk; (ii) more informative earnings announcements; (iii) a more volatile idiosyncratic component in their earnings; and (iv) more noise trading. The intuition behind all these four cases is that the benefit of collecting information outweighs its cost for these firms, which attracts more investor attention to their announcements.

Our model also predicts a higher risk premium and a steeper CAPM relation on days of heightened investor attention caused by high uncertainty. We demonstrate that the beta of a firm that announces earnings increases with the fraction of investors who pay attention to its announcement. Furthermore, when investors are collectively more attentive, resolving uncertainty, they are rewarded with a higher risk premium (Robichek and Myers, 1966; Epstein and Turnbull, 1980), which steepens the CAPM relation. While the increase in the market risk premium due to higher uncertainty is an obvious equilibrium outcome in asset pricing models, the steepening of the securities market line caused by heightened investor attention is novel. We also extend the model to show that the impact of uncertainty on attention is more significant for investors with lower information processing costs (e.g., institutional investors) and that our main results continue to hold in a dynamic setting.

To test the model's predictions, we use the VIX as a measure of economic uncertainty and SEC EDGAR downloads as a proxy for investor attention.¹ We find that investors pay more attention to earnings announcements on days with higher VIX, and that ERCs are larger for firms that announce on those days. We attribute this effect primarily to the increase in investor attention. Additionally,

we show that our ERC results are concentrated in firms with high CAPM beta (whose announcements are more likely to convey systematic information), higher institutional ownership (whose cost of information acquisition is likely lower), and more noise trading (whose stock prices are likely less informative). Furthermore, our findings indicate that investor attention is responsible for increased market betas on earnings announcement days, and we also find empirical support for a steeper CAPM relation on days with heightened investor attention.

Our study extends prior attention theories in two ways, providing rational explanations for several empirical findings in the literature previously attributed to behavioral factors. First, in our setting, firm-level announcements offer valuable information about both announcing firms and the overall economy. Existing theories overlook such *information spillovers* and restrict investors' attention to systematic or idiosyncratic news (e.g., Peng and Xiong, 2006; Kacperczyk et al., 2016). In our model, information spillovers result in a positive relationship between uncertainty and attention and influence firms' market betas on announcement days. Information spillovers also result in weaker ERCs with more firms announcing: as the number of announcements increases, prices convey more market-wide information for free, reducing attention incentives. This spillover effect contrasts with the explanation in Hirshleifer et al. (2009), which attributes weaker ERCs to multiple announcements competing for investors' limited attention (a cognitive constraint effect). Lastly, related theories investigate information spillovers in similar contexts (Patton and Verardo, 2012; Savor and Wilson, 2016), but do not address the interaction between information spillovers and investor attention or the impact of attention on ERCs and the CAPM equilibrium.

Second, the aggregate amount of attention in our economy fluctuates with incentives tied to economic uncertainty, whereas previous models bind attention to a fixed capacity constraint (Sims, 2003; Peng and Xiong, 2006; Kacperczyk et al., 2016). Our setup relaxes this constraint and instead assumes that investors face disclosure processing costs (Blankespoor et al., 2020). As a result, the aggregate amount of attention increases on days with higher uncertainty, which explains the steepening of the securities market line. Conversely, attention decreases on days with lower uncertainty or less informative announcements. This latter result offers an alternative and rational explanation for investors' inattention to Friday announcements (DellaVigna and Pollet, 2009; Louis and Sun, 2010; Michaely et al., 2016b). That is, Friday announcers may have different firm characteristics than non-Friday announcers, a prediction consistent with the empirical findings of Michaely et al. (2016a).

In a related empirical paper, Hirshleifer and Sheng (2022) also challenge the idea of fixed attention capacity constraints. They provide evidence that investors can potentially devote more or less attention to both macro and micro news (see also Eberbach et al., 2021). While our empirical findings are consistent with those in Hirshleifer and Sheng (2022), different from that study, we build a theory to explain these findings. In addition, we derive and analyze the cross-sectional

¹ As an alternative attention proxy, we confirm our results using Google stock ticker searches attributable to investors (deHaan et al., 2021).

implications of investors' rational responses to heightened uncertainty using EDGAR (Google) searches.²

Our study adds to the rapidly growing literature that documents a robust beta-return relation on various occasions: on macroeconomic announcement days; when investor attention is strong; in months after the U.S. midterm elections; on leading earnings announcement days; or overnight (Savor and Wilson, 2014; Ben-Rephael et al., 2021; Chan and Marsh, 2021a; 2021b). We contribute to this literature by showing theoretically that heightened investor attention leads to a steeper beta-return relation and increases firms' market betas on the days of their announcements.

Overall, our paper shows that economic uncertainty is a key driver of investors' attention to firm-level information. Rational attention behavior has critical asset pricing implications. First, investor attention to firm-level information varies by firms' characteristics, leading to differences in ERCs across firms. Second, heightened attention leads to higher market betas for announcing firms and a steeper securities market line. Consequently, investor attention may be an underappreciated factor in explaining the cross-section of asset returns.

2. Model

Consider an economy populated by a continuum of investors, indexed by $i \in [0, 1]$. The economy has three dates $t \in \{0, 1, 2\}$. At $t = 0$, each investor makes an information acquisition decision that we will describe below. At $t = 1$, investors trade competitively in financial markets. At $t = 2$, financial assets' payoffs are realized, and investors derive utility from consuming their terminal wealth. Investors trade a riskless asset and N risky assets indexed by $n \in \{1, \dots, N\}$. The riskless asset is in infinitely elastic supply and pays a gross interest rate of 1 per period. Each risky asset ("firm") has an equilibrium price P_n at $t = 1$ and pays a risky dividend at $t = 2$:

$$D_n = b_n f + e_n, \quad \text{for } n \in \{1, \dots, N\}. \quad (1)$$

The payoff D_n has a systematic component f and a firm-specific component e_n . The parameters b_n , which are heterogeneous across firms and known by investors, dictate the exposures of firms' payoffs to the systematic component. Without loss of generality, we assume that the average of b_n across firms is 1.

We denote by \mathbf{D} the $N \times 1$ vector of asset payoffs, by \mathbf{P} the $N \times 1$ vector of asset prices, and by $\mathbf{R}^e \equiv \mathbf{D} - \mathbf{P}$ the vector of dollar excess return of the risky assets.³ We set the total number of shares for all assets to \mathbf{M} (henceforth the *market portfolio*), an equal-weighting vector with all elements equal to $1/N$. The future market return is $R_M^e \equiv$

$\mathbf{M}'\mathbf{R}^e$. Assuming an equally-weighted market portfolio \mathbf{M} does not impact our results but aids in interpreting them in empirically measurable terms (elaborated below).

At $t = 0$, all investors have a common information set \mathcal{F}_0 that consists of the prior distributions of f and e_n :

$$f \sim \mathcal{N}(0, U^2) \quad (2)$$

$$e_n \sim \mathcal{N}(0, \sigma_{en}^2), \quad \text{for } n \in \{1, \dots, N\}. \quad (3)$$

We allow for variances σ_{en}^2 to vary in the cross-section of firms. Firm-specific components e_n are independent across firms, and f and e_n are independent $\forall n \in \{1, \dots, N\}$.

We refer to U as *uncertainty* for the rest of the paper. It represents investors' expected forecasting error conditional on information available at time 0, $U^2 \equiv \text{Var}[f|\mathcal{F}_0]$. As we will show below, in our model U is closely related to investors' pre-announcement uncertainty about the future return on the market, which helps us confront the theory with the data.

Defining U as uncertainty is the simplest way to derive theoretical predictions. Alternatively, we could be more specific about the information set \mathcal{F}_0 , without any impact on the results. Assuming, for instance, that *before* time 0 investors hold the prior $f \sim \mathcal{N}(0, \sigma_f^2)$, and that at time 0 they observe public information about f under the form of a signal $G = f + g$ with $g \sim \mathcal{N}(0, \sigma_g^2)$, Bayesian updating implies

$$U^2 = \text{Var}[f|\mathcal{F}_0] = \frac{\sigma_f^2 \sigma_g^2}{\sigma_f^2 + \sigma_g^2}. \quad (4)$$

A higher variance σ_f^2 of the fundamental or a higher variance σ_g^2 of the noise in public information increases investors' uncertainty at time 0. Thus, our results come through whether U measures uncertainty in macro fundamentals or captures noise in the available public information at time 0. We therefore keep our model agnostic about what determines U .⁴

A total of $A \leq N$ firms issue earnings announcements at $t = 1$. We denote the set of announcing firms by $\mathcal{A} \equiv \{1, \dots, A\}$. As in Teoh and Wong (1993), earnings announcements convey information about firms' future dividends:

$$E_a = D_a + \varepsilon_a, \quad \text{for } a \in \mathcal{A}, \quad (5)$$

where the earnings noise shocks ε_a are independently distributed, $\varepsilon_a \sim \mathcal{N}(0, \sigma_{\varepsilon a}^2)$, and drawn independently from f and e_n , $\forall a \in \mathcal{A}$ and $\forall n \in \{1, \dots, N\}$.

At $t = 0$, each investor i chooses whether or not to be attentive to the earnings announcements. Investor i can pay attention to announcements made by the firms in any of the 2^A possible subsets of \mathcal{A} . (The set of all subsets of \mathcal{A} represents the *power set* of \mathcal{A} , or $\mathcal{P}(\mathcal{A})$, and includes the empty set \emptyset and \mathcal{A} itself.) Thus, there are potentially 2^A investor types, indexed by $k \in \mathcal{P}(\mathcal{A})$. For instance, investors who choose to stay uninformed are of type $k = \emptyset$; investors

² Several recent studies use EDGAR data to explore different issues in corporate finance and asset pricing (e.g., Loughran and McDonald, 2011; DeHaan et al., 2015; Lee et al., 2015; Drake et al., 2015; Bauguess et al., 2018; Chen et al., 2020a; 2020b; Gao and Huang, 2020).

³ Throughout the paper, we will adopt the following notation: we use letters in plain font to indicate univariate variables and bold letters to indicate vectors and matrices; we use subscripts to indicate individual assets and superscripts to indicate individual investors. Appendix A.1 provides further details.

⁴ We discuss the introduction of an additional layer of information acquisition in Section 3.2 using a dynamic version of the model, and show that including this feature does not qualitatively change our results.

who pay attention to all earnings announcements are of type $k = \mathcal{A}$. We use the dummy variable I_a^k , with $a \in \mathcal{A}$ and $k \in \mathcal{P}(\mathcal{A})$, to indicate type k investor's decision to pay attention to E_a : if $a \in k$, then $I_a^k = 1$; otherwise, $I_a^k = 0$.

Each investor starts with zero initial wealth and maximizes expected utility at time 0,

$$\max_{k \in \mathcal{P}(\mathcal{A})} \mathbb{E}_0 \left[\max_{\mathbf{q}^k} \mathbb{E}_1^k \left[-e^{-\gamma (W^k - c|k|)} \right] \right], \quad (6)$$

where \mathbf{q}^k is the optimal portfolio of a type k investor and $|k|$ denotes the cardinality of the set k , or $|k| = \sum_{a \in \mathcal{A}} I_a^k$.

At time 0, investor i decides her type k , knowing that at time 1 she will choose an optimal portfolio based on the information set pertaining to the type k . The first optimization is a combinatorial discrete choice problem (see, e.g., Hu and Shi, 2019; Arkolakis et al., 2021, for recent examples in economics). The second optimization is a standard Markowitz (1952) portfolio choice problem, where γ is the risk aversion coefficient, $W^k = (\mathbf{q}^k)' \mathbf{R}^e$ is investor's final wealth at $t = 2$ (which depends on her type k), and c is the monetary cost of paying attention to one earnings announcement—e.g., an information-processing cost, or time and opportunity cost. The attention cost c is strictly positive and is the same across investors and firms. (We derive additional predictions in a model with heterogeneous costs across investors—e.g., retail versus institutional investors—in Section 3.)

At $t = 1$, investors build optimal portfolios:

$$\mathbf{q}^k = \frac{1}{\gamma} \text{Var}_1^k[\mathbf{D}]^{-1} (\mathbb{E}_1^k[\mathbf{D}] - \mathbf{P}), \quad \text{for } k \in \mathcal{P}(\mathcal{A}), \quad (7)$$

where the superscripts k in $\mathbb{E}_1^k[\cdot]$ and $\text{Var}_1^k[\cdot]$ read “under the information set of a type k investor.” That is, $\text{Var}_1^k[\mathbf{D}]$ is the $N \times N$ covariance matrix of assets' payoffs, conditioned on the type k investor's information set.

We assume that an unmodeled group of agents trades for non-informational reasons or liquidity needs. This is a common assumption in noisy rational expectations models, which ensures that equilibrium prices do not fully reveal investors' information. Consistent with much of the prior literature, we often interpret liquidity trading as noise (Grossman and Stiglitz, 1980; He and Wang, 1995). Liquidity traders have inelastic demands of \mathbf{x} shares, where each element of \mathbf{x} is normally and independently distributed, $x_n \sim \mathcal{N}(0, \sigma_{xn}^2)$.

Denoting by λ^k the fraction of type k investors, the prices of risky assets are determined in equilibrium by the market-clearing condition:

$$\sum_{k \in \mathcal{P}(\mathcal{A})} \lambda^k \mathbf{q}^k + \mathbf{x} = \mathbf{M}. \quad (8)$$

Before turning to the equilibrium analysis, we define the fraction of investors who observe the announcement E_a as

$$\Lambda_a \equiv \sum_{k \in \mathcal{P}(\mathcal{A})} \lambda^k I_a^k. \quad (9)$$

Importantly, in our model the attention capacity of investors is not constrained, in the sense that an equilibrium in which $\Lambda_a = 1 \forall a \in \mathcal{A}$ is possible, as we will describe below.

2.1. Equilibrium search for information

As is customary in noisy rational expectations models, prices take the linear form

$$\mathbf{P} = \boldsymbol{\alpha} \mathbf{E} + \boldsymbol{\xi} \mathbf{x} - \boldsymbol{\zeta} \mathbf{M}, \quad (10)$$

where $\mathbf{E} \equiv [E_1, E_2, \dots, E_A]'$, $\boldsymbol{\alpha}$ is a $N \times A$ matrix, and $\boldsymbol{\xi}$ and $\boldsymbol{\zeta}$ are $N \times N$ matrices.

Solving for the equilibrium price coefficients is not necessary to determine the equilibrium demand for information. Instead, it is sufficient to make the following conjecture (equivalent to Lemma 3.2 in Admati, 1985), which we will verify in Proposition 3.

Conjecture 1.

$$\hat{\mathbf{P}} \equiv \boldsymbol{\xi}^{-1} (\mathbf{P} + \boldsymbol{\zeta} \mathbf{M}) = \sum_{a=1}^A \frac{\Lambda_a}{\gamma \sigma_{\varepsilon a}^2} \mathbf{t}_a E_a + \mathbf{x}, \quad (11)$$

where $\hat{\mathbf{P}} \equiv [\hat{P}_1, \hat{P}_2, \dots, \hat{P}_N]'$ and \mathbf{t}_a is a standard basis vector of dimension N with all components equal to 0, except the a th, which is 1.

This conjecture transforms the equilibrium prices into simple signals about E_a , $a \in \mathcal{A}$. In equilibrium, all investors except the fully informed (of type $k = \mathcal{A}$) use prices to learn. Accordingly, the information sets of investors at time 1 are

$$\begin{cases} \mathcal{F}^k = \{E_a \mid a \in k\} \cup \hat{\mathbf{P}} & \text{if } k \in \mathcal{P}(\mathcal{A}) \setminus \mathcal{A}, \\ \mathcal{F}^k = \{E_a \mid a \in \mathcal{A}\} & \text{if } k = \mathcal{A}. \end{cases} \quad (12)$$

Before characterizing the information acquisition decision for each investor type, we define the following *learning coefficients*:

$$\ell_a^k = I_a^k + (1 - I_a^k) \ell_a, \quad \text{where } \ell_a \equiv \frac{\Lambda_a^2}{\Lambda_a^2 + \gamma^2 \sigma_{xa}^2 \sigma_{\varepsilon a}^2}. \quad (13)$$

If a type k investor observes the earnings announcement E_a , then $I_a^k = 1$ and the learning coefficient ℓ_a^k reaches its maximum value, 1. Without observing E_a , $I_a^k = 0$ and the investor relies on prices to learn, which yields $\ell_a^k = \ell_a < 1$. Prices are informative about E_a to the extent that *someone* pays attention to the signal E_a , that is, if $\Lambda_a > 0$. In this case, ℓ_a increases with the fraction of informed investors (investors learn more from prices when a higher fraction of them pay attention to E_a) and decreases with the amount of noise in supply σ_{xa} and the amount of noise in the earnings announcement $\sigma_{\varepsilon a}$ (investors learn less from prices when there is more noise in supply or when earnings announcements are noisier).

Investors' demand for information ultimately depends on the reduction in uncertainty achieved by observing new information. Because in our setup the vector of final payoffs \mathbf{D} is a multidimensional normally distributed random variable, the reduction in uncertainty from observing new information is conveniently measured using the notion of entropy: under the information set of any investor type $k \in \mathcal{P}(\mathcal{A})$, the vector \mathbf{D} has entropy

$$H^k[\mathbf{D}] = \frac{N}{2} \ln(2\pi + 1) - \frac{1}{2} \ln(\det(\text{Var}_1^k[\mathbf{D}]^{-1})). \quad (14)$$

From this definition, it follows that the uncertainty perceived by the investor decreases with the determinant of

the posterior precision matrix of \mathbf{D} (i.e., the inverse of the posterior covariance matrix $\text{Var}^k[\mathbf{D}]$, hereafter τ^k).

Defining $\text{Var}[\mathbf{D}] \equiv U^2 \mathbf{b}\mathbf{b}' + \text{Var}[\mathbf{e}]$, where \mathbf{e} is the vector of idiosyncratic components e_n in firms' payoffs given in (1), we can state the following proposition.

Proposition 1. *The posterior precision matrix for each investor type $k \in \mathcal{P}(A)$ is*

$$\tau^k \equiv \text{Var}_1^k[\mathbf{D}]^{-1} = \text{Var}[\mathbf{D}]^{-1} + \sum_{a=1}^A \frac{\ell_a^k}{\sigma_{\varepsilon a}^2} \mathbf{t}_a \mathbf{t}_a', \quad (15)$$

and its determinant is given by

$$\det(\tau^k) = \det(\text{Var}[\mathbf{D}]^{-1}) \left(\prod_{a=1}^A \frac{\ell_a^k \sigma_{\varepsilon a}^2 + \sigma_{\varepsilon a}^2}{\sigma_{\varepsilon a}^2} \right) \times \left(1 + U^2 \sum_{a=1}^A \frac{\ell_a^k b_a^2}{\ell_a^k \sigma_{\varepsilon a}^2 + \sigma_{\varepsilon a}^2} \right). \quad (16)$$

Proposition 1 shows how the heterogeneity in the learning coefficients ℓ_a^k across investors of different types $k \in \mathcal{P}(A)$ drives the heterogeneity in the determinants $\det(\tau^k)$. Because a higher determinant means less uncertainty (Eq. (14)), the determinants $\det(\tau^k)$ provide a clear ranking of the informational distances between the 2^A investor types. For instance the most informed investors (of type A) have the highest $\det(\tau^k)$ because $\ell_a^A = 1$, $\forall a \in A$, whereas the least informed investors (of type \emptyset) have the lowest $\det(\tau^k)$.

The ranking in $\det(\tau^k)$ dictated by **Proposition 1** allows for a simple characterization of the information market equilibrium. Consider a type k investor who decides whether to migrate to any alternative type in $\mathcal{P}(A) \setminus k$. The key quantity that regulates the investor's decision is the *benefit-cost ratio*, which we define as

$$B_{\theta}^k \equiv \frac{\det(\tau^k)}{\det(\tau^{\theta})} e^{-2\gamma c|k|}. \quad (17)$$

The ratio $\det(\tau^k)/\det(\tau^{\theta})$ in B_{θ}^k measures the gain in precision obtained from observing the earnings announcements made by all the firms in the set k , whereas $e^{-2\gamma c|k|}$ measures the cost of paying attention to these announcements. We can now formulate the following result.

Proposition 2. *A type k investor changes type from k to $k' \in \mathcal{P}(A) \setminus k$ if and only if*

$$\frac{B_{\theta}^{k'}}{B_{\theta}^k} > 1 \iff \frac{1}{2\gamma} \ln \frac{\det(\tau^{k'})}{\det(\tau^k)} > c(|k'| - |k|). \quad (18)$$

Assume, without loss of generality, that $|k'| - |k| > 0$. On the left-hand side of (18), $\frac{1}{2\gamma} \ln \frac{\det(\tau^{k'})}{\det(\tau^k)}$ measures the benefit of migrating from k to k' as a reduction in entropy divided by investor's risk aversion, $(H^k[\mathbf{D}] - H^{k'}[\mathbf{D}])/\gamma$; the right-hand side measures the attention cost. The type k investor changes type if and only if the benefit from the reduction in entropy achieved by becoming of type k' outweighs its cost. Risk aversion lowers the benefit of information: because more risk-averse investors trade less aggressively, they benefit less from paying attention to firm disclosures.

The ratio $\det(\tau^{k'})/\det(\tau^k)$ in (18) is greatly simplified by means of **Proposition 1**: all the heterogeneity pertaining to non-announcing firms enters only in $\det(\text{Var}[\mathbf{D}]^{-1})$ and thus vanishes in the ratio. To gain further insight into this ratio, let us focus on a simplified version where investors in aggregate pay attention to one firm only (i.e., there is only one announcing firm, a). In this case, a type \emptyset investor changes type to $\{a\}$ if and only if

$$\frac{1}{2\gamma} \ln \frac{1 + \frac{\text{Var}[D_a]}{\sigma_{\varepsilon a}^2}}{1 + \frac{\text{Var}[D_a]}{\sigma_{\varepsilon a}^2} \frac{\Lambda_a^2}{\Lambda_a^2 + \gamma^2 \sigma_{xa}^2 \sigma_{\varepsilon a}^2}} > c. \quad (19)$$

On the left-hand side the benefit of information increases with $\text{Var}[D_a]/\sigma_{\varepsilon a}^2$, which measures the *quality of information* provided by the earnings announcement; decreases with the fraction of informed investors Λ_a , in which case prices are more informative and the signal E_a becomes less valuable; increases with the amount of noise in supply σ_{xa} , in which case prices are less informative and the signal E_a becomes more valuable; and decreases with the risk aversion. (See also **Grossman and Stiglitz, 1980**, for similar tradeoffs.)

The same tradeoffs are at play when multiple firms are announcing, with the significant difference that heterogeneity in firms characteristics (b_a , $\sigma_{\varepsilon a}$, σ_{ea} , and σ_{xa}) yields heterogeneous information choices across firms. We will analyze this heterogeneity in **Section 2.4**, where we discuss the model's theoretical predictions. We focus here on the information market equilibrium, which we characterize in the following theorem.

Theorem 1. *There exist two positive values $c_{\min} < c_{\max}$, strictly increasing in U , such that:*

- (A) *If $c \in [c_{\max}, \infty)$, then the cost of information is prohibitive and no investor finds it optimal to pay attention to the earnings announcements: $\lambda^{\theta} = 1$.*
- (B) *If $c \in (c_{\min}, c_{\max})$, then there exists a set $\{\lambda^k \mid k \in \mathcal{P}(A)\}$ such that, in equilibrium: $\sum_{k \in \mathcal{P}(A)} \lambda^k = 1$; $\lambda^{\theta} < 1$; $\lambda^A < 1$; and the benefit-cost ratios $\{B_{\theta}^k \mid k \in \mathcal{P}(A)\}$ are determined such that for any pair $\{k, k'\} \in \mathcal{P}(A)$:*
 - (i) *If $\{\lambda^k > 0\} \wedge \{\lambda^{k'} > 0\}$, then $B_{\theta}^{k'}/B_{\theta}^k = 1$.*
 - (ii) *If $\{\lambda^k = 0\} \wedge \{\lambda^{k'} > 0\}$, then $B_{\theta}^{k'}/B_{\theta}^k \geq 1$.**Conditions Bi and Bii are both necessary and sufficient for the stability of the information market equilibrium when $c \in (c_{\min}, c_{\max})$.*
- (C) *If $c \in [0, c_{\min}]$, then the cost of information is small enough such that all investors pay attention to all the earnings announcements: $\lambda^A = 1$.*

Cases A and C are trivial equilibria in which the information cost is too high or too low. In these cases, investors unanimously choose to remain uninformed or to pay attention to all earnings announcements. Case B, which will be the focus of our analysis in **Section 2.4**, defines a set of conditions such that, in equilibrium, no investor can unilaterally improve their utility by changing their type. We explain in **Section 2.3** how investors arrive at this self-sustaining equilibrium, and describe an iterative algorithm

that converges to equilibrium from any initial conditions $\{\lambda_0^k > 0 \mid k \in \mathcal{P}(\mathcal{A})\}$.

2.2. Equilibrium prices and earnings response coefficients

We now aggregate investors' demands in order to solve for equilibrium prices. Define first the weighted average precision matrix for the population of informed investors as

$$\boldsymbol{\tau} \equiv \sum_{k \in \mathcal{P}(\mathcal{A})} \lambda^k \boldsymbol{\tau}^k. \quad (20)$$

Lemma 1. *The weighted average precision is given by*

$$\boldsymbol{\tau} = \text{Var}[\mathbf{D}]^{-1} + \begin{bmatrix} \text{diag}[\pi_a(\Lambda_a) \mid a \in \mathcal{A}] & \mathbf{0}_{A \times (N-A)} \\ \mathbf{0}_{(N-A) \times A} & \mathbf{0}_{(N-A) \times (N-A)} \end{bmatrix}, \quad (21)$$

where each coefficient $\pi_a(\Lambda_a)$ is a strictly increasing function of Λ_a ,

$$\pi_a(\Lambda_a) = \frac{\Lambda_a^2 + \Lambda_a \gamma^2 \sigma_{\varepsilon a}^2 \sigma_{\varepsilon a}^2}{\Lambda_a^2 \sigma_{\varepsilon a}^2 + \gamma^2 \sigma_{\varepsilon a}^4 \sigma_{\varepsilon a}^4}, \quad a \in \mathcal{A}, \quad (22)$$

and $\text{diag}[y_j \mid j \in \mathcal{Z}]$ is a diagonal matrix with $\{y_j \mid j \in \mathcal{Z}\}$ on its diagonal.

Each function $\pi_a(\Lambda_a)$ determines the aggregate precision gains from observing E_a . A key property of these functions, which will prove useful shortly, is that they depend on the economic uncertainty U only indirectly through Λ_a .

Proposition 3. *The equilibrium prices in this economy satisfy*

$$\boldsymbol{\tau} \mathbf{P} = \sum_{a=1}^A \pi_a(\Lambda_a) \boldsymbol{\iota}_a E_a + \gamma \begin{bmatrix} \text{diag} \left[\frac{\pi_a(\Lambda_a) \sigma_{\varepsilon a}^2}{\Lambda_a} \mid a \in \mathcal{A} \right] & \mathbf{0}_{A \times (N-A)} \\ \mathbf{0}_{(N-A) \times A} & \mathbf{I}_{N-A} \end{bmatrix} \mathbf{x} - \gamma \mathbf{M}, \quad (23)$$

where \mathbf{I}_z is the identity matrix of dimension z .

The earnings response coefficients (ERCs) measure the reactions of the equilibrium prices to the earnings announcements. In a simpler model with a sole announcer the ERC is the coefficient of E_a in the equilibrium price. In our model with N firms and A announcers, ERCs form the principal diagonal of the $N \times A$ matrix $\boldsymbol{\alpha}$ in the price conjecture (10). That is, ERCs measure the price reactions of the announcing firms to their own announcements. Denoting by \mathbf{D}_A the final payoffs of all announcing firms, we derive the following corollary.

Corollary 3.1. *The earnings response coefficients are given by the diagonal of the $A \times A$ matrix $\boldsymbol{\alpha}_A$, which solves:*

$$\boldsymbol{\alpha}_A = \mathbf{I}_A - (\mathbf{I}_A + \text{Var}[\mathbf{D}_A] \text{diag}[\pi_a(\Lambda_a) \mid a \in \mathcal{A}])^{-1}. \quad (24)$$

The $A \times A$ matrix $\boldsymbol{\alpha}_A$ is zero if $\Lambda_a = 0 \forall a \in \mathcal{A}$. An important separation result helps us interpret $\boldsymbol{\alpha}_A$: as shown in Lemma 1, the coefficients $\pi_a(\Lambda_a)$ do not directly depend on U . Therefore, in the following analysis, we can separately assess the effects of an increase in economic uncertainty on ERCs and, in particular, the additional effect that arises from changes in investor attention.

2.3. Illustration

To illustrate how investors' search for information converges to a stable equilibrium, it is helpful to write the individual optimization problem (6) in a more straightforward form. Appendix A.7 shows that at time 0, each investor makes the following choice:

$$\max_{k \in \mathcal{P}(\mathcal{A})} \ln B_{\theta}^k, \quad (25)$$

where the benefit-cost ratios B_{θ}^k have been defined in (17).

A key property of the function $f(k) = \ln B_{\theta}^k$ is submodularity—the difference in the incremental value of $f(k)$ that one element a makes when added to the type k decreases as the size of k increases. Submodularity can be interpreted as a property of *diminishing returns*. It implies that an individual investor's incentive to become more informed (e.g., to increase her type from k to $k \cup \{a\}$) decreases with her current level of attention. Furthermore, we show in Appendix A.7 that a migration of a positive mass of investors from any type k to a different type k' decreases the relative attractiveness of type k' with respect to type k , i.e., decreases the fraction $B_{\theta}^{k'}/B_{\theta}^k$. This implies that an individual investor's incentive to choose k' over k decreases if in aggregate more investors choose k' over k . Hence we recover the Grossman and Stiglitz (1980) result that individual action and the aggregate of (others) individual actions are strategic substitutes.

Hu and Shi (2019) and Arkolakis et al. (2021) derive an evolutionary learning algorithm that reaches the equilibrium of a submodular game from any initial point. Starting from a set of initial values $\{\lambda_0^k > 0 \mid k \in \mathcal{P}(\mathcal{A})\}$ such that $\sum_k \lambda_0^k = 1$, the algorithm allows some small fraction of the population of investors of a given type k to revise their strategy as the best response to the current total population strategy. This process is iterated over all types until it converges to a self-sustaining equilibrium in which no investor changes strategy, as in Theorem 1. We relegate the details of this algorithm to Appendix A.7 and focus here on a numerical example, which we illustrate in Fig. 1.

This numerical example considers an economy with three announcers. The announcing firms differ through their exposure to systematic risk, $b_1 > b_2 > b_3$, while other firm-level parameters are homogeneous across firms. The parameters that we chose are provided in the caption of the figure. Note that this example is only illustrative—in Section 4, we propose a realistic calibration with a larger number of announcers.

The dashed and solid lines in the figure depict the values c_{\min} and c_{\max} , respectively. The plot confirms the results of Theorem 1: (i) $c_{\min} < c_{\max}$ and (ii) c_{\min} and c_{\max} increase with the amount of uncertainty U . When $c \leq c_{\min}$, all investors are attentive to all earnings announcements, $\lambda^A = 1$; when $c \geq c_{\max}$, no investor pays attention to earnings announcements, $\lambda^{\emptyset} = 1$; when $c \in (c_{\min}, c_{\max})$, the two dotted lines that split the middle zone show that investors always find the announcement of firm 1 most valuable—they pay attention to E_1 in cases (B1), (B2), and (B3)—whereas the announcement of firm 3 least valuable—they pay attention to E_3 only in case (B3). Since $b_1 > b_2 > b_3$, E_1 is the most informative announcement about the

systematic factor f , and investors turn their attention first to firm 1. Thus, in this equilibrium investors behave as if they queue announcements based on their exposure to systematic risk. [Frederickson and Zolotoy \(2016\)](#) document a similar queuing result: investors devote more immediate attention to announcing firms that are comparatively more visible (i.e., larger firms, firms with more media coverage, higher advertising expense, or higher analyst coverage). In the case discussed here, attention queueing is based on firms' exposures to the systematic factor f . Indeed, as we show in the next section, firms' exposures to the systematic factor yield a clear ranking of investor attention across firms.

2.4. Implications for attention and earnings response coefficients

Building on the previous illustration, we derive several testable implications of the model. The first result that emerges from [Theorem 1](#) and [Fig. 1](#) is the effect of an increase in uncertainty on the information market equilibrium. Suppose uncertainty is low enough that all investors are inattentive—this corresponds to case A, depicted with the hashed area in the plot. Then, after an increase in uncertainty the equilibrium moves to the right, anywhere from case B to case C: a positive fraction of investors become attentive first to E_1 , and if the increase in uncertainty is sufficiently substantial, to E_2 and ultimately to E_3 . The main implication is that an increase in uncertainty triggers investor attention to firm-level information. Moreover, investors direct their attention to an increasing number of firms as uncertainty increases.

The previous implication refers to the *number of firms*: more announcing firms become the focus of investor attention as uncertainty increases. We now turn to the effect of uncertainty on the *number of investors* who pay attention to the earnings announcements. The fractions Λ_a of investors who observe each earnings announcement, defined in (9), are not apparent from [Fig. 1](#), which only shows when these fractions are positive or zero. To analyze how these fractions vary with uncertainty, assume for simplicity that no investor in the economy observes the announcement of firm a , or $\Lambda_a = 0$. Note that similar intuition holds without the assumption but with a more complicated expression. Then, for a type k investor the benefit of paying attention to E_a follows from (17):

$$\frac{\det(\tau^{k \cup \{a\}})}{\det(\tau^k)} = 1 + \frac{1}{\sigma_{\varepsilon a}^2} \left(\sigma_{\varepsilon a}^2 + \frac{b_a^2}{\frac{1}{U^2} + \sum_{\alpha=1, \alpha \neq a}^A \frac{b_\alpha^2 \ell_\alpha^k}{\ell_\alpha^k \sigma_{\varepsilon j}^2 + \sigma_{\varepsilon a}^2}} \right). \quad (26)$$

The first implication of (26) is that the benefit of paying attention to E_a strictly increases with uncertainty (this holds for all investor types and all announcing firms). Moreover, the benefit of attention is higher for firms with a stronger exposure b_a to the systematic component, a higher volatility $\sigma_{\varepsilon a}$ of their idiosyncratic component, and less noise $\sigma_{\varepsilon a}$ in their announcement. [Eq. \(26\)](#) also implies that the benefit of attention decreases with the amount of attention that investors pay to *other* earnings announce-

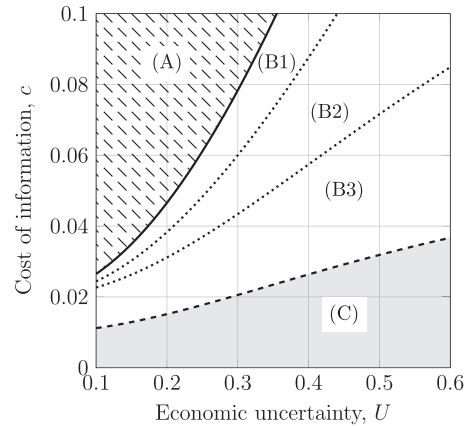


Fig. 1. Information market equilibrium.

This figure depicts the three cases of [Theorem 1](#), A, B, and C. We further split case B in three sub-cases: (B1) $\Lambda_1 > 0, \Lambda_2 = \Lambda_3 = 0$, in which investors only pay attention to the announcement of firm 1; (B2) $\Lambda_1 > 0, \Lambda_2 > 0, \Lambda_3 = 0$, in which investors pay attention to the announcements of firms 1 and 2 but not 3; (B3) $\Lambda_1 > 0, \Lambda_2 > 0, \Lambda_3 > 0$, in which investors pay attention to the announcements of all firms. The calibration used for this illustration is: $\gamma = 1, b_1 = 1.2, b_2 = 1, b_3 = 0.8, \sigma_{\varepsilon 1} = \sigma_{\varepsilon 2} = \sigma_{\varepsilon 3} = 0.2, \sigma_{\varepsilon 1} = \sigma_{\varepsilon 2} = \sigma_{\varepsilon 3} = 1$, and $\sigma_{x1} = \sigma_{x2} = \sigma_{x3} = 1$.

ments, as reflected in the summation term: if a large number of firms announce at the same time (i.e., A is high), and large fractions of investors are attentive (i.e., Λ_a is large, $\forall a \neq a$), then prices are highly informative about f and paying attention to E_a becomes less valuable. This implication is similar to the *investor distraction hypothesis* ([Hirshleifer et al., 2009](#)): when multiple announcements compete for investor attention, prices underreact to the new information. In our model, this result arises not because investors are distracted by the simultaneous announcements but because information spillovers increase aggregate price informativeness, diminishing the benefit of attention.

A critical implication of (26) emerges once we fix $b_a = 0$, which results in a constant benefit of paying attention to E_a . In this case, an increase in uncertainty does not lead to an increase in attention to firm-level information because no information spillover occurs from firm a to the rest of the economy. This implication, coupled with evidence from recent empirical work ([Hirshleifer and Sheng, 2022; Ben-Rephael et al., 2021; Chan and Marsh, 2021b](#)) and our empirical analysis in [Section 4](#), highlights the importance of information spillovers in theories of firm-level information acquisition.

Panel (a) of [Fig. 2](#) illustrates the impact of an increase in uncertainty in our calibrated economy with three announcers. The three lines depict the fractions of the population of investors attentive to each earnings announcement. This example assumes that $b_1 > b_2 > b_3$. Confirming [Eq. \(26\)](#), the fractions Λ_1, Λ_2 , and Λ_3 increase with U . We note that for low levels of economic uncertainty the fractions Λ_a are all zero for $a \in \{1, 2, 3\}$, which corresponds to case A of [Theorem 1](#). As uncertainty increases the economy moves successively to all the subcases of B and ultimately to case C.

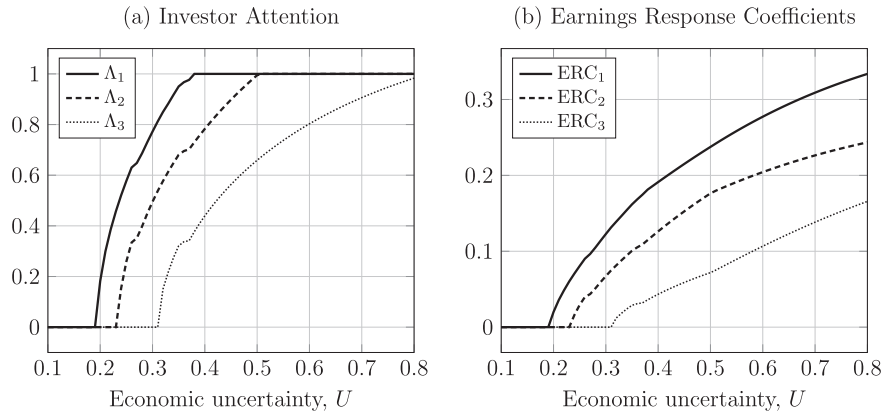


Fig. 2. The impact of economic uncertainty on investor attention and ERCs.

Panel (a) plots the fractions of attentive investors to each one of the three earnings announcements. Panel (b) plots the earnings response coefficients. In this economy, $b_1 > b_2 > b_3$, $c = 0.045$, and the rest of the calibration is provided in Fig. 1.

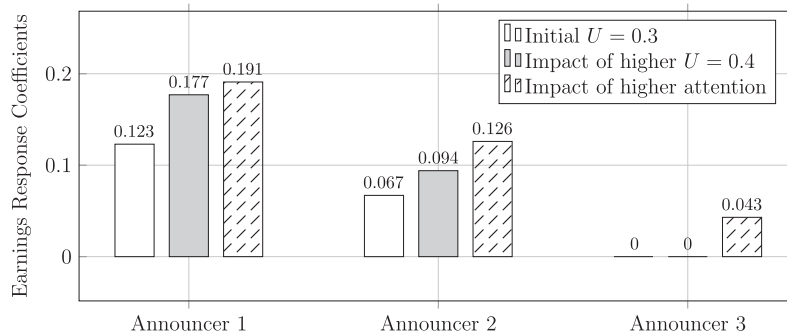


Fig. 3. The separate impact of an increase in uncertainty and an increase in investor attention on ERCs.

This figure plots the successive changes in ERCs of the announcing firms after an increase of economic uncertainty from 0.3 to 0.4. The grey bars plot ERCs resulting exclusively from the increase in U . The hashed bars plot the final ERCs, including also the impact of the increase in investor attention. In this economy, $b_1 > b_2 > b_3$, $c = 0.045$, and the rest of the calibration is provided in Fig. 1.

The increase in investor attention caused by an increase in uncertainty has additional implications for the response of prices to firm-level information. To gain more intuition, we write the ERC in an economy with a sole announcer (a particular case of Corollary 3.1):

$$ERC_a = 1 - \frac{1}{1 + (U^2 b_a^2 + \sigma_{ea}^2) \pi_a(\Lambda_a)}. \quad (27)$$

The ERC increases with uncertainty directly through an increase in the variance of the firm's payoff $\text{Var}[D_a] = U^2 b_a^2 + \sigma_{ea}^2$ and indirectly through an increase in investors' attention to the earnings announcement. Firms with a stronger exposure, b_a , to the systematic component, or a higher volatility, σ_{ea} , of their idiosyncratic component, observe a larger increase in their ERC as uncertainty and investor attention increase. Panel (b) of Fig. 2 revisits our economy with three announcers. It confirms that ERCs increase with uncertainty and that firms with stronger exposure to the systematic components have higher ERCs.

Eq. (27) implies that ERCs are driven both by the exogenous increase in uncertainty and the endogenous increase in investor attention and that the two effects compound each other. We disentangle these two effects in

Fig. 3. The gray bars depict the impact on ERCs of an increase in U . The hashed bars include the additional impact of the increase in investor attention, confirming the direct and indirect effects from (27). Note that in this example the ERC of the third announcer increases from zero to a positive value only through the indirect effect of an increase in attention.

We now turn to other dimensions of heterogeneity across firms and summarize the results in Fig. 4. Panels (a) and (d) analyze the effect of the volatility of the idiosyncratic component, $\sigma_{e1} > \sigma_{e2} > \sigma_{e3}$ (while all other parameters are constant across firms). Eqs. (26) and (27) imply that firms with higher σ_{ea} should observe stronger investor attention and ERCs to their announcements because the informativeness of an earnings announcement, $\text{Var}[D_a]/\sigma_{ea}^2$, is higher for firms with higher σ_{ea} . Thus, investors focus on those firms first after an increase in uncertainty. Panels (a) and (d) confirm these effects for the fractions of informed investors and ERCs.

Assuming that firms differ through the noise in their signals, $\sigma_{\varepsilon 1} < \sigma_{\varepsilon 2} < \sigma_{\varepsilon 3}$, implies that the signal of firm 1 is more valuable for investors for the same reason as above: E_1 is more informative about f than E_2 , which itself is

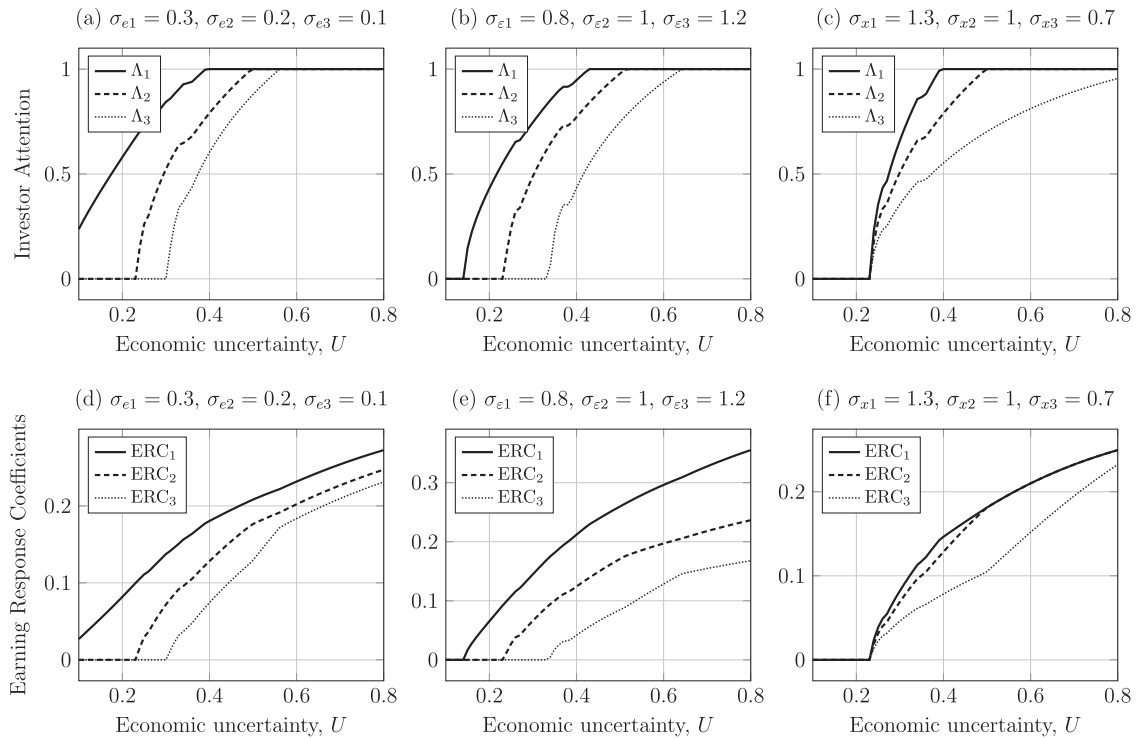


Fig. 4. The impact of economic uncertainty on investor attention and ERCs.

This figure plots the fractions of investors attentive to each earnings announcement (above) and ERCs (below), as functions of economic uncertainty, for different σ_{ea} , σ_{sa} , and σ_{xa} . The rest of the calibration is provided in Fig. 1, and $c = 0.045$.

more informative than E_3 . Panels (b) and (e) of Fig. 4 illustrate this. Finally, we also analyze the case of different noisiness of supply. Panels (c) and (f) consider an economy in which $\sigma_{x1} > \sigma_{x2} > \sigma_{x3}$ and show that after an increase in U , investors turn their attention more to firm 1, causing an increase in ERCs. The intuition stems from price informativeness: the equilibrium prices of firms with more substantial noise in supply reveal less information to investors, which increases the ex-ante incentive to acquire information from earnings (as in Grossman and Stiglitz, 1980). This intuition explains the greater attention and stronger ERCs for firms with higher σ_{xa} .

To summarize, the testable implications of our model concerning the impact of uncertainty on investor attention and on ERCs are: (i) when uncertainty increases, investors focus on earnings announcements of a larger number of firms, and more investors pay attention to each announcing firm; (ii) investors' incentives to pay attention to earnings announcements decrease with the number of firms that announce their earnings simultaneously; (iii) when uncertainty (investor attention) increases, ERCs strengthen for all announcing firms; and (iv) increases in ERCs caused by higher uncertainty (investor attention) are incrementally stronger for firms with higher b_a , higher σ_{ea} , lower σ_{sa} , and higher σ_{xa} .

2.5. Implications for firms' betas and the securities market line

In this section, we explore the impact of investor attention on the CAPM. Our goal is to show how cross-sectional

differences in attention levels among announcing firms affect individual firm betas, and how the aggregate attention of investors drives the resolution of uncertainty over time. Greater aggregate attention leads to a higher market risk premium, which, in the context of the CAPM, means a steeper securities market line (SML).

The derivation of a model-implied CAPM on earnings announcement days requires endogenous prices at time 0. Thus, maintaining the same model assumptions as in the previous analysis, we assume that at time 0 agents trade in the market and observe additional information. (The type of this information—public or private—is inconsequential for the results derived here.) As such, time 0 and time 1 represent the close of two consecutive trading days, with earnings being announced on the second day. Denoting equilibrium prices at times 0 and 1 by \mathbf{P}_0 and \mathbf{P}_1 , asset returns on the announcement day are $\mathbf{R}_1^e \equiv \mathbf{P}_1 - \mathbf{P}_0$.

At time 0, all agents observe a publicly available signal about the aggregate market payoff,

$$G = \mathbf{M}'\mathbf{D} + g, \quad \text{where } g \sim \mathcal{N}(0, \sigma_g^2), \quad (28)$$

where the noise in the public signal g is independent of all the random variables previously defined. In an economy with a large number of firms (i.e., when $N \rightarrow \infty$), one can interpret G as a signal about the systematic component f .

As in the baseline model, noise traders at time 0 have inelastic demands of \mathbf{x}_0 shares, with $x_{0,n} \sim \mathcal{N}(0, \sigma_{xn}^2)$, and we denote noise trading at time 1 by \mathbf{x}_1 , which is defined as before. Thus, the total supply of assets available for trading to informed investors is $\mathbf{M} - \mathbf{x}_0$ at time 0 and $\mathbf{M} -$

$\mathbf{x}_0 - \mathbf{x}_1$ at time 1. This follows He and Wang (1995) and Brennan and Cao (1997).

To summarize, in this slightly modified setup, investors trade before and after earnings announcements, making their information acquisition decision at any time between 0 and 1. The following proposition describes investor asset demands and risky asset prices at each market session in this model.

Proposition 4. *There exists a partially revealing rational expectations equilibrium in the two trading session economy in which*

- (i) *Individual asset demands for a type- k investor are given by:*

$$\mathbf{q}_0 = \frac{1}{\gamma} \tau_0 (\mathbb{E}_0[\mathbf{D}] - \mathbf{P}_0) \quad \text{and} \quad (29)$$

$$\mathbf{q}_1^k = \frac{1}{\gamma} \tau_1^k (\mathbb{E}_1^k[\mathbf{D}] - \mathbf{P}_1), \quad (30)$$

where $\tau_0 \equiv \text{Var}[\mathbf{D}|\mathcal{F}_0]^{-1}$ and $\tau_1^k \equiv \text{Var}[\mathbf{D}|\mathcal{F}_1^k]$, $\mathcal{F}_0 = \{G\}$, and $\mathcal{F}_1^k = \{G\} \cup \mathcal{F}^k$, with \mathcal{F}^k defined in (12).

- (ii) *The vectors of risky asset prices are given by*

$$\mathbf{P}_0 = \frac{1}{\sigma_g^2} \tau_0^{-1} \mathbf{M} \mathbf{G} - \gamma \tau_0^{-1} (\mathbf{M} - \mathbf{x}_0) \quad \text{and} \quad (31)$$

$$\mathbf{P}_1 = \tau_1^{-1} \sum_{k \in \mathcal{P}(\mathcal{A})} \lambda^k \tau_1^k \mathbb{E}_1^k[\mathbf{D}] - \gamma \tau_1^{-1} (\mathbf{M} - \mathbf{x}_0 - \mathbf{x}_1), \quad (32)$$

where $\tau_1 \equiv \sum_{k \in \mathcal{P}(\mathcal{A})} \lambda^k \tau_1^k$.

The proof is provided in Appendix A.8 and follows He and Wang (1995) and Brennan and Cao (1997), adapted to our Grossman and Stiglitz (1980) setup. Proposition 4 leads to a CAPM relation, which we describe in the following corollary.

Corollary 4.1. (CAPM) *Define the market excess return as $\mathbf{R}_M^e = \mathbf{M}' \mathbf{R}^e$. The following CAPM relation holds on an earnings announcement day:*

$$\mathbb{E}[\mathbf{R}^e] = \beta \mathbb{E}[\mathbf{R}_M^e], \quad \text{with } \beta = \frac{(\tau_0^{-1} - \tau_1^{-1}) \mathbf{M}}{U_0^2 - \mathbf{M}' \tau_1^{-1} \mathbf{M}}, \quad (33)$$

where the market risk premium is given by

$$\mathbb{E}[\mathbf{R}_M^e] = \gamma U_0^2 - \gamma \mathbf{M}' \tau_1^{-1} \mathbf{M}, \quad (34)$$

and $U_0^2 \equiv \mathbf{M}' \tau_0^{-1} \mathbf{M}$ represents the market-wide uncertainty (variance) that investors face before making information decisions and before the earnings announcements.

To understand how attention affects the CAPM, it is helpful to explain the equilibrium in this economy, as stated in Proposition 4 and Corollary 4.1. Expected returns from time 0 to time 2 are determined by the uncertainty that investors face at time 0, $\tau_0^{-1} = \text{Var}[\mathbf{D}|\mathcal{F}_0]$. Eq. (29) and market clearing at time 0 leads to $\mathbb{E}_0[\mathbf{D}] - \mathbf{P}_0 = \gamma \tau_0^{-1} (\mathbf{M} - \mathbf{x}_0)$. Taking the unconditional expectation of this relation and multiplying it with the market portfolio weights \mathbf{M}' yields the total risk premium from time 0 to time 2, $\mathbb{E}[\mathbf{D} - \mathbf{P}_0] = \gamma U_0^2$. Thus, the total risk premium

from time 0 to time 2 is fully determined by the aggregate uncertainty that investors face at time 0. This uncertainty increases with U and σ_g , as can be intuitively understood by considering an economy with a large number of firms:

$$\lim_{N \rightarrow \infty} U_0^2 = \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2}. \quad (35)$$

Investors' attention at time 1 determines how the total risk premium from time 0 to time 2 is allocated between these two periods. As attention influences the temporal resolution of uncertainty, it impacts the pattern of expected returns over time. Accordingly, Eq. (34), which describes the market risk premium during the earnings announcement day (time 0 to time 1), comprises two terms. The first term is the total risk premium from time 0 to time 2, γU_0^2 . Our focus lies on the second term, governed by investors' attention.

Without attention (if $\Lambda_a = 0 \forall a \in \mathcal{A}$), $\tau_1 = \tau_0$ and the market risk premium is zero, meaning that buying the market portfolio at time 0 and selling it at time 1 entails no risk. An analogy that illustrates this finding is the ship metaphor proposed by Robichek and Myers (1966).⁵ However, when investors are attentive, $\mathbf{M}' \tau_1^{-1} \mathbf{M}$ decreases with investor attention ($\partial \mathbf{M}' \tau_1^{-1} \mathbf{M} / \partial \Lambda_a < 0 \forall a \in \mathcal{A}$; see Appendix A.8) and yields a positive risk premium. Investors earn a risk premium by paying attention because they are rewarded for resolving uncertainty (Epstein and Turnbull, 1980). Eqs. (33) and (34) thus imply that heightened investor attention leads to a higher market risk premium.

In the CAPM framework, a higher market risk premium results in a steeper SML. This steepening effect of investor attention on the SML is independent of the initial level of uncertainty that prevails at time 0, U_0^2 . While an increase in the market risk premium due to higher U_0^2 is an expected equilibrium outcome—as evident in (34)—the higher risk premium due to greater investor attention, along with the corresponding SML steepening, represents a novel result. In this simple two-period model, attention raises the risk premium on announcement days through the temporal resolution of uncertainty.

Corollary 4.1 yields additional predictions about the market betas of announcing firms. These predictions emerge most transparently in a large economy in which,

⁵ In the metaphor, a ship begins a two-year journey, and the value of financial claims on the ship's cargo upon arrival is based on all available information at departure. If no new information is expected to permeate the market during the voyage, then buying these financial claims at departure and selling them after one year carries no risk. However, if new information is expected to emerge during the first year, selling these claims after the information becomes public entails risk, implying that the expected return for the first year exceeds the risk-free rate. In short, the timing of uncertainty resolution affects the pattern of expected returns over time. Epstein and Turnbull (1980) further formalize this concept in a general equilibrium framework. See also Kalay and Loewenstein (1985), fn. 3.

as $N \rightarrow \infty$, firms' market betas converge to:

$$\lim_{N \rightarrow \infty} \beta = \mathbf{b} + h \begin{bmatrix} \frac{\pi_1(\Lambda_1)\sigma_{e1}^2}{1 + \pi_1(\Lambda_1)\sigma_{e1}^2} b_1 \\ \frac{\pi_2(\Lambda_2)\sigma_{e2}^2}{1 + \pi_2(\Lambda_2)\sigma_{e2}^2} b_2 \\ \vdots \\ \frac{\pi_A(\Lambda_A)\sigma_{eA}^2}{1 + \pi_A(\Lambda_A)\sigma_{eA}^2} b_A \\ \mathbf{0}_{N-A} \end{bmatrix}, \quad (36)$$

where $h > 0$ and the scalars $\pi_a(\Lambda_a)$, $a \in \mathcal{A}$ are defined in Lemma 1 and are increasing in Λ_a . (The predictions do not hinge on taking the large economy limit, but the intuition is easier to convey in a large economy; see Appendix A.8.)

Eq. (36) has two predictions. First, betas are stronger for announcing firms. Consider two firms, one announcer and one non-announcer, with the same exposure to the systematic factor $b_a = b_n > 0$. The last term in (36) shows that the beta of the announcing firm increases on its announcement date (Patton and Verardo, 2012; Chan and Marsh, 2021b). Second, and more specific to our information acquisition setting, investor attention modulates the magnitude of the increase in the announcing firm's beta. Without attention, $\pi_a(0) = 0$, and the betas of the two firms remain the same. On the other hand, when attention is positive, the announcing firm's beta increases with the fraction of investors attentive to its announcement.

These predictions indicate that firms announcing earnings have higher expected returns than non-announcing firms. According to the CAPM relation in Corollary 4.1, this difference is attributed exclusively to the increased betas of the announcing firms. The rise in expected returns echoes the well-documented earnings announcement premium (Beaver, 1968; Chari et al., 1988; Ball and Kothari, 1991; Cohen et al., 2007; Frazzini and Lamont, 2007). However, our model attributes this phenomenon to an increase in systematic risk. A measured earnings announcement premium in abnormal (beta-adjusted) returns requires additional components, such as errors in empirical beta estimates (discussed in Andrei et al., 2023) or an augmented factor structure (illustrated by Savor and Wilson, 2016).

3. Additional implications and extensions

3.1. Heterogeneous attention costs

Our analysis so far has focused on an economy in which firms are heterogeneous, but investors are ex-ante identical. In reality, different investors may have different information acquisition costs. For instance, institutional owners presumably have lower information acquisition costs than retail investors. When choosing whether to pay attention to firm-level information, an institutional investor's alternative is generally to pay attention to a different financial signal or other job-related tasks (e.g., human resources, calling investors). In addition, institutional investors subscribe to services that lower the direct costs of information acquisition. In contrast, retail investors pay attention to a

primary job, family matter, hobby, or the back of their eyelids, which may carry higher personal opportunity costs.

To study the implications of heterogeneous information costs, we extend our model to two groups of investors, with information costs $c_l < c_h$. (These low-cost (c_l) and high-cost (c_h) investors can be thought of in different ways, such as institutions vs. individuals, local vs. non-local investors, or industry-focused vs. generalist investors.) The additional layer of heterogeneity requires re-writing the equilibrium conditions of Theorem 1 separately for each investor group. Importantly, $c_l < c_h$ implies that

$$B_{l,k}^{k \cup \{a\}} > B_{h,k}^{k \cup \{a\}}, \quad \forall k \in \mathcal{P}(\mathcal{A}) \text{ and } a \notin k, \quad (37)$$

where $B_{j,k}^{k \cup \{a\}} = \exp(-2\gamma c_j |k|) \det(\tau^{k \cup \{a\}}) / \det(\tau^k)$ for $j \in \{l, h\}$. In words, paying attention to one extra announcement has a larger net benefit for a low-cost investor than for a high-cost investor. Condition (37), labeled “monotonicity in types” by Hu and Shi (2019), guarantees the existence of an equilibrium and ensures that the solution method described in Appendix A.7 reaches the equilibrium.

Figure 5 plots the attention of low-cost (left) and high-cost (right) investors as functions of uncertainty. We use the same calibration with $b_1 > b_2 > b_3$ as in Fig. 1, split the population of investors into 50% low-cost and 50% high-cost (other splits lead to similar results), and fix $c_l = 0.045$ and $c_h = 0.055$. The two panels show that for any level of uncertainty, larger fractions of low-cost investors pay attention to the earnings announcements. The steeper lines in the left-side plot suggest that low-cost investors respond faster to the increase in uncertainty than high-cost investors, confirming the intuition from (37) that low-cost investors benefit comparatively more from increasing their attention.

Assuming different attention costs has further implications for ERCs. As shown in (27), ERCs increase with the amount of attention in the economy, which implies that the investor base of firms has an impact on ERCs: ERCs for firms with high ownership by low-cost investors should show a more robust response to an increase in uncertainty, through the stronger increase in attention. We test this theoretical implication in Section 4.

3.2. Dynamic model

Our main setup assumed a static information acquisition choice. However, in Appendix A.9, we consider a dynamic setup in which economic uncertainty varies over time. We examine how investors optimally acquire information in response to changes in uncertainty, investigate how this affects ERCs, and confirm that the comparative statics results from the one-period economy hold in this dynamic setup. We relegate the details of this extension to the appendix and discuss here its main implications.

In the dynamic version of the model, equilibrium prices depend on the weighted average of beliefs of informed and uninformed investors, consistent with previous work by Hirshleifer and Teoh (2003). When uncertainty increases, more investors pay attention, leading to a stronger impact of attention on prices and thus further strengthening the ERCs. Thus, we recover the intuition from the static model

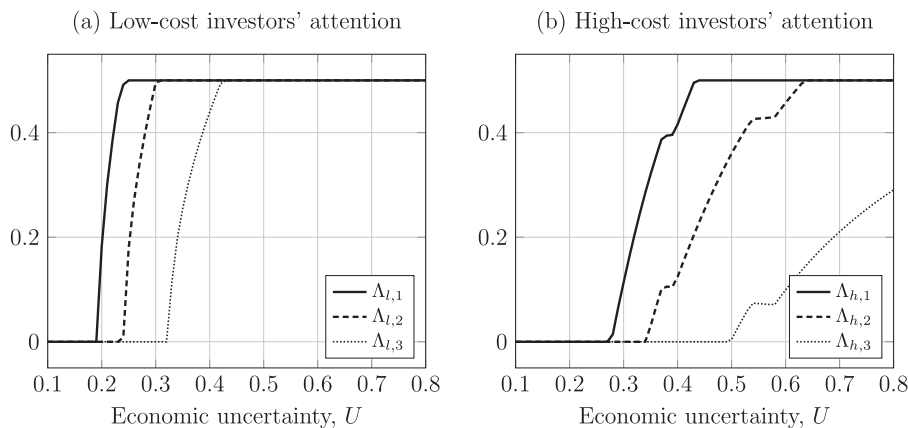


Fig. 5. The impact of economic uncertainty on investor attention in an economy with heterogeneous attention costs.

Each panel of the figure plots the fractions of attentive investors as functions of economic uncertainty, with low-cost investors in panel (a) and high-cost investors in panel (b). In this economy, $b_1 > b_2 > b_3$, $c_l = 0.045$, $c_h = 0.055$, the fractions of low-cost and high-cost investors are of equal size (50%), and the rest of the calibration is provided in Fig. 1.

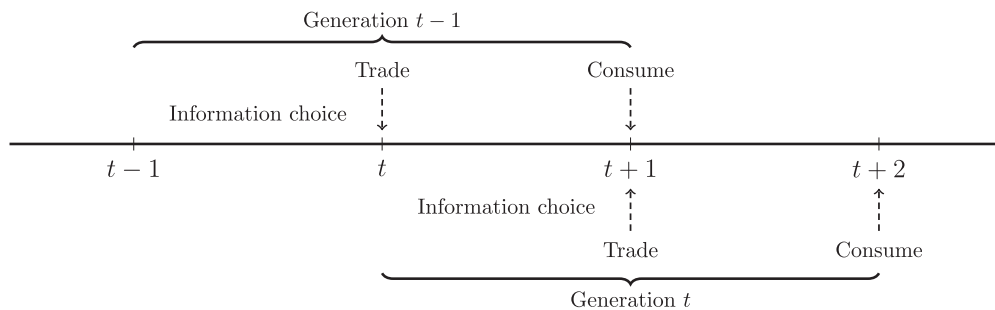


Fig. 6. Overlapping generations economy.

that ERCs increases with investor attention. Furthermore, this effect is more pronounced for firms with higher exposure to the systematic component or higher idiosyncratic volatility.

The dynamic model not only confirms the intuition from the one-period economy but also provides insight into the possibility that investors may increase their information acquisition before earnings announcements. In particular, the appendix demonstrates that, regardless of prior information acquisition decisions, heightened uncertainty at time t always increases the benefit of paying attention to the earnings announcement, leading to stronger ERCs. Thus, although investors' information search beforehand may partially offset the impact of increased uncertainty on ERCs, stronger attention still results in higher ERCs.⁶

⁶ As stated in Benamar et al. (2021), stronger investor attention in response to greater uncertainty does not fully neutralize the effect of uncertainty. This means that an increase in uncertainty at time $t - 1$ results in a higher uncertainty at t , despite investors paying closer attention at $t - 1$. Their conclusion is based on the idea that the cost of acquiring information increases with attention. As a result, the stronger attention at time $t - 1$ has only a partial offset on uncertainty at time t .

4. Empirical analyses

In this section, we conduct empirical tests of our theoretical predictions regarding the effect of aggregate uncertainty on investors' information acquisition, on ERCs, and on the CAPM. In our first set of tests, we examine the relation between uncertainty and investor attention around the announcement of quarterly earnings. That is, the unit of measurement in our analyses is the quarterly earnings announcement.

4.1. Variable definitions and summary statistics

We use the VIX, an option-based measure of expected S&P 500 volatility, to measure time-varying market-wide uncertainty. The VIX proxies for forward-looking stock market uncertainty, risk, or volatility, and its direct counterpart in our model is U (see Appendix A.8). To mitigate the potential for reverse causality, we use the closing VIX from the trading day prior to the earnings announcement.

We use the SEC's EDGAR download logs to capture investor information search. EDGAR is a public repository for company SEC filings. The SEC publicly discloses EDGAR search activity records (a "search" refers to accessing a specific filing). We employ the natural logarithm of two metrics: (1) the total volume of completed EDGAR searches per company-day (ESV) as a search-driven proxy for in-

vestor attention and (2) the number of downloads of a company's filings from unique IP addresses (*ESVU*) to measure the extensive margin of search, i.e., the number of investors accessing the firm's filings. EDGAR search records span February 14, 2003, to June 30, 2017.⁷ Note that a change in *ESV(U)* is equivalent to a change in $\log \Lambda_a$ in our model.⁸ As a secondary measure of investor search, we use the Investor Search Volume Index (*ISVI*), which is calculated based on investor searches for stock tickers via Google (generously provided by deHaan et al., 2021). We view *ISVI* as a secondary measure as it is available only from 2010 to 2018 and for a smaller sample of firms, and is a 0–100 index rather than a more easily interpretable raw count of searches.

As in prior studies (e.g., Livnat and Mendenhall, 2006; Hirshleifer et al., 2009; DellaVigna and Pollet, 2009; Even-Tov, 2017), we use standardized earnings surprise (*SUE*) deciles based on calendar-quarter sorts in our analyses of market reactions to earnings announcements. We measure earnings surprises as $SUE_{i,t} = (X_{i,t} - \mathbb{E}[X_{i,t}]) / P_{i,t}$, where i denotes firm, t denotes quarter, $X_{i,t}$ are IBES reported actual earnings, $\mathbb{E}[X_{i,t}]$ are expected earnings, taken as the latest median forecast from the IBES summary file (following Dai, 2020), and $P_{i,t}$ is the share price at the end of quarter t . We calculate earnings announcement returns, *EARET*, as the two-day size-adjusted returns, from the day of the earnings announcement through the day after.⁹

In our analyses of market reactions to earnings announcements, we use the following variables as controls, following prior literature (e.g., Hirshleifer et al., 2009): compound excess returns from ten to one days before the earnings announcement, *PreRet*; the market value of equity on the day of the earnings announcement, *Size*; the ratio of book value of equity to the market value of equity at the end of the quarter for which earnings are announced, *Book-to-Market*; earnings persistence based on estimated quarter-to-quarter autocorrelation in reported earnings, *EPersistence*; institutional ownership as a fraction of total shares outstanding at the end of the quarter for which the earnings are announced, *IO*; earnings volatility, *EVOL*; the reporting lag measured as the number of days from quarter end to the earnings announcement, *ERepLag*; analyst following defined as the number of analysts making quarterly earnings forecasts according to the IBES summary file, *#Estimates*; average monthly share turnover over

the preceding 12 months, *TURN*; an indicator variable for negative earnings, *Loss*; the number of other firms announcing earnings on the same day, *#Announcements*; year indicators; and day-of-week indicators.¹⁰

Our subsample analyses use partitions based on proxies for the underlying constructs. Although the exposures of firms' payoffs to the systematic factor f (the parameters b_n) are not perfectly observed in the data, they can be proxied by firms' CAPM betas. More precisely, in our model firms with larger exposures to f necessarily have higher market betas (we provide this link in Eq. (36)). We use forecast dispersion (*DISP*) and idiosyncratic volatility (*IDVOL*) as proxies for total earnings variance ($\text{Var}[E_a]$ in our model) and firm-specific payoff variance (σ_{ea}^2).¹¹ The variance of noise trade (σ_{xa}^2) is reflected in share turnover (*TURN*), though we caution that turnover also captures other constructs, such as information asymmetry and disagreement. Finally, we split the sample on institutional ownership (*IO*) to capture variation in the cost to investors of acquiring information (c), as these costs are likely to be lower for institutional than retail owners. We provide detailed variable definitions in Appendix B.

Our sample begins in 1995, as earnings announcement dates tended to be identified unreliably prior to 1995 (DellaVigna and Pollet, 2009; Hirshleifer et al., 2009). We further limit our sample to firms for which we can calculate analyst forecast-based earnings surprises, firms with a stock price greater than \$5, and firms with average monthly share turnover in the past year no lower than 1. The latter restrictions drop the smallest and least actively traded firms from the sample. Finally, we restrict the sample to observations for which data for all variables used in the respective analyses are available. This results in a sample of 224,675 firm-quarter observations for the analyses that do not require data on investor attention measures and 119,341 (62,757) for the analyses that require data availability on EDGAR (Google) searches. Table 1 provides the descriptive statistics for the variables used in our analyses.¹²

Table 2 provides correlations. Bold correlations are significant at the one percent level. *VIX* is negatively correlated with EDGAR search volume measures and *ISVI*, but these raw correlations do not account for other factors, such as time factors affecting both *VIX* and search volume (e.g., higher *VIX* and lower search in some years). *VIX* is generally not significantly related to earnings announcement returns or surprises, suggesting that prior-day economic uncertainty is not directly linked to firm-level earnings surprises.

⁷ Completed EDGAR searches refer to successful delivery of the requested document (code = 200), which is not an index page (idx = 0). EDGAR downloads can originate from humans or automated programs, as noted in prior research (e.g., Ryans, 2017). We use all downloads for three reasons: (1) automated downloads may serve information providers catering to investors; (2) automated downloads may be programmed to access EDGAR files conditional on other inputs to the program capturing, for instance, macroeconomic conditions; and (3) our use of year fixed effects in regressions controls for a secular trend of increasing robot downloads over time.

⁸ In the model, Λ_a can be approximated with Q_a/Q , where Q is a large number that measures the total population of investors and Q_a measures the number of investors who observe E_a . Hence, $\Delta \log \Lambda_a = \Delta \log Q_a$, and thus a change in $\log \Lambda_a$ is equivalent to a change in *ESV(U)*.

⁹ Our main results on earnings announcement window returns are robust to defining excess daily returns as firm-specific returns adjusted for either equal-weighted or value-weighted market returns.

¹⁰ To mitigate the influence of outliers among skewed/fat-tailed controls, we winsorize *Size*, *EPersistence*, and *EVOL* at the first and 99th percentiles.

¹¹ Note that Forecast Dispersion could be driven by variation and unpredictability in either earnings fundamentals ($\text{Var}[D_a] = b_a^2 \sigma_f^2 + \sigma_{ea}^2$) or earnings noise (σ_{ea}). As can be seen in a comparison of panels (d) and (e) of Fig. 4, σ_{ea}^2 and σ_{xa}^2 have opposing effects on the relation between uncertainty and ERCs.

¹² The number of observations for some variables in Table 1 is greater than 224,675, in part because we require some lagged variables to be non-missing in the regression tests but not in Table 1.

Table 1

Descriptive statistics.

This table reports descriptive statistics for the sample used in analyses of returns around earnings announcements. Detailed definitions of all variables are available in [Appendix B](#).

Variable	Obs. count	Mean	Std. dev.	25%	50%	75%
VIX	234,874	19.626	8.162	13.770	18.030	23.010
ESV	124,660	4.719	1.999	3.367	4.934	6.319
ESVU	124,660	3.664	1.506	2.639	3.912	4.883
ISVI	66,534	4.419	13.315	0.000	0.000	0.000
EARET	234,874	0.001	0.080	−0.033	0.001	0.037
SUE Decile	234,874	5.536	2.705	3.000	6.000	8.000
PreRet	234,851	0.002	0.081	−0.035	−0.001	0.035
Size	234,874	4973.899	13764.427	282.266	854.312	2980.429
Book-to-Market	234,727	0.534	0.382	0.274	0.458	0.701
EPersistence	234,206	0.226	0.398	−0.040	0.180	0.500
IO	225,437	0.633	2.288	0.430	0.666	0.842
EVOL	234,232	0.822	2.115	0.116	0.272	0.654
ERepLag	234,874	30.765	13.609	22.000	28.000	37.000
#Estimates	234,874	7.799	6.573	3.000	6.000	11.000
TURN	234,874	17.446	17.605	6.935	12.826	22.120
Loss	234,874	0.194	0.396	0.000	0.000	0.000
#Announcements	234,874	150.476	92.544	73.000	137.000	221.000

4.2. Attention and earnings response coefficients

As we elaborate on in [Section 2](#), our first hypotheses relate to the effects of economic uncertainty on investor attention to firm-level information, which we test for using investor searches and market reactions around earnings announcements.

Our first set of tests examines whether aggregate uncertainty affects firm-level search activity in and of itself. For these tests, we exploit the SEC EDGAR records of access to company-specific filings around quarterly earnings announcements as well as investors' Google searches captured by *ISVI*. We estimate the following regression equation:

$$SEARCH_{it} = c_0 + c_1 \times VIX_t + c_2 \times ESV_{it-1} + c_3 \times SUE_{it} + c_4 \times \text{abs}(SUE_{it}) + \gamma \cdot X_{it} + u_{it}, \quad (38)$$

where *SEARCH* is either the log of daily EDGAR search volume (*ESV*), the log of daily EDGAR search volume from unique IP addresses (*ESVU*), or *ISVI*. We also include the lagged dependent variable (*ESV*, *ESVU*, or *ISVI* on the previous earnings announcement), the standardized *SUE* decile, and the absolute value of the standardized *SUE* decile to control for differences in average search volume across firms and in response to earnings news. [Table 3](#) presents the results from the estimation of (38). In [Table 3](#) and the remaining tests, we standardize all variables to a mean of zero and unit variance for ease of interpretation.

The results in [Table 3](#) provide strong evidence for more active searching for firm-level information on days with higher *VIX*, as the coefficients of interest on *VIX* are positive and statistically significant for all three dependent variables. The coefficients of interest can be interpreted as the approximate percent change in search volume or unique searchers for a standard deviation change in the *VIX*. A one standard deviation change in *VIX* is associated with a 3.0 (3.4) percent increase in the number of EDGAR searches (from unique IP addresses) for the announcer's filings on the earnings announcement date and

a 1.8 percent increase in *ISVI* relative to its standard deviation (recall that *ISVI* is an index rather than a logged count variable such as *ESV(U)*).¹³ Lagged dependent variables are significantly associated with announcement day searches, as are the signed and absolute earnings surprise deciles (except for absolute *SUE* in the *ESV* specification). Coefficients on *#Announcements* are negative, though only statistically significant in the *ESV* and *ISVI* specifications, supporting the multiple-announcements effect shown in [Hirshleifer et al. \(2009\)](#). However, they also support our interpretation of [Eq. \(26\)](#): due to information spillovers, a higher number of announcements increases aggregate price informativeness, diminishing the benefit of attention. In the remainder, we focus on EDGAR search volume measures (*ESV* and *ESVU*), as these are available for a more extended period covering roughly twice the number of earnings announcements as *ISVI*.

Our next set of tests exploits the model's predictions regarding price reactions to firm-level information. We examine how economic uncertainty interacts with firm-level news in the price formation process. We focus on the association between size decile-adjusted stock returns in the two-day earnings announcement window and the earnings surprise, the *VIX*, the interaction between the *VIX* and the earnings surprise, and a set of controls. We interact each of these controls with our earnings surprise variable to mitigate concerns that a correlated omitted interaction drives the coefficient on our interaction of interest. Standard errors are clustered at the earnings announcement date level.

To test the hypotheses developed in [Section 2](#), we estimate the following regressions at the firm-quarter level:

¹³ In unreported analysis replacing *VIX* with *VIX* centile indicators in the specifications presented in [Table 3](#) (i.e., $SEARCH_{it} = c_0 + \sum_{j=1}^{100} c_{1j} \times VIXCentile_{tj} + \dots$), we find that the relation between *VIX* and attention measures is convex, consistent with the convexity in [Fig. 2\(a\)](#) when attention to announcements begins increasing from around $U \in [0.19, 0.32]$. We focus on the linear empirical effect identified by estimating [Eq. \(38\)](#) for ease of interpretation.

Table 2
Correlations. This table presents Spearman (Pearson) correlations above (below) the diagonal for all variables used in the analyses. Detailed definitions of all variables are available in Appendix B. Bold indicates significance at the one percent level.

Variable	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1 VIX																	
2 ESU	-0.025																
3 ESUV	-0.008	0.967															
4 ISVI	-0.050	0.153	0.168														
5 EARET	-0.002	-0.001	0.015	0.002													
6 SUE Decile	0.004	0.012	0.015	0.013	0.299												
7 PreRet	-0.007	0.004	0.003	0.003	-0.043	0.096											
8 Size	-0.050	0.215	0.242	0.169	0.002	0.002	0.007										
9 Book-to-Market	0.097	0.003	-0.005	-0.070	0.017	-0.019	-0.005	-0.142									
10 E-Persistence	0.028	-0.101	-0.097	0.000	0.001	0.004	0.010	0.010	-0.070								
11 IO	-0.007	0.016	0.015	0.058	0.000	0.002	-0.002	0.009	-0.008	0.000							
12 EVOL	-0.004	0.028	0.031	0.016	-0.018	0.003	-0.017	0.004	0.108	-0.113	0.004						
13 EReplag	-0.024	0.071	0.060	-0.019	-0.022	-0.049	-0.019	-0.128	0.032	-0.074	0.004	0.073					
14 #Estimates	-0.047	0.262	0.291	0.145	0.014	0.014	-0.004	0.534	-0.195	0.059	0.043	0.006	-0.140				
15 TURN	0.046	0.102	0.121	0.084	-0.022	0.032	0.018	-0.023	-0.086	0.077	0.037	0.100	0.041	0.298			
16 Loss	0.033	0.037	0.044	0.012	-0.091	-0.149	-0.040	-0.101	0.050	-0.055	-0.004	0.128	0.185	-0.070	0.157		
17 #Announcements	0.027	-0.095	-0.080	-0.034	0.002	0.025	0.004	-0.032	-0.019	0.012	0.004	0.008	-0.231	-0.019	0.000	-0.004	

$$EARET_{it} = c_0 + c_1 \times SUE_{it} + c_2 \times VIX_t + c_3 \times SUE_{it} * VIX_t + \gamma \cdot X_{it} + u_{it}, \text{ and}$$

$$EARET_{it} = c_0 + c_1 \times SUE_{it} + c_2 \times ESUV_t + c_3 \times SUE_{it} * ESUV_t + \gamma \cdot X_{it} + u_{it}, \quad (39)$$

where the dependent variable $EARET_{it}$ represents the announcement-window return and X_{it} represents a set of controls.

Column (1) of Table 4 reports our estimates of the first equation in (39). The coefficient on SUE decile is positive and significantly different from zero (0.204, $p < 0.01$), consistent with positive market responses to earnings surprises. Our coefficient of interest, the interaction between VIX and SUE , is also positive and significantly different from zero (0.015, $p < 0.01$). We infer from this that market responses to firm-level information are higher on days with greater uncertainty. Specifically, a one standard deviation change in VIX yields an ERC that is approximately seven percent higher than the average response to earnings surprises ($7\% = 0.015/0.204$).

Columns (2) and (3) of Table 4 explore the mediating role of attention. In column (2), we replace VIX with $ESUV$. The sample shrinks considerably because EDGAR search data is available for a shorter window (2003–2017 relative to the 1995–2020 earnings announcement sample). Even with the smaller sample, the coefficient on $ESUV * SUE$ is positive and significant (0.028, $p < 0.01$), consistent with earnings announcements that attract greater investor attention receiving stronger market reactions in the announcement window. In column (3), we include both VIX and $ESUV$ as well as their interactions with SUE . The coefficients of interest are both positive, although the $ESUV * SUE$ interaction (0.027, $p < 0.01$) is significant while the $VIX * SUE$ interaction (0.010, $p > 0.10$) becomes insignificant at traditional cutoffs. Overall, the coefficient pattern is consistent with the indirect effect of VIX on market responses, operating through investor attention allocation as reflected in EDGAR search activity, in line with the prediction of our model illustrated in Eq. (27) and Fig. 3.¹⁴

Our use of SUE deciles may raise a concern, as the dispersion of earnings surprises across SUE deciles could be larger during periods of high VIX . Empirically, we observe that the dispersion in the lowest SUE decile is higher on high VIX days. To ensure that this dispersion is not driving the higher ERCs, we conduct two additional analyses. First, we re-run the analyses in Table 4 using raw SUE s instead of SUE deciles. Second, we re-run the same analyses using a subsample that excludes the lowest SUE decile. Both tests (tabulated in IA.1 and IA.2, respectively, in the Internet Appendix) yield similar results for the $ESUV$ and SUE

¹⁴ Our results are consistent with Drake et al. (2015), who also find a positive association between EDGAR search volume and ERCs. Additionally, they present evidence that EDGAR search volume around the earnings announcement is associated with less post-earnings announcement drift (PEAD). While we do not find a similar effect on average, we find in untabulated analysis a moderate negative association between EDGAR search volume and near-term PEAD for firms with above-median CAPM beta. This is consistent with our theoretical prediction of stronger effects for firms with greater systematic risk and in line with the higher ERC for high-beta firms documented in our Table 6.

Table 3

Investor attention and economic uncertainty.

This table presents results of regressions of announcement-window EDGAR and (investor-driven) Google searches on prior day's closing value of *VIX* and controls (Eq. (38)). All variables are standardized to be mean-zero and unit-variance. Detailed definitions of all variables are available in Appendix B. Standard errors for the coefficients are clustered by date. ***, **, and * indicate statistical significance at the two-sided 1%, 5%, and 10% levels, respectively.

Dep. Var.	<i>ESV</i> (1)	<i>ESVU</i> (2)	<i>ISVI</i> (3)
<i>VIX</i>	0.030*** (0.005)	0.034*** (0.005)	0.018* (0.010)
lag(Dep. Var.)	0.455*** (0.013)	0.535*** (0.013)	0.238*** (0.008)
<i>SUE</i> Decile	0.005*** (0.001)	0.005*** (0.001)	0.009** (0.004)
abs(<i>SUE</i>) Decile	0.005 (0.003)	0.011*** (0.003)	0.015* (0.008)
<i>Size</i>	0.059*** (0.002)	0.059*** (0.003)	0.086*** (0.005)
<i>Book-to-Market</i>	−0.008*** (0.001)	−0.011*** (0.001)	−0.014*** (0.004)
<i>EPersistence</i>	−0.016*** (0.002)	−0.013*** (0.002)	0.005 (0.004)
<i>IO</i>	0.002 (0.001)	0.001 (0.001)	0.107*** (0.034)
<i>EVOL</i>	0.006*** (0.001)	0.007*** (0.001)	0.009* (0.005)
<i>ERepLag</i>	0.025*** (0.006)	0.016*** (0.006)	0.010** (0.005)
<i>#Estimates</i>	0.045*** (0.003)	0.045*** (0.003)	0.034*** (0.005)
<i>TURN</i>	0.027*** (0.002)	0.028*** (0.002)	0.055*** (0.006)
<i>Loss</i>	−0.001 (0.001)	0.004*** (0.001)	0.002 (0.004)
<i>#Announcements</i>	−0.024** (0.011)	−0.010 (0.012)	−0.019*** (0.005)
Year FE	Yes	Yes	Yes
Day-of-week FE	Yes	Yes	Yes
Obs. Count	119,341	119,341	62,757
<i>R</i> ²	0.803	0.817	0.122

interaction, while the *VIX* and *SUE* interaction becomes statistically weaker.

Next, we test additional theoretical predictions from our model using cross-sectional analysis. Our model demonstrates that the effect of economic uncertainty on ERCs is monotonic in CAPM beta (b_a), earnings variance ($\text{Var}[E_a]$), idiosyncratic volatility (σ_{ea}), noise trading (σ_{xa}), and investor attention costs (c). Accordingly, we predict that our findings will be stronger among firms with high beta, high earnings variance, high idiosyncratic volatility, high noise trading, and low processing costs, captured empirically by CAPM beta, forecast dispersion (*DISP*), idiosyncratic volatility (*IDVOL*), trailing share turnover (*TURN*), and institutional ownership (*IO*), respectively. Table 5 presents estimates from these cross-sectional splits, where the variable of interest is the *VIX***SUE* interaction.

In the CAPM beta split subsamples, the coefficient of interest has a positive sign but is not statistically significant for low-beta firms. In contrast, the coefficient for high-beta firms is positive and significantly different from both zero (0.022, $p < 0.01$) and the corresponding low-beta coefficient ($p < 0.10$). This is consistent with our result in

Table 4

ERCs, economic uncertainty and investor attention.

This table presents results of regressions of earnings announcement returns (*EARET*) on earnings surprise deciles interacted with the *VIX* (column 1), with *ESVU* (column 2), and with both the *VIX* and *ESVU* (column 3) (Eq. (39)). All variables are standardized to be mean-zero and unit-variance. Control variables include: *PreRet*, *Size*, *Book-to-Market*, *EPersistence*, *IO*, *EVOL*, *ERepLag*, *#Estimates*, *Turn*, *Loss*, *#Announcements*, year indicators, day-of-week indicators, and each of these interacted with *SUE* Decile. Detailed definitions of all variables are available in Appendix B. Standard errors for the coefficients are clustered by date. ***, **, and * indicate statistical significance at the two-sided 1%, 5%, and 10% levels, respectively.

	Dep. Var. = <i>EARET</i>		
	(1)	(2)	(3)
<i>VIX</i> * <i>SUE</i> Decile	0.015*** (0.005)		0.010 (0.007)
<i>ESVU</i> * <i>SUE</i> Decile		0.028*** (0.007)	0.027*** (0.007)
<i>SUE</i> Decile	0.204*** (0.012)	0.295*** (0.018)	0.297*** (0.018)
<i>VIX</i>	−0.007 (0.005)		−0.007 (0.006)
<i>ESVU</i>		−0.018*** (0.006)	−0.018*** (0.006)
lag(<i>ESVU</i>)		−0.001 (0.007)	−0.001 (0.007)
lag(<i>ESVU</i>)* <i>SUE</i> Decile		−0.011 (0.007)	−0.010 (0.007)
<i>PreRet</i>	−0.075*** (0.004)	−0.080*** (0.006)	−0.080*** (0.006)
<i>PreRet</i> * <i>SUE</i> Decile	−0.012*** (0.003)	−0.012** (0.005)	−0.011** (0.005)
Controls	Yes	Yes	Yes
Controls* <i>SUE</i> Decile	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Day-of-week FE	Yes	Yes	Yes
Obs. Count	224,675	119,332	119,332
<i>R</i> ²	0.111	0.143	0.143

Fig. 2, panel (b), that the effect of economic uncertainty on ERCs is greater for firms with larger exposures to systematic risk. In the subsamples split on forecast dispersion and idiosyncratic volatility, the coefficients of interest are both positive and significantly different from zero in above-median subsamples (0.017 and 0.014, $p < 0.05$). However, they are not significantly different from those in the corresponding below-median subsamples.

For the splits using share turnover to capture the expected magnitude of noise trade, σ_{xa} , the effects of economic uncertainty on ERCs are concentrated in subsamples with above-median *TURN*. The coefficient on *VIX***SUE* in the high-*TURN* sample is positive and significantly different from both zero (0.022, $p < 0.01$) and the coefficient in the low-*TURN* sample (0.04, $p < 0.10$ for the test of difference in coefficients). This plausibly captures the predicted positive effect shown in Fig. 4, panel (f), where the effect of economic uncertainty on ERCs is greater when the volatility of noise trade is larger. Similar to noise trade in our model, high turnover can make it difficult to infer fundamental information from price, making attention to earnings incrementally more valuable during periods of high uncertainty.

Our last sample splits are based on institutional ownership (*IO*). It is plausible to assume that retail investors

Table 5

Cross-sectional analyses: Economic uncertainty and ERCs.

This table presents results of regressions of earnings announcement returns (*EARET*) on earnings surprise deciles based on quarterly sorts (*SUE* Decile) interacted with the *VIX*. All variables are standardized to be mean-zero and unit-variance. Control variables include: *PreRet*, *Size*, *Book-to-Market*, *EPersistence*, *IO*, *EVOL*, *ERepLag*, *#Estimates*, *Turn*, *Loss*, *#Announcements*, year indicators, day-of-week indicators, and each of these interacted with *SUE* Decile. Detailed definitions of all variables are available in [Appendix B](#). Standard errors for the coefficients are clustered by date. ***, **, and * indicate statistical significance at the two-sided 1%, 5%, and 10% levels, respectively.

Sample:	Dep. Var. = <i>EARET</i>									
	Subsamples: based on within-year-of-earnings-announcement median splits on announcing firm characteristics									
	Low CAPM Beta	High CAPM Beta	Low <i>DISP</i>	High <i>DISP</i>	Low <i>IDVOL</i>	High <i>IDVOL</i>	Low <i>TURN</i>	High <i>TURN</i>	Low <i>IO</i>	High <i>IO</i>
<i>VIX</i> * <i>SUE</i> Decile	0.007 (0.006)	0.022*** (0.007)	0.022*** (0.007)	0.017** (0.007)	0.012** (0.005)	0.014** (0.007)	0.004 (0.006)	0.022*** (0.008)	0.007 (0.005)	0.024*** (0.008)
<i>SUE</i> Decile	0.206*** (0.015)	0.221*** (0.019)	0.296*** (0.019)	0.150*** (0.018)	0.180*** (0.013)	0.229*** (0.018)	0.360*** (0.019)	0.216*** (0.019)	0.237*** (0.016)	0.197*** (0.019)
<i>VIX</i>	−0.022*** (0.005)	0.009 (0.008)	−0.009* (0.005)	−0.007 (0.008)	−0.013*** (0.005)	−0.002 (0.008)	−0.015** (0.006)	−0.001 (0.008)	−0.008 (0.006)	−0.007 (0.006)
<i>PreRet</i>	−0.078*** (0.004)	−0.076*** (0.005)	−0.082*** (0.005)	−0.069*** (0.005)	−0.072*** (0.005)	−0.080*** (0.004)	−0.071*** (0.004)	−0.080*** (0.005)	−0.083*** (0.005)	−0.067*** (0.005)
<i>PreRet</i> * <i>SUE</i> Decile	−0.015*** (0.004)	−0.011*** (0.004)	−0.015*** (0.006)	−0.012*** (0.004)	−0.011** (0.004)	−0.014*** (0.003)	−0.018*** (0.004)	−0.011** (0.004)	−0.010*** (0.003)	−0.015*** (0.005)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Controls* <i>SUE</i> Decile	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Day-of-week FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Obs. Count	112,431	112,128	100,029	98,367	112,575	111,984	112,503	112,172	112,322	112,353
<i>R</i> ²	0.123	0.108	0.12	0.11	0.124	0.114	0.14	0.103	0.115	0.111

face greater opportunity costs than institutional investors when choosing whether to pay attention to firm-level information. Indeed, recent empirical evidence supports the view that retail investors are more susceptible to distractions than institutional investors (Israeli et al., 2022; Da et al., 2022). Consistent with this interpretation and our predictions illustrated in Fig. 5, we find that the effect of economic uncertainty on ERCs is concentrated in the high- IO subsample (0.024, $p < 0.01$), while the estimated effect for the low- IO subsample is insignificantly different from zero (0.007, $p > 0.10$). The difference in coefficients is large in percentage terms ($0.024/0.007 = 343\%$) and significantly different from zero at the 10% level, consistent with lower information acquisition costs amplifying the effects of economic uncertainty on ERCs.

Table 6 re-estimates the regressions from Table 5 with $ESVU$ replacing VIX , to provide evidence that the effects are attributable to attention rather than the VIX itself and other co-varying constructs, in line with Fig. 3 from our theoretical analysis. The pattern is generally similar, albeit weaker in some regressions, plausibly due to the smaller sample size. Interestingly, the results for the forecast dispersion and idiosyncratic volatility splits are stronger than those in Table 5, as the effect of $ESVU$ on ERCs is concentrated in the high forecast dispersion (0.036, $p < 0.01$) and idiosyncratic volatility (0.034, $p < 0.01$) subsamples. These coefficients are also significantly different from those in the corresponding below-median subsamples ($p < 0.10$ for both), consistent with heightened variance in earnings ($\text{Var}[E_a]$ or $\sigma_{\varepsilon_a}^2$) leading to stronger relations between attention and ERCs.

4.3. Calibration around earnings announcements

Can our model generate quantitatively similar attention responses to changes in economic uncertainty? To answer this question, we calibrate our model based on historical data. First, in our earnings announcement sample period from 1995 to 2020, the VIX averaged 20, with a daily standard deviation of 8.5. We define $U \equiv VIX/100$ (VIX values are quoted in percentage points) and standardize it, i.e., $\hat{U} \equiv (U - 0.2)/0.085$. In our illustration, we will allow U to take values between 0.1 and 0.4, since during our sample period the 10th and 90th percentile of VIX were 12 and 30, respectively.

In our sample the average number of firms per quarter is 2264, and the average number of announcements per trading day is 53, with a standard deviation of 67.¹⁵ To compare, Frederickson and Zolotoy (2016) report an average of 41 announcements per trading day with a standard deviation of 61, and Ferracuti and Lind (2021) report an average of 63 and a standard deviation of 83. Hirshleifer and Sheng (2022) report a higher average, 118, and a standard deviation of 79. These studies do not separately report the number of unique firms per year or quarter. Accordingly, we set the total number of firms in

the economy as $N = 3000$ and assume that between 10 and 100 firms announce their earnings on a given trading day. The remaining calibration parameters are: $\gamma = 10$; $\sigma_e = \sigma_\varepsilon = 0.4$ for all firms; the market portfolio \mathbf{M} is a vector whose values are all equal to $1/3000$; the volatility of noise in supply is $\sigma_x = 1/(3000 \times 4)$ for all firms (which ensures that the probability of having negative supplies is negligible); all the betas of the announcing firms are 1; and the cost of information is $c = 0.03$.¹⁶

Panel (a) of Fig. 7 plots the response of $\log \Lambda_a$ to a change in \hat{U} , or $\partial \log \Lambda_a / \partial \hat{U}$, in two cases: when 10 firms are announcing earnings (solid line) and when 100 firms are announcing (dashed line). On the horizontal axis we let U vary from 0.1 to 0.4, while the vertical axis measures the sensitivity of $\log \Lambda_a$ to changes in \hat{U} , consistent with the coefficient c_1 in (38). The plot shows that our calibrated model can match the numbers in Table 3. Furthermore, the model also correctly implies a lower coefficient when the number of announcers is higher (in which case price informativeness is higher), in line with the negative coefficients for $\#Announcements$ obtained in Table 3.

Panel (b) plots the model-implied ERCs as functions of U when 10 firms are announcing earnings (solid line) and when 100 firms are announcing (dashed line). Our model generates plausible magnitudes for ERCs, comparable with coefficients on SUE Decile in Table 4. The plot also shows that ERCs increase with U but are smaller when more firms announce earnings, consistent with panel (a) showing that attention is a substitute for price informativeness. (See also Chen et al., 2020b, who document a similar substitution effect between the acquisition of private information and the supply of public information.)

4.4. CAPM tests

We now turn to the predictions of our model for the CAPM. Corollary 4.1 shows that the market risk premium is increasing in both ex-ante uncertainty and investor attention, which implies a steeper SML. Furthermore, Eq. (36) implies that firms' betas increase on earnings announcement days, but only if investors pay attention to announcements. In order to test these predictions, we employ classical Fama and MacBeth (1973) two-step regressions to estimate firm and portfolio betas. The dataset utilized in our analysis comprises daily excess returns for individual firms available from CRSP, merged with the EDGAR search data. The sample period for the EDGAR data ranges from 2003-02-14 to 2017-06-30 and sets the limits for the final merged sample.

To conduct firm-level analysis, we estimate betas separately for earnings days and high-attention days. Specifically, we define four indicator variables for each firm i : $\mathbf{1}_{EA}^i$, which equals 1 on days when firm i announces earnings; $\mathbf{1}_{HighAtt}^i$, which equals 1 on days when investor attention to

¹⁵ Note that this differs from the mean $\#Announcements$ at the firm-announcement level reported in Table 1, as a trading day with N announcements would be counted once in an average across trading days but N times in an average across firm-announcements.

¹⁶ To the best of our knowledge, the only attempt in the literature to estimate the parameters of Hellwig (1980)'s noisy rational expectations model is Cho and Krishnan (2000). In line with the estimation in their Table 2, our calibration assumes that noise in supply is considerably smaller than noise in private information ($\sigma_x \ll \sigma_\varepsilon$), and also a reasonable value of ten for the coefficient of risk aversion.

Table 6

Cross-sectional analyses: Investor attention and ERCs.

This table presents results of regressions of earnings announcement returns (*EARET*) on earnings surprise deciles based on quarterly sorts (*SUE* Decile) interacted with *ESVU*. All variables are standardized to be mean-zero and unit-variance. Control variables include: *PreRet*, *Size*, *Book-to-Market*, *EPersistence*, *IO*, *EVOL*, *ERePlag*, *#Estimates*, *Turn*, *Loss*, *#Announcements*, year indicators, day-of-week indicators, and each of these interacted with *SUE* Decile. Detailed definitions of all variables are available in [Appendix B](#). Standard errors for the coefficients are clustered by date. ***, **, and * indicate statistical significance at the two-sided 1%, 5%, and 10% levels, respectively.

Sample:	Dep. Var. = <i>EARET</i>									
	Subsamples: based on within-year-of-earnings-announcement median splits on announcing firm characteristics									
	Low CAPM Beta	High CAPM Beta	Low <i>DISP</i>	High <i>DISP</i>	Low <i>IDVOL</i>	High <i>IDVOL</i>	Low <i>TURN</i>	High <i>TURN</i>	Low <i>IO</i>	High <i>IO</i>
<i>ESVU</i> * <i>SUE</i> Decile	0.014* (0.008)	0.035*** (0.011)	0.009 (0.011)	0.036*** (0.010)	0.012 (0.007)	0.034*** (0.010)	0.013* (0.008)	0.028** (0.012)	0.024*** (0.009)	0.026** (0.010)
<i>SUE</i> Decile	0.284*** (0.023)	0.313*** (0.025)	0.305*** (0.032)	0.261*** (0.027)	0.221*** (0.020)	0.348*** (0.027)	0.430*** (0.023)	0.304*** (0.030)	0.354*** (0.025)	0.240*** (0.027)
<i>ESVU</i>	−0.010 (0.008)	−0.025** (0.010)	−0.020** (0.008)	−0.021* (0.011)	−0.018*** (0.006)	−0.017 (0.011)	0.001 (0.007)	−0.035*** (0.012)	−0.016* (0.009)	−0.018* (0.010)
lag(<i>ESVU</i>)	−0.001 (0.008)	0.004 (0.011)	−0.010 (0.008)	0.011 (0.011)	−0.002 (0.007)	0.004 (0.011)	−0.005 (0.007)	0.014 (0.012)	0.004 (0.009)	−0.002 (0.010)
lag(<i>ESVU</i>)* <i>SUE</i> Decile	−0.008 (0.009)	−0.026** (0.012)	−0.021* (0.012)	−0.022** (0.011)	−0.009 (0.008)	−0.015 (0.011)	−0.001 (0.008)	−0.051*** (0.012)	−0.003 (0.010)	−0.029*** (0.011)
<i>PreRet</i>	−0.084*** (0.007)	−0.078*** (0.008)	−0.098*** (0.008)	−0.067*** (0.008)	−0.082*** (0.007)	−0.082*** (0.007)	−0.075*** (0.007)	−0.085*** (0.008)	−0.082*** (0.007)	−0.079*** (0.008)
<i>PreRet</i> * <i>SUE</i> Decile	−0.016** (0.007)	−0.009 (0.007)	−0.011 (0.010)	−0.011 (0.007)	−0.017** (0.007)	−0.010* (0.006)	−0.016*** (0.006)	−0.009 (0.007)	−0.011** (0.006)	−0.011 (0.009)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Controls* <i>SUE</i> Decile	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Day-of-week FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Obs. Count	59,131	60,089	54,710	53,347	60,313	58,907	59,827	59,505	59,158	60,174
<i>R</i> ²	0.148	0.144	0.158	0.138	0.156	0.145	0.171	0.134	0.148	0.142

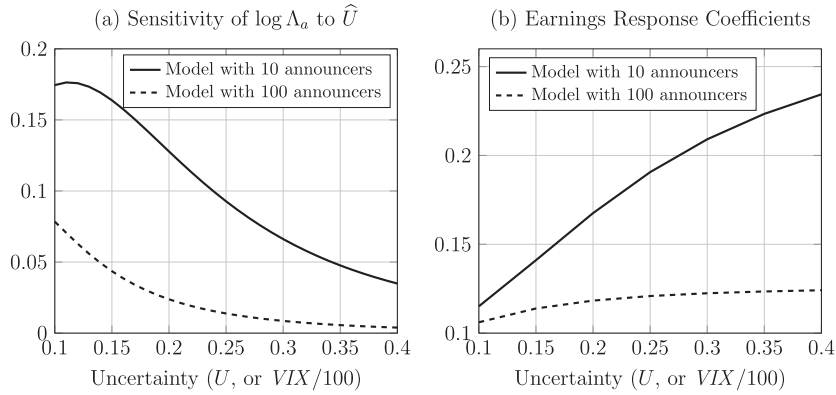


Fig. 7. Response of investor attention to changes in uncertainty and ERCs in an economy with 3000 firms.

Panel (a) plots the partial derivative of $\log \Lambda_a$ with respect to standardized uncertainty \hat{U} when $U \in [0.1, 0.4]$, and is thus the theoretical counterpart of the coefficient c_1 in (38) and in Table 3. Panel (b) plots ERCs implied by the theoretical model when $U \in [0.1, 0.4]$ and is thus the theoretical counterpart of the coefficient c_1 in (39) and in Table 4. Both panels consider two alternatives, one with 10 announcers (solid lines), and one with 100 announcers (dashed lines). See Section 4.3 for a detailed description of the calibration.

firm i (as measured by the time-detrended $ESV(U)$ of firm i) exceeds the full-sample median; $\mathbf{1}_{EA}^{high,i}$, which equals 1 when investor attention to firm i exceeds the median computed within the set of earnings announcement days; and $\mathbf{1}_{EA}^{low,i}$, which equals 1 when investor attention to firm i is below the median computed within the set of earnings announcement days. Subsequently, we estimate three time-series regressions for each firm:

$$r_{i,t}^e = \alpha_{Other}^i + \alpha_{\Delta EA}^i \mathbf{1}_{EA}^i + \beta_{Other}^i r_{M,t}^e + \beta_{\Delta EA}^i (\mathbf{1}_{EA}^i \times r_{M,t}^e) + \varepsilon_{i,t} \quad (40)$$

$$r_{i,t}^e = \alpha_{Other}^i + \alpha_{\Delta A}^i \mathbf{1}_{HighAtt}^i + \beta_{Other}^i r_{M,t}^e + \beta_{\Delta A}^i (\mathbf{1}_{HighAtt}^i \times r_{M,t}^e) + \varepsilon_{i,t} \quad (41)$$

$$r_{i,t}^e = \alpha_{Other}^i + \alpha_{\Delta EA}^{low,i} \mathbf{1}_{EA}^{low,i} + \alpha_{\Delta EA}^{high,i} \mathbf{1}_{EA}^{high,i} + \beta_{Other}^i r_{M,t}^e + \beta_{\Delta EA}^{low,i} (\mathbf{1}_{EA}^{low,i} \times r_{M,t}^e) + \beta_{\Delta EA}^{high,i} (\mathbf{1}_{EA}^{high,i} \times r_{M,t}^e) + \varepsilon_{i,t}, \quad (42)$$

where $r_{M,t}^e$ is the excess return on the market and $r_{i,t}^e$ is the excess return for firm i .

The first regression examines whether firm betas increase on earnings announcement days, with $\beta_{\Delta EA}^i$ in (40) representing the change in firm i 's beta on such days. The second regression tests whether firm betas vary with investor attention, with $\beta_{\Delta A}^i$ in (41) measuring the change in firm i 's beta on days when investor attention to the firm's information exceeds its sample median. The third regression directly tests (36), with $\beta_{\Delta EA}^{high,i}$ in (42) indicating the change in firm i 's beta on earnings announcement days with heightened investor attention.

We estimate (40)–(42) for each firm and calculate average betas and their standard errors across firms. Table 7 displays the estimates. Column (1) verifies Patton and Verardo (2012)'s finding that firm betas rise on earnings announcement days, with an average increase of 0.088 ($p < 0.05$). Columns (2) and (3) show that firm betas also

increase with investor attention on average, as measured by ESV and $ESVU$. When the detrended $ESV(U)$ surpasses its median, betas rise by 0.042 ($p < 0.01$) and 0.019 ($p < 0.01$), respectively.

Columns (4) and (5) divide earnings announcement days into high- and low-attention days, following (42). Betas increase on earnings announcement days *only* when investor attention is high. The average $\beta_{\Delta EA}^{high,i}$ is positive and statistically significant in both columns, while $\beta_{\Delta EA}^{low,i}$ is smaller and not statistically significant. Results hold for both attention measures, with average $\beta_{\Delta EA}^{high,i}$ being 0.141 ($p < 0.01$) and 0.123 ($p < 0.01$) using $ESV(U)$. Overall, our findings support our model's prediction that betas of announcing firms increase only when investors pay attention.

An alternative explanation of the results in Table 7 is that important announcements, such as those that reveal more news about aggregate economic developments, attract more attention. In other words, it is not the case that higher attention causes higher betas, but rather that announcements associated with higher betas attract more attention. Notably, our theoretical model aligns with this alternative explanation, as demonstrated in Section 2.1, Eq. (26), where we show that attention to high-beta firms is more valuable. The measured variation in betas could encompass both time-varying firm sensitivity to macro fluctuations and the added effect of heightened attention. We focus our discussion on the incremental effects of attention, which are a novel implication of our model, but we acknowledge that some identified effects could be due to increased attention to more informative announcements, i.e., naturally time-varying true betas (e.g., Ghysels, 1998).

In portfolio-level analysis, we investigate if the firm-level findings in Table 7 extend to the portfolio level. To accomplish this, we utilize all firms in our sample for which EDGAR search data is available to construct 10 beta-sorted portfolios and calculate their value-weighted daily excess returns. We categorize firms into portfolios based on their full-sample betas and do not rebalance portfolios, ensuring that each firm is assigned to a unique portfolio. For

Table 7

Firm betas and investor attention on earnings announcement days. This table reports average β coefficients across firms in three regressions that examine the effects of investor attention and earnings announcements on excess returns:

$$r_{i,t}^e = \alpha_{\text{Other}}^i + \alpha_{\Delta EA}^i \mathbf{1}_{EA}^i + \beta_{\text{Other}}^i r_{M,t}^e + \beta_{\Delta EA}^i (\mathbf{1}_{EA}^i \times r_{M,t}^e) + \varepsilon_{i,t} \quad (\text{i})$$

$$r_{i,t}^e = \alpha_{\text{Other}}^i + \alpha_{\Delta A}^i \mathbf{1}_{\text{HighAtt}}^i + \beta_{\text{Other}}^i r_{M,t}^e + \beta_{\Delta A}^i (\mathbf{1}_{\text{HighAtt}}^i \times r_{M,t}^e) + \varepsilon_{i,t} \quad (\text{ii})$$

$$r_{i,t}^e = \alpha_{\text{Other}}^i + \alpha_{\Delta EA}^{\text{low},i} \mathbf{1}_{EA}^{\text{low},i} + \alpha_{\Delta EA}^{\text{high},i} \mathbf{1}_{EA}^{\text{high},i} + \beta_{\text{Other}}^i r_{M,t}^e + \beta_{\Delta EA}^{\text{low},i} (\mathbf{1}_{EA}^{\text{low},i} \times r_{M,t}^e) + \beta_{\Delta EA}^{\text{high},i} (\mathbf{1}_{EA}^{\text{high},i} \times r_{M,t}^e) + \varepsilon_{i,t}, \quad (\text{iii})$$

where $\mathbf{1}_{EA}^i$ equals 1 on days when firm i announces earnings; $\mathbf{1}_{\text{HighAtt}}^i$ equals 1 on days when investor attention to firm i (the time-detrended $ESV(U)$ of firm i) is above the full-sample median; $\mathbf{1}_{EA}^{\text{high},i}$ equals 1 when attention to firm i is above its median computed within the set of EA days; and $\mathbf{1}_{EA}^{\text{low},i}$ equals 1 when attention to firm i is below its median computed within the set of EA days; $r_{M,t}^e$ is the excess return on the market; and $r_{i,t}^e$ is firm i 's excess return. The β coefficients are winsorized at the first and 99th percentiles to mitigate the influence of outliers. The analysis covers daily excess returns from February 2003 to June 2017, with standard errors in parentheses. ***, **, and * indicate statistical significance at the two-sided 1%, 5%, and 10% levels, respectively.

Regression Eq.: Attention measure:	(1) Eq. (i)	(2) Eq. (ii) ESV	(3) Eq. (ii) ESVU	(4) Eq. (iii) ESV	(5) Eq. (iii) ESVU
Average β_{Other}	1.027*** (0.011)	1.017*** (0.012)	1.027*** (0.012)	1.037*** (0.012)	1.037*** (0.012)
Average $\beta_{\Delta EA}$	0.088*** (0.029)				
Average $\beta_{\Delta A}$		0.042*** (0.004)	0.019*** (0.004)		
Average $\beta_{\Delta EA}^{\text{low}}$				0.062 (0.046)	0.040 (0.048)
Average $\beta_{\Delta EA}^{\text{high}}$				0.141*** (0.047)	0.123*** (0.047)
Average Adj. R^2	0.251	0.252	0.252	0.258	0.258
Nb. firms	1368	1260	1260	1260	1260

each portfolio j and on each trading day, we compute the attention variable $\bar{\mathbf{1}}_{\text{HighAtt},t}^j$ as the within-portfolio average of individual dummy variables $\mathbf{1}_{\text{HighAtt}}^i$ as previously defined. This portfolio-specific average can be interpreted as the fraction of firms in portfolio j to which investors pay heightened attention. Subsequently, we estimate the following regression:

$$r_{j,t}^e = \alpha_{\text{Other}}^j + \alpha_{\Delta A}^j \bar{\mathbf{1}}_{\text{HighAtt},t}^j + \beta_{\text{Other}}^j r_{M,t}^e + \beta_{\Delta A}^j (\bar{\mathbf{1}}_{\text{HighAtt},t}^j \times r_{M,t}^e) + \varepsilon_{j,t}, \quad (43)$$

where $r_{j,t}^e$ is the portfolio excess return. Our theory predicts a positive coefficient of $\beta_{\Delta A}^j$ for the interaction term between market excess returns and the aggregate attention towards the portfolio's constituent firms.

Table 8 presents separate panels for each attention measure. In Panel A, our model's prediction using the ESV measure is partially supported, with the interaction coefficient $\beta_{\Delta A}$ being positive and statistically significant in 7 out of 10 portfolios. The findings are more definitive in Panel B, which uses the $ESVU$ measure. Specifically, 9 out of 10 portfolios exhibit a positive change in betas, albeit statistically significant in 7 out of 10 portfolios. We observe a consistent pattern across both panels, where high-beta portfolios exhibit the largest increases in betas, supporting Eq. (36). According to this equation, the increase in betas on high-attention days should depend on the exposure

parameters, b , with a larger increase in betas for firms with greater initial exposure to systematic risk.¹⁷

Finally, we estimate day-specific CAPMs. Based on Corollary 4.1, both elevated ex-ante uncertainty and heightened investor attention result in an increase in the market risk premium and a steeper SML. Table 9 presents regression estimates for day-specific CAPMs, covering three portfolio sorts: the 10 beta-sorted portfolios developed earlier; 10 beta-sorted portfolios using the entire universe of US firms; and 25 size/BM-sorted portfolios.¹⁸ All estimates in Table 9 are reported in basis points per day. Our reference point is the all-days CAPM relationship, displayed in column (1) of each panel. Then, columns (2) and (3) classify trading days into subsamples with VIX_{t-1} above its median and in its top quartile, respectively. Next, columns (4) and (5) classify trading days into subsamples with detrended ESV above its median and in its top quartile, and columns (6) and (7) do the same for $ESVU$. Finally, columns (8) and (9) document the combined effect of high ex-ante uncertainty and heightened attention on days when the

¹⁷ In Table 8, each portfolio-day is assigned a unique attention measure $\bar{\mathbf{1}}_{\text{HighAtt},t}^j$. Although the test portfolios collectively form the market portfolio, which has a beta of 1 by definition, deviations in individual portfolio betas, $\beta_{\Delta A}^j$, are driven by different attention values $\bar{\mathbf{1}}_{\text{HighAtt},t}^j$ across the regressions, and are not expected to sum up to zero (on any given day, different portfolios have different attention measures).

¹⁸ Daily excess returns on 10 beta-sorted portfolios are available at <https://global-q.org/testingportfolios.html> and on 25 size/BM portfolios at https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Table 8

Portfolio betas and investor attention, 10 beta-sorted portfolios. This table reports the β coefficients for 10 beta-sorted portfolios in a regression of portfolio excess returns, $r_{j,t}^e$, on market excess returns, $r_{M,t}^e$, a variable that measures aggregate attention to the portfolio's constituent firms, $\bar{1}_{HighAtt,t}^i$, and an interaction term between market excess returns and the aggregate attention to the portfolio's constituent firms:

$$r_{j,t}^e = \alpha_{Other}^j + \alpha_{\Delta A}^j \bar{1}_{HighAtt,t}^i + \beta_{Other}^j r_{M,t}^e + \beta_{\Delta A}^j (\bar{1}_{HighAtt,t}^i \times r_{M,t}^e) + \varepsilon_{j,t}.$$

The portfolio attention variable $\bar{1}_{HighAtt,t}^i$ is calculated as the within-portfolio average of individual dummy variables $\mathbf{1}_{EA}^{high,i}$ ($\mathbf{1}_{EA}^{high,i}$ equals 1 when attention to firm i is above its median). Panel A (Panel B) reports results using the investor attention measure *ESV* (*ESVU*). The analysis covers daily excess returns from February 2003 to June 2017 for firms whose EDGAR search data is available, with standard errors in parentheses. ***, **, and * indicate statistical significance at the two-sided 1%, 5%, and 10% levels, respectively.

Panel A: 10 beta-sorted portfolios and high <i>ESV</i> -measured attention										
	Low	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	High
β_{Other}	0.218*** (0.015)	0.570*** (0.016)	0.731*** (0.014)	0.793*** (0.011)	1.022*** (0.011)	1.010*** (0.011)	1.161*** (0.012)	1.219*** (0.014)	1.326*** (0.021)	1.524*** (0.031)
$\beta_{\Delta A}$	0.088*** (0.028)	−0.060** (0.030)	−0.064** (0.026)	0.039* (0.020)	−0.119*** (0.021)	0.156*** (0.021)	0.077*** (0.023)	0.164*** (0.026)	0.203*** (0.038)	0.332*** (0.055)
R^2	0.330	0.665	0.818	0.909	0.927	0.941	0.936	0.928	0.890	0.842

Panel B: 10 beta-sorted portfolios and high <i>ESVU</i> -measured attention										
	Low	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	High
β_{Other}	0.199*** (0.014)	0.517*** (0.015)	0.696*** (0.013)	0.785*** (0.010)	0.992*** (0.011)	1.012*** (0.011)	1.188*** (0.012)	1.260*** (0.014)	1.384*** (0.020)	1.601*** (0.030)
$\beta_{\Delta A}$	0.135*** (0.029)	0.055* (0.031)	0.009 (0.026)	0.057*** (0.020)	−0.061*** (0.021)	0.161*** (0.022)	0.023 (0.024)	0.085*** (0.027)	0.091** (0.039)	0.187*** (0.056)
R^2	0.332	0.665	0.817	0.909	0.927	0.941	0.936	0.927	0.889	0.841

VIX_{t-1} and *ESV*(*U*) are both above their medians. The table also reports average market returns for each type of day (row 'Avg. R_M^e '), as well as the t -statistic of the difference between the CAPM slope estimate and the estimated market excess return (row 'Slope test').

Focusing on panels A and B, columns (2) and (3) display a steeper SML slope as ex-ante uncertainty increases. While the results are only statistically significant in panel B, we note that both the estimated slopes and market excess returns are higher on high-uncertainty days, and in panel B the slopes are not statistically different from the estimated market excess return. Turning to high-attention days, Columns (4)–(7) present statistically significant slope estimates in seven of eight cases across panels A and B, where these slopes are not statistically different from the estimated market excess returns. Finally, columns (8) and (9) of panels A and B exhibit notably stronger slope coefficients when heightened attention follows high ex-ante uncertainty, accompanied by higher estimated market excess returns.

Panel C of Table 9 yields similar inferences, albeit with weaker statistical significance. This is not too surprising given the well-documented result that the CAPM performs poorly in these portfolios (Fama and French, 1993; 1996; 2004; Cochrane, 2009). However, even in this case, columns (4)–(7) show evidence that the SML steepens on days with high aggregate attention. This effect strengthens with the level of attention. The slope of the SML on high-attention days is positive and statistically significant in three out of four cases, ranging from 4.44 ($p < 0.05$) to 10.04 ($p < 0.01$) basis points per day. These slope coefficients are not statistically different from estimated market returns on the same days. Comparing these results with the ones in columns (2) and (3), we notice that attention

has a stronger effect on the slope of the SML than uncertainty.¹⁹

Figure 8 provides a graphical representation of Table 9, panels B and C. It plots average daily excess returns in basis points against betas estimated from time-series regressions. The top (bottom) panels present results for 10 beta-sorted portfolios (25 size/BM-sorted portfolios). All plots are day-specific. The left panels plot the CAPM relation estimated on all days versus days when VIX_{t-1} is in its top quartile; the center and right panels plot the CAPM relation on all days versus days when the detrended aggregate *ESV*(*U*) measures are in their top quartiles.

The results in the top panels indicate that the SML is steeper on days with high uncertainty and heightened investor attention, based on analyzing 10 beta-sorted portfolios. However, the evidence is less strong when using *ESV* as an attention proxy. For the 25 size/BM portfolios (the bottom panels), the evidence of a steeper SML is stronger on days with high levels of investor attention, measured by both *ESV* and *ESVU*.

In summary, our theory proposes that heightened investor attention to firm-level news has two effects. First, it increases firm betas, as evidenced by Tables 7 and 8. Second, it resolves uncertainty, which in turn leads to a steeper SML, as shown in Table 9 and Fig. 8. Therefore, our paper provides a unified theoretical explanation for the relation between attention and the CAPM, demonstrating that investor attention to firm-level news is the

¹⁹ Although our model speaks directly to returns occurring in a similar latency as attention allocation, it seems plausible that short-window attention effects would be attenuated over longer return windows. Additional analysis (untabulated, available from the authors) supports the notion of daily attention effects attenuation when considering longer-horizon return windows.

Table 9

CAPM, economic uncertainty, and investor attention.

This table reports regression results for excess returns on 10 value-weighted beta-sorted portfolios of firms whose EDGAR search data is available (Panel A), 10 value-weighted beta-sorted portfolios of all firms (Panel B), and 25 value-weighted size/BM-sorted portfolios of all firms (Panel C). The regressions examine the relationship between portfolio excess returns and the market return, and are computed separately for days with various thresholds of VIX_{t-1} and detrended aggregate investor attention measures. The table also reports average market returns for each type of day (row 'Avg. R_M^e '), as well as the t -statistic of the difference between the CAPM slope estimate (row ' β ') and the estimated market excess return. Estimates are in basis points per day. The analysis covers daily excess returns from February 2003 to June 2017, with standard errors in parentheses. ***, **, and * indicate statistical significance at the two-sided 1%, 5%, and 10% levels, respectively.

Panel A: 10 beta-sorted portfolios (Only firms whose EDGAR search data is available)									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	All days	VIX > 50%	VIX > 75%	ESV > 50%	ESV > 75%	ESVU > 50%	ESVU > 75%	VIX > 50% ESV > 50%	VIX > 50% ESVU > 50%
Intercept	1.800 (1.057)	3.424 (2.007)	0.773 (3.127)	0.292 (1.159)	2.339 (1.739)	−0.538 (0.972)	−1.674 (1.428)	2.070 (1.780)	0.149 (1.585)
β	1.686 (0.979)	2.849 (1.829)	4.671 (2.793)	3.944*** (1.050)	0.827 (1.589)	3.769*** (0.891)	6.357*** (1.306)	5.809*** (1.580)	5.027*** (1.423)
Avg. R_M^e	4.137	6.932	6.171	5.136	4.168	3.908	5.736	8.579	5.410
Slope test	[−2.505]**	[−2.232]**	[−0.537]	[−1.136]	[−2.103]**	[−0.155]	[0.475]	[−1.753]*	[−0.269]
R-Square	0.271	0.233	0.259	0.638	0.033	0.691	0.748	0.628	0.609
Nb. days	3617	1810	905	1808	903	1808	903	995	902
Panel B: 10 beta-sorted portfolios (All firms)									
Intercept	2.767*** (0.625)	1.708* (0.795)	1.521 (1.257)	1.171 (0.773)	1.329 (0.741)	−0.357 (0.798)	−1.156 (1.428)	−0.131 (0.763)	−2.304 (1.333)
β	1.873** (0.566)	6.279*** (0.718)	6.202*** (1.127)	4.757*** (0.688)	4.048*** (0.672)	4.950*** (0.719)	7.732*** (1.305)	9.505*** (0.673)	8.324*** (1.192)
Avg. R_M^e	4.191	7.037	6.329	5.136	4.168	3.908	5.736	8.579	5.410
Slope test	[−4.097]***	[−1.056]	[−0.113]	[−0.551]	[−0.179]	[1.449]	[1.530]	[1.377]	[2.444]**
R-Square	0.578	0.905	0.791	0.857	0.819	0.855	0.814	0.961	0.859
Nb. days	3618	1811	905	1808	903	1808	903	995	902
Panel C: 25 size/BM-sorted portfolios									
Intercept	3.130* (1.735)	8.631*** (2.720)	5.301 (3.722)	1.355 (2.200)	0.202 (3.863)	−1.540 (2.385)	−3.034 (3.192)	7.375** (3.334)	3.247 (3.234)
β	1.671 (1.538)	−0.460 (2.432)	1.013 (3.327)	4.436** (1.902)	5.504 (3.323)	6.181*** (2.117)	10.044*** (2.799)	2.817 (2.889)	3.539 (2.891)
Avg. R_M^e	4.191	7.037	6.329	5.136	4.168	3.908	5.736	8.579	5.410
Slope test	[−1.638]	[−3.083]***	[−1.598]	[−0.368]	[0.402]	[1.074]	[1.539]	[−1.994]**	[−0.647]
R-Square	0.049	0.002	0.004	0.191	0.107	0.270	0.359	0.040	0.061
Nb. days	3618	1811	905	1808	903	1808	903	995	902

mechanism behind both the increase in betas on announcement days and the steepening of the CAPM relation.

More generally, because in our model heightened attention resolves uncertainty, we should observe a steeper CAPM relation on days with strong uncertainty resolution. A quick test of this statement is readily available: broadly defining days with high values of $\log(VIX_{t-1}/VIX_t)$ as days with strong uncertainty resolution, testing the CAPM on days when this proxy is high yields robust CAPM relations in any portfolio sorts and at the individual stock level. While this proxy for uncertainty resolution may be crude, the results nevertheless suggest that models incorporating uncertainty and attention fluctuations to generate time variation in uncertainty resolution (e.g., Andrei and Hasler, 2015; 2019; Benamar et al., 2021) could be useful for examining the cross-section of asset returns.

5. Conclusion

This paper examines the relationship between economic uncertainty and investor attention to firm-level earnings announcements. In a multi-firm equilibrium

model, we show that heightened economic uncertainty causes investors to allocate more attention to firm-level information. Investors pay incrementally more attention to the earnings announcements of high-beta firms, firms with more informative earnings announcements, higher idiosyncratic volatility of earnings, less informative prices, and lower information acquisition costs.

The central premise of our model is that investors learn valuable information about the economy from earnings announcements. Consequently, investors' learning intensifies when market-wide uncertainty is high. This implies a steeper beta-return relation on days of heightened investor attention. Moreover, our model predicts that betas of announcing firms increase with investors' attention to earnings announcements.

The data support these predictions. Using two proxies for investor attention to firm-level information (SEC EDGAR search traffic and Google stock ticker searches), we find that investors pay more attention to firm-level earnings announcements on days with high economic uncertainty. Our analysis further reveals that prices respond to earnings news more strongly when there is more significant

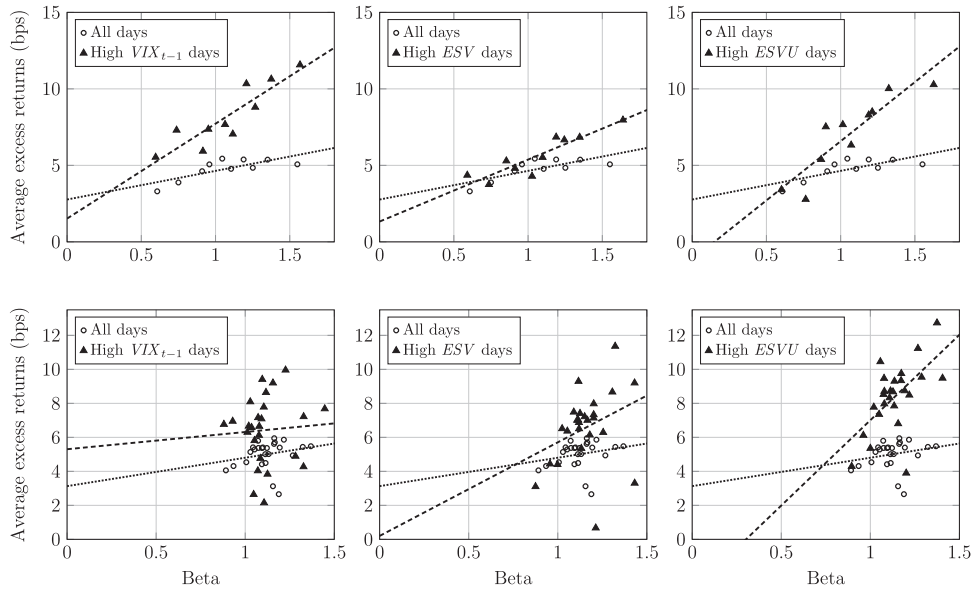


Fig. 8. The impact of uncertainty and attention on the CAPM.

This figure shows the relationship between the average daily value-weighted excess returns in basis points (bps) and full-sample betas for ten beta-sorted portfolios (top panels) and 25 portfolios sorted by size/BM (bottom panels). The estimates are presented separately for all days and days with the VIX_{t-1} in its top quartile (left panels), detrended aggregated ESV in its top quartile (middle panels), and detrended aggregated $ESVU$ in its top quartile (right panels). The lines depict day-specific CAPM relations. The data covers daily excess returns, VIX historical data, and EDGAR search records from February 2003 to June 2017.

economic uncertainty. These results are concentrated in firms with high CAPM beta, higher institutional ownership, and prior share turnover. We view these as consistent with our theoretical predictions related to cross-sectional variation in the benefit-to-cost ratio of information. **Finally, we find strong empirical support for higher betas on high-attention days and a steeper CAPM relation on days of heightened investor attention to firm-level information.**

In conclusion, these results suggest that economic uncertainty plays a vital role in shaping investor attention to firm-level information. Investors learn valuable information from earnings announcements, and their learning intensifies when market-wide uncertainty is high. This has implications for market risk pricing, as more reliable risk pricing occurs not only when uncertainty is high but also when investors respond to high uncertainty by intensifying their learning and processing of information. Thus, models that incorporate uncertainty and attention fluctuations to generate time variation in the resolution of uncertainty may provide crucial insights into the underlying mechanisms that drive asset prices.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors do not have permission to share some of the data that has been used.

Appendix A

A1. Proof of Proposition 1

Notation used thorough the Appendix:

- We denote \mathbf{I} as the identity matrix, $\mathbf{1}$ as a vector of ones, and $\mathbf{0}$ as a vector/matrix of zeros. These vectors and matrices are always assumed to have the conformable dimension, which we do not specify below in order to avoid overly cumbersome notation.
- The set of announcing firms is $\mathcal{A} = \{1, 2, \dots, A\}$. Within this set, firms are indexed by a .
- The set of investor types is the power set of \mathcal{A} , $\mathcal{P}(\mathcal{A})$, of dimension 2^A . Within this set, investor types are indexed by k .
- \bar{k} denotes the complement of an investor type $k \subseteq \mathcal{A}$, that is, $\bar{k} = \mathcal{A} \setminus k$.
- $|k|$ denotes the cardinality of the set k .
- ι_a is a standard basis vector of dimension N with all components equal to 0, except the a th, which is 1. ι_k ($\iota_{\bar{k}}$) represents the matrix with all the column vectors $\{\iota_a \mid a \in k\}$ ($\{\iota_a \mid a \in \bar{k}\}$).
- ι represents the matrix with all the column vectors $\{\iota_a \mid a \in \mathcal{A}\}$.
- $h_a \equiv \frac{\Delta_a}{\gamma \sigma_{\epsilon a}^2}$, for $a \in \mathcal{A}$. \mathbf{h}_k and $\mathbf{h}_{\bar{k}}$ denote the column vectors $\{h_a \mid a \in k\}$ and $\{h_a \mid a \in \bar{k}\}$.
- $\text{diag}[y_j \mid j \in \mathcal{Z}]$ denotes a diagonal matrix whose diagonal is $\{y_j \mid j \in \mathcal{Z}\}$. $\delta \mathbf{h}_k$ ($\delta \mathbf{h}_{\bar{k}}$) is a diagonal matrix whose diagonal is \mathbf{h}_k ($\mathbf{h}_{\bar{k}}$), e.g., $\delta \mathbf{h}_k = \text{diag}[\{h_a \mid a \in k\}]$.

- \mathbf{e}_k and $\mathbf{e}_{\bar{k}}$ denote the column vectors $\{\varepsilon_a \mid a \in k\}$ and $\{\varepsilon_a \mid a \in \bar{k}\}$, and $\mathbf{e} = \begin{bmatrix} \mathbf{e}_k \\ \mathbf{e}_{\bar{k}} \end{bmatrix}$. Similarly for \mathbf{x}_k , $\mathbf{x}_{\bar{k}}$, and \mathbf{x} .
- $\Sigma_{\varepsilon k}$ denotes the covariance matrix of the vector \mathbf{e}_k (a diagonal matrix whose elements are $\{\sigma_{\varepsilon a}^2 \mid a \in k\}$). $\Sigma_{\varepsilon \bar{k}}$ denotes the covariance matrix of the vector $\mathbf{e}_{\bar{k}}$. Σ_{xk} denotes the covariance matrix of the vector \mathbf{x}_k .

Learning for type k investors

Type k investors observe the earnings announcements $\{E_a \mid a \in k\}$, and learn from prices. [Conjecture 1](#) implies that the only prices useful for learning are $\{\hat{P}_a \mid a \in \bar{k}\}$. (If an investor observes E_a then the price signal \hat{P}_a is a noisy version of E_a and is redundant for learning.)

Group the information set of type k investors into two vectors, \mathbf{E}_k of dimension $|k|$ and $\hat{\mathbf{P}}_{\bar{k}}$ of dimension $|\bar{k}|$. Then we can write

$$\begin{bmatrix} \mathbf{D} \\ \mathbf{E}_k \\ \hat{\mathbf{P}}_{\bar{k}} \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{I}'_k \\ \delta \mathbf{h}'_{\bar{k}} \mathbf{I}'_{\bar{k}} \end{bmatrix} \mathbf{D} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \delta \mathbf{h}_{\bar{k}} \end{bmatrix} \begin{bmatrix} \mathbf{e}_k \\ \mathbf{e}_{\bar{k}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{I}'_{\bar{k}} \end{bmatrix} \mathbf{x}, \quad (\text{A.1})$$

and thus

$$\begin{bmatrix} \mathbf{D} \\ \mathbf{E}_k \\ \hat{\mathbf{P}}_{\bar{k}} \end{bmatrix} \sim \mathcal{N} \left(\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \text{Var}[\mathbf{D}] & \text{Var}[\mathbf{D}][\mathbf{I}_k] & \text{Var}[\mathbf{D}][\mathbf{I}'_{\bar{k}} \delta \mathbf{h}_{\bar{k}}] \\ \left[\begin{smallmatrix} \mathbf{I}'_k \\ \delta \mathbf{h}'_{\bar{k}} \mathbf{I}'_{\bar{k}} \end{smallmatrix} \right] \text{Var}[\mathbf{D}] & \left[\begin{smallmatrix} \mathbf{I}'_k \\ \delta \mathbf{h}'_{\bar{k}} \mathbf{I}'_{\bar{k}} \end{smallmatrix} \right] \text{Var}[\mathbf{D}][\mathbf{I}_k] & \left[\begin{smallmatrix} \mathbf{I}'_k \\ \delta \mathbf{h}'_{\bar{k}} \mathbf{I}'_{\bar{k}} \end{smallmatrix} \right] \text{Var}[\mathbf{D}][\mathbf{I}'_{\bar{k}} \delta \mathbf{h}_{\bar{k}}] + \begin{bmatrix} \Sigma_{\varepsilon k} \\ \mathbf{0} \\ \delta \mathbf{h}_{\bar{k}}^2 \Sigma_{\varepsilon \bar{k}} + \Sigma_{x\bar{k}} \end{bmatrix} \end{bmatrix} \right). \quad (\text{A.2})$$

We will apply the Projection Theorem, which we write here for convenience.

Projection Theorem. Consider the n -dimensional normal random variable

$$\begin{bmatrix} \boldsymbol{\theta} \\ \mathbf{s} \end{bmatrix} \sim \mathcal{N} \left(\begin{bmatrix} \boldsymbol{\mu}_{\theta} \\ \boldsymbol{\mu}_s \end{bmatrix}, \begin{bmatrix} \Sigma_{\theta\theta} & \Sigma_{\theta s} \\ \Sigma_{s\theta} & \Sigma_{ss} \end{bmatrix} \right). \quad (\text{A.3})$$

Provided Σ_{ss} is non-singular, the conditional density of $\boldsymbol{\theta}$ given \mathbf{s} is normal with conditional mean and conditional variance-covariance matrix:

$$\mathbb{E}[\boldsymbol{\theta}|\mathbf{s}] = \boldsymbol{\mu}_{\theta} + \Sigma_{\theta s} \Sigma_{ss}^{-1} (\mathbf{s} - \boldsymbol{\mu}_s) \quad (\text{A.4})$$

$$\text{Var}[\boldsymbol{\theta}|\mathbf{s}] = \Sigma_{\theta\theta} - \Sigma_{\theta s} \Sigma_{ss}^{-1} \Sigma_{s\theta}. \quad (\text{A.5})$$

Applied to [\(A.2\)](#), the Projection Theorem together with the Woodbury Matrix Identity imply:

$$\text{Var}^k[\mathbf{D}] = \left(\text{Var}[\mathbf{D}]^{-1} + \begin{bmatrix} \mathbf{I}_k & \mathbf{I}'_{\bar{k}} \delta \mathbf{h}_{\bar{k}} \end{bmatrix} \begin{bmatrix} \Sigma_{\varepsilon k}^{-1} & \mathbf{0} \\ \mathbf{0} & (\delta \mathbf{h}_{\bar{k}}^2 \Sigma_{\varepsilon \bar{k}} + \Sigma_{x\bar{k}})^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{I}'_k \\ \delta \mathbf{h}'_{\bar{k}} \mathbf{I}'_{\bar{k}} \end{bmatrix} \right)^{-1} \quad (\text{A.6})$$

$$= \left(\text{Var}[\mathbf{D}]^{-1} + \begin{bmatrix} \mathbf{I}_k & \mathbf{I}'_{\bar{k}} \end{bmatrix} \begin{bmatrix} \Sigma_{\varepsilon k}^{-1} & \mathbf{0} \\ \mathbf{0} & \delta \mathbf{h}_{\bar{k}}^2 (\delta \mathbf{h}_{\bar{k}}^2 \Sigma_{\varepsilon \bar{k}} + \Sigma_{x\bar{k}})^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{I}'_k \\ \delta \mathbf{h}'_{\bar{k}} \mathbf{I}'_{\bar{k}} \end{bmatrix} \right)^{-1} \quad (\text{A.7})$$

$$= \left(\text{Var}[\mathbf{D}]^{-1} + \mathbf{I} \text{diag} \left[\frac{\ell_a^k}{\sigma_{\varepsilon a}^2} \mid a \in \mathcal{A} \right] \mathbf{I}' \right)^{-1}, \quad (\text{A.8})$$

with ℓ_a^k defined in [\(13\)](#). We have thus obtained $\boldsymbol{\tau}^k \equiv \text{Var}^k[\mathbf{D}]^{-1}$ as in [Proposition 1](#). This simple form for $\boldsymbol{\tau}^k$ allows us to compute its determinant using the Matrix Determinant Lemma:

$$\det(\mathbf{A} + \mathbf{U}\mathbf{W}\mathbf{V}') = \det(\mathbf{W}^{-1} + \mathbf{V}'\mathbf{A}^{-1}\mathbf{U}) \det(\mathbf{W}) \det(\mathbf{A}), \quad (\text{A.9})$$

where $\mathbf{A} = \text{Var}[\mathbf{D}]^{-1}$, $\mathbf{U} = \mathbf{I}$, $\mathbf{W} = \text{diag} \left[\frac{\ell_a^k}{\sigma_{\varepsilon a}^2} \mid a \in \mathcal{A} \right]$, and $\mathbf{V}' = \mathbf{I}'$.

The Matrix Determinant Lemma implies

$$\det(\boldsymbol{\tau}^k) = \det(\text{Var}[\mathbf{D}]^{-1}) \left(\prod_{a=1}^A \frac{\ell_a^k}{\sigma_{\varepsilon a}^2} \right) \det \left(\text{diag} \left[\frac{\sigma_{\varepsilon a}^2}{\ell_a^k} \mid a \in \mathcal{A} \right] + \mathbf{I}' \text{Var}[\mathbf{D}] \mathbf{I} \right) \quad (\text{A.10})$$

$$= \det(\text{Var}[\mathbf{D}]^{-1}) \left(\prod_{a=1}^A \frac{\ell_a^k}{\sigma_{\varepsilon a}^2} \right) \det \left(\text{diag} \left[\frac{\sigma_{\varepsilon a}^2}{\ell_a^k} + \sigma_{\varepsilon a}^2 \mid a \in \mathcal{A} \right] + U^2 \mathbf{b}_{\mathcal{A}} \mathbf{b}_{\mathcal{A}}' \right), \quad (\text{A.11})$$

where $\mathbf{b}_{\mathcal{A}}$ is the vector of announcer firms' exposure to the systematic component f .

Further apply the Matrix Determinant Lemma to the last term:

$$\det(\boldsymbol{\tau}^k) = \det(\text{Var}[\mathbf{D}]^{-1}) \left(\prod_{a=1}^A \frac{\ell_a^k}{\sigma_{\varepsilon a}^2} \right) \left(\prod_{a=1}^A \left(\frac{\sigma_{\varepsilon a}^2}{\ell_a^k} + \sigma_{\varepsilon a}^2 \right) \right) \times \left(1 + \mathbf{b}_{\mathcal{A}}' \text{diag} \left[\frac{\ell_a^k}{\ell_a^k \sigma_{\varepsilon a}^2 + \sigma_{\varepsilon a}^2} \mid a \in \mathcal{A} \right] U^2 \mathbf{b}_{\mathcal{A}} \right) \quad (\text{A.12})$$

$$= \det(\text{Var}[\mathbf{D}]^{-1}) \left(\prod_{a=1}^A \frac{\ell_a^k \sigma_{\varepsilon a}^2 + \sigma_{\varepsilon a}^2}{\sigma_{\varepsilon a}^2} \right) \times \left(1 + U^2 \sum_{a=1}^A \frac{\ell_a^k b_a^2}{\ell_a^k \sigma_{\varepsilon a}^2 + \sigma_{\varepsilon a}^2} \right), \quad (\text{A.13})$$

which completes the proof of [Proposition 1](#). \square

A2. Proof of [Proposition 2](#)

The expected utility of a type \emptyset investor (uninformed) at time 1 is:

$$\mathcal{U}_1^\emptyset = \max_{\mathbf{q}^k} \mathbb{E}_1^\emptyset \left[-e^{-\gamma(W^\emptyset - c \sum_{a=1}^A I_a^\emptyset)} \right] = \max_{\mathbf{q}^k} \mathbb{E}_1^\emptyset \left[-e^{-\gamma(\mathbf{q}^\emptyset)' \mathbf{R}^e} \right]. \quad (\text{A.14})$$

Further replacing the optimal portfolio choice from [Eq. \(7\)](#) yields

$$\mathcal{U}_1^\emptyset = -\mathbb{E}_1^\emptyset \left[e^{-\mathbb{E}_1^\emptyset[\mathbf{R}^e]' \text{Var}_1^\emptyset[\mathbf{R}^e]^{-1} \mathbf{R}^e} \right] \quad (\text{A.15})$$

$$= -e^{-\frac{1}{2} \mathbb{E}_1^\emptyset[\mathbf{R}^e]' \text{Var}_1^\emptyset[\mathbf{R}^e]^{-1} \mathbb{E}_1^\emptyset[\mathbf{R}^e]}. \quad (\text{A.16})$$

Assume that a type \emptyset investor considers acquiring information and becoming of type $k \in \mathcal{P}(\mathcal{A})$, where $|k| > 0$. At time 1, from the perspective of the type \emptyset investor, $\mathbb{E}_1^k[\mathbf{R}^e]$ is a random vector. Denote this random vector by $\mathbf{z} + \mathbf{m}$, with mean \mathbf{m} and variance Σ (i.e., \mathbf{z} has mean $\mathbf{0}$ and variance Σ). By the law of iterated expectations,

$$\mathbf{m} \equiv \mathbb{E}_1^\emptyset[\mathbb{E}_1^k[\mathbf{R}^e]] = \mathbb{E}_1^\emptyset[\mathbf{R}^e], \quad (\text{A.17})$$

and by the law of total variance,

$$\Sigma \equiv \text{Var}_1^\emptyset[\mathbb{E}_1^k[\mathbf{R}^e]] = \text{Var}_1^\emptyset[\mathbf{R}^e] - \text{Var}_1^k[\mathbf{R}^e]. \quad (\text{A.18})$$

Therefore, for the type \emptyset investor, $-\frac{1}{2} \mathbb{E}_1^k[\mathbf{R}^e]' \text{Var}_1^k[\mathbf{R}^e]^{-1} \mathbb{E}_1^k[\mathbf{R}^e]$ (that is, the random exponent in [\(A.16\)](#), written for type k) is a random scalar that can be written as (define $\Sigma^\emptyset \equiv \text{Var}_1^\emptyset[\mathbf{R}^e]$ to simplify notation):

$$-\frac{1}{2} \mathbb{E}_1^k[\mathbf{R}^e]' \text{Var}_1^k[\mathbf{R}^e]^{-1} \mathbb{E}_1^k[\mathbf{R}^e] = -\frac{1}{2} (\mathbf{z} + \mathbf{m})' (\Sigma^\emptyset - \Sigma)^{-1} (\mathbf{z} + \mathbf{m}) \quad (\text{A.19})$$

$$= \underbrace{\mathbf{z}' \left(-\frac{1}{2} (\Sigma^\emptyset - \Sigma)^{-1} \right) \mathbf{z}}_{\mathbf{F}} + \underbrace{(-\mathbf{m}' (\Sigma^\emptyset - \Sigma)^{-1}) \mathbf{z}}_{\mathbf{G}'} + \underbrace{\mathbf{m}' \left(-\frac{1}{2} (\Sigma^\emptyset - \Sigma)^{-1} \right) \mathbf{m}}_{\mathbf{H}}. \quad (\text{A.20})$$

Our aim is to compute $\mathbb{E}_1^\emptyset[\mathcal{U}_1^k]$, i.e., the type \emptyset agent's expectation of what her expected utility will be if she changes type to k . We will apply the following Lemma ([Veldkamp, 2011](#), p. 102):

Lemma 2. Consider a random vector $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \Sigma)$. Then,

$$\mathbb{E} \left[e^{\mathbf{z}' \mathbf{F} \mathbf{z} + \mathbf{G}' \mathbf{z} + \mathbf{H}} \right] = \det(\mathbf{I} - 2\Sigma\mathbf{F})^{-\frac{1}{2}} e^{\frac{1}{2} \mathbf{G}' (\mathbf{I} - 2\Sigma\mathbf{F})^{-1} \Sigma \mathbf{G} + \mathbf{H}}. \quad (\text{A.21})$$

Compute first

$$\mathbf{I} - 2\Sigma\mathbf{F} = \mathbf{I} - 2\Sigma \left(-\frac{1}{2} (\Sigma^\emptyset - \Sigma)^{-1} \right) \quad (\text{A.22})$$

$$= \Sigma^\emptyset (\Sigma^\emptyset - \Sigma)^{-1}, \quad (\text{A.23})$$

which, using [\(A.18\)](#), leads to the determinant in [Lemma 2](#):

$$\det(\mathbf{I} - 2\Sigma\mathbf{F}) = \frac{\det(\Sigma^\emptyset)}{\det(\text{Var}_1^k[\mathbf{R}^e])} = \frac{\det(\boldsymbol{\tau}^k)}{\det(\boldsymbol{\tau}^\emptyset)}. \quad (\text{A.24})$$

The exponent in [Lemma 2](#) is:

$$\frac{1}{2} \mathbf{G}' (\mathbf{I} - 2\Sigma\mathbf{F})^{-1} \Sigma \mathbf{G} + \mathbf{H} \quad (\text{A.25})$$

$$= \frac{1}{2} (-\mathbf{m}' (\Sigma^\emptyset - \Sigma)^{-1}) (\Sigma^\emptyset - \Sigma) (\Sigma^\emptyset)^{-1} \\ \times \Sigma (-\mathbf{m}' (\Sigma^\emptyset - \Sigma)^{-1})' - \mathbf{m}' \frac{1}{2} (\Sigma^\emptyset - \Sigma)^{-1} \mathbf{m} \quad (\text{A.26})$$

$$= \frac{1}{2} \mathbf{m}' (\Sigma^\emptyset)^{-1} \Sigma (\Sigma^\emptyset - \Sigma)^{-1} \mathbf{m} - \frac{1}{2} \mathbf{m}' (\Sigma^\emptyset - \Sigma)^{-1} \mathbf{m} \quad (\text{A.27})$$

$$= \frac{1}{2} \mathbf{m}' ((\Sigma^\emptyset)^{-1} \Sigma - \mathbf{I}) (\Sigma^\emptyset - \Sigma)^{-1} \mathbf{m} \quad (\text{A.28})$$

$$= -\frac{1}{2} \mathbf{m}' (\Sigma^\emptyset)^{-1} \mathbf{m}. \quad (\text{A.29})$$

We can then use [Lemma 2](#) to write

$$\mathbb{E}_1^\emptyset[\mathcal{U}_1^k] = -e^{\gamma c |k|} \mathbb{E}_1^\emptyset \left[e^{-\frac{1}{2} \mathbb{E}_1^k[\mathbf{R}^e]' \text{Var}_1^k[\mathbf{R}^e]^{-1} \mathbb{E}_1^k[\mathbf{R}^e]} \right] \quad (\text{A.30})$$

$$= -e^{\gamma c |k|} \sqrt{\frac{\det(\boldsymbol{\tau}^\emptyset)}{\det(\boldsymbol{\tau}^k)}} e^{-\frac{1}{2} \mathbb{E}_1^\emptyset[\mathbf{R}^e]' \text{Var}_1^\emptyset[\mathbf{R}^e]^{-1} \mathbb{E}_1^\emptyset[\mathbf{R}^e]} \quad (\text{A.31})$$

$$= \mathcal{U}_1^\emptyset e^{\gamma c |k|} \sqrt{\frac{\det(\boldsymbol{\tau}^\emptyset)}{\det(\boldsymbol{\tau}^k)}}. \quad (\text{A.32})$$

At time $t = 0$, the type \emptyset investor compares $\mathbb{E}_0[\mathcal{U}_1^\emptyset]$ with $\mathbb{E}_0[\mathcal{U}_1^k]$ and acquires the additional signals if and only if

$$\mathbb{E}_0[\mathcal{U}_1^\emptyset] < \mathbb{E}_0[\mathcal{U}_1^k] = \mathbb{E}_0[\mathbb{E}_1^\emptyset[\mathcal{U}_1^k]], \quad (\text{A.33})$$

which, after replacement of [\(A.32\)](#), yields $e^{\gamma c |k|} \sqrt{\det(\boldsymbol{\tau}^\emptyset)/\det(\boldsymbol{\tau}^k)} < 1$ (the division by $\mathbb{E}_0[\mathcal{U}_1^\emptyset] < 0$ flips the inequality sign). Thus, an investor of type \emptyset changes type to k if and only if

$$B_\emptyset^k \equiv \frac{\det(\boldsymbol{\tau}^k)}{\det(\boldsymbol{\tau}^\emptyset)} e^{-2\gamma c |k|} > 1. \quad (\text{A.34})$$

Consider now two investor types k and k' as in [Proposition 2](#). The empty set \emptyset is the only common subset of both k and k' , for all $k, k' \in \mathcal{P}(\mathcal{A})$. Thus, the uninformed investor is a common reference point for type k and type k' investors, and therefore the investor with the lowest benefit-cost ratio among $\{B_\emptyset^k, B_\emptyset^{k'}\}$ will always choose to migrate to the other type. In other words, a type k investor changes type from k to $k' \in \mathcal{P}(\mathcal{A}) \setminus k$ if and only if

$$\frac{B_\emptyset^{k'}}{B_\emptyset^k} > 1 \iff \frac{1}{2\gamma} \ln \frac{\det(\boldsymbol{\tau}^{k'})}{\det(\boldsymbol{\tau}^k)} > c(|k'| - |k|). \quad (\text{A.35})$$

This holds regardless of the sign of $|k'| - |k|$. \square

A3. Proof of Theorem 1

An important property of the benefit-cost ratios B_{\emptyset}^k , for $k \in \mathcal{P}(\mathcal{A}) \setminus \emptyset$, is that they can be decomposed into the product of consecutive *one-step* benefit-cost ratios. Formally, let $k(i)$ be the i th element of k and $\kappa(i)$ the subset of k that contains all its elements up to and including $k(i)$. Using the convention $\kappa(0) = \emptyset$ and defining $B_{\kappa(i-1)}^{k(i-1) \cup \{k(i)\}} \equiv \frac{\det(\tau^{\kappa(i-1) \cup \{k(i)\}})}{\det(\tau^{\kappa(i-1)})} e^{-2\gamma c}$, we can write

$$B_{\emptyset}^k = \prod_{i=1}^{|k|} B_{\kappa(i-1)}^{k(i-1) \cup \{k(i)\}}. \quad (\text{A.36})$$

We first establish the following Lemma.

Lemma 3. Consider an announcer $a \in \mathcal{A}$ and any type $k \subseteq \mathcal{A} \setminus \{a\}$. Then

$$\arg \min_k B_{\emptyset}^{k \cup \{a\}} = \mathcal{A} \setminus \{a\} \quad (\text{A.37})$$

$$\arg \max_k B_{\emptyset}^{k \cup \{a\}} = \emptyset \quad (\text{A.38})$$

Lemma 3 states that the type k for which the one-step benefit-cost ratio $B_{\emptyset}^{k \cup \{a\}}$ attains its minimum is the highest cardinality type that excludes a , that is, $\mathcal{A} \setminus \{a\}$; and the type k for which $B_{\emptyset}^{k \cup \{a\}}$ attains its maximum is the empty set \emptyset . In other words, attention has diminishing returns: the lowest benefit from observing E_a belongs to the investor who already observes all the other earnings announcements; and the highest benefit belongs to the uninformed investor. The proof of **Lemma 3** follows from writing explicitly $B_{\emptyset}^{k \cup \{a\}}$ by means of **Proposition 1**,

$$B_{\emptyset}^{k \cup \{a\}} = \frac{\det(\tau^{k \cup \{a\}})}{\det(\tau^k)} e^{-2\gamma c} \quad (\text{A.39})$$

$$= \left(\frac{\sigma_{ea}^2 + \sigma_{\varepsilon a}^2}{\ell_a \sigma_{ea}^2 + \sigma_{\varepsilon a}^2} + \frac{b_a^2}{\frac{1}{U^2} + \sum_{\alpha=1}^A \frac{\ell_{\alpha}^k b_{\alpha}^2}{\ell_{\alpha} \sigma_{\varepsilon \alpha}^2 + \sigma_{\varepsilon \alpha}^2}} \frac{(1 - \ell_a) \sigma_{\varepsilon a}^2}{(\ell_a \sigma_{ea}^2 + \sigma_{\varepsilon a}^2)^2} \right) e^{-2\gamma c}, \quad (\text{A.40})$$

which is indeed minimized when $\ell_{\alpha}^k = 1$, $\forall \alpha \in \mathcal{A} \setminus \{a\}$, and maximized when $\ell_{\alpha}^k < 1$, $\forall \alpha \in \mathcal{A} \setminus \{a\}$. In the former case, k must be $\mathcal{A} \setminus \{a\}$; in the latter, k must be \emptyset . (NB: **Lemma 3** is a direct consequence of the fact that the function $\ln(B_{\emptyset}^k)$ is linearly related to the entropy defined in (14): $\ln(B_{\emptyset}^k) = 2(H^{\emptyset}[\mathbf{D}] - H^k[\mathbf{D}] - \gamma c|k|)$. By the submodularity property of the entropy, $\ln(B_{\emptyset}^k)$ is submodular and therefore $B_{\emptyset}^{k \cup \{a\}}$ has diminishing returns. See also **Appendix A.7.**)

Lemma 3, together with the multiplicative property (A.36), will allow us obtain the bounds c_{\min} and c_{\max} . We will first derive the lower bound c_{\min} . When the information cost is below c_{\min} , all investors are informed, i.e., $\lambda^{\mathcal{A}} = 1$. In order for this to be a stable equilibrium, the following conditions must hold simultaneously:

$$B_{\mathcal{A} \setminus \{a\}}^a \geq 1 \quad \forall a \in \mathcal{A}, \quad (\text{A.41})$$

meaning that no investor of type \mathcal{A} finds it optimal to renounce being attentive to any signal E_a . If these conditions

hold simultaneously, then one can easily show using the multiplicative property (A.36) and **Lemma 3** that

$$B_k^a \geq 1, \quad \text{for any type } k \in \mathcal{A}, \quad (\text{A.42})$$

meaning that no investor of type \mathcal{A} finds it optimal to be of any other possible type. (This can be shown by writing B_k^a as a product as in (A.36) and using **Lemma 3** for each individual term of the product; it is a direct consequence of the property of diminishing returns to attention.)

Conditions (A.41) further imply $\min_a B_{\mathcal{A} \setminus \{a\}}^a \geq 1$, which will pin down c_{\min} . Using the fact that $\lambda^{\mathcal{A}} = 1$, the definition of ℓ_a in Eq. (13) yields upper limits for all the learning coefficients ℓ_a ,

$$\bar{\ell}_a = \frac{1}{1 + \gamma^2 \sigma_{\varepsilon a}^2} \quad \forall a \in \mathcal{A}, \quad (\text{A.43})$$

and thus c_{\min} solves

$$e^{2\gamma c_{\min}} = \min_a \left(\frac{\sigma_{ea}^2 + \sigma_{\varepsilon a}^2}{\ell_a \sigma_{ea}^2 + \sigma_{\varepsilon a}^2} + \frac{b_a^2}{\frac{1}{U^2} + \sum_{\alpha=1}^A \frac{\ell_{\alpha} b_{\alpha}^2}{\ell_{\alpha} \sigma_{\varepsilon \alpha}^2 + \sigma_{\varepsilon \alpha}^2}} \frac{(1 - \bar{\ell}_a) \sigma_{\varepsilon a}^2}{(\bar{\ell}_a \sigma_{ea}^2 + \sigma_{\varepsilon a}^2)^2} \right). \quad (\text{A.44})$$

Since the right hand side equals $\min_a (\det(\tau^{\mathcal{A}}) / \det(\tau^{\mathcal{A} \setminus \{a\}}))$ and thus is always larger than one, Eq. (A.44) has a unique, strictly positive solution c_{\min} . It can be easily checked that c_{\min} is strictly increasing in U .

Consider now an equilibrium in which no investor is informed, or $\lambda^{\emptyset} = 1$. In order for this to be a stable equilibrium, the following conditions must hold simultaneously:

$$B_{\emptyset}^a \leq 1 \quad \forall a \in \mathcal{A}. \quad (\text{A.45})$$

If these conditions hold, then a consequence of the property of diminishing returns to attention is that $B_{\emptyset}^k \leq 1$ holds for any type $k \subseteq \mathcal{A}$. (This can be shown by writing B_{\emptyset}^k as a product as in (A.36) and using **Lemma 3** for each individual term of the product.)

Conditions (A.45) further imply $\max_a B_{\emptyset}^a \leq 1$, and $\lambda^{\emptyset} = 1$ leads to $\ell_a = 0 \quad \forall a \in \mathcal{A}$. Thus, c_{\max} solves

$$e^{2\gamma c_{\max}} = \max_a \left(1 + \frac{b_a^2 U^2 + \sigma_{\varepsilon a}^2}{\sigma_{\varepsilon a}^2} \right). \quad (\text{A.46})$$

This equation has a unique, strictly positive solution c_{\max} , which is strictly increasing in U . Furthermore, since $B_{\emptyset}^a > B_{\mathcal{A} \setminus \{a\}}^a \quad \forall a \in \mathcal{A}$ (by **Lemma 3**), it is clear that $\max_a B_{\emptyset}^a > \min_a B_{\mathcal{A} \setminus \{a\}}^a$ and therefore $c_{\max} > c_{\min}$. This completes the proofs of cases C and A of **Theorem 1**.

In case B of **Theorem 1**, the information cost is $c \in (c_{\min}, c_{\max})$. Clearly, when $c \in (c_{\min}, c_{\max})$ both conditions (A.41) and (A.45) are violated and thus the equilibrium cannot be $\lambda^{\emptyset} = 1$ or $\lambda^{\mathcal{A}} = 1$. Thus, in equilibrium there exists a set $\{\lambda^k \mid k \in \mathcal{P}(\mathcal{A})\}$ such that: $\sum_{k \in \mathcal{P}(\mathcal{A})} \lambda^k = 1$; $\lambda^{\emptyset} < 1$; and $\lambda^{\mathcal{A}} < 1$. Consider now all the pairs of types $\{k, k'\} \in \mathcal{P}(\mathcal{A})$. For each pair, there are four cases:

- (i) $\{\lambda^k > 0\} \wedge \{\lambda^{k'} > 0\}$: this can be a stable equilibrium (meaning that no investor has an incentive to migrate from type k to type k' or vice versa) only if $B_{\emptyset}^{k'} / B_{\emptyset}^k = 1$.

- (ii) $\{\lambda^k = 0\} \wedge \{\lambda^{k'} > 0\}$: this can be a stable equilibrium (meaning that no investor of type k' has an incentive to migrate to type k) only if $B_{\theta}^{k'}/B_{\theta}^k \geq 1$.
- (iii) $\{\lambda^k > 0\} \wedge \{\lambda^{k'} = 0\}$: this is the reversal of the previous case and requires $B_{\theta}^{k'}/B_{\theta}^k \leq 1$.
- (iv) $\{\lambda^k = 0\} \wedge \{\lambda^{k'} = 0\}$: in this case there is no condition on $B_{\theta}^{k'}/B_{\theta}^k$ since there are no investors of types k and k' .

Conditions i-iv are both necessary and sufficient for the stability of the information market equilibrium. See [Appendix A.7](#) for an algorithm that converges to the equilibrium for any set of positive initial values $\{\lambda_0^k > 0 \mid k \in \mathcal{P}(\mathcal{A})\}$ such that $\sum_k \lambda_0^k = 1$. \square

A4. Proof of [Lemma 1](#)

[Lemma 1](#) results directly after writing τ^k for each investor type under this form:

$$\tau^k = \text{Var}[\mathbf{D}]^{-1} + \iota \text{diag} \left[\frac{\ell_a^k}{\sigma_{\varepsilon a}^2} \mid a \in \mathcal{A} \right] \iota', \quad (\text{A.47})$$

where ι is a $N \times A$ matrix whose columns are the standard basis vectors ι_a for all the announcing firms (vectors having all components equal to 0, except the a th, which is 1).

The weighted average precision is then

$$\tau = \sum_{k \in \mathcal{P}(\mathcal{A})} \lambda^k \tau^k = \text{Var}[\mathbf{D}]^{-1} + \iota \text{diag} \left[\sum_{k \in \mathcal{P}(\mathcal{A})} \lambda^k \frac{\ell_a^k}{\sigma_{\varepsilon a}^2} \mid a \in \mathcal{A} \right] \iota', \quad (\text{A.48})$$

with ℓ_a^k defined in [\(13\)](#). Furthermore,

$$\begin{aligned} \sum_{k \in \mathcal{P}(\mathcal{A})} \lambda^k \frac{\ell_a^k}{\sigma_{\varepsilon a}^2} &= \frac{(1 - \Lambda_a) \ell_a}{\sigma_{\varepsilon a}^2} + \frac{\Lambda_a}{\sigma_{\varepsilon a}^2} \\ &= \frac{\Lambda_a^2 + \Lambda_a \gamma^2 \sigma_{\varepsilon a}^2 \sigma_{\varepsilon a}^2}{\Lambda_a^2 \sigma_{\varepsilon a}^2 + \gamma^2 \sigma_{\varepsilon a}^2 \sigma_{\varepsilon a}^4} = \pi_a(\Lambda_a), \end{aligned} \quad (\text{A.49})$$

which yields [\(21\)](#). \square

A5. Proof of [Proposition 3](#)

We will use the market clearing condition to solve for the undetermined price coefficients:

$$\sum_{k \in \mathcal{P}(\mathcal{A})} \lambda^k \frac{\text{Var}^k[\mathbf{D}]^{-1}}{\gamma} \mathbb{E}^k[\mathbf{D}] - \frac{\tau}{\gamma} \mathbf{P} + \mathbf{x} = \mathbf{M}. \quad (\text{A.50})$$

Using the Projection Theorem and $h_a \equiv \frac{\Lambda_a}{\gamma \sigma_{\varepsilon a}^2}$ we can compute

$$\begin{aligned} \text{Var}^k[\mathbf{D}]^{-1} \mathbb{E}^k[\mathbf{D}] &= \left(\text{Var}[\mathbf{D}]^{-1} + [\iota_k \quad \iota_k \delta \mathbf{h}_k] \begin{bmatrix} \Sigma_{\varepsilon k}^{-1} & \mathbf{0} \\ \mathbf{0} & (\delta \mathbf{h}_k^2 \Sigma_{\varepsilon \bar{k}} + \Sigma_{x \bar{k}})^{-1} \end{bmatrix} \begin{bmatrix} \iota_k' \\ \delta \mathbf{h}_k' \iota_k' \end{bmatrix} \right) \times \\ &\quad \times \text{Var}[\mathbf{D}] [\iota_k \quad \iota_k \delta \mathbf{h}_k] \left(\begin{bmatrix} \iota_k' \\ \delta \mathbf{h}_k' \iota_k' \end{bmatrix} \text{Var}[\mathbf{D}] [\iota_k \quad \iota_k \delta \mathbf{h}_k] + \begin{bmatrix} \Sigma_{\varepsilon k} & \mathbf{0} \\ \mathbf{0} & \delta \mathbf{h}_k^2 \Sigma_{\varepsilon \bar{k}} + \Sigma_{x \bar{k}} \end{bmatrix} \right)^{-1} \begin{bmatrix} \mathbf{E}_k \\ \hat{\mathbf{P}}_k \end{bmatrix}, \end{aligned} \quad (\text{A.51})$$

which simplifies to

$$\text{Var}^k[\mathbf{D}]^{-1} \mathbb{E}^k[\mathbf{D}] = [\iota_k \quad \iota_k \delta \mathbf{h}_k]$$

$$\times \begin{bmatrix} \Sigma_{\varepsilon k}^{-1} & \mathbf{0} \\ \mathbf{0} & (\delta \mathbf{h}_k^2 \Sigma_{\varepsilon \bar{k}} + \Sigma_{x \bar{k}})^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{E}_k \\ \hat{\mathbf{P}}_k \end{bmatrix} \quad (\text{A.52})$$

$$= [\iota_k \quad \iota_k] \begin{bmatrix} \Sigma_{\varepsilon k}^{-1} & \mathbf{0} \\ \mathbf{0} & \text{diag} \left[\frac{\gamma \Lambda_a}{\Lambda_a^2 + \gamma^2 \sigma_{\varepsilon a}^2 \sigma_{\varepsilon a}^2} \mid a \in \bar{k} \right] \end{bmatrix} \begin{bmatrix} \mathbf{E}_k \\ \hat{\mathbf{P}}_k \end{bmatrix}. \quad (\text{A.53})$$

According to [Conjecture 1](#),

$$\hat{\mathbf{P}}_k = \text{diag} \left[\frac{\Lambda_a}{\gamma \sigma_{\varepsilon a}^2} \mid a \in \bar{k} \right] \mathbf{E}_k + \mathbf{x}_{\bar{k}}, \quad (\text{A.54})$$

which, after replacement into [\(A.53\)](#), yields:

$$\begin{aligned} \text{Var}^k[\mathbf{D}]^{-1} \mathbb{E}^k[\mathbf{D}] &= \iota_k \text{diag} \left[\frac{1}{\sigma_{\varepsilon a}^2} \mid a \in k \right] \mathbf{E}_k \\ &\quad + \iota_k \text{diag} \left[\frac{\Lambda_a^2}{\Lambda_a^2 \sigma_{\varepsilon a}^2 + \gamma^2 \sigma_{\varepsilon a}^4 \sigma_{\varepsilon a}^2} \mid a \in \bar{k} \right] \mathbf{E}_{\bar{k}} \\ &\quad + \iota_k \text{diag} \left[\frac{\gamma \Lambda_a}{\Lambda_a^2 + \gamma^2 \sigma_{\varepsilon a}^2 \sigma_{\varepsilon a}^2} \mid a \in \bar{k} \right] \mathbf{x}_{\bar{k}}. \end{aligned} \quad (\text{A.55})$$

We now go back to [\(A.50\)](#), which we write as

$$\tau \mathbf{P} = \sum_{k \in \mathcal{P}(\mathcal{A})} \lambda^k \text{Var}^k[\mathbf{D}]^{-1} \mathbb{E}^k[\mathbf{D}] + \gamma \mathbf{x} - \gamma \mathbf{M}, \quad (\text{A.56})$$

which, after replacement of [\(A.55\)](#) becomes

$$\begin{aligned} \tau \mathbf{P} &= \begin{bmatrix} \text{diag}[\pi_a(\Lambda_a) \mid a \in \mathcal{A}] & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N-A} \end{bmatrix} \mathbf{E} \\ &\quad + \gamma \begin{bmatrix} \text{diag} \left[\frac{\pi_a(\Lambda_a) \sigma_{\varepsilon a}^2}{\Lambda_a} \mid a \in \mathcal{A} \right] & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N-A} \end{bmatrix} \mathbf{x} - \gamma \mathbf{M}, \end{aligned} \quad (\text{A.57})$$

where \mathbf{E} is the column vector of earnings announcements and the functions $\pi_a(\Lambda_a)$, $a \in \mathcal{A}$ are defined in [Lemma 1](#). We can now verify [Conjecture 1](#):

$$\begin{aligned} \hat{\mathbf{P}} &= \frac{1}{\gamma} \begin{bmatrix} \text{diag} \left[\frac{\Lambda_a}{\pi_a(\Lambda_a) \sigma_{\varepsilon a}^2} \mid a \in \mathcal{A} \right] & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N-A} \end{bmatrix} \\ &\quad \times \begin{bmatrix} \text{diag}[\pi_a(\Lambda_a) \mid a \in \mathcal{A}] & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N-A} \end{bmatrix} \mathbf{E} + \mathbf{x} \end{aligned} \quad (\text{A.58})$$

$$= \begin{bmatrix} \text{diag} \left[\frac{\Lambda_a}{\gamma \sigma_{\varepsilon a}^2} \mid a \in \mathcal{A} \right] & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N-A} \end{bmatrix} \mathbf{E} + \mathbf{x}, \quad (\text{A.59})$$

which completes the proof of [Proposition 3](#). \square

A6. Proof of [Corollary 3.1](#)

Define first $\mathbf{\Pi} \equiv \text{diag}[\pi_a(\Lambda_a) \mid a \in \mathcal{A}]$. From [\(A.57\)](#), the matrix of response coefficients to \mathbf{E} for all firms in the

economy, α , is given by

$$\alpha = \tau^{-1} \iota \Pi = (\text{Var}[\mathbf{D}]^{-1} + \iota \Pi \iota')^{-1} \iota \Pi, \quad (\text{A.60})$$

where ι represents the matrix with all the column vectors $\{\iota_a \mid a \in \mathcal{A}\}$. Multiplying with $\Pi \iota'$ and applying the Woodbury matrix identity yields:

$$\Pi \iota' \alpha = \Pi - (\Pi^{-1} + \iota' \text{Var}[\mathbf{D}] \iota)^{-1}. \quad (\text{A.61})$$

We recognize that $\iota' \alpha = \alpha_{\mathcal{A}}$ and $\iota' \text{Var}[\mathbf{D}] \iota = \text{Var}[\mathbf{D}_{\mathcal{A}}]$, where $\mathbf{D}_{\mathcal{A}}$ is the $A \times 1$ vector of payoffs for the announcing firms. Thus, after multiplication with Π^{-1} , we obtain Eq. (24):

$$\alpha_{\mathcal{A}} = \mathbf{I} - (\mathbf{I} + \text{Var}[\mathbf{D}_{\mathcal{A}}] \Pi)^{-1}. \quad (\text{A.62})$$

The earnings response coefficients of the announcing firms are given by the diagonal elements of the matrix $\alpha_{\mathcal{A}}$. We also note that Eq. (24) can alternatively be written $\alpha_{\mathcal{A}}^{-1} = \mathbf{I} + \Pi^{-1} \text{Var}[\mathbf{D}_{\mathcal{A}}]^{-1}$, by means of the Woodbury matrix identity. \square

A7. Equilibrium solution algorithm

We will first show that the maximization problem (6) is equivalent with the simplified form (25):

$$\begin{aligned} \max_{k \in \mathcal{P}(\mathcal{A})} \mathbb{E}_0 \left[\max_{\mathbf{q}^k} \mathbb{E}_1^k \left[-e^{-\gamma(W^k - c|k|)} \right] \right] \\ = \max_{k \in \mathcal{P}(\mathcal{A})} e^{\gamma c|k|} \mathbb{E}_0 \left[\max_{\mathbf{q}^k} \mathbb{E}_1^k \left[-e^{-\gamma(q^k)' \mathbf{R}^k} \right] \right] \end{aligned} \quad (\text{A.63})$$

$$= \max_{k \in \mathcal{P}(\mathcal{A})} e^{\gamma c|k|} \mathbb{E}_0 \left[-e^{-\frac{1}{2} \mathbb{E}_1^k[\mathbf{R}^k]' \text{Var}_1^k[\mathbf{R}^k]^{-1} \mathbb{E}_1^k[\mathbf{R}^k]} \right], \quad (\text{A.64})$$

which, after using (A.32) and the law of iterated expectations, yields

$$\begin{aligned} \max_{k \in \mathcal{P}(\mathcal{A})} e^{\gamma c|k|} \mathbb{E}_0 \left[\sqrt{\frac{\det(\tau^\theta)}{\det(\tau^k)}} \mathcal{U}_1^\theta \right] \\ = \max_{k \in \mathcal{P}(\mathcal{A})} e^{\gamma c|k|} \sqrt{\frac{\det(\tau^\theta)}{\det(\tau^k)}} \mathbb{E}_0[\mathcal{U}_1^\theta]. \end{aligned} \quad (\text{A.65})$$

We notice that $\mathbb{E}_0[\mathcal{U}_1^\theta]$ is a constant that does not depend on the individual choice of the investor. Dividing by this (negative) constant yields

$$\begin{aligned} \max_{k \in \mathcal{P}(\mathcal{A})} \frac{1}{2} \ln(\det(\tau^k)) - \frac{1}{2} \ln(\det(\tau^\theta)) - \gamma c|k| \\ = \max_{k \in \mathcal{P}(\mathcal{A})} \frac{1}{2} \ln B_\theta^k, \end{aligned} \quad (\text{A.66})$$

and therefore the optimization problem at time 0 for each investor in this economy is (25).

To prove that the function $\ln B_\theta^k$ is submodular, consider two types $k, k' \in \mathcal{P}(\mathcal{A})$ with $k \subseteq k'$ and $a \in \mathcal{A} \setminus k'$, then use (A.39) and (A.40) to compute

$$\ln B_\theta^{k \cup \{a\}} - \ln B_\theta^k = \ln B_\theta^{k' \cup \{a\}} - \ln B_\theta^{k'}, \quad (\text{A.67})$$

$$= \ln \left(\frac{\sigma_{ea}^2 + \sigma_{\varepsilon a}^2}{\ell_a \sigma_{ea}^2 + \sigma_{\varepsilon a}^2} + \frac{b_a^2}{\frac{1}{U^2} + \sum_{\alpha=1}^A \frac{\ell_\alpha^k b_\alpha^2}{\ell_\alpha^k \sigma_{ea}^2 + \sigma_{\varepsilon a}^2}} \frac{(1 - \ell_a) \sigma_{\varepsilon a}^2}{(\ell_a \sigma_{ea}^2 + \sigma_{\varepsilon a}^2)^2} \right) - 2\gamma c. \quad (\text{A.68})$$

The same difference is lower when written for k' instead of k , due to the term $\sum_{\alpha=1}^A \frac{\ell_\alpha^k b_\alpha^2}{\ell_\alpha^k \sigma_{ea}^2 + \sigma_{\varepsilon a}^2}$ in the denominator (this term is larger when written for k' because $k \subseteq k'$). Therefore,

$$\ln B_\theta^{k \cup \{a\}} - \ln B_\theta^k \geq \ln B_\theta^{k' \cup \{a\}} - \ln B_\theta^{k'}, \quad (\text{A.69})$$

and thus the function $\ln B_\theta^k$ is indeed submodular. We further prove the following Lemma.

Lemma 4. For any two types $k, k' \in \mathcal{P}(\mathcal{A})$ and $\lambda^k > 0$, a migration of a positive mass of investors $z < \lambda^k$ from k to k' decreases $B_\theta^{k'}/B_\theta^k$.

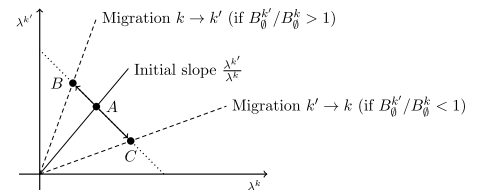
Proof. Consider a type $k \in \mathcal{P}(\mathcal{A})$ and its complement $\bar{k} = \mathcal{A} \setminus k$. Using Proposition 1, write

$$\begin{aligned} \det(\tau^k) = \det(\text{Var}[\mathbf{D}]^{-1}) \left(\prod_{a \in k} \frac{\sigma_{ea}^2 + \sigma_{\varepsilon a}^2}{\sigma_{\varepsilon a}^2} \right) \left(\prod_{a \in \bar{k}} \frac{\ell_a \sigma_{ea}^2 + \sigma_{\varepsilon a}^2}{\sigma_{\varepsilon a}^2} \right) \\ \times \left(1 + U^2 \sum_{a \in k} \frac{b_a^2}{\sigma_{ea}^2 + \sigma_{\varepsilon a}^2} + U^2 \sum_{a \in \bar{k}} \frac{\ell_a b_a^2}{\ell_a \sigma_{ea}^2 + \sigma_{\varepsilon a}^2} \right). \end{aligned} \quad (\text{A.70})$$

A migration from $k \rightarrow k'$ increases the terms $\prod_{a \in \bar{k}} \frac{\ell_a \sigma_{ea}^2 + \sigma_{\varepsilon a}^2}{\sigma_{\varepsilon a}^2}$ and $\sum_{a \in \bar{k}} \frac{\ell_a b_a^2}{\ell_a \sigma_{ea}^2 + \sigma_{\varepsilon a}^2}$, while all the other terms of the decomposition (A.70) remain constant. Thus, $\det(\tau^k)$ increases. One can similarly show that $\det(\tau^{k'})$ decreases, and therefore $B_\theta^{k'}/B_\theta^k$ decreases. \square

The submodularity property of the function $\ln B_\theta^k$, coupled with the monotonicity of $B_\theta^{k'}/B_\theta^k$ implied by Lemma 4, justify the use of an iterative algorithm that converges towards a stable equilibrium. The algorithm is adapted from Hu and Shi (2019) and Arkolakis et al. (2021) and consists of the following steps:

1. Start from any set of positive initial values $\{\lambda_0^k > 0 \mid k \in \mathcal{P}(\mathcal{A})\}$ such that $\sum_k \lambda_0^k = 1$. Compute the benefit-cost ratios $\{B_\theta^k \mid k \in \mathcal{P}(\mathcal{A})\}$.
2. For any two types $k, k' \in \mathcal{P}(\mathcal{A})$, compute $B_\theta^{k'}/B_\theta^k$:
 - (a) if $B_\theta^{k'}/B_\theta^k = 1$, no further changes in λ^k and $\lambda^{k'}$ are needed at this step.
 - (b) if $B_\theta^{k'}/B_\theta^k > 1$, then allow a small fraction of the population of type k investors to migrate to type k' , which will decrease $B_\theta^{k'}/B_\theta^k$ (Lemma 4). In the illustration below, the dot A depicts the initial values $\{\lambda^k, \lambda^{k'}\}$, located on a line with slope $\lambda^{k'}/\lambda^k$. The algorithm multiplies the slope of the line by $m > 1$ and finds two new values λ_{new}^k and $\lambda_{new}^{k'}$ such that $\lambda_{new}^k + \lambda_{new}^{k'} = \lambda^k + \lambda^{k'}$ and $\lambda_{new}^k < \lambda^k$, thus reaching the dot B :



After the multiplication, the new values for λ^k and $\lambda^{k'}$ are given by

$$\begin{aligned}\lambda_{new}^k &= \lambda^k \frac{\lambda^k + \lambda^{k'}}{\lambda^k + m\lambda^{k'}} \quad \text{and} \\ \lambda_{new}^{k'} &= \lambda^{k'} \frac{\lambda^k + \lambda^{k'}}{\lambda^k/m + \lambda^{k'}}.\end{aligned}\quad (\text{A.71})$$

To ensure stability of the solution, m is set to increase with $(B_{\theta}^{k'}/B_{\theta}^k - 1)$. Finally, compute the benefit-cost ratios $\{B_{\theta}^k | k \in \mathcal{P}(A)\}$ using the new values $\{\lambda_{new}^k, \lambda_{new}^{k'}\}$.

(c) if $B_{\theta}^{k'}/B_{\theta}^k < 1$, apply a similar procedure as in the previous step, moving from A to C .

3. Iterate step 2 until the algorithm has converged to the desired accuracy and the conditions of [Theorem 1](#) are satisfied. Convergence is guaranteed by [Lemma 4](#).

A.8. (CAPM) Proofs of [Proposition 4](#) and [Corollary 4.1](#)

Investors' learning and uncertainty at time 0 Given information at time 0, investors form beliefs about \mathbf{D} , $\mathbb{E}_0[\mathbf{D}]$ and $\text{Var}_0[\mathbf{D}]$. The prior variance of \mathbf{D} is $\text{Var}[\mathbf{D}] = U^2 \mathbf{b}\mathbf{b}' + \text{Var}[\mathbf{e}]$. Based on investors' information set $\mathcal{F}_0 = \{G\}$, we apply the Projection Theorem (page 56), with:

$$\Sigma_{\theta\theta} = \text{Var}[\mathbf{D}] = U^2 \mathbf{b}\mathbf{b}' + \text{Var}[\mathbf{e}] \quad (\text{A.72})$$

$$\Sigma_{\theta s} = \text{Cov}[\mathbf{D}, G] = \text{Var}[\mathbf{D}]\mathbf{M} = U^2 \mathbf{b} + \text{Var}[\mathbf{e}]\mathbf{M} \quad (\text{A.73})$$

$$\Sigma_{s\theta} = \text{Cov}[G, \mathbf{D}] = \mathbf{M}'\text{Var}[\mathbf{D}] = U^2 \mathbf{b}' + \mathbf{M}'\text{Var}[\mathbf{e}] \quad (\text{A.74})$$

$$\Sigma_{ss} = \text{Var}[G] = \mathbf{M}'\text{Var}[\mathbf{D}]\mathbf{M} + \sigma_g^2 = U^2 + \mathbf{M}'\text{Var}[\mathbf{e}]\mathbf{M} + \sigma_g^2, \quad (\text{A.75})$$

where \mathbf{b} is the $N \times 1$ vector of firms' exposures to the systematic component f , $\mathbf{M}\mathbf{b} = \mathbf{1}$ by assumption, and \mathbf{e} is the vector of idiosyncratic components in firms' payoffs. Then

$$\begin{aligned}\text{Var}_0[\mathbf{D}] &= U^2 \mathbf{b}\mathbf{b}' + \text{Var}[\mathbf{e}] - (U^2 \mathbf{b} + \text{Var}[\mathbf{e}]\mathbf{M}) \\ &\quad \times \frac{1}{U^2 + \mathbf{M}'\text{Var}[\mathbf{e}]\mathbf{M} + \sigma_g^2} (U^2 \mathbf{b}' + \mathbf{M}'\text{Var}[\mathbf{e}]),\end{aligned}\quad (\text{A.76})$$

or, using the Woodbury Matrix Identity:

$$\tau_0 \equiv \text{Var}_0[\mathbf{D}]^{-1} = \text{Var}[\mathbf{D}]^{-1} + \frac{1}{\sigma_g^2} \mathbf{M}\mathbf{M}'. \quad (\text{A.77})$$

Investors' posterior expectation at time 0 is:

$$\mathbb{E}_0[\mathbf{D}] = \text{Var}[\mathbf{D}]\mathbf{M}(\sigma_g^2 + \mathbf{M}'\text{Var}[\mathbf{D}]\mathbf{M})^{-1}G = \frac{1}{\sigma_g^2} \tau_0^{-1} \mathbf{M}G. \quad (\text{A.78})$$

where the second equality results from multiplying the first equality with $\tau_0^{-1} \tau_0$ and simplifying.

The market-wide uncertainty at time 0 is defined as $U_0^2 \equiv \text{Var}_0[\mathbf{M}\mathbf{D}]$:

$$U_0^2 = \mathbf{M}'\text{Var}_0[\mathbf{D}]\mathbf{M}$$

$$= \sigma_g^4 \frac{1}{\sigma_g^2} \mathbf{M}' \left(\text{Var}[\mathbf{D}]^{-1} + \mathbf{M} \frac{1}{\sigma_g^2} \mathbf{M}' \right)^{-1} \mathbf{M} \frac{1}{\sigma_g^2} \quad (\text{A.79})$$

$$\begin{aligned}&= \sigma_g^4 \left[\frac{1}{\sigma_g^2} - (\sigma_g^2 + \mathbf{M}'\text{Var}[\mathbf{D}]\mathbf{M})^{-1} \right] \\ &= \frac{\sigma_g^2 \mathbf{M}'\text{Var}[\mathbf{D}]\mathbf{M}}{\sigma_g^2 + \mathbf{M}'\text{Var}[\mathbf{D}]\mathbf{M}} = \frac{1}{\frac{1}{\mathbf{M}'\text{Var}[\mathbf{D}]\mathbf{M}} + \frac{1}{\sigma_g^2}}.\end{aligned}\quad (\text{A.80})$$

Since $\mathbf{M}'\text{Var}[\mathbf{D}]\mathbf{M} = U^2 + \mathbf{M}'\text{Var}[\mathbf{e}]\mathbf{M}$, U_0 increases if U increases or if σ_g increases. Furthermore, $\lim_{N \rightarrow \infty} \mathbf{M}'\text{Var}[\mathbf{D}]\mathbf{M} = U^2$ and we recover [Eq. \(35\)](#) in the text.

Equilibrium To solve for the equilibrium prices, conjecture the following linear forms:

$$\mathbf{P}_0 = \Gamma_0 G + \xi_{00} \mathbf{x}_0 - \zeta_0 \mathbf{M} \quad (\text{A.81})$$

$$\mathbf{P}_1 = \Gamma_1 G + \xi_{01} \mathbf{x}_0 - \zeta_1 \mathbf{M} + \alpha_1 \mathbf{E} + \xi_1 \mathbf{x}_1 \quad (\text{A.82})$$

Noise traders hold \mathbf{x}_0 at time 0 and $\mathbf{x}_0 + \mathbf{x}_1$ at time 1.

Time 1 At time 1, investors' learning follows [Appendix A.1](#), with the addition that all investors observe G from the previous period and the conjecture (11) must change to take this into account (Note: investors' information at time 0 is public, and thus \mathbf{x}_0 is observed):

$$\hat{\mathbf{P}}_1 \equiv \xi_1^{-1} (\mathbf{P}_1 - \Gamma_1 G - \xi_{01} \mathbf{x}_0 + \zeta_1 \mathbf{M}) = \sum_{a=1}^A \frac{\Lambda_a}{\gamma \sigma_{\varepsilon a}^2} \iota_a E_a + \mathbf{x}_1. \quad (\text{A.83})$$

The posterior variance $\text{Var}_1^k[\mathbf{D}]$ is now

$$\tau_1^k \equiv \text{Var}_1^k[\mathbf{D}]^{-1} = \tau_0 + \sum_{a=1}^A \frac{\ell_a^k}{\sigma_{\varepsilon a}^2} \iota_a \iota_a', \quad (\text{A.84})$$

where τ_0 is defined in [\(A.77\)](#). The weighted average precision at time 1 is then

$$\begin{aligned}\tau_1 &= \sum_{k \in \mathcal{P}(A)} \lambda^k \tau_1^k = \tau_0 \\ &\quad + \begin{bmatrix} \text{diag}[\pi_a(\Lambda_a) | a \in \mathcal{A}] & \mathbf{0}_{A \times (N-A)} \\ \mathbf{0}_{(N-A) \times A} & \mathbf{0}_{(N-A) \times (N-A)} \end{bmatrix},\end{aligned}\quad (\text{A.85})$$

where $\pi_a(\Lambda_a)$ is defined as in [\(22\)](#). We can then write a modified version of [\(A.55\)](#):

$$\begin{aligned}\tau_1^k \mathbb{E}_1^k[\mathbf{D}] &= \iota_k \text{diag} \left[\frac{1}{\sigma_{\varepsilon a}^2} | a \in k \right] \mathbf{E}_k \\ &\quad + \iota_{\bar{k}} \text{diag} \left[\frac{\Lambda_a^2}{\Lambda_a^2 \sigma_{\varepsilon a}^2 + \gamma^2 \sigma_{\varepsilon a}^4 \sigma_{xa}^2} | a \in \bar{k} \right] \mathbf{E}_{\bar{k}} \\ &\quad + \iota_{\bar{k}} \text{diag} \left[\frac{\gamma \Lambda_a}{\Lambda_a^2 + \gamma^2 \sigma_{\varepsilon a}^2 \sigma_{xa}^2} | a \in \bar{k} \right] \mathbf{x}_{\bar{k}} + \frac{1}{\sigma_g^2} \mathbf{M}G,\end{aligned}\quad (\text{A.86})$$

which leads to a new market clearing condition (the counterpart of [\(8\)](#) in the baseline setup):

$$\sum_{k \in \mathcal{P}(A)} \lambda^k \mathbf{q}_1^k + \mathbf{x}_0 + \mathbf{x}_1 = \mathbf{M}, \quad \text{where } \mathbf{q}_1^k = \frac{1}{\gamma} \tau_1^k (\mathbb{E}_1^k[\mathbf{D}] - \mathbf{P}_1). \quad (\text{A.87})$$

Thus, prices at time 1 solve a modified version of (A.56) in the baseline setup, and one can check that they verify the new conjecture (A.83). We thus obtain (32) in Proposition 4:

$$\tau_1 \mathbf{P}_1 = \sum_{k \in \mathcal{P}(A)} \lambda^k \tau_1^k \mathbb{E}_1^k[\mathbf{D}] + \gamma \mathbf{x}_0 + \gamma \mathbf{x}_1 - \gamma \mathbf{M}. \quad (\text{A.88})$$

Time 0 Consider an investor who at time 0 knows that she will be of type k at time 1. We prove here that knowing her future type does not change her portfolio choice at time 0,

$$\mathbf{q}_0 = \frac{1}{\gamma} \tau_0 (\mathbb{E}_0[\mathbf{D}] - \mathbf{P}_0), \quad (\text{A.89})$$

which is Eq. (29) in Proposition 4. The proof of this statement follows Brennan and Cao (1997), adapted to our Grossman and Stiglitz (1980) setup with information acquisition.

The final wealth of a type- k investor at time 2 is (taking into account the cost of information):

$$W^k = (\mathbf{q}_0^k)'(\mathbf{P}_1 - \mathbf{P}_0) - c|k| + (\mathbf{q}_1^k)'(\mathbf{D} - \mathbf{P}_0), \quad (\text{A.90})$$

and he expected utility at time 1 for this investor is then given by

$$\mathcal{U}_1^k = -\exp \left[-\gamma (\mathbf{q}_0^k)'(\mathbf{P}_1 - \mathbf{P}_0) + \gamma c|k| - \frac{1}{2} (\mathbb{E}_1^k[\mathbf{D}] - \mathbf{P}_1)' \tau_1^k (\mathbb{E}_1^k[\mathbf{D}] - \mathbf{P}_1) \right]. \quad (\text{A.91})$$

Defining

$$\mathbf{a}^k \equiv \mathbb{E}_1^k[\mathbf{D}] - \mathbf{P}_0 \quad (\text{A.92})$$

$$\mathbf{c}^k \equiv \mathbb{E}_1^k[\mathbf{D}] - \mathbf{P}_1, \quad (\text{A.93})$$

we can write the expected utility at time 1 as

$$\mathcal{U}_1^k = -\exp \left[-\gamma (\mathbf{q}_0^k)'(\mathbf{a}^k - \mathbf{c}^k) + \gamma c|k| - \frac{1}{2} (\mathbf{c}^k)' \tau_1^k \mathbf{c}^k \right]. \quad (\text{A.94})$$

To compute the expected utility at time 0, $\mathbb{E}_0[\mathcal{U}_1^k]$, we need the joint distribution at time 0 of \mathbf{a}^k and \mathbf{c}^k (both of them are random for a time 0 investor). The law of iterated expectations implies:

$$\mathbb{E}_0[\mathbf{a}^k] = \mathbb{E}_0[\mathbf{D}] - \mathbf{P}_0. \quad (\text{A.95})$$

Using that $\mathbb{E}_0[\mathbf{x}_1] = \mathbf{0}$, Eq. (A.88) implies

$$\tau_1 \mathbb{E}_0[\mathbf{P}_1] = \sum_{k \in \mathcal{P}(A)} \lambda^k \tau_1^k \mathbb{E}_0[\mathbf{D}] + \gamma \mathbf{x}_0 - \gamma \mathbf{M} \quad (\text{A.96})$$

$$= \tau_1 \mathbb{E}_0[\mathbf{D}] - \gamma (\mathbf{M} - \mathbf{x}_0), \quad (\text{A.97})$$

and thus

$$\mathbb{E}_0[\mathbf{P}_1] = \mathbb{E}_0[\mathbf{D}] - \gamma \tau_1^{-1} (\mathbf{M} - \mathbf{x}_0), \quad (\text{A.98})$$

which leads to

$$\mathbb{E}_0[\mathbf{c}^k] = \mathbb{E}_0[\mathbf{D}] - \mathbb{E}_0[\mathbf{D}] + \gamma \tau_1^{-1} (\mathbf{M} - \mathbf{x}_0) = \gamma \tau_1^{-1} (\mathbf{M} - \mathbf{x}_0). \quad (\text{A.99})$$

We now compute variances and covariances of \mathbf{a}^k and \mathbf{c}^k :

$$\begin{aligned} \text{Var}_0[\mathbf{a}^k] &= \text{Var}_0[\mathbb{E}_1^k[\mathbf{D}]] = \text{Var}_0[\mathbf{D}] - \text{Var}_1^k[\mathbf{D}] \\ &= \tau_0^{-1} - (\tau_1^k)^{-1}, \end{aligned} \quad (\text{A.100})$$

and, defining $\mathbf{\Omega} \equiv \text{Var}_0[\mathbf{D} - \mathbf{P}_1]$,

$$\text{Var}_0[\mathbf{c}^k] = \text{Var}_0[\mathbb{E}_1^k[\mathbf{D} - \mathbf{P}_1]] = \mathbf{\Omega} - (\tau_1^k)^{-1}. \quad (\text{A.101})$$

Finally, the covariance $\text{Cov}_0[\mathbf{a}^k, \mathbf{c}^k]$ is

$$\text{Cov}_0[\mathbf{a}^k, \mathbf{c}^k] = \text{Cov}_0[\mathbb{E}_1^k[\mathbf{D}], \mathbb{E}_1^k[\mathbf{D}] - \mathbf{P}_1] \quad (\text{A.102})$$

$$= \text{Var}_0[\mathbb{E}_1^k[\mathbf{D}]] - \text{Cov}_0[\mathbb{E}_1^k[\mathbf{D}], \mathbf{P}_1] \quad (\text{A.103})$$

$$= \tau_0^{-1} - (\tau_1^k)^{-1} - \text{Cov}_0[\mathbb{E}_1^k[\mathbf{D}], \mathbf{P}_1]. \quad (\text{A.104})$$

To solve for $\text{Cov}_0[\mathbb{E}_1^k[\mathbf{D}], \mathbf{P}_1]$, consider the most informed type, denoted by \tilde{k} . Then

$$\text{Cov}_0[\mathbb{E}_1^{\tilde{k}}[\mathbf{D}], \mathbf{P}_1] = \underbrace{\text{Cov}_1^{\tilde{k}}[\mathbb{E}_1^{\tilde{k}}[\mathbf{D}], \mathbf{P}_1]}_{=0} + \text{Cov}_0[\mathbb{E}_1^{\tilde{k}}[\mathbf{D}], \mathbf{P}_1], \quad (\text{A.105})$$

and thus all the covariances $\text{Cov}_0[\mathbb{E}_1^k[\mathbf{D}], \mathbf{P}_1]$ take the same value, $\text{Cov}_0[\mathbb{E}_1^{\tilde{k}}[\mathbf{D}], \mathbf{P}_1]$.

Then, using (A.88):

$$\begin{aligned} \text{Cov}_0[\mathbb{E}_1^{\tilde{k}}[\mathbf{D}], \mathbf{P}_1] &= \text{Cov}_0 \\ &\times \left[\mathbb{E}_1^{\tilde{k}}[\mathbf{D}], \tau_1^{-1} \left(\sum_{k \in \mathcal{P}(A)} \lambda^k \tau_1^k \mathbb{E}_1^k[\mathbf{D}] + \gamma \mathbf{x}_0 + \gamma \mathbf{x}_1 - \gamma \mathbf{M} \right) \right], \end{aligned} \quad (\text{A.106})$$

and since \tilde{k} is the most informed investor, she does not learn from prices and therefore $\mathbb{E}_1^{\tilde{k}}[\mathbf{D}]$ does not depend on \mathbf{x}_1 . Thus, we can further write the covariance above as

$$\text{Cov}_0[\mathbb{E}_1^{\tilde{k}}[\mathbf{D}], \mathbf{P}_1] = \tau_1^{-1} \sum_{k \in \mathcal{P}(A)} \lambda^k \tau_1^k \text{Cov}_0[\mathbb{E}_1^{\tilde{k}}[\mathbf{D}], \mathbb{E}_1^k[\mathbf{D}]] \quad (\text{A.107})$$

$$= \tau_1^{-1} \sum_{k \in \mathcal{P}(A)} \lambda^k \tau_1^k \left(\underbrace{\text{Cov}_1^{\tilde{k}}[\mathbb{E}_1^{\tilde{k}}[\mathbf{D}], \mathbb{E}_1^k[\mathbf{D}]]}_0 + \text{Cov}_0[\mathbb{E}_1^{\tilde{k}}[\mathbf{D}], \mathbb{E}_1^k[\mathbf{D}]] \right) \quad (\text{A.108})$$

$$= \tau_1^{-1} \sum_{k \in \mathcal{P}(A)} \lambda^k \tau_1^k (\tau_0^{-1} - (\tau_1^k)^{-1}) \quad (\text{A.109})$$

$$= \tau_0^{-1} - \tau_1^{-1}, \quad (\text{A.110})$$

and thus, going back to (A.104), we obtain

$$\text{Cov}_0[\mathbf{a}^k, \mathbf{c}^k] = \tau_0^{-1} - (\tau_1^k)^{-1} - (\tau_0^{-1} - \tau_1^{-1}) = \tau_1^{-1} - (\tau_1^k)^{-1}. \quad (\text{A.111})$$

Eqs. (A.95), (A.99), (A.100), (A.101), and (A.111) imply the joint distribution of \mathbf{a}^k and \mathbf{c}^k :

$$\begin{aligned} \begin{bmatrix} \mathbf{a}^k \\ \mathbf{c}^k \end{bmatrix} &\sim \mathcal{N} \left(\begin{bmatrix} \mathbb{E}_0[\mathbf{D}] - \mathbf{P}_0 \\ \gamma \tau_1^{-1} (\mathbf{M} - \mathbf{x}_0) \end{bmatrix}, \right. \\ &\left. \begin{bmatrix} \tau_0^{-1} - (\tau_1^k)^{-1} & \tau_1^{-1} - (\tau_1^k)^{-1} \\ \tau_1^{-1} - (\tau_1^k)^{-1} & \mathbf{\Omega} - (\tau_1^k)^{-1} \end{bmatrix} \right). \end{aligned} \quad (\text{A.112})$$

We are now ready to compute

$$\mathbb{E}_0[\mathcal{U}_1^k] = \mathbb{E}_0\left[-\exp\left(-\gamma(\mathbf{q}_0^k)'(\mathbf{a}^k - \mathbf{c}^k) + \gamma c|k| - \frac{1}{2}(\mathbf{c}^k)' \boldsymbol{\tau}_1^k \mathbf{c}^k\right)\right] \quad (\text{A.113})$$

using Lemma 2. To simplify notation, denote by $\mathbb{E}_0[\mathbf{a}^k] = \mathbf{m}_a$ and $\mathbb{E}_0[\mathbf{c}^k] = \mathbf{m}_c$ (these do not depend on k) and \mathbf{z} the demeaned vector, $\begin{bmatrix} \mathbf{a}^k \\ \mathbf{c}^k \end{bmatrix} = \begin{bmatrix} \mathbf{z}_a \\ \mathbf{z}_c \end{bmatrix} + \begin{bmatrix} \mathbf{m}_a \\ \mathbf{m}_c \end{bmatrix}$. The exponent above is

$$\begin{aligned} & \mathbf{z}' \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & -\boldsymbol{\tau}_1^k/2 \end{bmatrix}}_{\mathbf{F}} \mathbf{z} + \underbrace{\left[-\gamma(\mathbf{q}_0^k)' \quad \gamma(\mathbf{q}_0^k)' - \mathbf{m}_c' \boldsymbol{\tau}_1^k\right]}_{\mathbf{G}'} \mathbf{z} \\ & + \underbrace{\gamma c|k| + \gamma(\mathbf{q}_0^k)'(\mathbf{m}_c - \mathbf{m}_a) - \frac{1}{2} \mathbf{m}_c' \boldsymbol{\tau}_1^k \mathbf{m}_c}_{\mathbf{H}}. \quad (\text{A.114}) \end{aligned}$$

Let $\boldsymbol{\Sigma}$ be the covariance matrix in (A.112). Then,

$$\mathbf{I} - 2\boldsymbol{\Sigma}\mathbf{F} = \mathbf{I} - 2 \begin{bmatrix} 0 & -(\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}) \frac{\boldsymbol{\tau}_1^k}{2} \\ 0 & -(\boldsymbol{\Omega} - (\boldsymbol{\tau}_1^k)^{-1}) \frac{\boldsymbol{\tau}_1^k}{2} \end{bmatrix} \quad (\text{A.115})$$

$$= \mathbf{I} - \begin{bmatrix} 0 & -\boldsymbol{\tau}_1^{-1} \boldsymbol{\tau}_1^k + \mathbf{I} \\ 0 & -\boldsymbol{\Omega} \boldsymbol{\tau}_1^k + \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \boldsymbol{\tau}_1^{-1} \boldsymbol{\tau}_1^k - \mathbf{I} \\ 0 & \boldsymbol{\Omega} \boldsymbol{\tau}_1^k \end{bmatrix}. \quad (\text{A.116})$$

Use block inversion to obtain $(\mathbf{I} - 2\boldsymbol{\Sigma}\mathbf{F})^{-1}$. The diagonal blocks are both invertible, and thus

$$\begin{aligned} (\mathbf{I} - 2\boldsymbol{\Sigma}\mathbf{F})^{-1} &= \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & (\boldsymbol{\Omega} \boldsymbol{\tau}_1^k)^{-1} \end{bmatrix} \\ &\times \begin{bmatrix} \mathbf{I} & -(\boldsymbol{\tau}_1^{-1} \boldsymbol{\tau}_1^k - \mathbf{I})(\boldsymbol{\Omega} \boldsymbol{\tau}_1^k)^{-1} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \quad (\text{A.117}) \end{aligned}$$

$$= \begin{bmatrix} \mathbf{I} & [(\boldsymbol{\tau}_1^k)^{-1} - \boldsymbol{\tau}_1^{-1}] \boldsymbol{\Omega}^{-1} \\ \mathbf{0} & (\boldsymbol{\tau}_1^k)^{-1} \boldsymbol{\Omega}^{-1} \end{bmatrix}, \quad (\text{A.118})$$

and thus $(\mathbf{I} - 2\boldsymbol{\Sigma}\mathbf{F})^{-1} \boldsymbol{\Sigma}$ equals

$$\begin{bmatrix} \boldsymbol{\tau}_0^{-1} - (\boldsymbol{\tau}_1^k)^{-1} - [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}] \boldsymbol{\Omega}^{-1} [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}] & [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}] \boldsymbol{\Omega}^{-1} (\boldsymbol{\tau}_1^k)^{-1} \\ (\boldsymbol{\tau}_1^k)^{-1} \boldsymbol{\Omega}^{-1} [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}] & (\boldsymbol{\tau}_1^k)^{-1} [\mathbf{I} - \boldsymbol{\Omega}^{-1} (\boldsymbol{\tau}_1^k)^{-1}] \end{bmatrix}. \quad (\text{A.119})$$

In Lemma 2, the term $\frac{1}{2} \mathbf{G}' (\mathbf{I} - 2\boldsymbol{\Sigma}\mathbf{F})^{-1} \boldsymbol{\Sigma} \mathbf{G}$ equals

$$\begin{aligned} & \frac{1}{2} \begin{bmatrix} -\gamma(\mathbf{q}_0^k)' & \gamma(\mathbf{q}_0^k)' - \mathbf{m}_c' \boldsymbol{\tau}_1^k \end{bmatrix} \\ & \times \begin{bmatrix} \boldsymbol{\tau}_0^{-1} - (\boldsymbol{\tau}_1^k)^{-1} - [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}] \boldsymbol{\Omega}^{-1} [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}] & [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}] \boldsymbol{\Omega}^{-1} (\boldsymbol{\tau}_1^k)^{-1} \\ (\boldsymbol{\tau}_1^k)^{-1} \boldsymbol{\Omega}^{-1} [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}] & (\boldsymbol{\tau}_1^k)^{-1} [\mathbf{I} - \boldsymbol{\Omega}^{-1} (\boldsymbol{\tau}_1^k)^{-1}] \end{bmatrix} \\ & \times \begin{bmatrix} -\gamma \mathbf{q}_0^k \\ \gamma \mathbf{q}_0^k - \boldsymbol{\tau}_1^k \mathbf{m}_c \end{bmatrix}, \quad (\text{A.120}) \end{aligned}$$

and thus it has the following form

$$\begin{aligned} & \frac{1}{2} \mathbf{G}' (\mathbf{I} - 2\boldsymbol{\Sigma}\mathbf{F})^{-1} \boldsymbol{\Sigma} \mathbf{G} + \mathbf{H} = \frac{1}{2} \begin{bmatrix} g_1' & g_2' \end{bmatrix} \begin{bmatrix} a & b \\ b' & d \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \\ & = \frac{1}{2} g_1' a g_1 + g_1' b g_2 + \frac{1}{2} g_2' d g_2 + \mathbf{H}, \quad (\text{A.121}) \end{aligned}$$

with,

$$\frac{1}{2} g_1' a g_1 \equiv \frac{1}{2} \gamma^2 (\mathbf{q}_0^k)' (\boldsymbol{\tau}_0^{-1} - (\boldsymbol{\tau}_1^k)^{-1} - [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}])$$

$$\times \boldsymbol{\Omega}^{-1} [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}] \mathbf{q}_0^k \quad (\text{A.122})$$

$$g_1' b g_2 \equiv -\gamma (\mathbf{q}_0^k)' [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}] \boldsymbol{\Omega}^{-1} (\boldsymbol{\tau}_1^k)^{-1} [\gamma \mathbf{q}_0^k - \boldsymbol{\tau}_1^k \mathbf{m}_c] \quad (\text{A.123})$$

$$\begin{aligned} \frac{1}{2} g_2' d g_2 &\equiv \frac{1}{2} [\gamma (\mathbf{q}_0^k)' - \mathbf{m}_c' \boldsymbol{\tau}_1^k] (\boldsymbol{\tau}_1^k)^{-1} [\mathbf{I} - \boldsymbol{\Omega}^{-1} (\boldsymbol{\tau}_1^k)^{-1}] \\ &\times [\gamma \mathbf{q}_0^k - \boldsymbol{\tau}_1^k \mathbf{m}_c]. \quad (\text{A.124}) \end{aligned}$$

Taking the first order condition with respect to \mathbf{q}_0^k yields (using matrix differentiation rules: $\partial x' A x / \partial x = (A + A')x$ and $\partial x' A / \partial x = A$):

$$\begin{aligned} \frac{\partial \frac{1}{2} g_1' a g_1}{\partial \mathbf{q}_0^k} &= \gamma^2 (\boldsymbol{\tau}_0^{-1} - (\boldsymbol{\tau}_1^k)^{-1} - [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}]) \\ &\times \boldsymbol{\Omega}^{-1} [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}] \mathbf{q}_0^k \quad (\text{A.125}) \end{aligned}$$

$$\begin{aligned} \frac{\partial g_1' b g_2}{\partial \mathbf{q}_0^k} &= -2\gamma^2 [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}] \boldsymbol{\Omega}^{-1} (\boldsymbol{\tau}_1^k)^{-1} \mathbf{q}_0^k \\ &+ \gamma [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}] \boldsymbol{\Omega}^{-1} (\boldsymbol{\tau}_1^k)^{-1} \boldsymbol{\tau}_1^k \mathbf{m}_c \quad (\text{A.126}) \end{aligned}$$

$$\begin{aligned} &= -2\gamma^2 [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}] \boldsymbol{\Omega}^{-1} (\boldsymbol{\tau}_1^k)^{-1} \mathbf{q}_0^k \\ &+ \gamma [\boldsymbol{\tau}_1^{-1} - (\boldsymbol{\tau}_1^k)^{-1}] \boldsymbol{\Omega}^{-1} \mathbf{m}_c \quad (\text{A.127}) \end{aligned}$$

$$\begin{aligned} \frac{\partial \frac{1}{2} g_2' d g_2}{\partial \mathbf{q}_0^k} &= \gamma^2 (\boldsymbol{\tau}_1^k)^{-1} [\mathbf{I} - \boldsymbol{\Omega}^{-1} (\boldsymbol{\tau}_1^k)^{-1}] \mathbf{q}_0^k \\ &- \frac{1}{2} \gamma (\boldsymbol{\tau}_1^k)^{-1} [\mathbf{I} - \boldsymbol{\Omega}^{-1} (\boldsymbol{\tau}_1^k)^{-1}] \boldsymbol{\tau}_1^k \mathbf{m}_c \\ &- \frac{1}{2} \gamma [\mathbf{I} - (\boldsymbol{\tau}_1^k)^{-1} \boldsymbol{\Omega}^{-1}] \mathbf{m}_c \quad (\text{A.128}) \end{aligned}$$

$$= \gamma^2 (\boldsymbol{\tau}_1^k)^{-1} [\mathbf{I} - \boldsymbol{\Omega}^{-1} (\boldsymbol{\tau}_1^k)^{-1}] \mathbf{q}_0^k - \gamma [\mathbf{I} - (\boldsymbol{\tau}_1^k)^{-1} \boldsymbol{\Omega}^{-1}] \mathbf{m}_c \quad (\text{A.129})$$

$$\frac{\partial \mathbf{H}}{\partial \mathbf{q}_0^k} = \gamma (\mathbf{m}_c - \mathbf{m}_a). \quad (\text{A.130})$$

All the terms with \mathbf{q}_0^k sum up to (there are 3 terms; add first term with first half of second term; add third term

with second half of second term; take total; $\tau_1^{-1}\Omega^{-1}(\tau_1^k)^{-1}$ is symmetric):

$$\gamma^2(\tau_0^{-1} - \tau_1^{-1}\Omega^{-1}\tau_1^{-1})\mathbf{q}_0^k, \quad (\text{A.131})$$

whereas all the terms without \mathbf{q}_0^k sum up to (there are 3 terms):

$$\gamma\tau_1^{-1}\Omega^{-1}\mathbf{m}_c - \gamma\mathbf{m}_a, \quad (\text{A.132})$$

and thus the first order condition with respect to \mathbf{q}_0^k is

$$(\tau_0^{-1} - \tau_1^{-1}\Omega^{-1}\tau_1^{-1})\mathbf{q}_0^k = \frac{1}{\gamma}(\mathbf{m}_a - \tau_1^{-1}\Omega^{-1}\mathbf{m}_c). \quad (\text{A.133})$$

From (A.95) and (A.99), we know that $\mathbf{m}_a = \mathbb{E}_0[\mathbf{D}] - \mathbf{P}_0$ and $\mathbf{m}_c = \gamma\tau_1^{-1}(\mathbf{M} - \mathbf{x}_0)$. Since none of them depend on k , it follows that \mathbf{q}_0^k is independent on k and thus

$$\begin{aligned} \tau_0^{-1}\mathbf{q}_0 - \tau_1^{-1}\Omega^{-1}\tau_1^{-1}\mathbf{q}_0 \\ = \frac{1}{\gamma}(\mathbb{E}_0[\mathbf{D}] - \mathbf{P}_0) - \frac{1}{\gamma}\tau_1^{-1}\Omega^{-1}\gamma\tau_1^{-1}(\mathbf{M} - \mathbf{x}_0). \end{aligned} \quad (\text{A.134})$$

The last terms on each side cancel out by market clearing. We therefore obtain (A.94):

$$\mathbf{q}_0 = \frac{1}{\gamma}\tau_0(\mathbb{E}_0[\mathbf{D}] - \mathbf{P}_0). \quad (\text{A.135})$$

Using (A.78) and market clearing yields (31) and completes the proof of Proposition 4. \square

Proof of Corollary 4.1 (CAPM) Eqs. (A.135) and (A.99) imply $\mathbb{E}_0[\mathbf{D} - \mathbf{P}_0] = \gamma\tau_0^{-1}(\mathbf{M} - \mathbf{x}_0)$ and $\mathbb{E}_0[\mathbf{D} - \mathbf{P}_1] = \gamma\tau_1^{-1}(\mathbf{M} - \mathbf{x}_0)$. Thus,

$$\mathbb{E}_0[\mathbf{P}_1 - \mathbf{P}_0] = \gamma(\tau_0^{-1} - \tau_1^{-1})(\mathbf{M} - \mathbf{x}_0). \quad (\text{A.136})$$

Taking unconditional expectation and defining $\mathbf{R}^e \equiv \mathbf{P}_1 - \mathbf{P}_0$ yields

$$\mathbb{E}[\mathbf{R}^e] = \gamma(\tau_0^{-1} - \tau_1^{-1})\mathbf{M}, \quad (\text{A.137})$$

which, written for the market portfolio is

$$\mathbb{E}[\mathbf{R}_M^e] = \gamma(\mathbf{M}'\tau_0^{-1}\mathbf{M} - \mathbf{M}'\tau_1^{-1}\mathbf{M}) = \gamma(U_0^2 - \mathbf{M}'\tau_1^{-1}\mathbf{M}), \quad (\text{A.138})$$

where U_0^2 is the market-wide uncertainty at time 0, defined in (A.80). The second term in brackets, $\mathbf{M}'\tau_1^{-1}\mathbf{M}$, decreases with Λ_a , $\forall a$. To see this, we know from (A.85) that

$$\tau_1 = \tau_0 + \begin{bmatrix} \text{diag}[\pi_a(\Lambda_a) \mid a \in \mathcal{A}] & \mathbf{0}_{A \times (N-A)} \\ \mathbf{0}_{(N-A) \times A} & \mathbf{0}_{(N-A) \times (N-A)} \end{bmatrix}, \quad (\text{A.139})$$

and that $\pi_a(\Lambda_a)$ increases in Λ_a (Lemma 1). Thus

$$\frac{\partial \mathbf{M}'\tau_1^{-1}\mathbf{M}}{\partial \pi_a(\Lambda_a)} = \mathbf{M}' \frac{\partial \tau_1^{-1}}{\partial \pi_a(\Lambda_a)} \mathbf{M} \quad (\text{A.140})$$

$$= -\mathbf{M}'\tau_1^{-1} \frac{\partial \tau_1}{\partial \pi_a(\Lambda_a)} \tau_1^{-1} \mathbf{M} \quad (\text{A.141})$$

$$= -\mathbf{M}'\tau_1^{-1} \iota_a \iota_a' \tau_1^{-1} \mathbf{M} < 0, \quad (\text{A.142})$$

where we have used that the derivative of the inverse of a matrix K is $-K^{-1}K^d K^{-1}$ (d meaning derivative: start with $\mathbf{I}^d = (KK^{-1})^d = K^d K^{-1} + K(K^{-1})^d$ and solve for $(K^{-1})^d$).

Thus, $\mathbf{M}'\tau_1^{-1}\mathbf{M}$ decreases when investors are more attentive and Λ_a increases.

From (A.137), (A.138), we obtain the CAPM in Corollary 4.1:

$$\mathbb{E}[\mathbf{R}^e] = \frac{(\tau_0^{-1} - \tau_1^{-1})\mathbf{M}}{U_0^2 - \mathbf{M}'\tau_1^{-1}\mathbf{M}} \mathbb{E}[\mathbf{R}_M^e]. \quad \square \quad (\text{A.143})$$

Firms' betas: proof of Eq. (36) To understand how market betas relate to firms' exposures \mathbf{b} to the systematic factor, and how betas are governed by investor attention, we start by analyzing the numerator in (A.143), or $(\tau_0^{-1} - \tau_1^{-1})\mathbf{M}$. From (A.85), $\tau_1 = \tau_0 + \iota \Pi \iota'$, where Π is a $A \times A$ diagonal matrix with the scalars $\pi_a(\Lambda_a)$ on its diagonal, $\Pi = \text{diag}[\pi_a(\Lambda_a) \mid a \in \mathcal{A}]$. This yields

$$\tau_1^{-1} = \tau_0^{-1} - \tau_0^{-1} \iota (\Pi^{-1} + \iota' \tau_0^{-1} \iota)^{-1} \iota' \tau_0^{-1}, \quad (\text{A.144})$$

and thus

$$(\tau_0^{-1} - \tau_1^{-1})\mathbf{M} = \tau_0^{-1} \iota (\Pi^{-1} + \iota' \tau_0^{-1} \iota)^{-1} \iota' \tau_0^{-1} \mathbf{M}. \quad (\text{A.145})$$

The term $\tau_0^{-1} \iota$ represents the first A columns of $\tau_0^{-1} = \text{Var}_0[\mathbf{D}]$. Using (A.76), removing all terms that vanish when $N \rightarrow \infty$, and denoting by $\mathbf{b}_A = [b_1 \ b_2 \ \dots \ b_A]'$,

$$\tau_0^{-1} \iota = \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b} \mathbf{b}_A' + \text{Var}[\mathbf{e}]\iota. \quad (\text{A.146})$$

This implies (using $\mathbf{b}'\mathbf{M} = 1$ and further removing vanishing terms):

$$\iota' \tau_0^{-1} \mathbf{M} = \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A \quad (\text{A.147})$$

$$\Pi^{-1} + \iota' \tau_0^{-1} \iota = \Pi^{-1} + \text{Var}[\mathbf{e}_A] + \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A \mathbf{b}_A', \quad (\text{A.148})$$

where $\mathbf{e}_A = [e_1 \ e_2 \ \dots \ e_A]'$. Using (A.146)–(A.148), the term $(\tau_0^{-1} - \tau_1^{-1})\mathbf{M}$ is then

$$\begin{aligned} & \left(\frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b} \mathbf{b}_A' + \text{Var}[\mathbf{e}]\iota \right) \\ & \times \left(\Pi^{-1} + \text{Var}[\mathbf{e}_A] + \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A \mathbf{b}_A' \right)^{-1} \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A \end{aligned} \quad (\text{A.149})$$

$$\begin{aligned} & = \mathbf{b} \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A' \left(\Pi^{-1} + \text{Var}[\mathbf{e}_A] + \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A \mathbf{b}_A' \right)^{-1} \\ & \times \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A \end{aligned} \quad (\text{A.150})$$

$$+ \left[\text{Var}[\mathbf{e}_A] \right] \left(\Pi^{-1} + \text{Var}[\mathbf{e}_A] + \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A \mathbf{b}_A' \right)^{-1} \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A \quad (\text{A.151})$$

$$\begin{aligned} & = \mathbf{b} \left[\frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} - \left(\frac{U^2 + \sigma_g^2}{U^2 \sigma_g^2} + \mathbf{b}_A' (\Pi^{-1} + \text{Var}[\mathbf{e}_A])^{-1} \mathbf{b}_A \right)^{-1} \right] \\ & \quad \text{a strictly positive scalar, } \equiv \omega_1 \end{aligned} \quad (\text{A.152})$$

$$+ \left[\begin{array}{c} \text{Var}[\mathbf{e}_A] \\ \mathbf{0} \end{array} \right] \left(\Pi^{-1} + \text{Var}[\mathbf{e}_A] + \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A \mathbf{b}_A' \right)^{-1} \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A \quad (\text{A.153})$$

The scalar ω_1 is strictly positive because the diagonal matrix $\Pi^{-1} + \text{Var}[\mathbf{e}_A]$ is positive definite. The last term above equals (solving only for its non-zero part):

$$\text{Var}[\mathbf{e}_A] \left[\left(\Pi^{-1} \text{Var}[\mathbf{e}_A]^{-1} + \mathbf{I} + \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A \mathbf{b}_A' \text{Var}[\mathbf{e}_A]^{-1} \right) \text{Var}[\mathbf{e}_A] \right]^{-1} \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A \quad (\text{A.154})$$

$$= \left(\underbrace{\Pi^{-1} \text{Var}[\mathbf{e}_A]^{-1} + \mathbf{I}}_{\text{an } A \times A \text{ matrix, } A} + \mathbf{b}_A \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A' \text{Var}[\mathbf{e}_A]^{-1} \right)^{-1} \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbf{b}_A \quad (\text{A.155})$$

$$= \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2} \mathbb{A}^{-1} \mathbf{b}_A \left[1 - \left(\frac{1}{\frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2}} + \underbrace{\mathbf{b}_A' \text{Var}[\mathbf{e}_A]^{-1} \mathbb{A}^{-1} \mathbf{b}_A}_{\text{a strictly positive scalar, } \omega_2} \right)^{-1} \mathbf{b}_A' \text{Var}[\mathbf{e}_A]^{-1} \mathbb{A}^{-1} \mathbf{b}_A \right] \quad (\text{A.156})$$

$$= \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2 + \omega_2 U^2 \sigma_g^2} \begin{bmatrix} \frac{\pi_1(\Lambda_1) \sigma_{e1}^2}{1 + \pi_1(\Lambda_1) \sigma_{e1}^2} b_1 \\ \frac{\pi_2(\Lambda_2) \sigma_{e2}^2}{1 + \pi_2(\Lambda_2) \sigma_{e2}^2} b_2 \\ \vdots \\ \frac{\pi_A(\Lambda_A) \sigma_{eA}^2}{1 + \pi_A(\Lambda_A) \sigma_{eA}^2} b_A \end{bmatrix} \quad (\text{A.157})$$

The scalar ω_2 is strictly positive because the diagonal matrix $\text{Var}[\mathbf{e}_A]^{-1} \mathbb{A}^{-1}$ is positive definite. We can then write market betas, $\beta = \frac{(\tau_0^{-1} - \tau_1^{-1}) \mathbf{M}}{\mathbf{M}'(\tau_0^{-1} - \tau_1^{-1}) \mathbf{M}}$, as

$$\frac{1}{\omega_1 + \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2 + \omega_2 U^2 \sigma_g^2} \sum_{a=1}^A \frac{\pi_a(\Lambda_a) \sigma_{ea}^2}{1 + \pi_a(\Lambda_a) \sigma_{ea}^2} \frac{b_a}{N}} \left(\omega_1 \mathbf{b} + \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2 + \omega_2 U^2 \sigma_g^2} \begin{bmatrix} \frac{\pi_1(\Lambda_1) \sigma_{e1}^2}{1 + \pi_1(\Lambda_1) \sigma_{e1}^2} b_1 \\ \frac{\pi_2(\Lambda_2) \sigma_{e2}^2}{1 + \pi_2(\Lambda_2) \sigma_{e2}^2} b_2 \\ \vdots \\ \frac{\pi_A(\Lambda_A) \sigma_{eA}^2}{1 + \pi_A(\Lambda_A) \sigma_{eA}^2} b_A \\ \mathbf{0}_{N-A} \end{bmatrix} \right) \quad (\text{A.158})$$

In a large economy ($N \rightarrow \infty$), the denominator in the first term converges to ω_1 and thus we recover Eq. (36) in the text, with $h > 0$ defined as:

$$h \equiv \frac{1}{\omega_1} \frac{U^2 \sigma_g^2}{U^2 + \sigma_g^2 + \omega_2 U^2 \sigma_g^2} \quad (\text{A.159})$$

The result that the announcing firms' betas increase with attention does not depend on taking the limit $N \rightarrow \infty$. In unreported analysis, we verify this result through simulations in a smaller economy. We find that the result always holds in our simulations, which we have performed for a wide range of parameter values. \square

A9. Dynamic model

The dynamic setup comprises an overlapping-generations economy, where a new generation of investors is born each period. We denote the investors born at time t as *generation* t . Each generation is present in the economy for three dates and, as in the static model, engages in information acquisition and trading activities sequentially. In particular, investor $i \in [0, 1]$ of generation $t - 1$ chooses to acquire information between $t - 1$ and t , trades securities at t , and consumes final wealth at $t + 1$. Subsequently, the investors of generation $t - 1$ liquidate their positions at time $t + 1$ by selling them to the investors of generation t at prevailing market prices. The timeline is depicted in Fig. 6.

We assume that investors trade a single risky asset and a riskless asset. (Considering multiple risky assets would significantly complicate the analysis without providing any additional insights.) The riskless asset is in infinitely elastic supply and pays a gross interest rate of $R_f > 1$ per period. The risky asset pays a risky dividend per period,

$$D_{t+1} = b f_{t+1} + e_{t+1}, \quad (\text{A.160})$$

which, as in (1), has two components: a systematic component, $f_{t+1} \sim \mathcal{N}(0, U_t^2)$, and a firm-specific component, $e_t \sim \mathcal{N}(0, \sigma_e^2)$.

Uncertainty (U_t) takes $S \geq 2$ possible values, u_s , $s \in \{1, \dots, S\}$, and we denote the probability of the event $U_t = u_s$ by p_s . Furthermore, U_t is observable to generation $t - 1$ investors, who make an information acquisition choice between $t - 1$ and t and trade in the market at t . One could assume, for instance, that U_t is revealed at time $t - \epsilon$, where ϵ is very small (e.g., a fraction of a second).

This assumption preserves the sequence of the information acquisition and trading decisions, as in Grossman and Stiglitz (1980).

At time t , the firm issues an earnings announcement,

$$E_t = D_{t+1} + \varepsilon_t, \quad (\text{A.161})$$

with $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$. We denote the investors who pay attention to E_t as I investors, and those who decide to remain uninformed as \emptyset investors. The indicator variable I^k takes the value 1 if $k = I$ and 0 if $k = \emptyset$. The cost of paying attention to E_t is $c > 0$.

Each investor $i \in [0, 1]$ of generation $t - 1$ starts with zero initial wealth and maximizes expected utility:

$$\max_{k \in \{I, \emptyset\}} \mathbb{E}_{t-1} \left[\max_{q_t^k} \mathbb{E}_t^k \left[-e^{-\gamma (W_{t+1}^k - c^k)} \right] \right], \quad (\text{A.162})$$

where $W_{t+1}^k \equiv q_t^k (D_{t+1} + P_{t+1} - R_f P_t) \equiv q_t^k R_{t+1}^e$ is type k investor's terminal wealth.

The risky asset demand of liquidity (noise) traders equals x_t , with x_t being independently and identically distributed, $x_t \sim \mathcal{N}(0, \sigma_x^2)$. We conjecture the following linear structure for the price, which is the dynamic equivalent of (10) from the static version of the model:

$$P_t = \alpha_t E_t + \xi_t x_t. \quad (\text{A.163})$$

The equilibrium in the dynamic model follows the same steps as the static model but with added complexity due to time variation in uncertainty, which creates a non-linearity. Specifically, the distribution of the future price, P_{t+1} , becomes non-Gaussian. To restore linearity, we use a commonly employed approximation method in the literature (Vayanos and Weill, 2008; Gârleanu, 2009). It is important to note that dynamic models of this type have multiple equilibria (e.g., Banerjee, 2011; Andrei, 2018), and this model exhibits two equilibria: a low-volatility and high-volatility equilibrium. The results presented in this paper hold for both equilibria.

The following proposition characterizes investors' attention decision.

Proposition 5.

- (a) *Investor i is attentive to the earnings announcement if and only if*

$$\frac{\text{Var}_t^\emptyset[R_{t+1}^e]}{\text{Var}_t^I[R_{t+1}^e]} > e^{2\gamma c}. \quad (\text{A.164})$$

- (b) *The benefit of information, $\text{Var}_t^\emptyset[R_{t+1}^e]/\text{Var}_t^I[R_{t+1}^e]$, increases in $\text{Var}_t[D_{t+1}] = b^2 U_t^2 + \sigma_\varepsilon^2$.*

Proof. We start by making the following conjecture for equilibrium prices:

$$\hat{P}_t \equiv \xi_t^{-1} P_t = \frac{\Lambda_t}{\gamma \sigma_\varepsilon^2} Z_t E_t + x_t, \quad (\text{A.165})$$

where Λ_t is the fraction of informed investors and Z_t will be determined in equilibrium below.

Learning for the informed investor: For the informed investor, the only informative signal at time t is E_t . Application of the Projection Theorem yields

$$\text{Var}_t^I[D_{t+1}] = \frac{\text{Var}_t[D_{t+1}]\sigma_\varepsilon^2}{\text{Var}_t[D_{t+1}] + \sigma_\varepsilon^2} = \left(\text{Var}_t[D_{t+1}]^{-1} + \frac{1}{\sigma_\varepsilon^2} \right)^{-1}, \quad (\text{A.166})$$

and

$$\mathbb{E}_t^I[D_{t+1}] = \frac{\text{Var}_t[D_{t+1}]}{\text{Var}_t[D_{t+1}] + \sigma_\varepsilon^2} E_t = \frac{\text{Var}_t^I[D_{t+1}]}{\sigma_\varepsilon^2} E_t. \quad (\text{A.167})$$

Learning for the uninformed investor: The uninformed investor learns from the price signal \hat{P}_t , and thus the Projection Theorem implies:

$$\text{Var}_t^\emptyset[D_{t+1}] = \frac{\text{Var}_t[D_{t+1}]\sigma_\varepsilon^2}{\frac{\Lambda_t^2 Z_t^2}{\Lambda_t^2 Z_t^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_x^2} \text{Var}_t[D_{t+1}] + \sigma_\varepsilon^2}$$

$$= \left(\text{Var}_t[D_{t+1}]^{-1} + \frac{\Lambda_t^2 Z_t^2}{\Lambda_t^2 Z_t^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_x^2} \frac{1}{\sigma_\varepsilon^2} \right)^{-1}, \quad (\text{A.168})$$

and

$$\mathbb{E}_t^\emptyset[D_{t+1}] = \text{Var}_t^\emptyset[D_{t+1}] \frac{\gamma \Lambda_t Z_t}{\Lambda_t^2 Z_t^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_x^2} \hat{P}_t \quad (\text{A.169})$$

$$= \frac{\Lambda_t^2 Z_t^2}{\Lambda_t^2 Z_t^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_x^2} \frac{\text{Var}_t^\emptyset[D_{t+1}]}{\sigma_\varepsilon^2} E_t + \text{Var}_t^\emptyset[D_{t+1}] \frac{\gamma \Lambda_t Z_t}{\Lambda_t^2 Z_t^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_x^2} x_t. \quad (\text{A.170})$$

Equilibrium: When forming optimal portfolios at time t , both I and \emptyset investors form expectations about $P_{t+1} + D_{t+1}$. Using that $\mathbb{E}_t^k[P_{t+1}] = 0 \quad \forall k \in \{I, \emptyset\}$, informed investors' beliefs are:

$$\mathbb{E}_t^I[P_{t+1} + D_{t+1}] = \mathbb{E}_t^I[D_{t+1}] \quad (\text{A.171})$$

$$\text{Var}_t^I[P_{t+1} + D_{t+1}] = \text{Var}_t^I[D_{t+1}] + \sum_{s=1}^S p_s [\alpha_{s,t+1}^2 (b^2 U_s^2 + \sigma_\varepsilon^2 + \sigma_\varepsilon^2) + \xi_{s,t+1}^2 \sigma_x^2], \quad (\text{A.172})$$

where p_s represents the probability of reaching the state U_s . The last term in (A.172) is the variance of the future price, $\text{Var}_t[P_{t+1}]$, which is the same for I and \emptyset investors, and does not change over time (the information that investors have at t becomes irrelevant at $t + 1$; furthermore, at any time t investors face the same probability distribution over future values of U_s , and thus over the values of the price coefficients at time $t + 1$). Thus, we denote the last term in (A.172) by $\text{Var}[P_{t+1}]$.

Similar reasoning leads to uninformed investors' beliefs:

$$\mathbb{E}_t^\emptyset[P_{t+1} + D_{t+1}] = \mathbb{E}_t^\emptyset[D_{t+1}] \quad (\text{A.173})$$

$$\text{Var}_t^\emptyset[P_{t+1} + D_{t+1}] = \text{Var}_t^\emptyset[D_{t+1}] + \text{Var}[P_{t+1}]. \quad (\text{A.174})$$

Consider now the optimization problem that all investors face:

$$\max_{k \in \{I, \emptyset\}} \mathbb{E}_{t-1} \left[e^{\gamma c^k} \max_{q_t^k} \mathbb{E}_t^k \left[-e^{-\gamma q_t^k (P_{t+1} + D_{t+1} - R_f P_t)} \right] \right], \quad (\text{A.175})$$

which leads to the following portfolio choice problem of $k \in \{I, \emptyset\}$ investors:

$$\max_{q_t^k} \mathbb{E}_t^k \left[-e^{-\gamma q_t^k (P_{t+1} + D_{t+1} - R_f P_t)} \right]. \quad (\text{A.176})$$

In the expectation above, the future price P_{t+1} is normally distributed conditional on the future value of U_s . One can write the expectation as

$$\mathbb{E}_t^k \left[-e^{-\gamma q_t^k (P_{t+1} + D_{t+1} - R_f P_t)} \right] = \sum_{s=1}^S p_s \mathbb{E}_t^k \left[-e^{-\gamma q_t^k (P_{s,t+1} + D_{t+1} - R_f P_t)} \right], \quad (\text{A.177})$$

where $P_{s,t+1}$ is the future price in the state U_s . Defining $R_{s,t+1}^e \equiv P_{s,t+1} + D_{t+1} - R_f P_t$, the expectation can be further written as

$$\mathbb{E}_t^k \left[-e^{-\gamma q_t^k (P_{t+1} + D_{t+1} - R_f P_t)} \right] = \sum_{s=1}^S p_s \left(-e^{-\gamma q_t^k \mathbb{E}_t^k [R_{s,t+1}^e] + \frac{1}{2} \gamma^2 (q_t^k)^2 \text{Var}_t^k [R_{s,t+1}^e]} \right). \quad (\text{A.178})$$

We resort to an approximation of this function (Vayanos and Weill, 2008; Gârleanu, 2009). This approximation preserves risk aversion towards diffusion risks, but creates risk neutrality towards discrete jump risks. The approximation is very accurate in this setting, particularly because $\mathbb{E}_t^k [R_{s,t+1}^e] = \mathbb{E}_t^k [D_{t+1}] + \mathbb{E}_t^k [P_{s,t+1}] - R_f P_t$ does not vary across future states ($\mathbb{E}_t^k [P_{s,t+1}] = 0 \forall s$), and thus the future distribution of prices remains symmetric, unimodal, and elliptical (only the variance $\text{Var}_t^k [R_{s,t+1}^e]$ changes across future states). First, define $\overline{\text{Var}}_t^k [R_{s,t+1}^e] \equiv \gamma \text{Var}_t^k [R_{s,t+1}^e]$ and replace this above to obtain a function of γ :

$$f(\gamma) = \sum_{s=1}^S p_s \left(-e^{-\gamma q_t^k \mathbb{E}_t^k [R_{s,t+1}^e] + \frac{1}{2} \gamma (q_t^k)^2 \overline{\text{Var}}_t^k [R_{s,t+1}^e]} \right). \quad (\text{A.179})$$

The Taylor expansion of $f(\gamma)$ around zero is given by $f(\gamma) = f(0) + \gamma f'(0) + \mathcal{O}(\gamma)$, where $\mathcal{O}(\gamma)$ represents higher-order terms that go to zero faster than γ as $\gamma \rightarrow 0$. Therefore

$$f(\gamma) \approx -1 + \sum_{s=1}^S p_s \left(\gamma q_t^k \mathbb{E}_t^k [R_{s,t+1}^e] - \frac{1}{2} \gamma (q_t^k)^2 \overline{\text{Var}}_t^k [R_{s,t+1}^e] \right) \quad (\text{A.180})$$

$$= -1 + \sum_{s=1}^S p_s \left(\gamma q_t^k \mathbb{E}_t^k [R_{s,t+1}^e] - \frac{1}{2} \gamma^2 (q_t^k)^2 \text{Var}_t^k [R_{s,t+1}^e] \right) \quad (\text{A.181})$$

$$= -1 + \gamma q_t^k \mathbb{E}_t^k [R_{t+1}^e] - \frac{1}{2} \gamma^2 (q_t^k)^2 \text{Var}_t^k [R_{t+1}^e]. \quad (\text{A.182})$$

The first order condition with respect to q_t^k leads to the optimal portfolio of the informed and uninformed investors:

$$q_t^I = \frac{\mathbb{E}_t^I [R_{t+1}^e]}{\gamma \text{Var}_t^I [R_{t+1}^e]} \quad \text{and} \quad q_t^\theta = \frac{\mathbb{E}_t^\theta [R_{t+1}^e]}{\gamma \text{Var}_t^\theta [R_{t+1}^e]}. \quad (\text{A.183})$$

Market clearing requires:

$$\Lambda_t \frac{\mathbb{E}_t^I [D_{t+1} + P_{t+1}] - R_f P_t}{\gamma \text{Var}_t^I [R_{t+1}^e]} + (1 - \Lambda_t) \frac{\mathbb{E}_t^\theta [D_{t+1} + P_{t+1}] - R_f P_t}{\gamma \text{Var}_t^\theta [R_{t+1}^e]} = -x_t, \quad (\text{A.184})$$

and since the price conjecture (A.163) implies $\mathbb{E}_t^I [P_{t+1}] = \mathbb{E}_t^\theta [P_{t+1}] = 0$, this yields

$$\frac{\Lambda_t \mathbb{E}_t^I [D_{t+1}]}{\text{Var}_t^I [R_{t+1}^e]} + \frac{(1 - \Lambda_t) \mathbb{E}_t^\theta [D_{t+1}]}{\text{Var}_t^\theta [R_{t+1}^e]} - \left(\frac{\Lambda_t}{\text{Var}_t^I [R_{t+1}^e]} + \frac{1 - \Lambda_t}{\text{Var}_t^\theta [R_{t+1}^e]} \right) R_f P_t = -\gamma x_t, \quad (\text{A.185})$$

and we recognize the weighted average precision across investors, denoted hereafter by τ_t :

$$\tau_t \equiv \frac{\Lambda_t}{\text{Var}_t^I [R_{t+1}^e]} + \frac{1 - \Lambda_t}{\text{Var}_t^\theta [R_{t+1}^e]}. \quad (\text{A.186})$$

Eq. (A.185) further leads to

$$\tau_t R_f P_t = \frac{\Lambda_t \mathbb{E}_t^I [D_{t+1}]}{\text{Var}_t^I [R_{t+1}^e]} \frac{\text{Var}_t^I [D_{t+1}]}{\text{Var}_t^I [R_{t+1}^e]} + \frac{(1 - \Lambda_t) \mathbb{E}_t^\theta [D_{t+1}]}{\text{Var}_t^\theta [R_{t+1}^e]} \frac{\text{Var}_t^\theta [D_{t+1}]}{\text{Var}_t^\theta [R_{t+1}^e]} + \gamma x_t. \quad (\text{A.187})$$

After replacement of (A.166), (A.167) and (A.168)–(A.170), we obtain

$$P_t = \frac{\tau_t^{-1}}{R_f} \left(\frac{\Lambda_t}{\sigma_\varepsilon^2} \frac{\text{Var}_t^I [D_{t+1}]}{\text{Var}_t^I [R_{t+1}^e]} + \frac{1 - \Lambda_t}{\sigma_\varepsilon^2} \frac{\Lambda_t^2 Z_t^2}{\Lambda_t^2 Z_t^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_x^2} \frac{\text{Var}_t^\theta [D_{t+1}]}{\text{Var}_t^\theta [R_{t+1}^e]} \right) E_t + \frac{\tau_t^{-1}}{R_f} \left(\gamma + (1 - \Lambda_t) \frac{\gamma \Lambda_t Z_t}{\Lambda_t^2 Z_t^2 + \gamma^2 \sigma_\varepsilon^2 \sigma_x^2} \frac{\text{Var}_t^\theta [D_{t+1}]}{\text{Var}_t^\theta [R_{t+1}^e]} \right) x_t, \quad (\text{A.188})$$

which determines the coefficients in the price conjecture $P_t = \alpha E_t + \xi x_t$. Moreover, the conjecture (A.165), which requires that $\frac{\alpha_t}{\xi_t} = \frac{\Lambda_t}{\gamma \sigma_\varepsilon^2} Z_t$, together with (A.188) imply that Z_t must be

$$Z_t = \frac{\text{Var}_t^I [D_{t+1}]}{\text{Var}_t^I [R_{t+1}^e]}. \quad (\text{A.189})$$

We now solve for the equilibrium Λ_t in the dynamic model. The approximated expected utility of uninformed investors in (A.182), after replacement of the optimal portfolio choice (A.183), is

$$\mathcal{U}_t^\theta = -1 + \gamma \frac{\mathbb{E}_t^\theta [R_{t+1}^e]}{\gamma \text{Var}_t^\theta [R_{t+1}^e]} \mathbb{E}_t^\theta [R_{t+1}^e] - \frac{1}{2} \gamma^2 \left(\frac{\mathbb{E}_t^\theta [R_{t+1}^e]}{\gamma \text{Var}_t^\theta [R_{t+1}^e]} \right)^2 \text{Var}_t^\theta [R_{t+1}^e] \quad (\text{A.190})$$

$$= \frac{1}{2} \frac{\mathbb{E}_t^\theta [R_{t+1}^e]^2}{\text{Var}_t^\theta [R_{t+1}^e]} - 1 \approx -e^{-\frac{1}{2} \frac{\mathbb{E}_t^\theta [R_{t+1}^e]^2}{\text{Var}_t^\theta [R_{t+1}^e]}}. \quad (\text{A.191})$$

where we have used the approximation $x - 1 \approx -e^{-x}$. This approximation restores the expected utility in exponential form and is highly accurate when $\mathbb{E}_t^\theta [R_{t+1}^e]^2 / (2 \text{Var}_t^\theta [R_{t+1}^e])$ is small, which is likely to be the case: $\mathbb{E}_t^\theta [R_{t+1}^e]^2 / \text{Var}_t^\theta [R_{t+1}^e]$ represents the squared Sharpe ratio of the stock from the perspective of uninformed investors.

Similarly, for an informed investor,

$$\mathcal{U}_t^I \approx -e^{\gamma c} e^{-\frac{1}{2} \frac{\mathbb{E}_t^I [R_{t+1}^e]^2}{\text{Var}_t^I [R_{t+1}^e]}}. \quad (\text{A.192})$$

For an uninformed investor, $\mathbb{E}_t^I [R_{t+1}^e]$ is a normally distributed random variable with mean $\mathbb{E}_t^\theta [R_{t+1}^e]$ (by the law of iterated expectations) and variance $\Sigma_t \equiv \text{Var}_t^\theta [R_{t+1}^e] - \text{Var}_t^I [R_{t+1}^e]$ (by the law of total variance). Taking expectation at $t - 1$ of (A.192) as in (A.175) and applying Lemma 2 yields

$$\mathbb{E}_{t-1} \left[-e^{\gamma c} e^{-\frac{1}{2} \frac{\mathbb{E}_t^I [R_{t+1}^e]^2}{\text{Var}_t^I [R_{t+1}^e]}} \right] = \mathcal{U}_t^\theta e^{\gamma c} \left(\frac{\text{Var}_t^I [R_{t+1}^e]}{\text{Var}_t^\theta [R_{t+1}^e]} \right)^{1/2}. \quad (\text{A.193})$$

Since $\mathcal{U}_t^\theta < 0$, the uninformed investor is attentive to the earnings announcement if and only if

$$\frac{\text{Var}_t^\theta[R_{t+1}^e]}{\text{Var}_t^I[R_{t+1}^e]} > e^{2\gamma c}, \quad (\text{A.194})$$

which proves part (a) of [Proposition 5](#). Using [\(A.166\)](#) and [\(A.168\)](#), the benefit of information is

$$\frac{\text{Var}_t^\theta[R_{t+1}^e]}{\text{Var}_t^I[R_{t+1}^e]} = \frac{\text{Var}[P_{t+1}] + \frac{\text{Var}_t[D_{t+1}]\sigma_\varepsilon^2}{\frac{\Lambda_t^2 Z_t^2}{\Lambda_t^2 Z_t^2 + \gamma^2 \sigma_x^2 \sigma_\varepsilon^2} \text{Var}_t[D_{t+1}] + \sigma_\varepsilon^2}}{\text{Var}[P_{t+1}] + \frac{\text{Var}_t[D_{t+1}]\sigma_\varepsilon^2}{\text{Var}_t[D_{t+1}] + \sigma_\varepsilon^2}}. \quad (\text{A.195})$$

Since $\frac{\Lambda_t^2 Z_t^2}{\Lambda_t^2 Z_t^2 + \gamma^2 \sigma_x^2 \sigma_\varepsilon^2} < 1$, $\frac{\text{Var}_t^\theta[R_{t+1}^e]}{\text{Var}_t^I[R_{t+1}^e]}$ increases in $\text{Var}_t[D_{t+1}]$, proving part (b) of [Proposition 5](#). \square

[Proposition 5](#) recovers the same result as in the static model: the benefit of paying attention to E_t increases with economic uncertainty. Moreover, the benefit of attention is higher when b is higher and when the volatility σ_ε of the idiosyncratic component is higher.

Proposition 6. The earnings response coefficient in this economy is given by

$$\text{ERC}_t = \frac{w_t}{R_f} \frac{\text{Var}_t[D_{t+1}]}{\text{Var}_t[D_{t+1}] + \sigma_\varepsilon^2} + \frac{1 - w_t}{R_f} \frac{\text{Var}_t[D_{t+1}]}{\text{Var}_t[D_{t+1}] + \sigma_\varepsilon^2 / \ell_t}, \quad (\text{A.196})$$

where $w_t \in [0, 1]$, $\ell_t \in [0, 1]$. Both w_t and ℓ_t are increasing with the fraction Λ_t of investors who pay attention to E_t . Thus, the earnings response coefficient increases in Λ_t .

Proof. The ERC (i.e., the sensitivity α_t of the price P_t to the earnings announcement E_t) follows directly from [\(A.186\)](#) to [\(A.188\)](#):

$$\alpha_t = \frac{1}{R_f} \frac{1}{\frac{\Lambda_t}{\text{Var}_t^I[R_{t+1}^e]} + \frac{1 - \Lambda_t}{\text{Var}_t^\theta[R_{t+1}^e]}} \left(\frac{\Lambda_t \text{Var}_t^I[D_{t+1}]}{\text{Var}_t^I[R_{t+1}^e] \sigma_\varepsilon^2} + \frac{(1 - \Lambda_t) \text{Var}_t^\theta[D_{t+1}]}{\text{Var}_t^\theta[R_{t+1}^e] \sigma_\varepsilon^2} \frac{\Lambda_t^2 Z_t^2}{\Lambda_t^2 Z_t^2 + \gamma^2 \sigma_x^2 \sigma_\varepsilon^2} \right), \quad (\text{A.197})$$

which, after defining w_t and ℓ_t as

$$w_t = \frac{\frac{\Lambda_t}{\text{Var}_t^I[R_{t+1}^e]}}{\frac{\Lambda_t}{\text{Var}_t^I[R_{t+1}^e]} + \frac{1 - \Lambda_t}{\text{Var}_t^\theta[R_{t+1}^e]}} \quad \text{and} \quad \ell_t = \frac{\Lambda_t^2 Z_t^2}{\Lambda_t^2 Z_t^2 + \gamma^2 \sigma_x^2 \sigma_\varepsilon^2} < 1, \quad (\text{A.198})$$

yields [\(A.196\)](#).

[Proposition 6](#) shows that the ERC is a weighted average. According to [\(A.198\)](#), a higher fraction Λ_t increases the weight placed on the ratio $\frac{\text{Var}_t[D_{t+1}]}{\text{Var}_t[D_{t+1}] + \sigma_\varepsilon^2}$, which, due to the fact that $\ell_t < 1$, raises the weighted average. Moreover, [Eq. \(A.198\)](#) indicates that a higher Λ_t increases ℓ_t , which further boosts $\frac{\text{Var}_t[D_{t+1}]}{\text{Var}_t[D_{t+1}] + \sigma_\varepsilon^2 / \ell_t}$ and, therefore, the weighted average. Hence, both effects confirm that a higher fraction of attentive investors leads to an increase in the ERC. \square

In [Proposition 6](#), ℓ_t is the dynamic counterpart of the learning coefficient defined in [\(13\)](#) for the static model. Two effects take place when uncertainty increases. The first effect is an increase in both terms of [\(A.196\)](#) through

$\text{Var}_t[D_{t+1}]$. The second effect follows from [Proposition 5](#): the increase in economic uncertainty increases investor attention, and therefore both w_t and ℓ_t increase, further strengthening the ERC. We thus recover the intuition from the static model: the ERC increases with economic uncertainty, both directly through an increase in the variance of the firm's payoff $\text{Var}_t[D_{t+1}]$ and indirectly through an increase in investor attention. The two effects are stronger for firms with a higher b or idiosyncratic volatility σ_ε .

Finally, [Proposition 5](#) shows that regardless of prior information acquisition decisions, the benefit of paying attention to the earnings announcement increases with uncertainty at time t . Thus, although investors' search for information beforehand may dampen the effect of an increase in uncertainty on the conditional variance $\text{Var}_t[D_{t+1}]$, greater investor attention increases w_t and ℓ_t , strengthening the ERC. These effects guarantee that heightened investor attention increases ERCs.

Appendix B. Variable definitions

Variable	Description
VIX	Closing value of VIX on the trading day prior to the earnings announcement. Source: CRSP.
ESV	Log daily number of EDGAR downloads of the company's filings from SEC EDGAR. Source: SEC.
ESVU	Log daily number of EDGAR downloads of the company's filings from unique IP addresses. Source: SEC.
ISVI	Investor Search Volume Index based on investors' Google searches of stock tickers. Source: DeHaan, Lawrence, and Litjens (2021).
EARET	Compound excess return over the size decile portfolio for earnings announcement trading date and one trading day after. Source: CRSP.
SUE Decile	Earnings surprise relative to analyst consensus forecasts deflated by quarter-end share price. Source: IBES Summary File, CRSP.
abs(SUE Decile)	Absolute value of standardized (mean-zero and unit-variance) SUE Decile. Source: IBES Summary File, CRSP.
PreRet	Compound excess return over the size decile portfolio for earnings announcement trading date -10 to -1. Source: CRSP.
Size	Market value of equity on the earnings announcement date in \$M. Source: CRSP.
Book-to-Market	Book to market ratio at the end of quarter for which earnings are announced. Source: Compustat.
EPersistence	Earnings persistence based on AR(1) regression with at least 4, up to 16 quarterly earnings. Source: Compustat.
IO	Institutional ownership as a fraction of total shares outstanding as of the latest calendar quarter (13F reporting date) prior to the earnings announcement. Source: Thomson-Reuters 13F Data, CRSP.
EVOL	Standard deviation of seasonally differenced quarterly earnings over the prior 16 (at least 4) quarters. Source: Compustat.
ERepLag	Days from quarter-end to earnings announcement. Source: Compustat.
#Estimates	Number of analysts making quarterly earnings forecasts. Source: IBES Summary File.

(continued on next page)

Variable	Description
TURN	Average monthly share turnover for the 12 months preceding the earnings announcement. Source: CRSP.
Loss	Indicator for negative earnings. Source: Compustat.
#Announcements	Number of concurrent earnings announcements. Source: Compustat, IBES.
CAPM Beta	CAPM Beta estimated using the CRSP value-weighted market return index for the 250 (at least 60) trading days prior to the earnings announcement. Source: CRSP
IDVOL	Idiosyncratic volatility estimated using the CAPM model with the CRSP value-weighted market return index for the 250 (at least 60) trading days prior to the earnings announcement. Source: CRSP
DISP	Earnings forecast dispersion calculated as standard deviation of analyst forecasts deflated by mean absolute forecast. Source: IBES Summary File.

Appendix C. CAPM tests: data description and robustness checks

The analysis in Section 4.4 starts by merging by GVKEY and DATE the database that contains firm daily excess returns with the EDGAR search database. The individual returns sample limits are from January 2002 to December 2020. The EDGAR sample limits are from 2003-02-14 to 2017-06-30, which dictates the final limits of the merged sample. This initial merged dataset consists of 11,097,305 observations and 4497 distinct firms.

To identify high/low attention days, we build detrended time series of log search data at the individual firm level. Then, we add the value of 1 to the EDGAR search data to be able to take the log on days with zero EDGAR search. (These days commonly occur at the beginning of the sample; as another option, we have removed the first five years of data, from 2003 to 2007, and the results are robust to this alternative.) After detrending the log EDGAR search data, we split the residuals according to their sample median, with high-attention days ($1_{\text{HighAtt}}^i = 1$) being the days whose residuals are above the median.

Before estimating the regressions (40)–(42) for Table 7, we clean up the data as follows:

- (i) Using the Thomson Reuters I/B/E/S database, we remove announcements recorded after 4:00 PM on a given date. While one can measure investor attention (EDGAR downloads) on days when these announcements are released, investors trading on a U.S. exchange will react to the announcements only on the following trading day. This non-synchronicity prevents us from properly matching EA days and high-attention days. (The results are robust and even gain statistical significance if we do not remove these announcements.)
- (ii) We remove firms that have less than 20 earnings announcements. This ensures that there are enough earnings announcement days for the regression (42), which further splits the earnings announcement days into low/high attention days. (The results are similar if we use a tighter threshold, e.g., 40.)

- (iii) We remove firms that have more than 500 zero EDGAR search values. (The results are similar if we use a tighter threshold, e.g., 250.)

Additionally, Table 7 results remain robust to alternative divisions of the earnings announcement days in (42). Instead of using breakpoints for high/low attention within the set of EA days (dependent split), we can base the division on the full sample median (independent split).

The analysis that yields Tables 8, 9 and Fig. 8 uses return data from 2003-02-14 to 2017-06-30, which corresponds to the sample limits for the EDGAR data. Finally, the results in Table 9 are robust to several alternative specifications. The first robustness check concerns our definition of high-attention days. Rather than using the raw detrended $ESV(U)$ measures, we regress $ESV(U)$ on VIX and use the residuals instead, with similar results. Second, the results are stronger in panels A–B and remain confirmatory in panel C after removing the first five years of data, years during which EDGAR search numbers are relatively lower.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:[10.1016/j.jfineco.2023.05.003](https://doi.org/10.1016/j.jfineco.2023.05.003).

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