**Problem 1**

1. : All possible functions:

F1 : { (a,0), (b,0), (c,0) } F5 : { (a,1), (b,0), (c,0) }

F2 : { (a,0), (b,0), (c,1) } F6 : { (a,1), (b,0), (c,1) }

F3 : { (a,0), (b,1), (c,0) } F7 : { (a,1), (b,1), (c,0) }

F4 : { (a,0), (b,1), (c,1) } F8 : { (a,1), (b,1), (c,1) }

(b) : Pow(X) = { A : A ⊆ X } and |Pow(X)| = 2|X|

🡺 Pow({a, b, c}) = 2|{a, b, c}|=23=8

We can consider like this : “0” and “1” represent “exist” and “not exist”, so if the element in { a, b, c } all point to 0, they represent an empty set. Therefore, the subset of { a, b, c } can represent the existence and non-existence of a, b, c. So the number of functions is equal to |Pow({a, b, c})|.

🡺when |co-domain| = 2, Pow({a, b, c}) = the answer for (a).

(c) : if card(A) = m and card(B) = n:

1. Number of fuctions from A to B : nm
2. For the symmetric relation, you can analyze half triangle in the relation matrix. Every elements in the matrix can exist and not exist. So the number of relations from A to B : |Pow(m x n)| = 2|m x n|
3. For the symmetric relation, you can analyze half triangle in the relation matrix, and it can be concluded that half triangle has (1+m) /2 elements, So the number of symmetric relations from A to B is 2 (1+m)/2 .

**Problem 2**

1. S 2,-3 = { 2m - 3n : m, n ∈ Z}

1 (where m = 2, n = 1); 3 (where m = 3, n = 1);

5 (where m = 4, n = 1); 7 (where m = 5, n = 1);

4 (where m = 5, n = 2);

1. S 12,16 = { 12m + 16n : m, n ∈ Z}

0 (where m = 0, n = 0); 12 (where m = 1, n = 0);

16 (where m = 0, n = 1); 28 (where m = 1, n = 1);

24 (where m = 2, n = 0);

1. ∵ d = gcd(x , y) and x, y ∈ Z ∴ x = k1\*d, for some k1 ∈ Z;

Same reason for y = k2\*d, for some k2 ∈ Z;

∴ S x,y = { (k1 \* d) m + (k2 \* d) n : m, n ∈ Z }, for some k1, k2 ∈ Z;

= { (k1 \* m+ k2 \* n) d : m, n ∈ Z };

And then we can see that (k1 \* m+ k2 \* n) ∈ Z because of k1, k2, m and n ∈ Z;

Then, {n : n ∈ Z and d | n} = {n : n = k\*d and k ∈ Z and n ∈ Z };

∵ { (k1 \* m+ k2 \* n) } ⊆ Z

∴ { (k1 \* m+ k2 \* n) d : m, n ∈ Z } ⊆ {n : n = k\*d and k ∈ Z and n ∈ Z } and that is : S x,y ⊆ {n : n ∈ Z and d|n}.

1. ∵ {n : n ∈ Z and z | n} ;

∴ {n : n ∈ Z and n = p\*z, for some p∈ Z };

∵ z is the smallest positive number in Sx,y, so z can be standed for (p1 \* m+ p2 \* n) d, for some p1, p2, m and n, making z be the smallest positive number in Sx,y, that is let (p1 \* m+ p2 \* n) be the smallest positive number since d ≥ 0 and z > 0.

∴ {n : n = p\*(p1 \* m+ p2 \* n) d and m, n, p, k1, k2∈ Z }

∵ p1, p2 are specific two numbers to satisfy z to be the smallest positive number in Sx,y.

∴ (p1 \* m+ p2 \* n) ∈{ (k1 \* m+ k2 \* n) : m, n, k1, k2∈ Z }

∵ p \* (p1 \* m+ p2 \* n) is a multiple of (p1 \* m+ p2 \* n)

∴ { p \* (p1 \* m+ p2 \* n) } ⊆ { (k1 \* m+ k2 \* n) } for some p ∈ Z

∴ { p\*(p1 \* m+ p2 \* n) d } ⊆ { (k1 \* m+ k2 \* n) d }

And that is {n : n ∈ Z and z|n} ⊆ Sx,y

1. ∵z is the smallest positive number in Sx,y, so we can express z as z = (k1 \* m+ k2 \* n) \* d ＞ 0

∵ d = gcd(x , y) > 0

∴ (k1 \* m+ k2 \* n) > 0 and (k1 \* m+ k2 \* n) ∈ Z

∵ z is the smallest positive number in Sx,y

∴ (k1 \* m+ k2 \* n) = 1 that is z / d = 1

∴ z = d

∴ z ≥ d

1. According to the conclusion of (e):

∵ z = d

∴ z ≤ d

**Problem 3**

1. (A ∗ B) ∗ (A ∗ B)

= (AC∪ BC) ∗ (AC∪ BC) (definition)

= (AC∪ BC) C∪(AC∪ BC) C (definition)

= ((AC)C∩ (BC)C)∪((AC)C∩ (BC)C) (de Morgan’s Laws)

= (A∩B)∪(A∩B) (double complementation)

= A∩(B∪B) (distribution)

= A∩B (idempotence)

1. AC = AC ∪ AC  (idempotence)

= A ∗ A (definition)

1. Φ = AC ∩ A (Complementation)

= (AC ∗ A) ∗ (AC ∗ A) (by conclusion (a))

= ((A ∗ A) ∗ A) ∗ ((A ∗ A) ∗ A) (by conclusion (b))

1. A \ B = A∩ BC (by definition)

= A∩ (B ∗ B) (by conclusion (b))

= (A ∗ (B ∗ B)) ∗ (A ∗ (B ∗ B)) (by conclusion (a))

**Problem 4**

1. w = a, v = ba;

∵ ba ≠ az, for z ∈ Σ∗ and a ≠ baz, for z ∈ Σ∗

1. By definition of R ←(B) :

R←({aba}) is a set satisfying a condition that:

R←({aba}) = { w : aba = wz, z ∈ Σ ∗ };

🡺 R←({aba}) = { λ, a, ab, aba }

1. let z be λ:

Reflexivity: For all w∈Σ∗ , (w, w) ∈ R for all w ( due to w = wλ ).

Antisymmetry: For all w, v∈Σ∗ , if (w, v) ∈ R and (v, w) ∈ R:

That is : v = wλ and w = vλ 🡺 v = w

Transitivity: For all w, v∈Σ∗ , if (w, v) ∈ R and (v, t) ∈ R:

That is : v = wλ and t = vλ 🡺 t = wλλ = w

So (t, w) ∈ R

**Problem 5**

Case x = 0:

If x = 0: ∵ x|yz ∴ yz = 0

∵ gcd(x, y) = gcd(0, y) = 1 ∴ y = 1

∴ z = x = 0

∴ x|z

Case x ≠ 0:

As we can see in problem 2, so d = gcd(x, y) can be represented in the form of (mx + ny) for some m, n ∈ Z.

So we can write gcd(x , y) = 1 = (m0x + n0y) for some m0, n0∈ Z.

And x|yz 🡺 yz = kx, for some k ∈ Z

Multiply both sides of this equation by z, and we can get :

z = m0xz + n0yz, and we substitude yz by kx: z = m0xz + n0kx

∴ z = (m0z + n0k)x ∵ m0，z，n0，k ∈ Z

∴ (m0z + n0k) is an integer ∴ x|z