Can Center-of-Inertia Model be Identified From Ambient Frequency Measurements?

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***Abstract*—This letter analyzes the difficulty of estimating power system inertia under ambient conditions using the Center-of-Inertia (CoI) system model. We show that the main obstacle to doing this is a difficulty in detecting a peak in the Power Spectral Density (PSD) of the frequency trace. This is due to a combination of two factors: (i) the Ornstein-Uhlenbeck (OU) process with a high mean-reversion time, which models the load disturbance and is mathematically equivalent to a low-pass filter with a high time constant; (ii) the CoI dominant mode is highly damped. This observation also explains why it is possible to estimate system inertia under ambient conditions using wide-area PMU measurements by exploiting information about inter-area oscillations which have lower damping than the dominant mode of the CoI model. We validated those findings by using the PSD of the actual 2-hour frequency trace for Great Britain from 00:00 to 02:00 of January 01, 2019.**

***Index Terms*—Frequency domain analysis, frequency measurement, load modeling, power system parameter estimation, signal processing, Stochastic processes, system inertia.**

I. Introduction

THE challenge of estimating the aggregated inertia of a power grid is gaining attention due to the replacement of conventional synchronous generators by power electronics-interfaced sources such as solar and wind. This causes not only a significant reduction in the inertia but also its value becomes highly variable, e.g., system inertia of Great Britain (GB) could vary by over 50 percent during a day [1]. Hence, accurately estimating the current value of the grid inertia is critical for assessing system security - as confirmed by the GB outage in August 2019 when the inaccurate inertia estimation was seen as a contributing factor [2]. System inertia can be relatively easily estimated using the center-of-inertia (CoI) power system model from a ringdown response triggered by a large disturbance [3], [4]. However, large disturbances rarely happen, so there is a need to develop a methodology for inertia estimation using ambient measurements at normal operation.

A naive approach to estimate inertia would be to assume a Center-of-Inertia (CoI) model such as the one corresponding to the middle gray Power System Model box in Fig. 1, which represents a generic model of an isolated (single-area) system such as GB. Estimating inertia, or any other power system parameters, would then be done in three steps. First, Fourier transform is applied to an ambient frequency trace to detect a peak in Power Spectral Density (PSD). Second, the corresponding CoI dominant mode (frequency and damping) is identified. The CoI dominant mode is strongly affected by system inertia, and, as we demonstrate later, it usually has the frequency of $\sim 0.04 - 0.25$ Hz and is well-damped. The third step would consist of estimating inertia by matching the parameters of the CoI model to the dominant mode, which is a standard problem in structural engineering [5].

However, the inherent assumption behind the first step (peak identification) is that the input is white noise while the aggregated load disturbance is more accurately described by the Ornstein-Uhlenbeck (OU) process, which is a stationary process suitable for modeling the ambient noise [6], [7]. We prove that it is difficult to detect a PSD peak due to the two factors: (i) the OU process with high mean-reversal time is effectively a low-pass filter with a high time constant suppressing oscillations in the frequency range of interest 0.04–0.25 Hz; (ii) the CoI dominant mode is well damped. This explains why it is impossible to directly estimate inertia from ambient measurements using the CoI model. We have validated those findings by comparing PSD of actual GB frequency trace with a simulated PSD of a simple CoI power system model over a range of mean-reversion times. Such a high time constant makes it impossible to extract the peak from the PSD. Next, we explain why inertia estimation is possible using a multi-machine model and utilizing wide-area ambient measurements, as reported, e.g., in [8], [9].

II. Center-of-Inertia Model

In this section, we will analyze the dynamics of the CoI model using an approximate model of GB system shown in Fig. 1 with parameters given in Table I. It consists of three blocks: the

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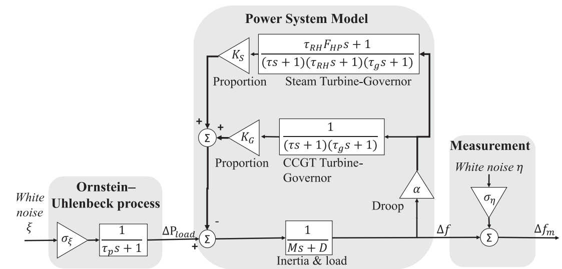

Fig. 1. The center-of-inertia model subjected to load fluctuations.

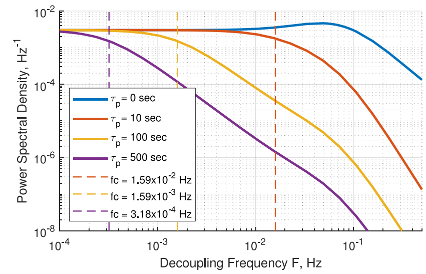

Fig. 2. Theoretical power spectral density of the center-of-inertia model with various τ_p and model parameters from Table I. The cut-off frequencies $f_c = \frac{1}{2\pi\tau_p}$ are represented by dashed lines.

TABLE I
PARAMETERS OF CENTER-OF-INERTIA MODEL

Parameter	Description	Value	Range
α	inverse droop $1/R$	11 p.u.	6–15 p.u.
M	inertia constant	10 s	9–15 s
D	damping coefficient	0.8 p.u.	0–1.5 p.u
τ/τ_g	turbine/governor time constant	0.5/0.2 s	0.2–1/0.1–0.5 s
F_{HP}	fraction of turbine power generated by high pressure section	0.33	0.2–0.5
τ_{RH}	time constant of reheater	6 s	4–11 s
K_G/K_S	proportion of CCGT/steam turbines	0.42/0.22	0.1–0.6/0.1–0.4

stochastic OU dynamics of the load disturbance, a simple power system model, and a measurement error. The power system model includes only the primary frequency control, i.e., the droop, as the secondary control is slow and does not affect the mode of interest. The parameters of the OU process are τ_p, the mean-reversion time (s), which is the characteristic time constant of the process drifting to mean [6], [7], σ_ξ, the noise amplitude (p.u.), and ξ, white noise with $\mathbf{E}[\xi(t)] = 0$, $\mathbf{E}[\xi^2(t)] = 1$. The load disturbance output, ΔP_{load} (p.u.), from the OU process is then fed into the power system model. Here, we assume that the main source of fluctuations is load disturbance, but fluctuations could also come from other sources, e.g., renewable generation. However, if fluctuations from renewables start to dominate in a system, an additional analysis might be needed since the OU process can be inappropriate to model those fluctuations. The model parameters are listed in Table I, assuming the equivalent inverse droop $\alpha = 11$ p.u. [10], the average portions of CCGT and steam turbines respectively $K_G = 0.42, K_S = 0.22$ with the range corresponding to min and max fractions for 2019 [11] (the remaining portion mainly consists of renewables not participating in the frequency regulation), and other parameters are taken from [12]. Despite the minimum K_G for GB system being about 10%, we have additionally verified our analysis assuming $K_G = 0$ as some countries may not have CCGT. The inaccuracy of frequency measurements is modeled using white noise η with 1 mHz standard deviation such that $\sigma_\eta = \frac{10^{-3}\,\text{Hz}}{50\,\text{Hz}} = 2 \times 10^{-5}$ (p.u.). Note that the measurement inaccuracy is independent of load disturbance ξ. The oscillatory mode of the Power System Model (and of the whole CoI model) is found to be $-0.21 \pm 0.29j$, corresponding to a modal natural frequency ω_n of 0.05 Hz and a damping ratio ζ of 0.58.

Using the above model from Fig. 1, we will now demonstrate the difficulties of mode extraction from ambient measurements. First, we relate the power spectral densities (PSD) of the given model with the transfer function of the linear time-invariant (LTI) system as follows:

$$S_{\Delta f}(F) = |H(j2\pi F)|^2 \sigma_\xi^2 + \sigma_\eta^2 \tag{1}$$

where F is the decomposition frequency in Hz,[1] $H(j2\pi F)$ is the transfer function of the OU with the equivalent system model given in Fig. 1. The PSD $S_{\Delta f}(F)$ is calculated using the parameters from Table I, and plots for different values of τ_p are illustrated in Fig. 2. The blue curve corresponds to $\tau_p = 0$ s, i.e., when the system is excited by the white noise, exhibiting a peak at the mode frequency ≈ 0.05 Hz. This matches the previously calculated theoretical dominant modal frequency of the CoI model. However, this peak vanishes when the input is modeled by the OU process with $\tau_p \geq 10$ s, which makes it difficult to deduce the modal frequency. Those observations were made for specific values of the parameters in Table I but we have checked them to be valid for a practical range of parameter values $M \in [9, 15]$, $\alpha \in [6, 15]$ (corresponding to the range of 30% - 75% generation participating in the primary frequency regulation [11]). This shows that the reason for the peak absence is the combination of low-pass filtering properties of the OU process with a high damping ζ of the CoI dominant mode.

To generalize our observations, we derive the peak existence conditions here. The CoI dominant pole is approximated by a harmonic oscillator. Further, the PSD of the low pass filter and the harmonic oscillator excited by the white noise with a standard deviation of 1 has the following form:

$$S_y(\Omega) = \left| \frac{1}{[\tau_p(j\Omega) + 1][(j\Omega)^2 + 2\zeta\omega_n(j\Omega) + \omega_n^2]} \right|^2 \tag{2}$$

where $\Omega = 2\pi F$ is the decoupling frequency in rad. Further, one can analyze zeros of the derivative $S_y'(\Omega)$ to obtain the peak existence conditions as follows,

$$\begin{cases} \forall \tau_p > 0, & \text{if } \zeta < 0.26 \\ 0 < \tau_p \leq \frac{\sqrt{2(1-2\zeta^2)}}{\omega_n}, & \text{if } 0.26 \leq \zeta \leq 0.70. \end{cases} \tag{3}$$

[1] F and f are both in Hz, but the former refers to the Fourier transform, and the latter to the power system frequency

The ω_n of the CoI model has a typical range of 0.04–0.25 Hz with a corresponding ζ between 0.4–0.7. Under these values and referring to (3), a τ_p higher than 10 seconds will prevent an effective parameters estimation. In practice, much higher values of τ_p have been reported in [6], [13].

The question now arises whether or not it is at all possible to estimate the system inertia from ambient measurements. While we have shown that it is difficult to do it directly using the single-machine CoI model, inertia estimation is possible by utilizing information about rotor oscillations obtained from wide-area ambient measurements that can catch inter-area modes that typically have low damping ζ of around 5% [14]). According to the conditions outlined in (3), their spectral peak will not be suppressed by the low-pass filter of the OU process. Inertia estimation is then possible by utilizing a multi-machine system model that includes a transmission network. Examples of such approaches include [8], where the inertia of each area is estimated by observing the dynamics between changes in active power and the frequency deviations. In [15], modal information extracted from ambient PMU measurements is used to estimate the equivalent inertia of each area, and in [9] to estimate the inertia of individual generators.

III. Verification Using Actual Frequency Measurements

In essence, the OU process is a phenomenological model. Therefore, both the mean-reversion time τ_p and σ_ξ cannot be measured directly but only roughly estimated from the observed system frequency measurements. In this section, we use the actual 2-hour trace of frequency (00:00–02:00, January 01, 2019) at a 1-second resolution[2] in GB system available at [16] to test the following hypothesis: frequency fluctuations comply with the system model given in Fig. 1 when the OU process has a high $\tau_p \gg 10$ s. We would like to emphasize that our intention was **not** to obtain an accurate validation of the observed trace using an accurate GB CoI model. That would be a big task on its own. Instead, we have initially used a simple generic CoI model with reasonable values of parameters given in Table I as our aim was only to confirm that τ_p is high. Later on, we verified our findings using a more detailed system model. Similarly, we were **not** aiming to estimate the exact value of τ_p, but rather to confirm that it is high enough to filter out the frequencies of interest. As we have already demonstrated, $\tau_p > 10$ s obscures the spectral peak for a wide range of system parameters, so there was no need to use accurate values of GB system parameters.

In Fig. 3, the blue signal illustrates the PSD of the actual frequency trace, while the dashed yellow curve shows the best match of the simulated PSD of the system model, which was obtained when $\tau_p = 400$ s. These two signals fit reasonably well, confirming that τ_p has indeed a high value $\gg 10$ seconds. The match is not perfect because, as we have already mentioned, the Power System Model in Fig. 1 is only a rough representation of the actual GB system. The red dashed curve shows the

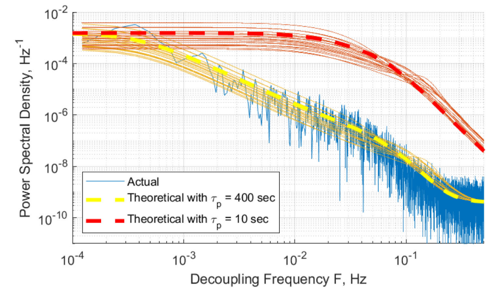

Fig. 3. Actual and theoretical PSDs for GB system over a 2 h window with 1 s resolution.

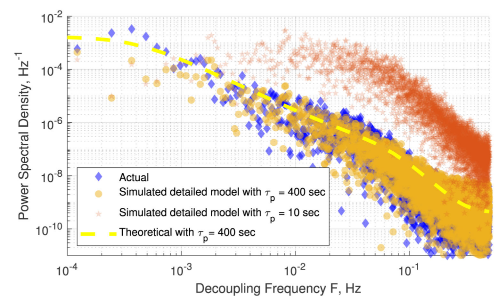

Fig. 4. Actual and simulated PSDs using the detailed model. The simulated (yellow dots) PSD overlaps with the actual (blue diamonds) PSD.

simulated PSD for $\tau_p = 10$, which clearly does not match the actual spectrum.

To verify that the inaccuracies in the model parameters do not change the order of magnitude of τ_p, Fig. 3 shows additional 40 PSDs with various model parameters randomly taken from the range given in the last column of Table I: 20 PSDs for both τ_p equal to 400 s (yellow curves) and 10 s (red curves). The result shows that the variations in model parameters do not change the spectrum shape significantly, which justifies our conclusion of high $\tau_p \gg 10$ s.

As the assumed system model shown in Fig. 1 is quite simple, we have verified the results by additionally simulating different types of power stations participating in the primary frequency control, each with their more detailed non-linear turbine-governor models.[3] Their participation in the primary frequency control was set to be equal to the GB generation mix during the 2 h period: 26% nuclear, 4% biomass, 25% Combined Cycle Gas Turbines (CCGT), 1.6% hydro [11]. The governor models used were: IEEEG1 for steam turbine (nuclear and biomass), IEEEG3 for hydro turbine, and GAST2A for CCGT; the details and typical parameters could be found in [17]. The comparison of actual and simulated PSDs given in Fig. 4 demonstrates a very good fit for $\tau_p = 400$ s and no match with smaller $\tau_p = 10$ s. Therefore, simulations of the detailed model also support our conclusions.

Finally, to confirm that the input to the system is indeed OU process, let us test a hypothesis that the input is the white noise instead. Then the estimated time constant of 400 s would not correspond to τ_p, but to $\frac{M}{D+\alpha} \approx \frac{M}{\alpha}$. This is the approximated

[2]A relatively low 1 Hz sampling rate is sufficient as the dominant mode of the CoI model usually has a frequency of about 0.04–0.25 Hz

[3]The model was posted at https://github.com/goriand/COI_PSD

dominant pole of the power system model in Fig. 1 with neglected turbine and governor time constants. Such a high ratio $\frac{M}{\alpha}$ would be well outside the reasonable range of any practical systems, confirming that it would not be appropriate to assume white noise as the input signal.

IV. Conclusion

This letter has analyzed the difficulty of estimating power system inertia using the CoI system model by extracting the dominant mode of oscillation from the PSD of the frequency trace under ambient conditions. We have shown that the difficulty of detecting a peak in the PSD in the frequency range of interest of $0.04 - 0.25$ Hz is due to a combination of two factors: the OU process, which models the input disturbance, is effectively a low-pass filter with a high time constant (mean-reversion time); and that the dominant mode is highly damped. This observation also explains why it is possible to estimate system inertia using ambient wide-area PMU measurements and a multi-machine system model by exploiting information about inter-area oscillations, which have lower damping than the dominant mode of the CoI model. The PSDs fit reasonably well when the mean reversion time is high (400 s). Such a high time constant makes it difficult to extract the peak from PSD.

References

[1] "Operating a low inertia system. a system operability framework document," National Grid ESO, Tech. Rep., Feb. 2020.

[2] J. Bialek, "What does the GB power outage on 9 august 2019 tell us about the current state of decarbonised power systems?," *Energy Policy*, vol. 146, 2020, Art. no. 111821.

[3] J. Zhao, Y. Tang, and V. Terzija, "Robust online estimation of power system center of inertia frequency," *IEEE Trans. Power Syst.*, vol. 34, no. 1, pp. 821–825, Jan. 2019.

[4] P. M. Ashton, C. S. Saunders, G. A. Taylor, A. M. Carter, and M. E. Bradley, "Inertia estimation of the GB power system using synchrophasor measurements," *IEEE Trans. Power Syst.*, vol. 30, no. 2, pp. 701–709, Mar. 2015.

[5] J. E. Mottershead, M. Link, and M. I. Friswell, "The sensitivity method in finite element model updating: A tutorial," *Mech. Syst. Signal Process.*, vol. 25, no. 7, pp. 2275–2296, 2011.

[6] C. Roberts, E. M. Stewart, and F. Milano, "Validation of the ornstein-uhlenbeck process for load modeling based on μpmu measurements," in *Proc. Power Syst. Comput. Conf.*, 2016, pp. 1–7.

[7] P. Vorobev, D. M. Greenwood, J. H. Bell, J. W. Bialek, P. C. Taylor, and K. Turitsyn, "Deadbands, droop, and inertia impact on power system frequency distribution," *IEEE Trans. Power Syst.*, vol. 34, no. 4, pp. 3098–3108, Jul. 2019.

[8] K. Tuttelberg, J. Kilter, D. Wilson, and K. Uhlen, "Estimation of power system inertia from ambient wide area measurements," *IEEE Trans. Power Syst.*, vol. 33, no. 6, pp. 7249–7257, Nov. 2018.

[9] A. Gorbunov, A. Dymarsky, and J. Bialek, "Estimation of parameters of a dynamic generator model from modal PMU measurements," *IEEE Trans. Power Syst.*, vol. 35, no. 1, pp. 53–62, Jan. 2020.

[10] J. B. Ekanayake, N. Jenkins, and G. Strbac, "Frequency response from wind turbines," *Wind Eng.*, vol. 32, no. 6, pp. 573–586, 2008.

[11] *Data Maximum Ltd*, "Gb power generation by fuel type," 2019. [Online]. Available: https://datamaximum.co.uk/datasets

[12] P. Kundur, N. J. Balu, and M. G. Lauby, *Power System Stability and Control*. New York, NY, USA: McGraw-hill, 1994.

[13] B. Schäfer, C. Beck, K. Aihara, D. Witthaut, and M. Timme, "Non-gaussian power grid frequency fluctuations characterized by lévy-stable laws and superstatistics," *Nature Energy*, vol. 3, no. 2, pp. 119–126, 2018.

[14] G. Rogers, *Power System Oscillations*. Springer Science & Business Media, 2012.

[15] D. Yang *et al.*, "Ambient-data-driven modal-identification-based approach to estimate the inertia of an interconnected power system," *IEEE Access*, vol. 8, pp. 118799–118807, 2020.

[16] National Grid ESO, "Historic frequency data," 2019. [Online]. Available: https://www.nationalgrideso.com/balancing-services/frequency-response-services/historic-frequency-data

[17] "Dynamic models for turbine-governors in power system studies," IEEE PES Task Force, IEEE Tech. Rep. PES-TR1, 2013.