`measure_adj`: A software package for making measurements and computing adjoint sources for seismic tomography

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```
SVN directory    : /seismo/3D/measure_adj/USER_MANUAL/
This file        : measure_adj_manual.pdf
Additional notes : multitaper_notes.pdf, multitaper_vala.pdf
```

Contents

1 Introduction

NOTE: This is a preliminary user manual with many modifications to come.

The open-source software package measure_adj is designed for making measurements for seismic tomography. Several misfit functions are available, such as a simple waveform difference, a simple cross-correlation traveltime difference, or a frequency dependent multitaper traveltime difference. For each misfit function, the code also provides adjoint sources, which are needed for tomographic inversions using adjoint methods (e.g., Tromp et al., 2005; Liu & Tromp, 2006; Tape et al., 2007). This code was used to make measurements and adjoint sources for the (large-scale) tomographic inversion of Tape et al. (2009) for the southern California crust.

MEAS–ADJ is designed to complement FLEXWIN (Maggi et al., 2009), which is designed to pick time windows for measurement. In particular, FLEXWIN has the option of providing a MEASUREMENT.WINDOWS file that can be directly read into measure_adj. However, FLEXWIN is not necessary to run measure_adj.

There are two additional sets of notes on multitaper measurements and adjoint sources, multitaper_notes.pdf and multitaper_vala.pdf. Some sections of multitaper_notes.pdf were published as "Multitaper measurements for adjoint tomography" in Tape (2009, Appendix C).

Please email Carl Tape (carltape@fas.harvard.edu) with suggestions or corrections.

2 Getting started

2.1 System requirements

In order to install and run, measure_adj program requires:

- UNIX operating system (Linux, Solaris, MacOS . . .)
- GNU make
- a fortran compiler (gfortran, ifort, etc...)
- other packages : SAC (Seismic Analysis Code, available from IRIS); GMT (Generic Mapping Tools) for the plotting scripts

MEAS–ADJ requires the following libraries external to the package in order to compile and run: libsacio.a and libsac.a. Both libraries are distributed by IRIS as part of the SAC package (version 101.2 and above). The IRIS download site (as of 21-Oct-2009) is here: http://www.iris.edu/software/sac/sac.request.htm. (To check your version, type sac.)

3 Obtaining the code

(Use SVN for now.)

The package contains the MEAS–ADJ code and documentation, as well as a set of test data, examples of user files for different scenarios, and a set of utility scripts that may be useful for running measure_adj on large datasets.

4 Compilation

If your compiler of choice is gfortran, then you should be able to use the `Makefile` with only minor modifications (notably you may need to change the search path for the `libsacio.a` library). If you prefer another compiler, you should modify the OPT and FC lines in the makefiles accordingly. We tested the code using gfortran version 4.3.3. (To check your version, type `gfortran --version`.)

Steps to compile the MEAS–ADJ package:

1. `make clean`
2. `make`

You should end up with the `measure_adj` executable.

4.1 Running the test case

After compiling, execute the following two commands:

```
measure_adj
csh -f run_mt_measure_adj.csh
```

If all goes well, you should generate the figure

`PLOTS/9818433_T006_T030_MPM_CI_m16_mtm_win_adj.pdf`

A set of auxilliary programs/files are required for the successful run of post-processing and plotting scripts, including

- Southern California faults xy files in `PLOTS/plot_win_adj.pl`:

```
$dir0 = "/opt/seismo/datalib/SC/";
$fault_file = "$dir0/jennings.xy";
```

- Dr. Lupei Zhu's `saclst` program
- Dr. Lupei Zhu's `pssac2` plotting program (`pssac` may work as well).

5 Running MEAS–ADJ

Two input files are read in when `mt_measure_adj` is executed.

5.1 Input file 1: `MEASUREMENT.WINDOWS`

```
3
DATA/9818433.CI.MPM.BHR.sac.d.T006_T030
SYN/MPM.CI.BHR.semd.sac.m16.T006_T030
           3
   11.2770    58.4770
   58.4770    81.3770
   81.3770   101.6770
DATA/9818433.CI.MPM.BHT.sac.d.T006_T030
SYN/MPM.CI.BHT.semd.sac.m16.T006_T030
           1
   49.3270    95.3270
DATA/9818433.CI.MPM.BHZ.sac.d.T006_T030
SYN/MPM.CI.BHZ.semd.sac.m16.T006_T030
           1
   60.9770    99.3770
```

The first line contains the number of pairs of records to be read in. Each pair of files is followed by the number of windows within which measurements will be made, followed by the time intervals for each window. Note that data and synthetics require to have exactly the same `t0, dt, npts` for the code to run through. Therefore no interpolation of `dt` is really necessary at any point.

5.2 Input file 2: MEASUREMENT.PAR

```
 -0.585 0.0110   18200  # tstart, DT, npts: time vector for simulations
                     7  # imeas (1-8; see manual)
                    BH  # channel of synthetics: BH or LH
     30.000      6.000  # TLONG and TSHORT: band-pass periods for records
               .false.  # RUN_BANDPASS: use band-pass on records
                .true.  # DISPLAY_DETAILS
                .true.  # OUTPUT_MEASUREMENT_FILES
                .true.  # COMPUTE_ADJOINT_SOURCE
    -4.5000     4.5000  # TSHIFT_MIN; TSHIFT_MAX
    -1.5000     1.5000  # DLNA_MIN; DLNA_MAX
                 0.690  # CC_MIN
                     1  # ERROR_TYPE -- 0 none; 1 CC, MT-CC; 2 MT-jack-knife (all for MTM!)
                 1.000  # DT_SIGMA_MIN: water level of traveltime uncertainty
                 0.500  # DLNA_SIGMA_MIN: water level of dlnA uncertainty
                     1  # ITAPER:1 multi-taper; 2 cosine; 3 boxcar taper type for freq-dep. m
          0.020  2.50  # WTR, NPI (ntaper = 2*NPI)
                 2.000  # DT_FAC (following 4 pars are for MTM --> CC check)
                 2.500  # ERR_FAC
                 3.500  # DT_MAX_SCALE
                 1.500  # NCYCLE_IN_WINDOW
```

The names below correspond to the variable names in `mt_measure_adj` (see subroutine `read_par_file` in `mt_sub.f90`).

- Time vector: `tt` (start time), `dtt` (time step), `nn` (number of points). These values are particularly important if adjoint sources are requested. Based on the convention in SPECFEM3D, for example, it is best to select the start time to be $t_0 = -1.5\tau_{\rm h}$, where $\tau_{\rm h}$ is the half duration of the CMT source. The time step and the number of time steps should exactly match the time file of the SPECFEM3D synthetics.
- Option for measurement: `imeas0` = 1–8 (see section 6).
- Channel: `chan` is BH or LH (based on options for SEM synthetics).
- Bandpass range: `TLONG` (long-period), `TSHORT` short-period
- `RUN_BANDPASS`: whether to bandpass the seismograms.
- `OUTPUT_MEASUREMENT_FILES`: whether to output measurement files for each window.
- Limits on cross-correlation measurement: `TSHIFT_MIN`; `TSHIFT_MAX`. These can optionally be read in from the `PAR_FILE` used in the FLEXWIN algorithm.
- Limits on amplitude measurement: `DLNA_MIN`; `DLNA_MAX`. These can optionally be read in from the `PAR_FILE` used in the FLEXWIN algorithm.

- Limits on the cross-correlation measurement: CC_MIN. This can optionally be read in from the PAR_FILE used in the FLEXWIN algorithm.

- ERROR_TYPE = 0–2: Type of estimated errors associated with multi-taper measurement (and adjoint source). No error (0), cross-correlation error estimate (1), multitaper jack-knife error estimate (2). The cross-correlation error estimate is presented in Tape et al. (2010, Appendix A). For imeas=5,6 (cc), sigma_dt/dlnA_cc is automatically used. For ERROR_TYPE = 0, all sigma_tau and sigma_dlnA are set to be 1 for multitaper adjoint sources.

- DT_SIGMA_MIN: water-level minimum cross-correlation traveltime difference uncertainty estimate.

- DLNA_SIGMA_MIN: water-level minimum amplitude difference uncertainty estimate.

- Multitaper parameter, ITAPER: Type of taper to use in constructing the transfer function between synthetics and data. Taper options are multitaper (1), cosine taper (2), and boxcar taper (3). For the single-taper options (2–3) the transfer function is not used, as the adjoint sources are constructed directly from the synthetic seismograms. For the multitaper option, the number of tapers is fixed to be twice NPI (see multitaper_vala.pdf).

- Multitaper parameters, WTR, NPI (see multitaper_vala.pdf). Note WTR is also used to determine i_right corresponding to the maximum frequency of valid frequency-dependent measurements. It requires that the power at this frequency is above 10 times the WTR*max_syn_power.

- Multitaper parameter, DT_FAC (see section 5.2.1).

- Multitaper parameter, ERR_FAC (see section 5.2.1).

- Multitaper parameter, DT_MAX_SCALE (see section 5.2.1).

- Multitaper parameter, NCYCLE_IN_WINDOW, (see section 5.2.1).

5.2.1 Multitaper parameters

The selection of the multitaper parameters DT_FAC, ERR_FAC, DT_MAX_SCALE, and NCYCLE_IN_WINDOW are not easy for each particular dataset. These parameters determine whether a multitaper measurement is reasonable enough to retain (otherwise revert to a cross-correlation measurement). The key subroutine is mt_measure_select.f90.

MTMs are rejected (user_trace = .false.) based on:

- number of cycles in the window

- number of frequency points given within [fstart,fend]. Even when RUN_BANDPASS is turned off, the TLONG and TSHORT are still converted to fstart,fend. They are best set to the values used in FLEXWIN

- tshift_cc <= dt: too small a time shift

- `abs(dtau_w(j)) > Tvec(j)/DT_FAC`: τ at a specific frequency is too high
- `err_dt(j) > Tvec(j)/ERR_FAC`: error for a specific frequency is too high
- `abs(dtau_w(j)) > DT_MAX_SCALE*abs(tshift))` τ at specific frequency deviates too much for global `tshift_cc`

Note that the cross-correlation measurements are then run through `cc_measure_select()` to determine if they are usable or not (`tshift, dlnA = 0`).

Two example sets of parameters are given

```
! ********* adjust following for global traces ***********************
  ! ratio of current period with respect to dt and err_dt measurements
  double precision, parameter :: DT_FAC = 1.0
  double precision, parameter :: ERR_FAC = 8.0
  ! max time shift allowed at all freq should be DT_MAX_SCALE * Tshift_xc
  double precision, parameter :: DT_MAX_SCALE = 5.0
  ! few cycles for surfaces waves
  double precision, parameter :: NCYCLE_IN_WINDOW = 2.0 ! window length > 2.*50
!*********************************************************************

! ********* adjust following for socal 3-30s traces *******************
  ! ratio of current period with respect to dt and err_dt measurements
  double precision, parameter :: DT_FAC = 2.0
  double precision, parameter :: ERR_FAC = 2.5
  ! max time shift allowed at all freq should be DT_MAX_SCALE * Tshift_xc
  double precision, parameter :: DT_MAX_SCALE = 3.5
  ! use 3 cycles for surfaces waves
  double precision, parameter :: NCYCLE_IN_WINDOW = 1.5
!*********************************************************************
```

5.3 Output files

There are two output files related to measurements in the local run directory:

1. `window_chi`, a comprehensive output file of misfit values, with one line per window.

```
! KEY: write misfit function values to file (for each window)
! Here are some columns of values in window_chi (add 8 for the actual column number)
!  1: MT-TT chi (imeas=7),    2: MT-dlnA chi (imeas=8)
!  3: XC-TT chi (imeas=5),    4: XC-dlnA chi (imeas=6)
!  5: MT-TT meas (freq-average),   6: MT-dlnA meas,   7: XC-TT meas,   8: XC-dlnA meas
!  9: MT-TT error, 10: MT-dlnA error, 11: XC-TT error, 12: XC-dlnA error
! WINDOW     : 13: data power, 14: syn power, 15: (data-syn) power, 16: window duration
! FULL RECORD: 17: data power, 18: syn power, 19: (data-syn) power, 20: record duration
! WINDOW     : 21: tr_chi,       22: am_chi
```

```
! Example of a reduced file: awk '{print $2,$3,$4,$5,$6,$29}' window_chi > window_tr_chi

write(13,'(a14,a8,a3,a5,i4,i4,2e14.6,20e14.6,2e14.6,2f14.6)') &
   file_prefix0,sta,net,chan_syn,j,imeas,&
   tstart,tend,window_chi(:),tr_chi,am_chi,T_pmax_dat,T_pmax_syn

! misfit function value
if(is_mtm==1) window_chi(1) = 0.5 * 2.0 * df * sum( (dtau_w(1:i_right))**2 * wp_taper(1:i_
if(is_mtm==1) window_chi(2) = 0.5 * 2.0 * df * sum( (dlnA_w(1:i_right))**2 * wq_taper(1:i_
window_chi(3) = 0.5 * (tshift/sigma_dt_cc)**2 ! tr_chi
window_chi(4) = 0.5 * (dlnA/sigma_dlnA_cc)**2 ! amp_chi
! cc/averaged-mt tshift/dlnA measurement
if(is_mtm==1) window_chi(5)  = sum( dtau_w(1:i_right) * w_taper(1:i_right) ) / sum(w_taper
if(is_mtm==1) window_chi(6)  = sum( dlnA_w(1:i_right) * w_taper(1:i_right) ) / sum(w_taper
window_chi(7) = tshift
window_chi(8) = dlnA
! estimated measurement uncertainties
if(is_mtm==1) window_chi(9) = sigma_dt
if(is_mtm==1) window_chi(10) = sigma_dlnA
window_chi(11) = sigma_dt_cc
window_chi(12) = sigma_dlnA_cc
! for normalization, divide by duration of window
window_chi(13) = 0.5 * sum(dat_dtw(:)**2)
window_chi(14) = 0.5 * sum(syn_dtw(:)**2)
window_chi(15) = 0.5 * sum((dat_dtw-syn_dtw)**2) ! tr/amp_chi for imeas=1/2
window_chi(16) = nlen*dt
window_chi(17) = 0.5 * sum( data**2 ) ! power of entire trace
window_chi(18) = 0.5 * sum( syn**2 )
window_chi(19) = 0.5 * sum( (data-syn)**2 )
window_chi(20) = npts*dt
```

2. `window_index`, an abbreviated output file with the indexing for each window. The columns are

   ```
   write(12,'(a3,a8,a5,a5,3i5,2f12.3)') net,sta,chan_syn,chan_dat,nwin,ipair,j,tstart,tend
   ```

   where `nwin` is the overall window counter, `ipair` is the seismogram (pair) counter, and `j` is the local window counter. For example, for the test case run:

   ```
   CI MPM     BHR  BHR      1    1    1      11.277      58.477
   CI MPM     BHR  BHR      2    1    2      58.477      81.377
   CI MPM     BHR  BHR      3    1    3      81.377     101.677
   CI MPM     BHT  BHT      4    2    1      49.327      95.327
   CI MPM     BHZ  BHZ      5    3    1      60.977      99.377
   ```

With COMPUTE_ADJOINT_SOURCE = .true.,

1. adjoint source files (e.g., MPM.CI.BHZ.iker07.adj) will appear in OUTPUT_FILES/. These are ascii files with the time columne defined by tstart,DT,npts in the parameter file. (if DO_RAY_DENSITY_SOURCE = .true., adjoint file names are MPM.CI.BHZ.iker7.density.adj).

2. window_chi_sum in local directory, the sum of misfit values (CC or MTM) of all windows, with weights taken into account if DO_WEIGHTING = .true.. This scalar value literally corresponds to all the adjoint sources produced in OUTPUT_FILES/. If input includes all windows for a particular event, this scalar value is the event misfit. More comprehensive sum of misfit can be found in the script

   /ADJOINT_TOMO/iterate_adj/matlab/compute_misfit.m

With OUTPUT_MEASUREMENT_FILES = .true. in MEASUREMENT.PAR, you should find numerous output files in OUTPUT_FILES/. Other files include (prefix=MPM.CI.BHZ.01.mtm, prefix0=MPM.CI.BHZ)

```
prefix.recon_syn_cc.sac            # syn_dtw_cc
prefix.recon_syn_cc_dt.sac         # syn_dtw_cc_dt
prefix.dt/dlnA_average/cc          # dtau/dlnA_meas, dtaul/dlnA_sigma ('average' means 'mtm')
prefix.ph/abs/dlnA/ph_cor/dt       # transfer fun.:  phi/abs/log(abs)/phi(corr)/dtau(idf_new:i
                                   #                 tshift/dlnA from cc are added to dtau/dln
prefix.recon_syn.sac               # recon. syn with both dtau(om) and dlnA(om) applied for al
prefix.recon_syn_dt.sac            # reconstructed syn with only dtau(om) applied
prefix.err_ph/abs/dt/dlnA[_full]   # err_phi/abs/dt/dlnA_mtm(idf_new:idf_new:i_right)
prefix.freq_limits                 # fstart/fend (input),df, fstart/fend(adjusted for given wi
prefix0.recon.sac                  # recon. syn after applying CC or MTM for the entire data t
```

IF DISPLAY_MORE_DETAILS = .true. in the parameter file, following files will be written for every window pair

```
prefix.obs (original data/syn)
prefix.syn
prefix.dat.sac (time windowed data/syn)
prefix.syn.sac
prefix.obs.power % abs(dat_dtwo(1:i_right)), i.e., dat_dtw_cc
prefix.syn.power % abs(syn_dtwo(1:i_right)), i.e., syn_dtw
```

5.4 Scripts

The three scripts in the run directory are

1. write_par_file.pl. Write the file MEASUREMENT.PAR from a given set of parameters. This is called within the run scripts in scripts_tomo.

2. prepare_adj_src.pl. This script reads in a set of adjoint sources made from Z-R-T records and outputs a set of adjoint sources in Z-E-N that can be used in SPECFEM3D.

```
prepare_adj_src.pl -m CMT -z BH -s STATION -o OUTDIR -i [rotation] all_adj_ZRT_files
prepare_adj_src.pl -m CMTSOLUTION_9818433 -s PLOTS/STATIONS_TOMO -o ADJOINT_SOURCES \
                   OUTPUT_FILES/*adj
```

It loops over all the stations with adjoint sources present in `OUTPUT_FILES`, and write an associate `STATION_ADJOINT` to local dir. It also rotates adjoint sources to E-N-Z, and deposits them into `ADJOINT_SOURCES` dir. The key is that it will create an all-zeros record if no measurement is made on a particular component (say, Z), but IS made on another component (say, R or T). It uses the following fortran program:

```
rotate_adj_src baz(radian!) zfile tfile rfile efile nfile
```

The perl script also outputs `STATIONS_ADJOINT` file for all the stations with adjoint sources.

3. `combine_adj_src.pl`. This script combines two sets of adjoint sources (for example, two different bandpassed versions), and outputs the new adjoint sources, along with a new `STATIONS_ADJOINT` file to be used.

```
Usage: combine_adj_src.pl DIR_1 DIR_2 DIR_NEW imeas1 imeas2 chan
```

Adjoint sources from `dir1` and `dir2` with the same station/net/component names are added up; station files from `dir1/STATIONS_ADJOINT` and `dir2/STATIONS_ADJOINT` are combined into `dir_new/STATIONS_ADJOINT`.

Scripts in the `PLOT/` directory:

- The primary plotting script is `PLOTS/plot_win_adj.pl`.

```
plot_win_adj_all.pl -m CMTFILE -a STATION_FILE -n chan -b iboth -l tmin/tmax \
    -k imeas/iadj -d data_dir -s syn_dir -c recon_dir -w winfile -i smodel -j Tmin/Tmax
plot_win_adj_all.pl -l -10/200 -m ../CMTSOLUTION_9818433 -n BH -b 0 -k 7/1 \
    -a STATIONS_ADJOINT -d DATA -s SYN -c RECON -w MEASUREMENT.WINDOWS -i m16 -j 6/30
```

which uses another script

```
plot_win_adj.pl -m $cmtfile -n $sta/$net/$chan -b $iboth -l $tcuts -k $opt_k \
   -a $station_list -d $data_dir -s $syn_dir -c $recon_dir -w $winfile -i $smodel -j $Ts
plot_win_adj.pl -m ../CMTSOLUTION_9818433 -n MPM/CI/BH -b 0 -l -10/200 -k 7/1 \
   -a STATIONS_ADJOINT -d DATA -s SYN -c RECON -w MEASUREMENT.WINDOWS -i m16 -j 6/30
```

gives plots of detailed information on data, syn, reconstructed syn and corresponding adjoint sources, as well as source/station distributions on a map (Figure 8 for MTM, `imeas=7`). Some tweaking is probably necessary before successful application to a different dataset, including: setting evid in CMT file (correspond to data name evid.net.sta), setting corresponding kstnam,knetwk,kcmpnm for both data and synthetics, setting o header properly, and all synthetics need to be present (data/recon-syn not needed).

- plot the statistics of window files (check file structure before use)

```
plot_win_stats_all.pl Tmin/Tmax model iplot [plot histogram or not]
plot_win_stats.pl Tmin/Tmax eid name meas_file sta_file eid_text cmtall_psmeca
```

- organize `plot_win_adj_all.pl` output by station or event (???)

```
make_pdf_by_event.pl
make_pdf_by_station.pl
```

- For large datasets, we have included an additional set of scripts in the directory `scripts_tomo`. Each user will have to make some modifications to these scripts.
- For plotting output files for individual measurements, we have included a set of scripts in `scripts_meas`. These scripts have not been extensively tested.

6 Measurement options

There are several choices of measurements (or definitions of misfit functions), and each choice leads to a different adjoint source, as illustrated in Tromp et al. (2005). The user must specify a value of `imeas` in `MEASUREMENT.PAR` with the following options:

1. `imeas = 1`, normalized waveform difference. Adjoint source is constructed from the *data only*, with the form $-d(t)/\|d(t)\|^2$.

$$\begin{aligned} \phi &= \frac{1}{2}\frac{\int [d(t)-s(t)]^2\,dt}{\int d^2(t)\,dt} \quad \text{at} \quad s(t)=0 \\ f^\dagger(t) &= -\frac{d(t)}{\|d(t)\|^2} \end{aligned}$$

   If `NO_WAVEFORM_DIFFERENCE = .true.`, this becomes (why???)

$$f^\dagger(t) = d(t)$$

2. `imeas = 2`, waveform difference, $s(t)-d(t)$.

$$\begin{aligned} \phi &= \frac{1}{2}\int [d(t)-s(t)]^2\,dt \\ f^\dagger(t) &= s(t)-d(t) \end{aligned}$$

3. `imeas = 3`, cross-correlation traveltime for a (banana-doughtnut) sensitivity kernel. The

measurement between data and synthetics *is not used* in constructing the adjoint source.

$$\begin{aligned}
\phi &= T_{syn} \\
\delta\phi &= \delta T_{syn} = -\frac{\int w(t)\dot{s}(t)\delta s(t)\,dt}{\int w(t)\dot{s}^2(t)\,dt} \qquad \text{TTL (43)} \\
f^\dagger(t) &= -\frac{w(t)\dot{s}(t)}{\int w(t)\dot{s}^2(t)\,dt}
\end{aligned}$$

where $w(t)$ is the time window over which the measurement is made. Note the $-$ sign!.

4. `imeas = 4`, amplitude difference for a (banana-doughtnut) sensitivity kernel. The measurement between data and synthetics *is not used* in constructing the adjoint source.

$$\begin{aligned}
\phi &= \ln A_{syn} \\
\delta\phi &= \delta ln A_{syn} = \frac{\int w(t)s(t)\delta s(t)\,dt}{\int w(t)s^2(t)\,dt} \qquad \text{TTL (64)} \\
f^\dagger(t) &= \frac{w(t)s(t)}{\int w(t)s^2(t)\,dt}
\end{aligned}$$

5. `imeas = 5`, cross-correlation traveltime difference for an event kernel.

$$\begin{aligned}
\phi &= \frac{1}{2}\sum_{rp}\left[\frac{T_{obs}-T_{syn}}{\sigma_{\Delta T}}\right]^2 \\
\delta\phi &= -\sum_{rp}\frac{T_{obs}-T_{syn}}{\sigma_{\Delta T}^2}\,\delta T_{syn} = \sum_{rp}\frac{\Delta T_{syn}}{\sigma_{\Delta T}^2}\frac{\int w(t)\dot{s}(t)\delta s(t)\,dt}{\int w(t)\dot{s}^2(t)\,dt} \\
f^\dagger(t) &= \sum_{rp}\frac{w(t)\dot{s}(t)}{\int w(t)\dot{s}^2(t)\,dt}\frac{\Delta T_{syn}}{\sigma_{\Delta T}^2}
\end{aligned}$$

where the traveltime *delay* of observed data w.r.t. synthetics $\Delta T_{syn} = T_{obs} - T_{syn}$ *is used* in constructing the adjoint source. The summation is performed over receivers (r) and phases (p).

Note the cross-correlation related measurements, including `tshift, dlnA, cc_max` given by `compute_syn_cc()` correspond to

$$cc_i = \frac{\sum_j s_j d_{j+i}}{\sum_j s_j s_j}, \quad \Delta T = i_{max} * dt, \quad \Delta \ln A = \frac{1}{2}\ln\left[\frac{\int d^2(t)dt}{\int s^2(t)dt}\right], \tag{1}$$

and average error estimates are given by

$$\sigma_{\Delta T} = \sqrt{\frac{\int [d(t)-s_c(t)]^2\,dt}{\int \dot{s}_c^2(t)\,dt}}, \quad \sigma_{\Delta lnA} = \sqrt{\frac{\int [d(t)-s_c(t)]^2\,dt}{\int s_{ct}^2(t)\,dt}} \tag{2}$$

where $s_c(t)$ is the reconstructed synthetics after applying ΔT and $\Delta \ln A$ corrections.

6. `imeas = 6`, amplitude difference for an event kernel. As

$$d(t) \sim \exp(\Delta \ln A)\, s(t) \sim (1 + \Delta \ln A_{syn})\, s(t),$$

the misfit of amplitude anomaly may be defined by

$$\begin{aligned}
\phi &= \frac{1}{2}\sum_{rp}\left[\frac{\ln A_{obs} - \ln A_{syn}}{\sigma_{\Delta lnA}}\right]^2 = \sum_{rp}\frac{1}{2}\left[\frac{\Delta \ln A}{\sigma_{\Delta lnA}}\right]^2 \\
\delta\phi &= -\sum_{rp}\frac{\Delta \ln A}{\sigma^2_{\Delta lnA}}\,\delta lnA = -\sum_{rp}\frac{\Delta \ln A}{\sigma^2_{\Delta lnA}}\frac{\int w(t)s(t)\delta s(t)\,dt}{\int w(t)s^2(t)\,dt} \\
f^\dagger(t) &= -\sum_{rp}\frac{w(t)s(t)}{\int w(t)s^2(t)\,dt}\frac{\Delta \ln A}{\sigma^2_{\Delta lnA}}
\end{aligned}$$

where the measurement amplitude anomaly between data and synthetics $\Delta \ln A = \ln A_{obs} - \ln A_{syn} \sim A_{obs}/A_{syn} - 1$ *is used* in constructing the adjoint source.

7. `imeas = 7`, multitaper traveltime difference for an event kernel. The measurement between data and synthetics *is used* in constructing the adjoint source. See `multitaper_notes.pdf`.

$$\begin{aligned}
\phi_P(\mathbf{m}) &= \frac{1}{2}\sum_{rp}\int W_{P_{rp}}(\omega)\left[\tau^{obs}_{rp}(\omega) - \tau_{rp}(\omega,\mathbf{m})\right]^2 d\omega \\
\delta\phi_P &= -\sum_{rp}\int W_{P_{rp}}(\omega)\Delta\tau_{rp}(\omega,\mathbf{m})\delta\tau_{rp}(\omega,\mathbf{m})\,d\omega \\
f^\dagger_P(t) &= \sum_{rp}\sum_j h_j(t)P_j(t), \quad P_j(\omega) = W_{P_{rp}}(\omega)\Delta\tau_{rp}(\omega)p_j(\omega), \\
p_j(\omega) &= \frac{i\omega s_j(\omega)}{\sum_k |(i\omega)s_k(\omega)|^2}, \quad s_j(t) = s(t,\mathbf{m})h_j(t)
\end{aligned}$$

where $\Delta\tau_{rp}(\omega,\mathbf{m}) = \tau^{obs}_{rp}(\omega) - \tau_{rp}(\omega,\mathbf{m})$ is the frequency-dependent traveltime (*delay*) measurements,

$$W_{P_{rp}}(\omega) = \frac{W_{rp}(\omega)}{\sigma^2_{P_{rp}}(\omega)\int W_{rp}(\omega)d\omega}$$

is the frequency domain taper scaled by error estimates, and $h_j(t)$ is the j'th time-domain single taper applied to synthetics. The frequency domain taper between $[f_1, f_2]$ may be defined by

$$W_{rp}(f) = 1 - \left[\cos\frac{\pi(f - f_1)}{f_2 - f_1}\right]^\gamma, \quad \gamma = 10 \tag{3}$$

8. `imeas = 8`, multitaper amplitude difference for an event kernel. The measurement between

data and synthetics *is used* in constructing the adjoint source. See `multitaper_notes.pdf`.

$$
\begin{aligned}
\phi_Q(\mathbf{m}) &= \frac{1}{2}\sum_{rp}\int W_{Q_{rp}}(\omega)\left[\ln A^{obs}_{rp}(\omega) - \ln A_{rp}(\omega,\mathbf{m})\right]^2 d\omega \\
\delta\phi_Q &= -\sum_{rp}\int W_{Q_{rp}}(\omega)\Delta\ln A_{rp}(\omega,\mathbf{m})\delta\ln A_{rp}(\omega,\mathbf{m})\,d\omega \\
f^\dagger_Q(t) &= \sum_{rp}\sum_j h_j(t)Q_j(t), \quad Q_j(\omega) = W_{Q_{rp}}(\omega)\Delta\ln A_{Q_{rp}}(\omega)q_j(\omega), \\
& \qquad q_j(\omega) = -\frac{s_j}{\sum_k |s_k|^2} = i\omega p_j(\omega)
\end{aligned}
$$

where $\Delta\ln A_{rp}(\omega,\mathbf{m}) = \ln A^{obs}_{rp}(\omega) - \ln A_{rp}(\omega,\mathbf{m})$ is the frequency-dependent amplitude anomaly measurements, and

$$
W_{Q_{rp}}(\omega) = \frac{W_{rp}(\omega)}{\sigma^2_{Q_{rp}}(\omega)\int W_{rp}(\omega)d\omega}
$$

Both ϕ_P and ϕ_Q are dimensionaless numbers, and the adjoint sources $f^\dagger_P(t)$ and $f^\dagger_Q(t)$ have the units of $1/[\text{syn}\cdot\text{time}]$.

7 More Notes

7.1 NO_WAVEFORM_DIFFERENCE

Carl: please add more here.

7.2 DO_WEIGHTING

It is also possible to weigh windowed measurements of different categories if weighting is set in `ma_weighting` module (`DO_WEIGHTING`):

$$
\Phi = \sum_\alpha W_\alpha \sum_i \phi_{\alpha,i} \tag{4}
$$

For instance, weights can be given to windows in `P_SV/SH/Rayleigh/Love - Z/R/T`, 6 different categories of measurements separately. They are only used to change the traveltime adjoint sources (and corresponding ϕ values) at this point. How about weighting related to the noise-level of the data trace itself?

7.3 Time-domain taper

Time domain tapers are first applied to windowed data and synthetics (no matter what other options are)

$$
w(t) = 1 - \cos^a(2\pi t/T), \quad 0 \le t \le T \tag{5}
$$

where a is an even integer (i.e., $a = 10$) or a Welch window

$$w(t) = 1 - \frac{(t - T/2)^2}{(T/2)^2} \tag{6}$$

7.4 Reonstructed synthetics

$$\tilde{s}(\omega) = s(\omega)e^{\Delta \ln A(\omega) - i\omega\Delta\tau(\omega)} \tag{7}$$

with the FFT convention in D&T (p109)

7.5 Post-processing of adjoint sources

Note since data and synthetics have actually been pre-filtered, therefore the adjoint source that satisfies

$$\delta\phi = \int f^\dagger(t)s^f(t)dt = \frac{1}{2\pi}\int f^\dagger(\omega)B^*(\omega)s^*(\omega)\,d\omega = \int F^{-1}[f^\dagger(\omega)B^*(\omega)](t)s(t)\,dt \tag{8}$$

i.e., the adjoint source also needs to be filtered by the same band-pass filter to produce the exact Fréchet derivatives.

7.6 DO_RAY_DENSITY_SOURCE

Both `sigma_tau` and `sigma_dlnA`, as well as measurements `Delta_dtau/DlnA` are set to be 1, and carried all the way to the adjoint source `tr/amp_adj_src(:)`. `DO_RAY_DENSITY_SOURCE` automatically sets `Error_type=0`. It provides a good indication of the volumetric coverage of the model domain by the given measurement sets.

7.7 MTM

`dtau_mtm(:)` starts from $i > 1$, i.e., ignore the time shift at very long periods, and equivalently, the first point of `err_dt(1) = LARGE` so that it does not contribute to adjoint sources and chi values.

7.8 Questions

Should we weigh data by their noise level (before the first arrival)?

8 Miscellaneous

8.1 Bug reports and suggestions for improvements

To report bugs or suggest improvements to the code, please send an email to the CIG Computational Seismology Mailing List (cig-seismo@geodynamics.org) or Carl Tape (carltape@fas.harvard.edu), and/or use our online bug tracking system Roundup (www.geodynamics.org/roundup).

8.2 Notes and Acknowledgments

This software package initiated as a effort of Jeroen Tromp's research group at Caltech starting in 2006. The main developers of the MEAS-ADJ source code are Qinya Liu, Carl Tape, and Ying Zhou. The multitaper measurement capability originated from codes used by Ying Zhou (Zhou et al., 2004, 2005). The multitaper adjoint sources were implemented by Carl Tape (Tape, 2009, Appendix C). The organizational structure of the package was made by Qinya Liu, with adaptations by Carl Tape. The following individuals have also contributed to the development of the source code or related scripts: Vala Hjörleifsdóttir, Min Chen. The following individuals contributed to this manual: Carl Tape.

The `measure_adj` code makes use of filtering and enveloping algorithms that are part of SAC (Seismic Analysis Code, Lawerence Livermore National Laboratory) provided for free to IRIS members. We thank Brian Savage for adding interfaces to these algorithms in recent SAC distributions.

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8.3 License

(See FLEXWIN manual for possible options.)

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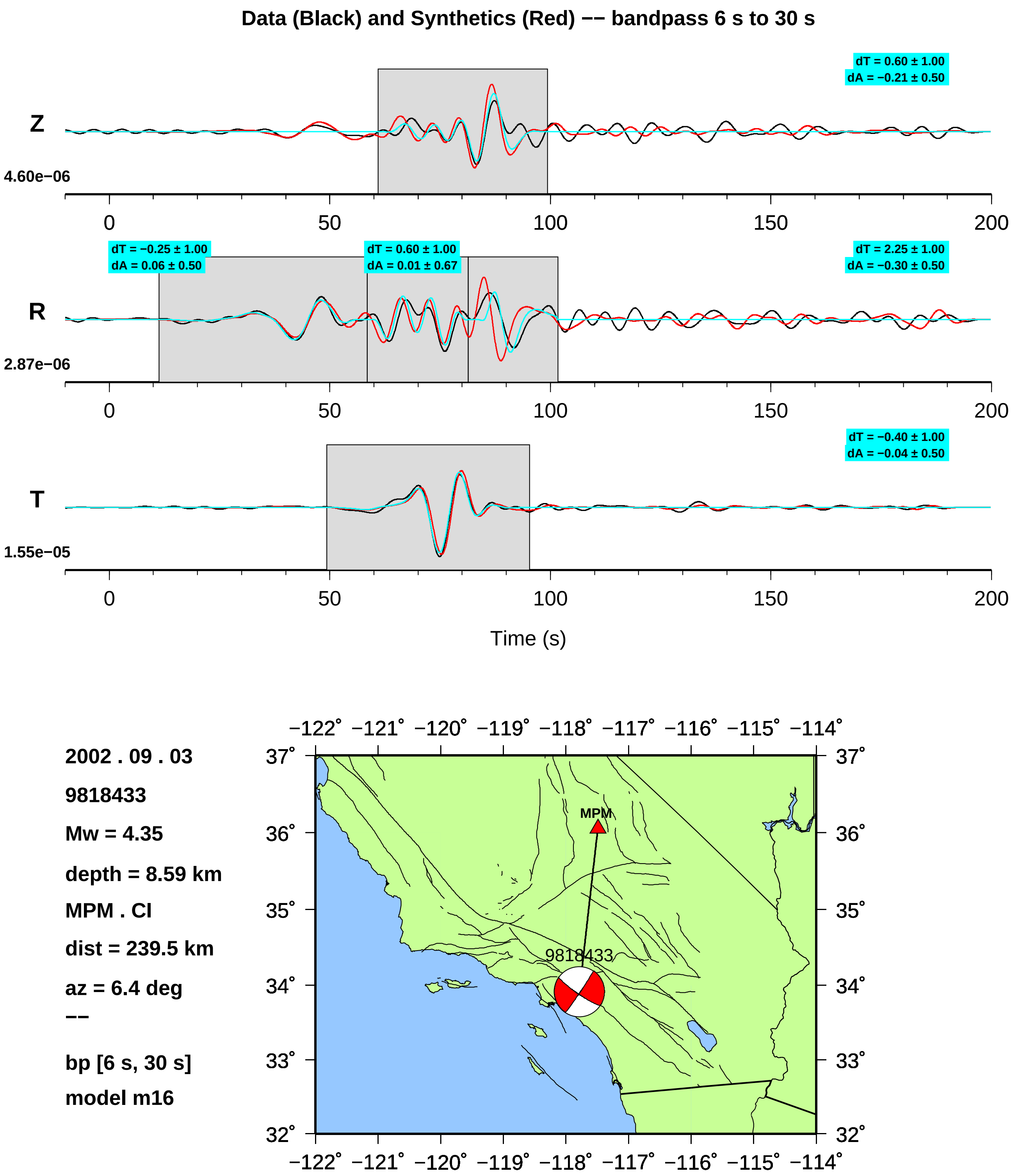


Figure 1: Test case seismograms with `COMPUTE_ADJOINT_SOURCE = .false.`. Plot shows a three-component seismogram: Z = vertical, R = radial, T = transverse. Black is the observed records, red is the synthetics, and blue is the reconstructed synthetics, made by applying the ΔT and $\Delta \ln A$ measurements within each window to the synthetics. No adjoint sources are plotted.

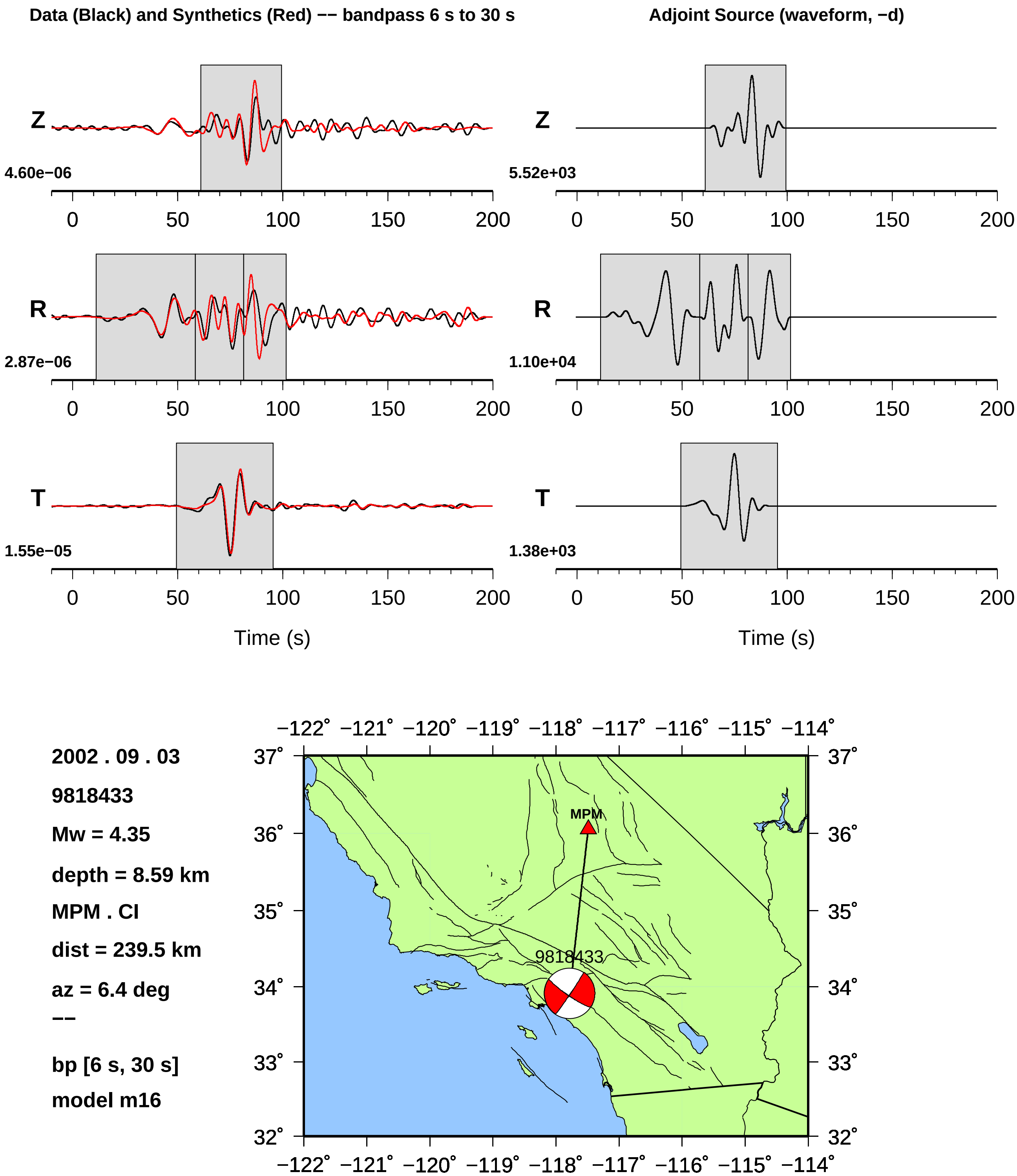


Figure 2: *Left:* Data (black), synthetics (red), and measurement windows. *Right:* Adjoint sources constructed from the data alone (`imeas = 1`), $-d(t)/\|d(t)\|^2$.

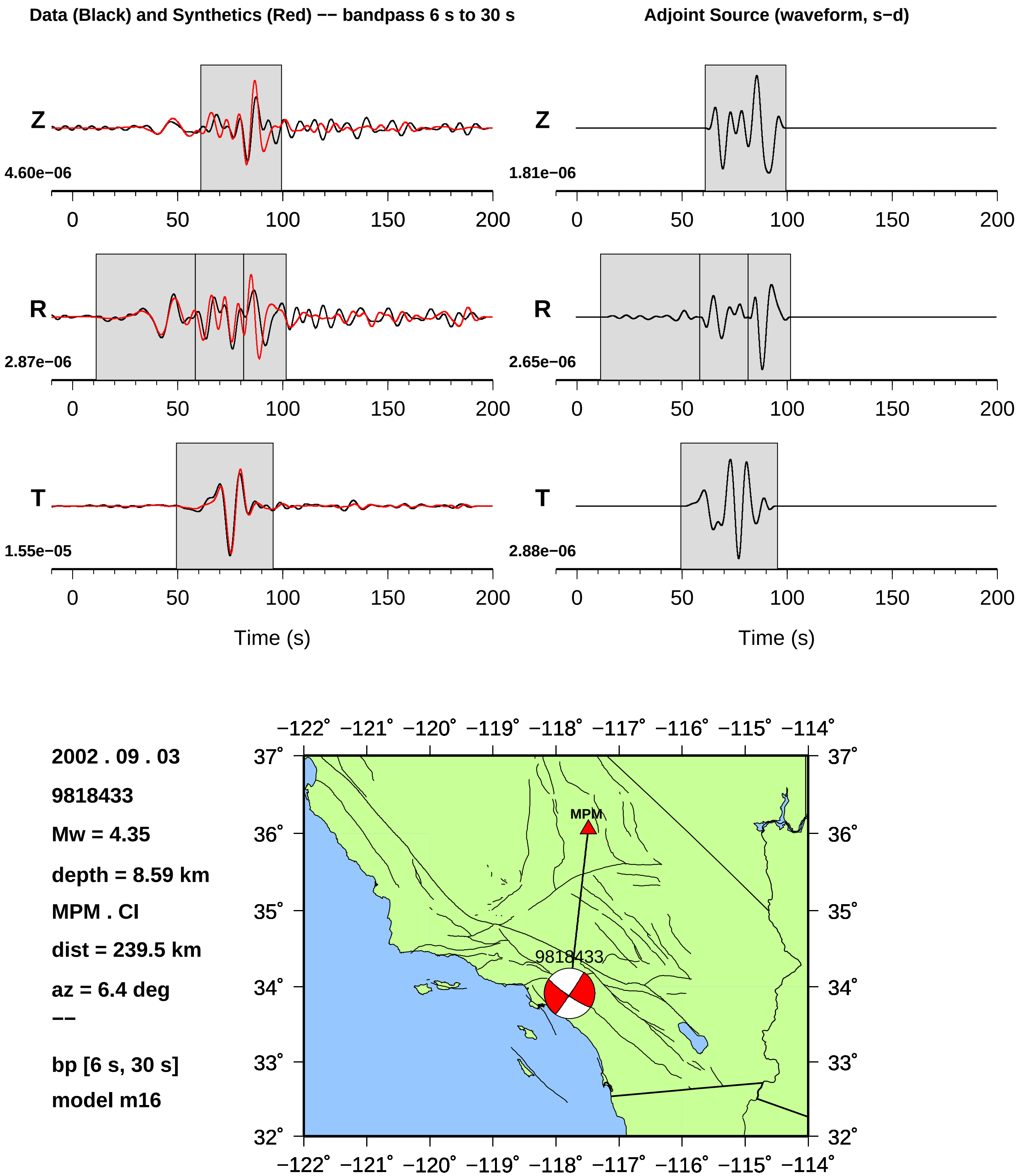


Figure 3: *Left:* Data (black), synthetics (red), and measurement windows. *Right:* Adjoint sources for a waveform difference measurement (`imeas = 2`), $\mathbf{s} - \mathbf{d}$.

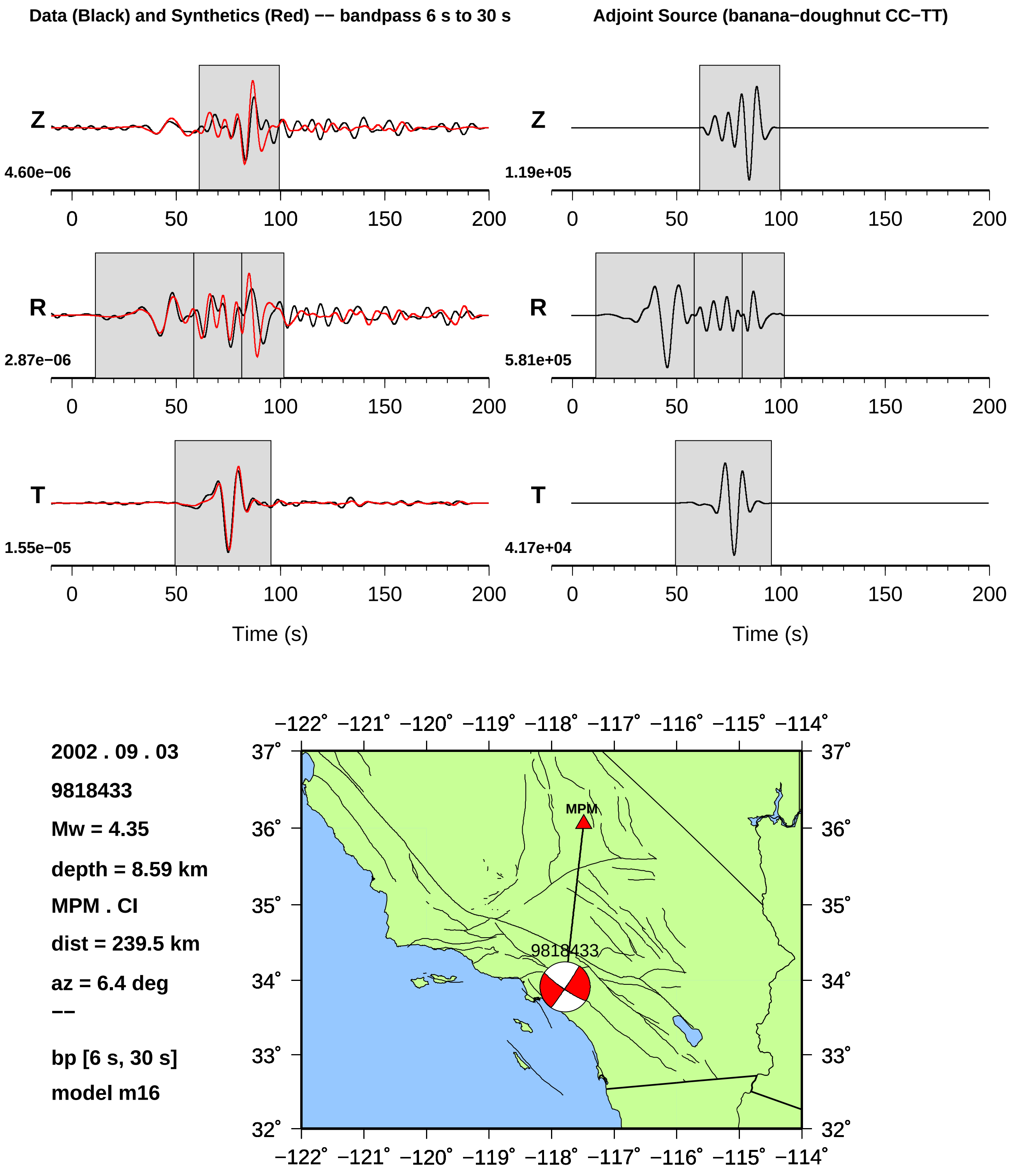


Figure 4: *Left:* Data (black), synthetics (red), and measurement windows. *Right:* Adjoint sources for a sensitivity kernel based on a cross-correlation traveltime difference, ΔT (`imeas = 3`).

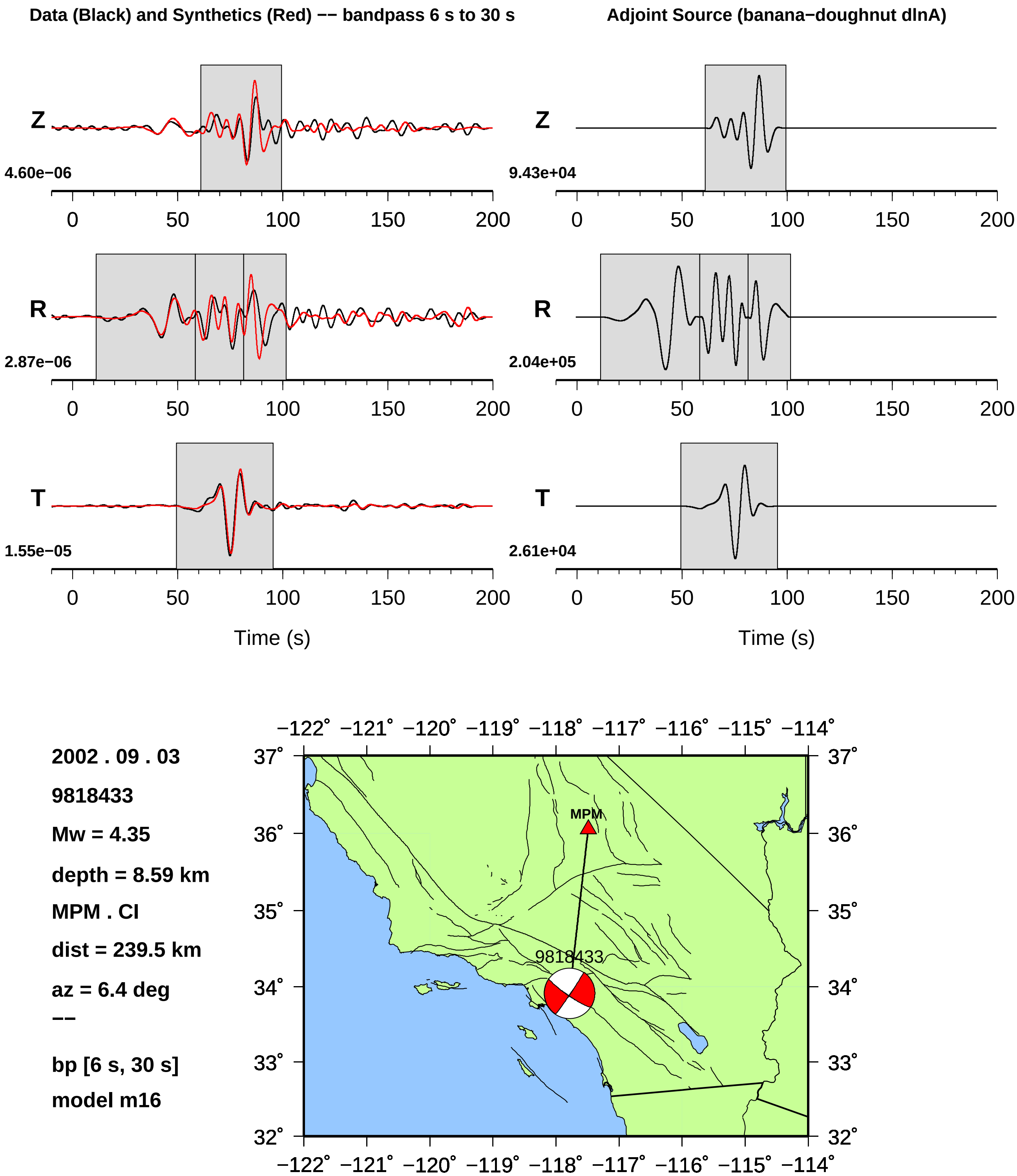


Figure 5: *Left:* Data (black), synthetics (red), and measurement windows. *Right:* Adjoint sources for a sensitivity kernel based on an amplitude difference, $\Delta \ln A$ (`imeas = 4`).

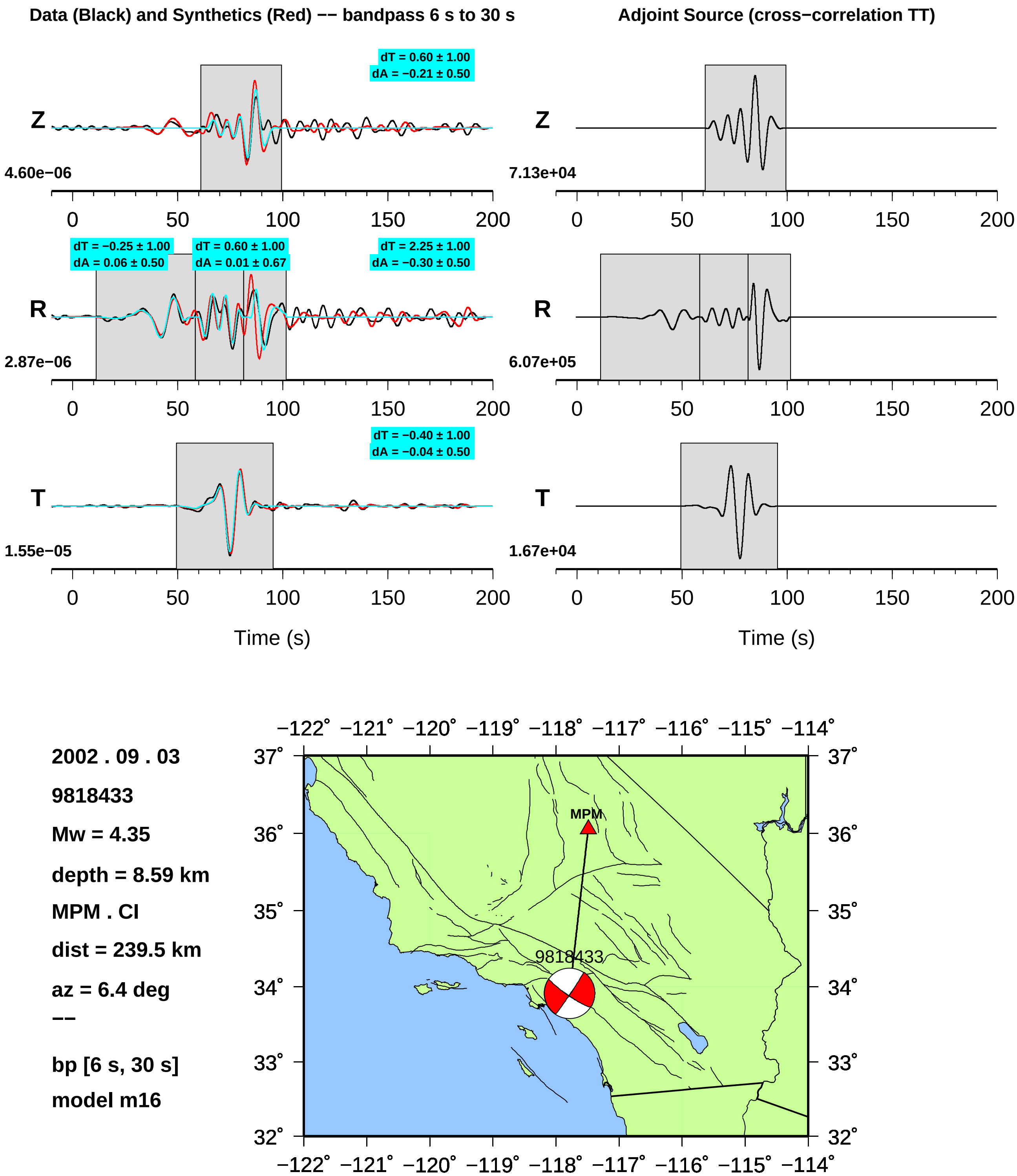


Figure 6: *Left:* Data (black), synthetics (red), and measurement windows. *Right:* Adjoint sources for a cross-correlation traveltime difference, ΔT (`imeas = 5`).

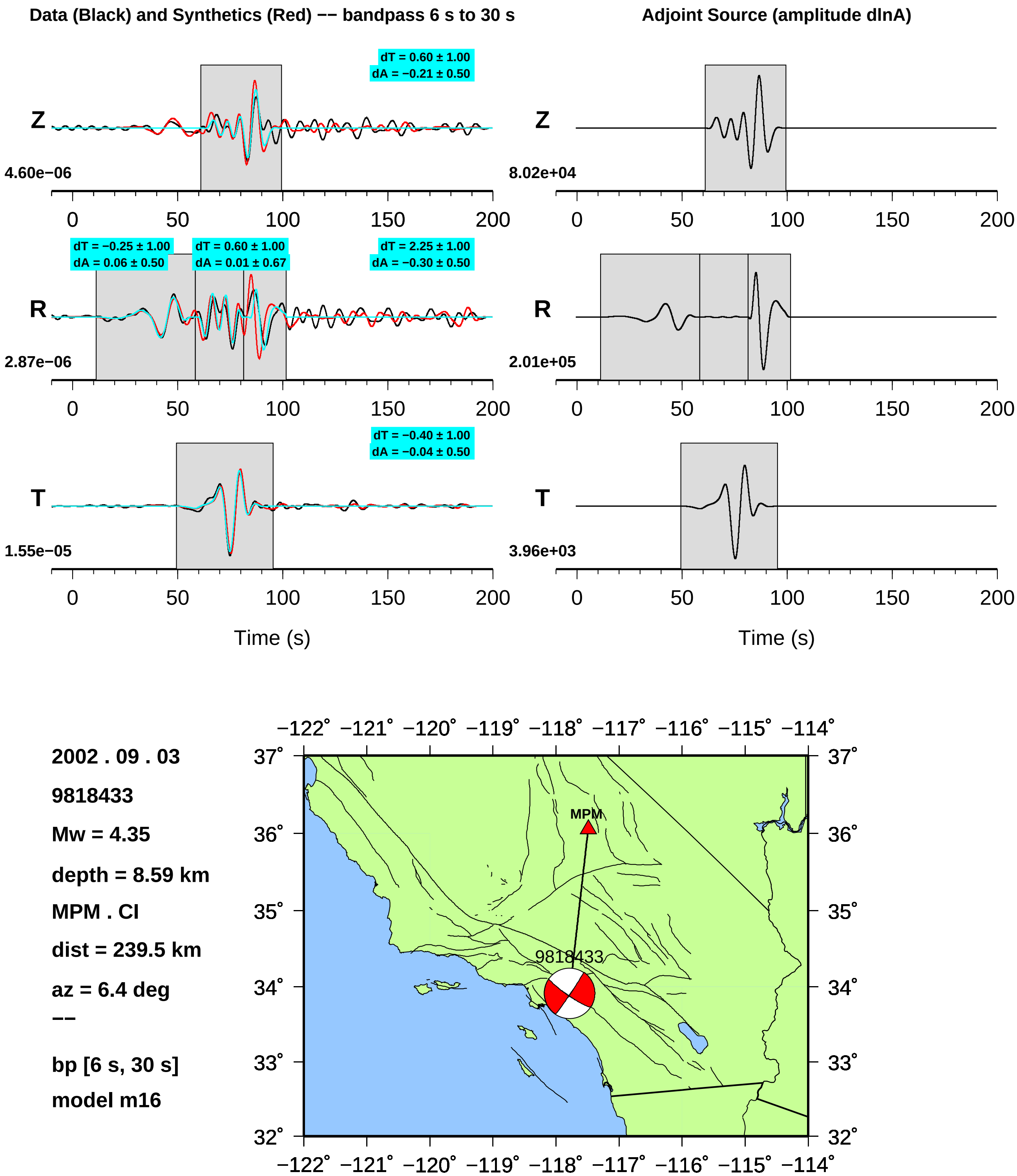
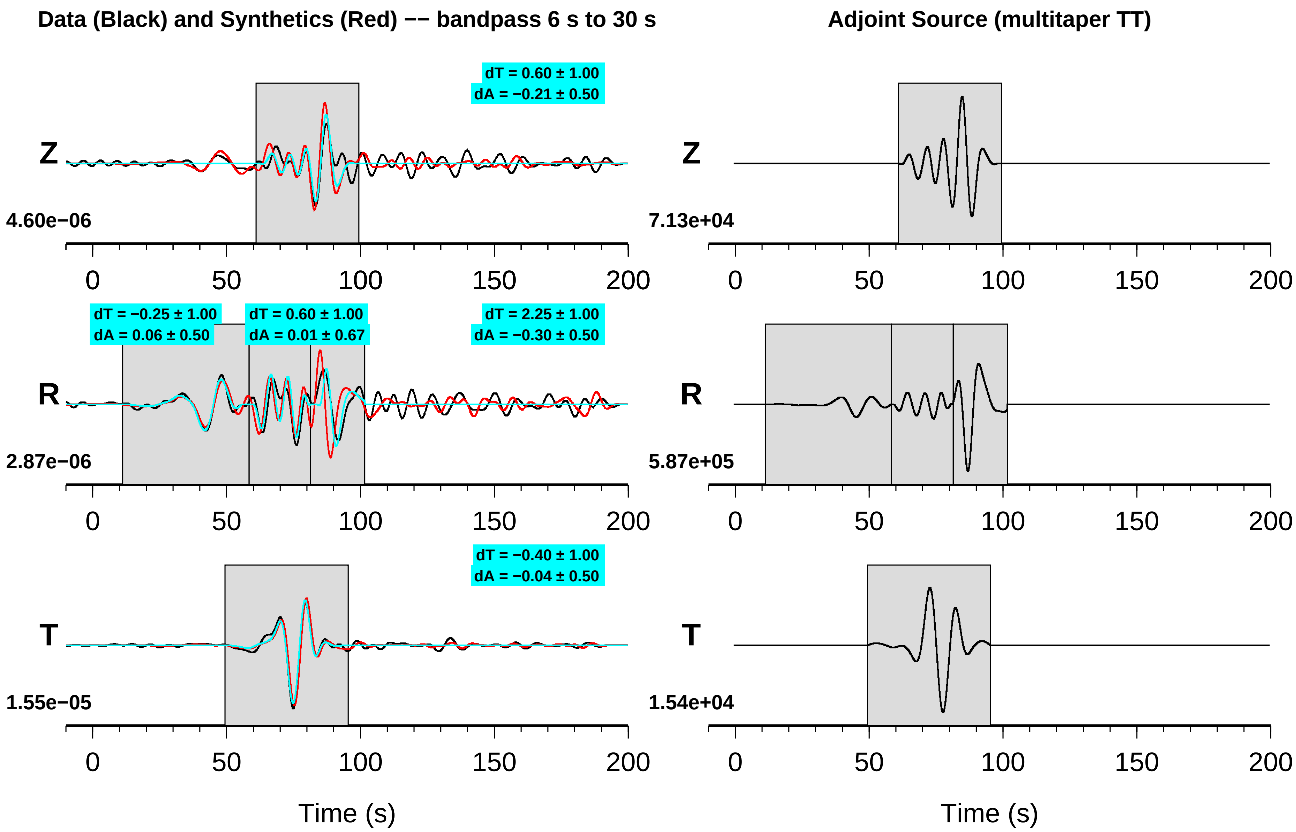


Figure 7: *Left:* Data (black), synthetics (red), and measurement windows. *Right:* Adjoint sources for an amplitude difference, $\Delta \ln A$ (`imeas = 6`).



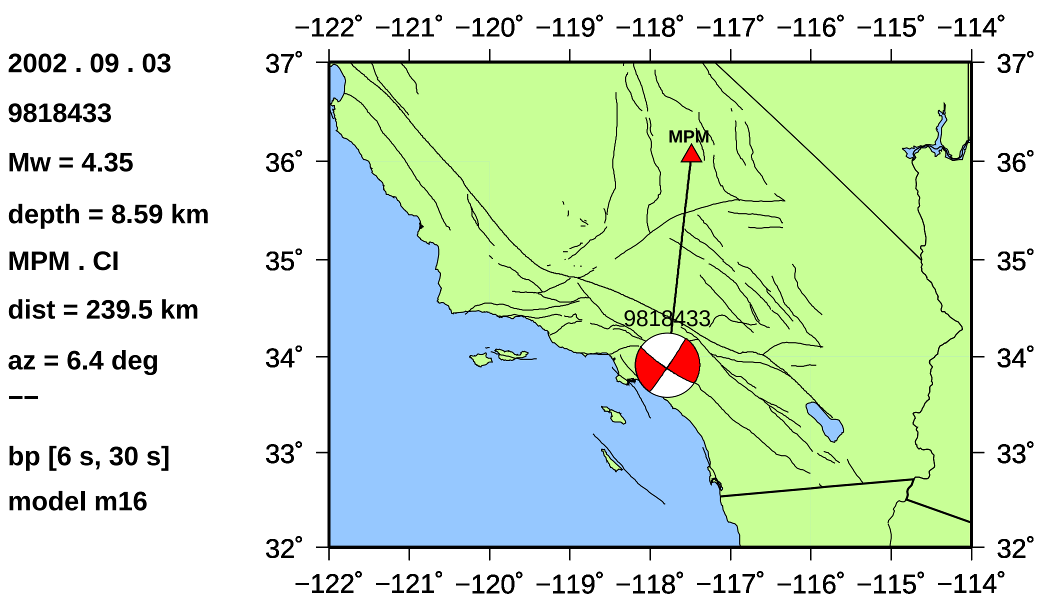


Figure 8: *Left:* Data (black), synthetics (red), and measurement windows. *Right:* Adjoint sources for a multitaper traveltime difference (`imeas = 7`).

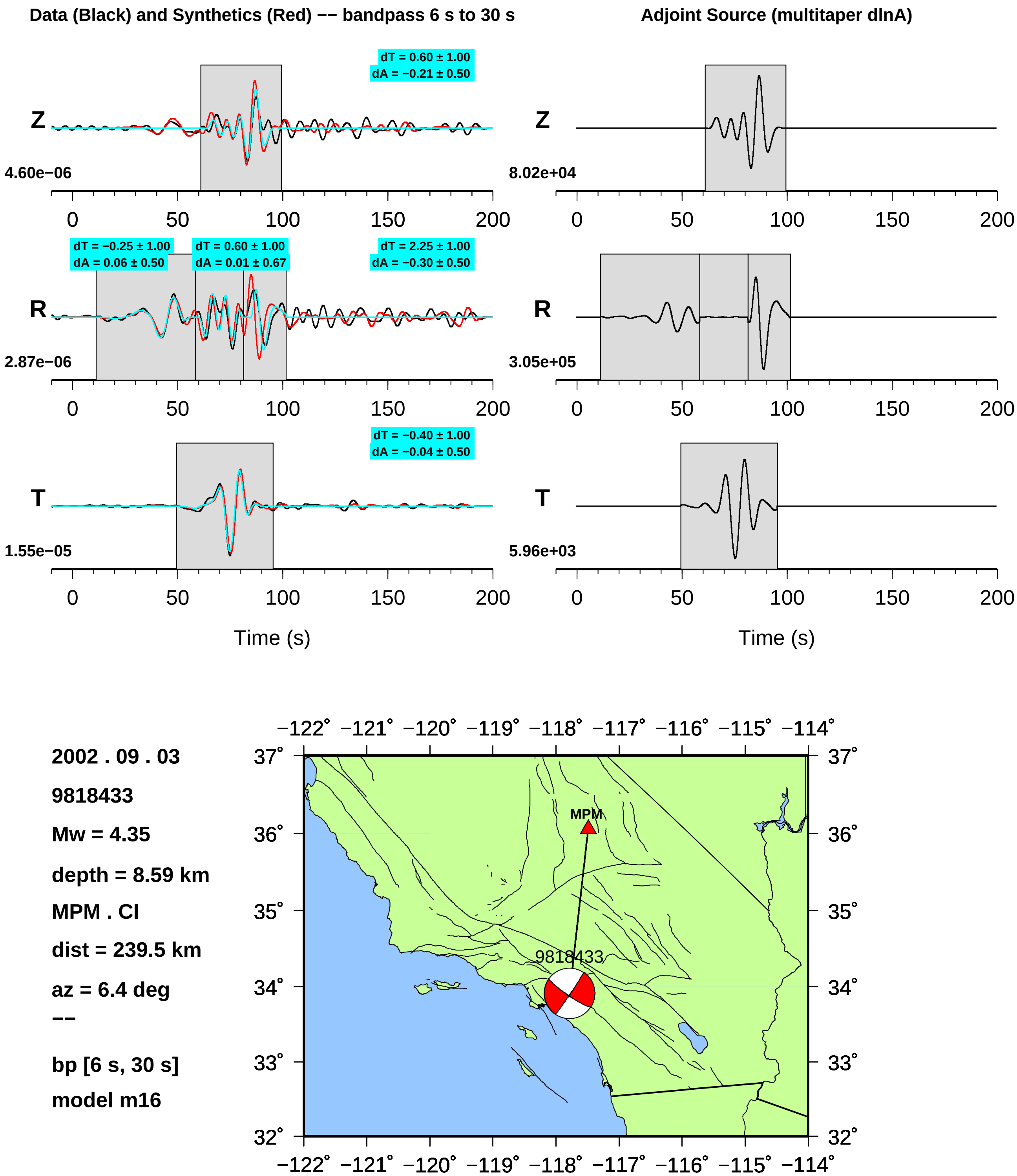


Figure 9: *Left:* Data (black), synthetics (red), and measurement windows. *Right:* Adjoint sources for a multitaper amplitude difference (`imeas = 8`).