

Combinatorics and Probability

Problem 1

- (a) In how many ways can the letters a, b, c, d, e, f be arranged so that the letters a and b are next to each other?
 - (b) In how many ways can the letters a, b, c, d, e, f be arranged so that the letters a and b are not next to each other?
 - (c) In how many ways can the letters a, b, c, d, e, f be arranged so that the letters a and b are next to each other but a and c are not?
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Problem 2

A 4-letter word is selected at random from Σ^4 , where $\Sigma = \{a, b, c, d, e\}$.

- (a) What is the probability that the letters in the word are distinct?
 - (b) What is the probability that there are no vowels in the word?
 - (c) What is the probability that the word begins with a vowel?
 - (d) What is the **expected number of vowels** in the word?
 - (e) Let x be the answer to the previous question. What is the probability of the word having $\lceil x \rceil$ or more vowels?
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Problem 3

A black die and a red die are tossed. What is the probability that

- (a) the sum of the values is even?
 - (b) the number on the red die is bigger than the number on the black die?
 - (c) the number on the red die is twice the number on the black die?
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Problem 4

Team α faces team β in a 5-match series. Matches are either won or lost, i.e., there are no draws. It takes 3 wins to win the series. Team α has probability p ($0 < p < 1$) of winning a match. Consider each of the following situations and calculate the probability that they will lose the whole series.

- (a) They have lost the first match of the series already.
- (b) They have lost one of the first two matches of the series already.

- (c) They have lost the first two matches of the series already.
 - (d) They have lost one of the first three matches of the series already.
 - (e) They have lost two of the first three matches of the series already.
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Problem 5

Let E_1, E_2 be two events. Prove that $P(E_1 \setminus E_2) = P(E_1) - P(E_2)$ implies $P(E_2 \setminus E_1) = 0$.