Tutorial 4

Name: Zhaokun Su

Breakout group: 3

Zid: z5235878

5. Determine whether the following sentences are valid (i.e. tautologies) using truth tables.

   (i) $((P \lor Q) \land \neg P) \to Q$

   (ii) $((P \to Q) \land \neg(P \to R)) \to (P \to Q)$

   (iii) $\neg(\neg P \land P) \land P$

   (iv) $(P \lor Q) \to \neg(\neg P \land \neg Q)$

We test using our tableau_prover.py to test our propositions:

```
tableau_test('((P | Q) & -P) -> Q')
tableau_test('((P -> Q) & -(P -> R)) -> (P -> Q)')
tableau_test('-(-P & P) & P')
tableau_test('(P | Q) -> -(-P & -Q)')
```

For (i), (ii), (iii), (iv) respectively:

```
-(((P | Q) & -P) -> Q)
   -Q
      ((P | Q) & -P)
         -P
            (P | Q)
               P
                  CLOSED
               Q
                  CLOSED
 |- (((P | Q) & -P) -> Q): True
```

```
-(((P -> Q) & -(P -> R)) -> (P -> Q))
   ((P -> Q) & -(P -> R))
      -(P -> Q)
         P
            -Q
               -(P -> R)
                  P
                     -R
                        (P -> Q)
                           -P
                              CLOSED
                           Q
                              CLOSED
 |- (((P -> Q) & -(P -> R)) -> (P -> Q)): True
```

```
-(-(-P & P) & P)
   --(-P & P)
      (-P & P)
         P
            -P
               CLOSED
   -P
      AGENDA EMPTY
 |- (-(-P & P) & P): False
```

```
-((P | Q) -> -(-P & -Q))
   --(-P & -Q)
      (-P & -Q)
         -Q
            -P
               (P | Q)
                  P
                     CLOSED
                  Q
                     CLOSED
 |- ((P | Q) -> -(-P & -Q)): True
```

Which gives us answers: (i),(ii)and(iv) are tautologies, (iii) are not.

6. Repeat Question 5 using resolution. In this case, try to show

(i) $\vdash ((P \vee Q) \wedge \neg P) \to Q$

(ii) $\vdash ((P \to Q) \wedge \neg(P \to R)) \to (P \to Q)$

(iii) $\vdash \neg(\neg P \wedge P) \wedge P$

(iv) $\vdash (P \vee Q) \to \neg(\neg P \wedge \neg Q)$

i) $\vdash ((P \vee Q) \wedge \neg P) \to Q$

Convert to CNF:

$\neg(((P \vee Q) \wedge \neg P) \to Q)$

$\Leftrightarrow \neg(\neg((P \vee Q) \wedge \neg P) \vee Q)$

$\Leftrightarrow \neg\neg((P \vee Q) \wedge \neg P) \wedge \neg Q$

$\Leftrightarrow ((P \vee Q) \wedge \neg P) \wedge \neg Q$

$\Leftrightarrow (P \vee Q) \wedge \neg P \wedge \neg Q$

proof:

1. $P \vee Q$ [Negated Query]
2. $\neg P$ [Negated Query]
3. $\neg Q$ [Negated Query]
4. Q ~~[Negated Query]~~ [1, 2 resolution]
5. ϕ [3, 4 resolution]

$\Downarrow$

~~$\neg(((P \vee Q) \wedge \neg P) \to Q)$ is a tautology~~

$((P \vee Q) \wedge \neg P) \to Q$ is a tautology

iii) $\vdash ((P \to Q) \land \neg(P \to R)) \to (P \to Q)$

Converting to CNF

$(\neg\, ((P \to Q) \land \neg(P \to R)) \to (P \to Q))$

$\Leftrightarrow \neg\, (\neg((P \to Q) \land \neg(P \to R)) \lor (P \to Q))$

$\Leftrightarrow ((\neg P \lor Q) \land \neg(\neg P \lor R)) \land \neg(\neg P \lor Q)$

$\Leftrightarrow (\neg P \lor Q) \land (P \land \neg R) \land P \land \neg Q$

Proof:

1. $\neg P \lor Q$ $\left.\right\}$ Negated Query
2. P
3. $\neg R$
4. $\neg Q$
5. Q 1, 2 resolution
6. ϕ 4, 5 resolution

$\Downarrow$

$((P \to Q) \land \neg(P \to R)) \to (P \to Q)$ is a tautology

iii) $\vdash \neg(\neg P \wedge P) \wedge P$

Converting to CNF:

$$\neg(\neg(\neg P \wedge P) \wedge P)$$

$\Leftrightarrow \quad (\neg P \wedge P) \vee \neg P$

$\Leftrightarrow \quad (\neg P \vee \neg P) \wedge (\neg P \vee P)$

$\Leftrightarrow \quad (\neg P \vee \neg P)$

$\Leftrightarrow \quad \neg P$

proof: 1. $\neg P$ [Negated Query]

$\Downarrow$

cannot derive empty clause using resolution $\Rightarrow \neg(\neg P \wedge P) \wedge P$ is not tautology

iv) $\vdash (P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$

Converting to CNF:

$$\neg((P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q))$$

$\Leftrightarrow \neg(\neg(P \vee Q) \vee \neg(\neg P \vee \neg Q)) \Leftrightarrow (P \vee Q) \wedge (\neg P \wedge \neg Q)$

proof:

1. $P \vee Q$
2. $\neg P$
3. $\neg Q$

} Negated Query

4. Q 1, 2 resolution
5. $\emptyset$ 3, 4 resolution

$\Rightarrow (P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$ is a tautology