Estimation of Dynamic Grid Flexibility Using Matrix Perturbation Theory

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***Abstract*—In this letter, a numerically-based method is proposed to estimate the metrics of dynamic grid flexibility, namely the inertial index, and the flexibility index. The novelty of the proposed method lies in using matrix perturbation theory, through which the sensitivity of a system to any perturbation is determined quantitatively. This notion is applied to the power system scenario under small disturbances, through which the inertial and the flexibility indices are calculated directly from the system matrix. This makes the proposed method immune to modeling complexities, and enables to implicitly understand the impact of any used generator or controller model. The applicability of the method is tested for the IEEE 39-bus system.**

***Index Terms*—Locational grid flexibility, perturbation theory, small-signal stability, inertia distribution, power system dynamics.**

I. Introduction

TWO METRICS of dynamic grid flexibility (DGF), namely the inertial index and the flexibility index were introduced in [1], [2] as network sensitivity and eigenvalue sensitivity problem, respectively. These model-based metrics were developed for planning studies to quantify the impact of fluctuating nodal power generation or demand, on grid dynamics. These previously proposed analytical methods explicitly captured the effects of parametric variations and topological changes, but were prone to modeling complexities, as any analytical formulation would be. Hence, any changes in modeling details, or inclusion of additional controller dynamics require mathematical modification of these indices.

An alternative and novel approach, using a numerical-based method for defining these indices to describe the system dynamic behavior after a perturbation, is proposed in this letter. Using the concept of matrix perturbation theory that defines the small-signal behavior of a system after a perturbation [3], [4], and the notion of condition numbers which in turn define the boundedness of these variations, the metrics of DGF are quantitatively derived. Hence, the proposed method in this letter impoves the concept of DGF further by proposing a numerically efficient way to calculate the inertial and flexibility indices. The novel contributions of this work are threefold: (a) applying the concept of matrix perturbation theory in power system flexibility studies, (b) unlike other perturbation-based methods for inertial distribution calculation [5], [6] where step changes in loads were made in every bus of their individual systems, the proposed method provides more computational simplicity than any other previous perturbation-based methods as both the inertial and flexibility indices are calculated directly from the system matrix and thus improving its practical applicability, and (c) for any given modeling detail for the generators and their associated controllers, the inertial and flexibility indices can be calculated from just the updated system matrix information, without going into the sensitivity-based approach of the previous methods [1], [2], which makes the proposed method relatively immune to modeling complexities. For a general planning problem, the model information is either available or estimable with considerable accuracy. The implications of the proposed method could be instrumental in detailed analysis of such planning studies, and to understand implicitly, the impact of any specific generator model or controller dynamics on the grid flexibility metrics.

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II. Proposed Methodology

The generalized definition of operational flexibility is provided in [7], [8] where it is described as an inherent property of the power system, which defines its *capability* to absorb *disturbances* in order to maintain its secure operation. In [8], the term *locational* flexibility was further introduced which indicates the flexibility at a given bus in the grid. By quantitatively measuring this system capability in terms of small-signal stability (particularly for small disturbances arising due to continual fluctuations in generation or demand at a particular location), the flexibility of the system can be assessed with respect to its dynamics. This explains the physical meaning of dynamic grid flexibility [1], [2].

The linearized model of an *n*-machine power system after eliminating the simultaneous algebraic variables is given in vector notation by

$$\frac{d}{dt}\Delta \boldsymbol{x} \; = \boldsymbol{A_{sys}}\, \Delta \boldsymbol{x} + \boldsymbol{B}\Delta \boldsymbol{u} \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^{t\times 1}$ is state vector, $\boldsymbol{u} \in \mathbb{R}^{c\times 1}$ is the input vector, $\boldsymbol{B} \in \mathbb{R}^{t\times c}$ is the input matrix, and $\boldsymbol{A_{sys}} \in \mathbb{R}^{t\times t}$ represents the state matrix. $\Delta(\cdot)$ denotes the incremental change in the variables post linearization around an initial operating point. DGF involves incorporating the operational flexibility of the grid, due to variations in active power generation or load into the small-signal stability problem. Therefore, the perturbations of interest are only due to the nodal active power fluctuations. Let $\lambda \in \mathbb{C}$ be an eigenvalue of $\boldsymbol{A_{sys}}$ and $\boldsymbol{v} \in \mathbb{C}^{t\times 1}$ be its associated eigenvector, so $\boldsymbol{A_{sys}}\boldsymbol{v} = \lambda \boldsymbol{v}$. For any active power perturbation in the *j*th bus (ΔP_j), let the system be represented by $\boldsymbol{A_{sys}}^* = \boldsymbol{A_{sys}} + \boldsymbol{E}$, where $\boldsymbol{E}$ denotes the effect of perturbation, i.e., $\boldsymbol{E} = f(\Delta P_j)$. The eigenvalue of the perturbed system becomes $\lambda^* = \lambda + \sigma$, and its associated eigenvector $\boldsymbol{v}^* = \boldsymbol{v} + \boldsymbol{z}$, which satisfies

$$(\boldsymbol{A_{sys}} + \boldsymbol{E})(\boldsymbol{v} + \boldsymbol{z}) = (\lambda + \sigma)(\boldsymbol{v} + \boldsymbol{z}) \tag{2}$$

The objective is to quantitatively evaluate how close $(\boldsymbol{A_{sys}} + \boldsymbol{E})$ is to $\boldsymbol{A_{sys}}$, which suggests how sensitive the system is to the perturbation.

To establish this the notion of condition number is used. Any nonsingular matrix $\boldsymbol{M}$ will have a finite condition number given by [9]

$$\kappa\ (\boldsymbol{M}) = \|\boldsymbol{M}\|\ .\|\boldsymbol{M}^{-1}\| \tag{3}$$

It is evident from (3) that the matrix condition number $\kappa(\boldsymbol{M})$ is a function of the used norm. Observing the variations in $\kappa(\boldsymbol{M})$ to the used norm is not the objective of this paper, hence in the rest of the paper only the 2-norm or spectral norm is used. Through eigen decomposition $\boldsymbol{A_{sys}}$ can be represented as $\boldsymbol{A_{sys}} = \boldsymbol{S}\boldsymbol{\Lambda}\boldsymbol{S}^{-1}$, where $\boldsymbol{S}$ is the right eigenvector matrix and $\boldsymbol{\Lambda}$ is the diagonal matrix containing the eigenvalues as its diagonal elements. A practical power system will not have null damping [10], and for such a damped system, $\boldsymbol{A_{sys}}$ will be either full rank or one-rank deficient, depending on whether the rotor angle referenced model is used or not, but with distinct eigenvalues in both cases. Hence $\boldsymbol{A_{sys}}$ is diagonalizable, and $\boldsymbol{S}$ is a full-rank matrix with a finite condition number. As $\boldsymbol{A_{sys}}$ is diagonalizable, using the Bauer-Fike theorem [11] the following inequality is established

$$|\sigma| = |\gamma - \lambda|\ \le \kappa(\boldsymbol{S}).\|\boldsymbol{E}\| \tag{4}$$

which is applicable for all eigenvalues of the system. From (4), it is implied that the changes in all eigenvalues of $\boldsymbol{A_{sys}}$ due to a perturbation are upper bounded by the condition number of its eigenvector matrix. In other words, (4) provides the information of how badly the system will behave due to a perturbation. In the context of a power system, the boundedness in the variation of the system modes can be quantified by calculating the condition number of the modeshape matrix $\kappa(\boldsymbol{S})$. It is known that the system inertial distribution measures the locational capability of the system to resist any nodal fluctuation after a disturbance [2]. Hence, for a perturbation in the power injection level at the *j*th node or bus in the grid, if the condition number of the modeshape matrix of the perturbed system $\kappa(\boldsymbol{S}^*)$ is low (or high), the variations in the eigenvalues $|\sigma|$ will be less (or more), indicating that the inertial content of that node is high (or low), which also indicates that the node is near to (or distant from) the center of inertia (COI) of the system.

The Bauer-Fike theorem gives a broad overview of the post-perturbation system behavior. It can however be desirable in power system oscillation studies, to find the sensitivity of a particular eigenvalue to perturbations, especially for dominant or critical inter-area modes. The flexibility index, which is an eigenvalue sensitivity problem, performs that task. For such mode-specific problem, the condition number of a simple eigenvalue (whose algebraic multicity is equal to 1) [3] can be used.

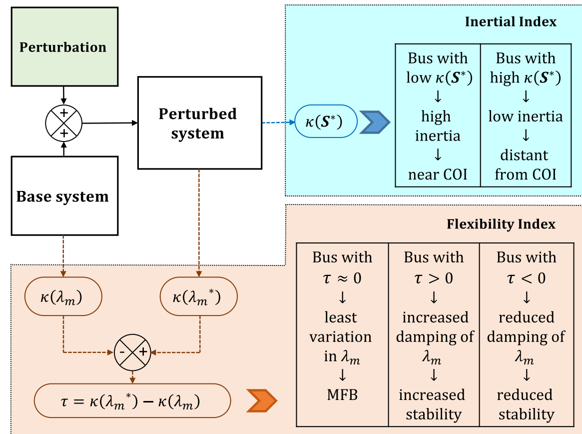

Fig. 1. The schematic diagram of the proposed methodology.

Consider $\lambda_m \in \mathbb{C}$ to be the eigenvalue associated with the most critical inter-area mode of $\boldsymbol{A_{sys}}$, and $\boldsymbol{w}_m \in \mathbb{C}^{1\times t}$ and $\boldsymbol{v}_m \in \mathbb{C}^{t\times 1}$ to be its left and right eigenvectors respectively. As the perturbation magnitude is small, so $\epsilon = \|\boldsymbol{E}\|/\|\boldsymbol{A_{sys}}\| \ll 1$, $|\sigma| = O(\epsilon)$, and $\|\boldsymbol{z}\| = O(\epsilon)$. Simplifying (2) in terms of the critical mode yields [4]

$$\sigma_m = \frac{1}{\boldsymbol{w}_m{}^H \boldsymbol{v}_m} \boldsymbol{w}_m{}^H \boldsymbol{E} \boldsymbol{v}_m + O\left(\epsilon^2\right) \tag{5}$$

where $(\cdot)^H$ is the conjugate transpose operator, and $O(\epsilon^2)$ denote the product of two $O(\epsilon)$ terms which can be neglected. For normalized eigenvectors, i.e., $\|\boldsymbol{w}_m\| = 1$, $\|\boldsymbol{v}_m\| = 1$, and $\boldsymbol{w}_m{}^H\ \boldsymbol{v}_m = |\boldsymbol{w}_m{}^H \boldsymbol{v}_m|$, and from (5) the following inequality is derived

$$|\sigma_m| \le \kappa(\lambda_m)\|\boldsymbol{E}\| \tag{6}$$

where

$$\kappa\ (\lambda_m) = \frac{1}{|\boldsymbol{w}_m{}^H \boldsymbol{v}_m|} \tag{7}$$

Here $\kappa(\lambda_m)$ is the condition number of the critical eigenvalue. From (6), it is implied that the variation of an eigenvalue due to the perturbation is bounded by its condition number which are calculated using its left and right eigenvectors. For a practical multi-area power system, the locational flexibility of a bus is determined by its tolerance to fluctuations in active power injections on the damping of the critical inter-area mode [2]. For a perturbation in the power injection level at the *j*th bus, the relative condition number is given by the difference of the condition number of the critical eigenvalue in the perturbed state $\kappa(\lambda_m{}^*)$ with the original state $\kappa(\lambda_m)$, i.e.,

$$\tau = \kappa(\lambda_m{}^*) - \kappa(\lambda_m) \tag{8}$$

Hence, if $\tau \approx 0$, then $\kappa(\lambda_m{}^*) \approx \kappa(\lambda_m)$ and therefore $|\sigma_m{}^*| \approx |\sigma_m|$, i.e., that the variation of λ_m is minimal due to the disturbance, which indicates that the bus is the most flexible bus (MFB) of the grid. If $\tau > 0$ (or $\tau < 0$) then $|\sigma_m{}^*| > |\sigma_m|$ (or $|\sigma_m{}^*| < |\sigma_m|$) which indicates that the damping of the critical mode of the post perturbation system will be more (or less) than that of the basecase, and therefore the system stability will improve (or reduce).

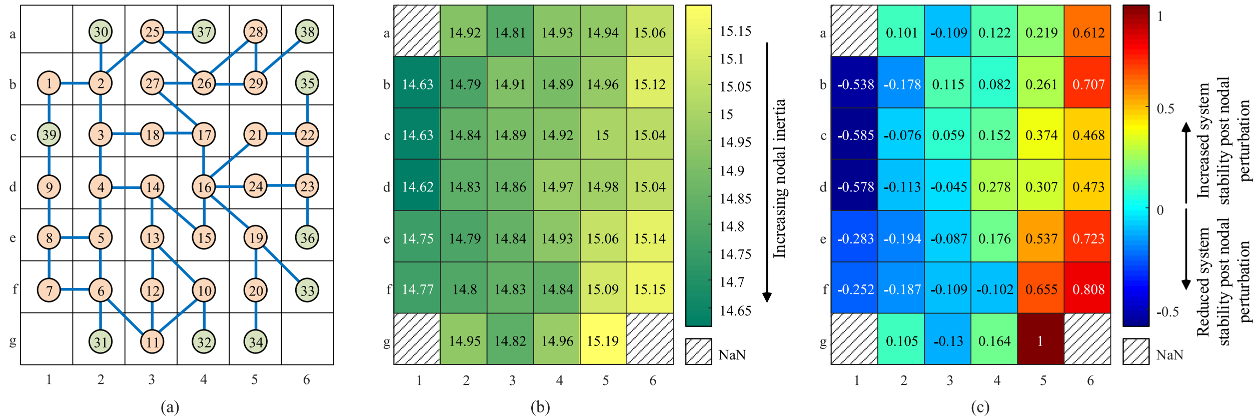

Fig. 2. Results on the IEEE 39-bus system (a) a graph-wise schematic diagram of the system, (b) heat map of the inertial distribution, and (c) heat map of the flexibility index.

The schematic diagram of the proposed methodology in calculating the system inertia and flexibility indices using only the system matrix information, and some mathematical tools like eigenvalue analysis, critical mode identification, and condition number calculation is shown in Fig. 1.

III. Case Study

The proposed methodology discussed in the previous section was implemented on the IEEE 39-bus system. To emulate the perturbations, a load increase of 60 MW (approximately 1% of the total system active power load) was temporarily added to every bus to produce a library of 39 perturbed systems (test cases). From the system matrix of the basecase and the perturbed systems, the eigenvector matrix was calculated, and the most critical mode was identified for all the cases individually. Finally, by calculating $\kappa(\boldsymbol{S}^*)$ and τ using (3) and (8) respectively, the nodal inertial and the flexibility indices of the system were estimated. The results on the 39-bus system are demonstrated in detail in Fig 2.

In Fig. 2(a), a rearranged meshed schematic of the 39-bus system is shown where every bus can be spatially identified. All the generators were initially modeled in their classical representation with mechanical damping $D = 0.1$ p.u. First, the inertial index was estimated by calculating $\kappa(\boldsymbol{S}^*)$ for every perturbed test case, as shown by a heat map in Fig. 2(b). It was observed that the locations in deep green have the least $\kappa(\boldsymbol{S}^*)$ value, indicating that the boundedness of eigenvalue variations at these locations are less than the rest of the grid, thereby making these buses closest to the COI. On the other hand, the buses highlighted in yellow have a higher bound, which make these buses the most distant locations from the COI. It should be noted that the numerical value of $\kappa(\boldsymbol{S}^*)$ is trivial, and it changes with any given operating conditions which may vary due to parametric variations, or modeling changes. For a given system model and operating condition, only the bus ranking obtained through these numerical values hold significance. In Fig. 2(c), the heat map of the estimated flexibility distribution is shown spatially. The relative condition number τ was calculated for every test case and normalized for better clarity. It was observed that for the near COI buses shown in deep blue, τ is negative which indicates that any increase in active power loads in these locations will reduce the damping of the critical inter-area mode. For buses that are most distant from the COI, shown in brown and red, τ is positive which implies that an increase of load in these locations will improve the damping of the critical mode. For bus 14, τ is minimal. Hence, the variation of critical mode due to the load perturbation at this bus will be least, which makes this location the most tolerant to power fluctuations, and thereby the MFB of the system. These results of bus rank ordering in terms of their inertial and flexibility indices are identical to the analytically estimated results in [1], but requires far less computational efforts and mathematical complexity.

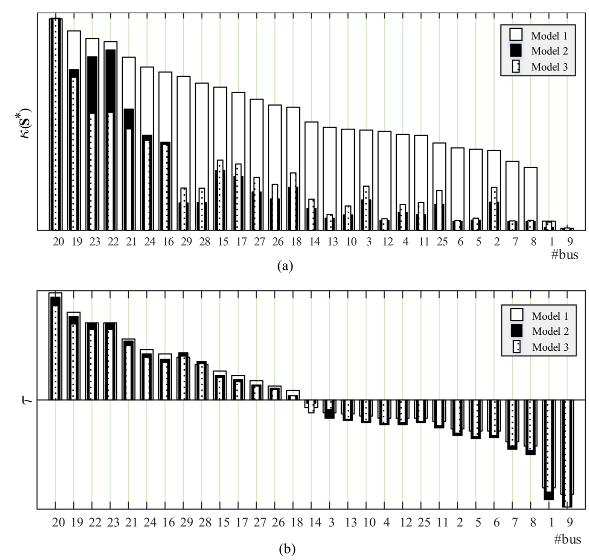

Fig. 3. Impact of modeling changes on (a) inertial index, (b) flexibility index.

The tractability of the proposed method in capturing the impact of any modeling modification like change of machine model, or inclusion of controllers on the inertial and flexibility distribution is highlighted in Fig. 3. Here, the previously used system model where all machines are classically represented is denoted as Model 1. For Model 2, only the machine connected to bus 33 was modified to be represented by the flux decay model [12]. For Model 3, the same machine at generator 33 is now represented by flux decay model with fast exciter having parameters: $K_a = 100$, and $T_a = 0.15$. The subtle variations in the bus ranks in the inertial and flexibility indices can be seen in Fig. 3(a), and Fig. 3(b), respectively, where the buses are ordered with respect to Model 1 ranking. Hence, capturing the dynamics implicitly of any new system model, without requiring any modification in the mathematical tool, is the primary advantage of the proposed method over other analytical methods.

IV. Conclusion

A new methodology in estimating the metrics of DGF is proposed, where the notion of matrix perturbation theory is implemented to the power system small-signal stability problem. The rationale of using the condition number of the eigenvector matrix, and of a simple eigenvalue to calculate the inertial and flexibility indices is established. Using only the system matrix information and some simple mathematical tools, the proposed method provides the simplest and accurate estimate of the locational inertial and flexibility distribution in the grid with no analytical complicacies.

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