Functional Dependency

Functional Dependency

A “good” database schema should not lead to *update anomalies*.

- update anomalies,
- functional dependencies,
- Armstrong Axioms,
- closures.

Update Anomalies

Redundancy in a database means storing a piece of data more than once.

Redundancy is often useful for efficiency and semantic reasons, but creates the potential for consistency problems.

A poor *redundancy control* may cause update anomalies.

Consider the example relation below (adapted from "An Introduction to Database Systems" by Desai):

STUDENTS					
Name	Course	Phone_no	Major	Prof	Grade
Jones	353	237-4539	Comp Sci	Smith	A
Ng	329	427-7390	Chemistry	Turner	B
Jones	328	237-4539	Comp Sci	Clark	B
Martin	456	388-5183	Physics	James	A
Dulles	293	371-6259	Decision Sci	Cook	C
Duke	491	823-7293	Mathematics	Lamb	B
Duke	356	823-7293	Mathematics	Bond	UN
Jones	492	237- 4539	Comp Sci	Cross	UN
~~Baxter~~	~~379~~	~~839-0827~~	~~English~~	~~Broes~~	~~C~~

Modification anomalies: e.g. Jones's phone number appears 3 times. When a phone number is changed, it must be changed in all 3 places, or the data will be inconsistent.

Update Anomalies

Insertion anomalies:

- If Jones enrolls in another course, and a different phone number is entered, again the data will be inconsistent.

- Also, if the only way that the association between course and professor is stored in this relation, we can only enter the association when someone enrolls in the course.

Deletion anomalies: If the last student in a course is deleted, the association between professor and course is lost.

Functional dependencies

A function f from S_1 to S_2 has the property

$$if\ x, y \in S_1 \text{ and } x = y, \text{then } f(x) = f(y)\ .$$

A generalization of keys to avoid design flaws violating the above rule.

Let X and Y be sets of attributes in R.

X (*functionally*) determines Y , $X \rightarrow Y$, iff $t_1[X] = t_2[X]$ implies $t_1[Y] = t_2[Y]$.

i.e., f (t(X)) = t [Y]

We also say $X \rightarrow Y$ is a *functional* dependency, and that Y is *functionally* dependent on X.

X is called the *left side*, Y the *right side* of the dependency.

Examples

- For every Name, there is a unique Phone_no and Major, assume Name is unique;
- For every Course, there is a unique Prof;
- For every Name and Course, there is a unique Grade.

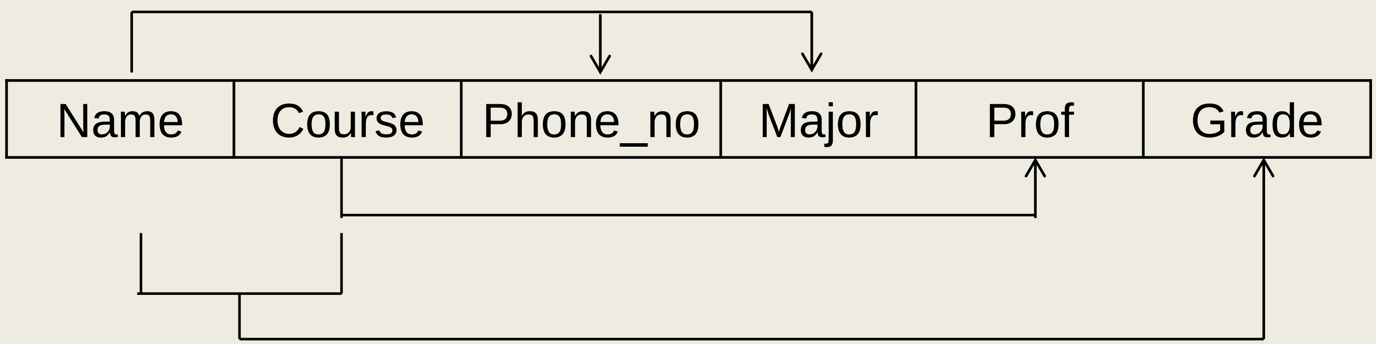
In this example:

$$\{Name\} \rightarrow \{Phone\_no\ ,\ Major\}$$

$$\{Course\} \rightarrow \{Prof\}$$

$$\{Name\ ,\ Course\} \rightarrow \{Grade\}$$

We can also show these in a diagram like this one:



Notice that other FD's follow from these:

$$\{Name\} \rightarrow \{Major\}$$

$$\{Course\ ,\ Grade\} \rightarrow \{Prof\ ,\ Grade\}$$

Functional dependencies

Let F be a set of FD's.

Definition 1: $X \rightarrow Y$ is inferred from F (or that F infers $X \rightarrow Y$), written in

$$F \models X \rightarrow Y$$

if any relation instance satisfying F must also satisfy $X \rightarrow Y$.

Impossible to list every relation to verify if $X \rightarrow Y$ is inferred from F.

A set ρ of derivation rules are required, such that:

a $X \rightarrow Y$ is inferred from F according to Definition 1 iff it can be

derived using ρ.

Armstrong’s axioms (1974)

Notation: If X and Y are sets of attributes, we write XY for their union.

e.g. X = {A, B}, Y = {B, C}, XY = {A, B, C}

F1 (Reflexivity) If $X \supseteq Y$ then $X \rightarrow Y$.

F2 (Augmentation) $\{X \rightarrow Y\} \models XZ \rightarrow YZ$.

F3 (Transitivity) $\{X \rightarrow Y, Y \rightarrow Z\} \models X \rightarrow Z$.

F4 (Additivity) $\{X \rightarrow Y, X \rightarrow Z\} \models X \rightarrow YZ$.

F5 (Projectivity) $\{X \rightarrow YZ\} \models X \rightarrow Y$.

F6 (Pseudotransitivity) $\{X \rightarrow Y, YZ \rightarrow W\} \models XZ \rightarrow W$.

Example: Given F = $\{A \rightarrow B, A \rightarrow C, BC \rightarrow D\}$, derive $A \rightarrow D$:

1. $A \rightarrow B$ (given)

2. $A \rightarrow C$ (given)

3. $A \rightarrow BC$ (by F4, from 1 and 2)

4. $BC \rightarrow D$ (given)

5. $A \rightarrow D$ (by F3, from 3 and 4)

F4 (Additivity) $\{X \rightarrow Y, X \rightarrow Z\} \models X \rightarrow YZ$.

F5 (Projectivity) $\{X \rightarrow YZ\} \models X \rightarrow Y$.

F6 (Pseudotransitivity) $\{X \rightarrow Y, YZ \rightarrow W\} \models XZ \rightarrow W$.

In fact, F4, F5, and F6 can be derived from F1-F3.

Example: Prove $\{X \rightarrow Y, X \rightarrow Z\} \models X \rightarrow YZ$.

1) $X \rightarrow Y$ is given.

2) $XX \rightarrow XY$ (by F2); that is, $X \rightarrow XY$

3) $X \rightarrow Z$ is given.

4) $XY \rightarrow YZ$ (by F2)

5) $X \rightarrow YZ$ (by F3, 2) and 4))

Armstrong's axioms

We can prove that Armstrong's axioms are sound and complete:

Sound: if F derives $A \rightarrow B$ by using Armstrong's axioms, then F $\models A \rightarrow B$ by Definition 1.

Complete: if F $\models M \rightarrow N$ by Definition 1, then F derives $M \rightarrow N$ by using Armstrong's axioms.

Algorithm to Check a FD

Given F, how do we check if $X \rightarrow Y$ is in F^+?

F^+ denotes the smallest set of FD's that

- contains F, and
- is *closed* under Armstrong's axioms.

F^+ is the *closure* of F.

$F = \{ A \rightarrow B, B \rightarrow C, A \rightarrow C \}$

F^+ = {AB -> A, AB -> B, AB -> C, AC -> A, AC -> B, AC -> C, ABC -> A, ABC -> B, ABC -> C, AB -> AB, AB -> BC, AB -> AC, …….}

F^+ always has an exponential size regarding |F|.

Too expensive to compute F^+ to verify a membership.

Instead we can compute the *closure* X^+ of X under F, X^+ is the largest set of attributes functionally determined by X.

It can be proven (using additivity) that

S1: $$\boxed{X^+ = \cup_{\forall X \to A \in F^+} A.}$$

S2: $$\boxed{X \to Y \in F^+ \text{ iff (if and only if) } Y \subseteq X^+.}$$

Example:

$F = \{ A \rightarrow B, BC \rightarrow D, A \rightarrow C \}$, compute $\{A\}^+$

1st scan of F:

$X^+ := \{A\}$

$X^+ := \{A, B\}$

$X^+ := \{A, B, C\}$

2nd scan of F:

$X^+ := \{A, B, C, D \}$

3rd scan of F: no change, therefore the algorithm terminates.

$\{A\}^+ := \{A, B, C, D \}$

Algorithm to compute X^+

$X^+ := X$;

change := true;

while change do

    begin

    change := false;

    for each FD $W \rightarrow Z$ in F do

        begin

        if ($W \subseteq X^+$) and ($Z \nsubseteq X^+$) then do

            begin

            $X^+ := X^+ \cup Z$;

            change := true;

            end

        end

    end

Algorithm to Compute a Candidate Key

Given a relational schema *R* and a set *F* of functional dependencies on R.

A key *X* of *R* must have the property that $X^+ = R$.

Algorithm to compute a candidate key

Step 1: Assign *X* a superkey in F.

Step 2: Iteratively remove attributes from *X* while retaining the property $X^+ = R$ till no reduction on *X*.

The remaining *X* is a key.

Example:

$R = \{A, B, C, D\}$ and $F = \{ A \rightarrow B, BC \rightarrow D, A \rightarrow C \}$

$X = \{A, B, C\}$ if the left hand side of F is a super key.

A cannot be removed because $\{BC\}^+ = \{B, C, D\} \neq R$

B can be removed because $\{AC\}^+ = \{A, B, C, D\} = R$

$\Rightarrow X = \{ A, C\}$

C can be further removed because $\{A\}^+ = \{A, B, C, D\}$

$\Rightarrow X = \{A\}$

Algorithm to Compute All the Candidate Keys

Given a relational schema R and a set F of functional dependencies on R, the algorithm to compute all the candidate keys is as follows:

T := ∅

Main:

X := S where S is a super key which does not contain any candidate key in T

remove := true

While remove do

For each attribute A ∈ X

Compute $\{X\text{-}A\}^+$ with respect to F

If $\{X\text{-}A\}^+$ contains all attributes of R then

X := X – {A}

Else

remove := false

T :=T ∪ X

Repeat *Main* until no available S can be found. Finally, T contains all the candidate keys.

Algorithm to Compute All the Candidate Keys

Example:

R(A, B, C, D, E)

F = {$A \rightarrow B$, $BC \rightarrow A$, $D \rightarrow E$}

Algorithm to Compute All the Candidate Keys

Example:

R(A, B, C, D, E)

F = $\{A \rightarrow B, BC \rightarrow A, D \rightarrow E\}$

Step 1:

Let X := {A, B, C, D}

Step 2:

Try to remove A

$\{B, C, D\}^+ = \{A, B, C, D, E\}$

Thus X := {B, C, D}

Algorithm to Compute All the Candidate Keys

Step 3:

Try to remove B, C, D

$\{C, D\}^+ = \{C, D, E\}$

$\{B, D\}^+ = \{B, D, E\}$

$\{B, C\}^+ = \{A, B, C\}$

Thus cannot be removed

So {B, C, D} is a candidate key and add to T

Step 4:

Find another superkey

Let X := {A, C, D}

Algorithm to Compute All the Candidate Keys

Step 5:

Try to remove A, C, D

$\{C, D\}^+ = \{C, D, E\}$

$\{A, D\}^+ = \{A, B, D, E\}$

$\{A, C\}^+ = \{A, B, C\}$

Thus cannot be removed

So {A, C, D} is another candidate key and add to T

Step 6:

Cannot find any other super keys,

So candidate keys are {B, C, D} and {A, C, D}