A Method Integrating Deterministic and Stochastic Approaches for the Simulation of Voltage Unbalance in Electric Power Distribution Systems

Yaw-Juen Wang, *Member, IEEE* and Lambert Pierrat

***Abstract*—This paper deals with modeling, analysis and Monte Carlo simulation of three-phase voltage unbalance based on data measured from a 69/11-kV distribution substation. Random variation of the voltage unbalance factor is modeled with the aid of correlated Gaussian random variables that represent random variations in three-phase active and reactive powers. Also presented in this paper is a network reduction technique using multiple-port Thévenin equivalent circuits that allows the Monte Carlo simulation to be carried out faster. Comparison of simulated and recorded probability distributions of voltage unbalance factors is given, and good agreement has been obtained.**

***Index Terms*—Measurement, Monte Carlo simulation, network reduction, random processes, voltage unbalance.**

I. INTRODUCTION

AMONG a number of factors that impair the quality of electric power, voltage unbalance has become an issue that merits particular attention. For instance, the effects of voltage unbalance on the performance of induction motors [1], [2], on harmonic contents of power electronics converters [3], [4], on line losses [5] and on railway power systems [6], [7], have been active topics studied by electrical engineers.

Unbalance of three-phase voltages results from asymmetry of line/cable impedances and from inequality of loads in three phases [8]. The former is related to the structure of the electric power system in which geometric allocation of lines/cables significantly influences their impedances. Efforts are in general made to reduce the asymmetry of transmission line impedances by transposition. On the other hand, voltage unbalance caused by uneven distribution of loads over three phases is more difficult to mitigate.

Large single-phase loads such as single-phase induction motors, traction locomotives, induction heating, etc., are typical examples that cause voltage unbalance in industrial electric power systems. In low voltage residential and/or commercial systems, single-phase loads account for the majority of power consumption. Wherever possible, efforts are made to distribute the single-phase loads uniformly over three phases. From a statistical point of view, however, uniformly distributing single-phase loads over three phases only ensures their respective expected values of loads to be approximately equal. But it is unlikely that at a given instant loads in the three phases are balanced because they vary in a random manner. In other words, even if the average loads in the three phases are kept the same, the instantaneous power demands in the three phases differ from each other, leading to unbalanced voltages at the point of common coupling.

The level of unbalance of a set of three-phase voltages is often measured by the ratio of their negative- to positive-sequence component. This ratio is termed as the voltage unbalance factor (*VUF*). By assuming the magnitudes of three-phase voltage phasors being Gaussian random variables and no phase angle deviations (i.e., 120° out of phase with each other), Pierrat and Morrison [9] developed a probabilistic model of the VUF and showed that the VUF would have a Gaussian distribution if the unbalance were mainly caused by asymmetry in system impedances, and a Rayleigh distribution if system impedances were symmetric. This model, though capable of providing physical insights into the causes of voltage unbalance, is somewhat restricted by its assumptions.

Instead of assuming normally distributed voltage magnitudes, it seems more plausible to assume that the loads (including active and reactive powers) in the three phases, at a given moment, approach a jointly normal distribution as far as the number of electric appliances supplied by the power system is large enough (and it is usually the case). This assumption has its theoretical base supported by the central limit theorem, and hence allows the restrictions of the model proposed by [9] to be released and more accurate results to be obtained. Another advantage of using a jointly normal distribution model lies in its ability of handling correlation between all Gaussian random variables. This ability is especially important in modeling three-phase active and reactive powers since they are usually strongly correlated. In this paper, we try to separate three-phase active and reactive powers in to deterministic and stochastic components. The former is represented by simple time functions while the latter is modeled by correlated Gaussian random variables.

This paper combines numerical techniques of load flow and Monte Carlo simulation and proposes a novel method of simulating voltage unbalance caused by random fluctuation of loads. The simulation begins with generation of six *correlated* Gaussian random variables representing three-phase active and reactive powers. This is followed by a load flow calculation to determine the three-phase voltage phasors at the point of common coupling. The VUF can then be obtained by a

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Y.-J. Wang is with the Department of Electrical Engineering, National Yunlin University of Science and Technology, Tou-Liu, Yun-Lin 640, Taiwan (e-mail: wangyj@yuntech.edu.tw).

L. Pierrat is with the Laboratory of Geophysical and Industrial Flows, University of Grenoble, 38041 Grenoble, France.

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symmetrical component conversion. The variation in time and the probability density functions (PDFs) and the cumulative distribution functions (CDFs) of the VUF are compared with data measured from a 69/11 kV substation for validation.

II. Formulation

A. Generation of Correlated Gaussian Random Variables

Let P_a, P_b and P_c be the total active powers and Q_a, Q_b and Q_c the total reactive powers absorbed by a cluster of loads in phases a, b and c of an electric power distribution system. For sufficiently large number of loads, P_a, P_b, P_c, Q_a, Q_b and Q_c have approximately a jointly normal distribution. If the distribution system supplies only three-phase loads, random variables P_a, P_b, P_c, Q_a, Q_b and Q_c are *strongly correlated*, owing to the symmetry of variation in each phase. On the contrary, if all loads are (statistically) independent single-phase loads, the six random variables are *uncorrelated*. A realistic distribution system supplies a mixture of three-phase and single-phase loads. Hence, the real situation may be somewhere between the two extreme cases just mentioned.

In order to simulate random variations of active and reactive powers in the three phases, correlated Gaussian random numbers must be generated. A Monte Carlo method proposed by Wang and Pierrat [10] for generating six correlated Gaussian random variables using Box and Muller's method [11] and Cholesky decomposition [12], is used in this paper for generating random variables P_a, P_b, P_c, Q_a, Q_b and Q_c.

B. Network Reduction and Load Flow Study

Monte Carlo method is a powerful tool for the simulation of randomly varying loads, and it can be combined with the conventional deterministic load flow calculation to ascertain random variation of voltages at the load buses. To do so, the deterministic three-phase load flow calculation is repeated a sufficiently large number of times. Each time the nodal active and reactive powers are changed by a generation and transformation of random numbers, as mentioned in the previous section. The probability distributions of nodal voltages, and thus that of the VUF, can then be obtained. However, the amount of calculation would be prohibitive for any practical distribution system if repetitive three-phase load flow calculations were to be carried out by considering random variations of active and reactive powers at all the load buses.

In fact, it is often the case that the voltage unbalance problem occurs only at some particular substations or feeders that supply large single-phase loads (e.g., railway traction locomotives) or unbalanced three-phase loads (e.g., arc furnaces). In this case, one may wish to confine his/her study of voltage unbalance to those buses to which unbalanced loads are connected. The notion of *network reduction* helps one reach this goal by representing a large network with its Norton or Thévenin equivalence while retaining the buses of special interests. The *network reduction* is important when Monte Carlo simulation and three-phase load flow study are to be combined together to calculate the probability distribution of the VUF at a particular load bus, since both methods are computationally intensive. For a $(k+m)$-bus system in which k buses are to be eliminated and m buses to be retained, its nodal equations in matrix format can be written as

$$\boldsymbol{I} = \boldsymbol{Y}\boldsymbol{V} \tag{1}$$

where $\boldsymbol{V}$, $\boldsymbol{I}$, and $\boldsymbol{Y}$ are defined as shown at the bottom of the page.

Note that in (1) the buses to be retained are numbered last m nodes. Eq. (1) is partitioned as shown above to obtain

$$\left[\begin{array}{c|c} \boldsymbol{Y}_{EE} & \boldsymbol{Y}_P \\ \hline \boldsymbol{Y}_P^T & \boldsymbol{Y}_{PP} \end{array}\right] \left[\begin{array}{c} \boldsymbol{V}_E \\ \hline \boldsymbol{V}_P \end{array}\right] = \left[\begin{array}{c} \boldsymbol{I}_E \\ \hline \boldsymbol{I}_P \end{array}\right]. \tag{2}$$

Eliminating $\boldsymbol{V}_E$ yields

$$(\boldsymbol{Y}_{PP} - \boldsymbol{Y}_P^T \boldsymbol{Y}_{EE}^{-1} \boldsymbol{Y}_P)\boldsymbol{V}_P = \boldsymbol{I}_P - \boldsymbol{Y}_P^T \boldsymbol{Y}_{EE}^{-1} \boldsymbol{I}_E. \tag{3}$$

Eq. (3) can be rewritten as

$$\boldsymbol{Y}_{eq}\boldsymbol{V}_P = \boldsymbol{I}_{eq}^{(0)} \tag{4}$$

where

$$\boldsymbol{Y}_{eq} = \boldsymbol{Y}_{PP} - \boldsymbol{Y}_P^T \boldsymbol{Y}_{EE}^{-1} \boldsymbol{Y}_P, \tag{5}$$

$$\boldsymbol{V} = [V_1 \quad V_2 \quad \cdots \quad V_k \,|\, V_{k+1} \quad \cdots \quad V_{k+m}]^T,$$
$$\boldsymbol{I} = [I_1 \quad I_2 \quad \cdots \quad I_k \,|\, I_{k+1} \quad \cdots \quad I_{k+m}]^T, \quad \text{and}$$

and

$$\boldsymbol{Y} = \left[\begin{array}{cccc|ccc} Y_{11} & Y_{12} & \cdots & Y_{1k} & Y_{1,\,k+1} & \cdots & Y_{1,\,k+m} \\ Y_{21} & Y_{22} & \cdots & \vdots & Y_{2,\,k+1} & \cdots & Y_{2,\,k+m} \\ \vdots & & \ddots & \vdots & \vdots & \ddots & \vdots \\ Y_{k1} & \cdots & \cdots & Y_{kk} & Y_{k,\,k+1} & \cdots & Y_{k,\,k+m} \\ \hline Y_{k+1,\,1} & Y_{k+1,\,2} & \cdots & Y_{k+1,\,k} & Y_{k+1,\,k+1} & \cdots & Y_{k+1,\,k+m} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ Y_{k+m,\,1} & Y_{k+m,\,2} & \cdots & Y_{k+m,\,k} & Y_{k+m,\,1} & \cdots & Y_{k+m,\,k+m} \end{array}\right].$$

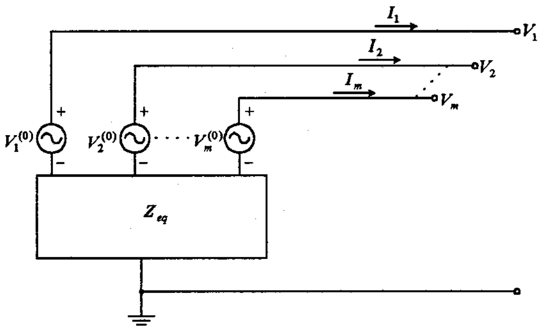


Fig. 1. An m-port Thévenin equivalent circuit of the distribution system.

$$\boldsymbol{I}_{eq}^{(0)} = \boldsymbol{I}_P - \boldsymbol{Y}_P^T \boldsymbol{Y}_{EE}^{-1} \boldsymbol{I}_E. \tag{6}$$

It is noted that in (5) and (6), $\boldsymbol{Y}_{eq}$ represents the Norton equivalent admittance matrix and $\boldsymbol{I}_{eq}^{(0)}$ the Norton current vector. The corresponding Thévenin impedance matrix and voltage source vector can be given by

$$\boldsymbol{Z}_{eq} = \boldsymbol{Y}_{eq}^{-1} \tag{7}$$

$$\boldsymbol{V}_{eq}^{(0)} = \boldsymbol{Y}_{eq}^{-1} \boldsymbol{I}_{eq}^{(0)}. \tag{8}$$

An m-port Thévenin equivalent circuit is shown in Fig. 1, in which m buses are retained for the study of voltage unbalance problem. In the figure, the currents $I_1, I_2, \ldots, I_m$ can be written in the matrix format as

$$\boldsymbol{I} = [I_1 \quad I_2 \quad \cdots \quad I_m]^T = \left\{ \left[\frac{S_1}{V_1} \quad \frac{S_2}{V_2} \quad \cdots \quad \frac{S_m}{V_m} \right]^T \right\}^* \tag{9}$$

where $S_1, S_2, \ldots, S_m$ refer to the complex powers drawn at each corresponding bus and the asterisk $(*)$ to the conjugate. The voltage vector can be given by

$$\mathbf{V} = [V_1 \quad V_2 \quad \cdots \quad V_m]^T = \boldsymbol{V}_{eq}^{(0)} - \boldsymbol{Z}_{eq}\boldsymbol{I}. \tag{10}$$

Eqs. (9) and (10) lend themselves to the iterative method of finding the solutions of $\boldsymbol{V}$ for known complex powers $S_1, S_2, \ldots, S_m$. In this paper, Gauss–Seidel iterative method [13] is used to determine the bus voltages.

C. Voltage Unbalance Factor

The VUF is defined as the ratio of negative-sequence to positive-sequence component

$$\mathrm{VUF} = |\boldsymbol{V}^- / \boldsymbol{V}^+| \times 100\% \tag{11}$$

where $j = \sqrt{-1}$, and $\boldsymbol{V}^-$ and $\boldsymbol{V}^+$ refer to negative-sequence and positive-sequence voltages. $\boldsymbol{V}^-$ and $\boldsymbol{V}^+$ are given using Fortescue transformation [14] by

$$\boldsymbol{V}^- = (\boldsymbol{V}_{AT} + a^2 \boldsymbol{V}_{BT} + a\boldsymbol{V}_{CT})/3 \tag{12}$$

$$\boldsymbol{V}^+ = (\boldsymbol{V}_{AT} + a\boldsymbol{V}_{BT} + a^2 \boldsymbol{V}_{CT})/3 \tag{13}$$

where $\boldsymbol{a} = \exp(j2\pi/3)$, and $\boldsymbol{V}_{AT}$, $\boldsymbol{V}_{BT}$ and $\boldsymbol{V}_{CT}$ refer to the three-phase voltage phasors.

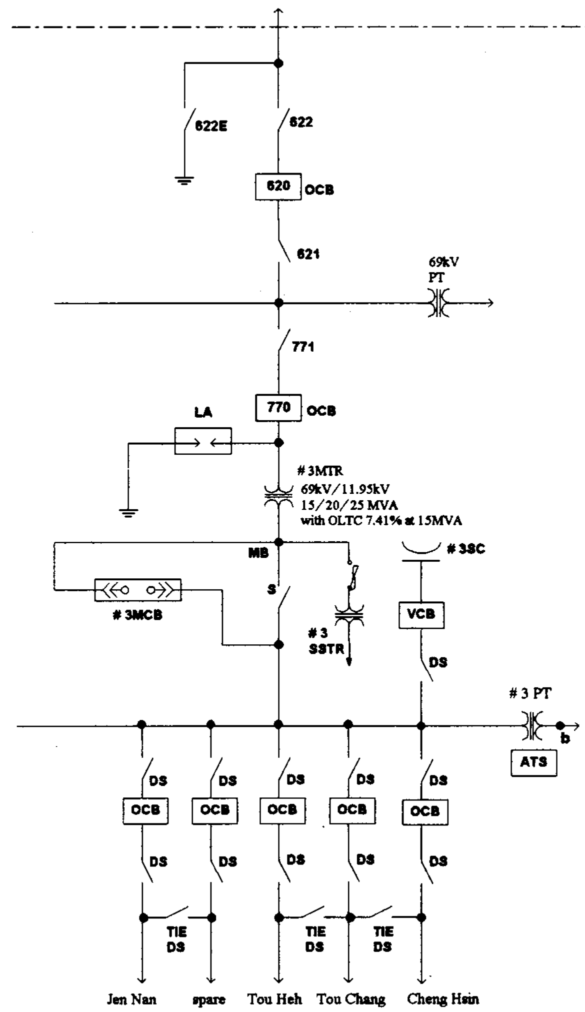


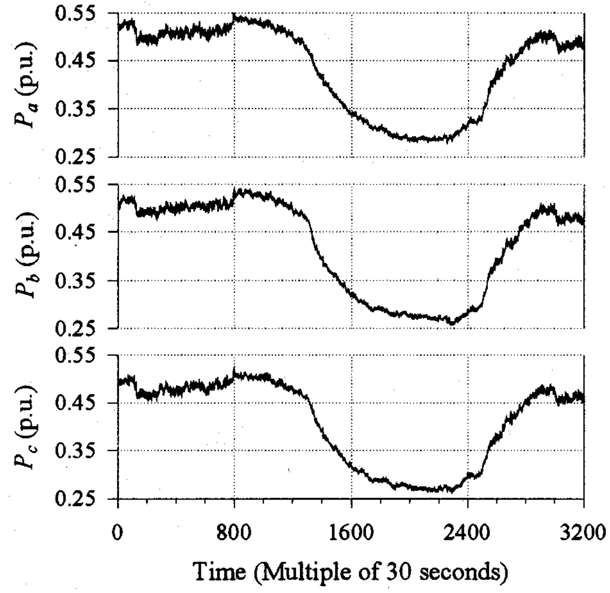
Fig. 2. One-line diagram showing a distribution transformer connected to the 69 kV and the 11 kV buses.

III. Numerical Examples

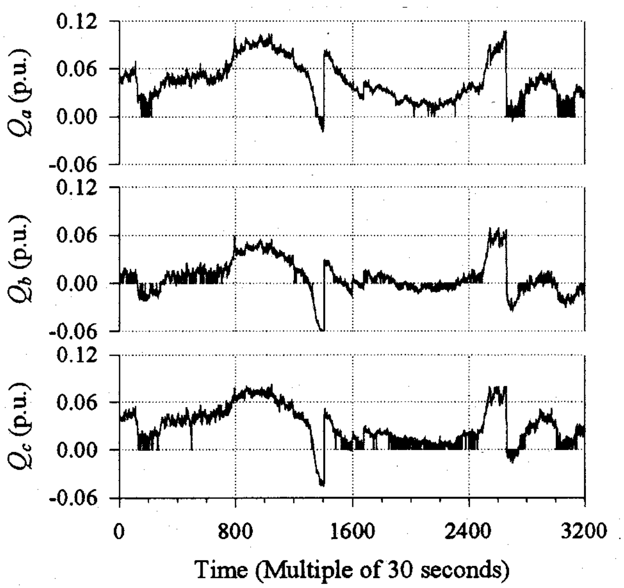
An example is given for illustration. The example is taken from the Tou–Liu district distribution substation of the Taiwan Power Company which is a 69/11 kV substation. The example studies the probability distribution and time varying characteristics of the VUF at the secondary terminals of a distribution transformer using the field recorded three-phase power data.

A. Description of the 69/11 kV Distribution Substation

Fig. 2 shows the one-line diagram of a 69/11 kV substation containing three distribution transformers. Only the part of transformer #3 is shown in the figure. The low-tension side of the transformer is connected to five feeders supplying electricity to several load centers in Tou-Liu city. The three-phase active and reactive powers flowing through the transformer have been recorded using a digital power analyzer. The measurement was carried out in late autumn of 1998 during several days. Because of memory capacity limit of the power analyzer, an appropriate sampling period must be selected to achieve successive data recording without interruption. Several sessions of measurement were conducted. Each session lasted either 27 hours at a sampling period of 30 seconds, or 7 days at a



(a)



(b)

Fig. 3. Recorded three-phase power carried by the transformer. (a) Three-phase active powers. (b) Three-phase reactive powers.

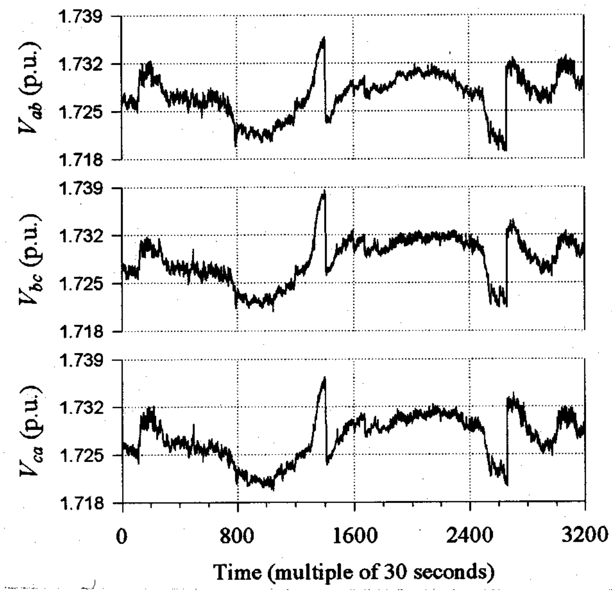
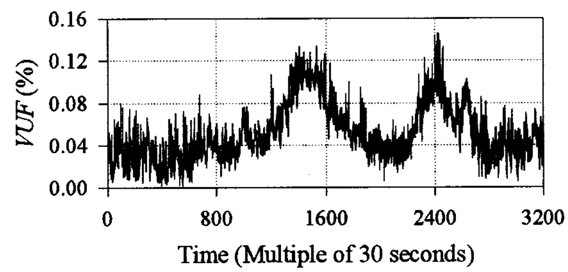
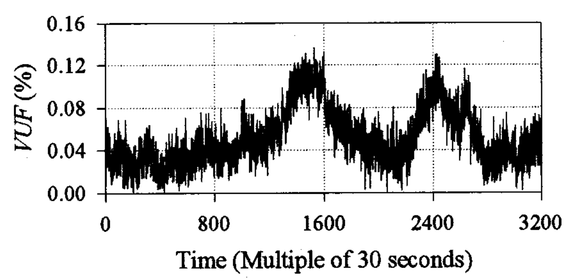


Fig. 4. Evolutions of the three-phase line-to-line voltages at the low voltage side of the transformer.



(a)



(b)

Fig. 5. (a) Evolution of the VUF obtained using voltage data shown in Fig. 3. (b) Evolution of the VUF obtained using Monte Carlo simulation method.

sampling period of 5 minutes. The data to be studied using the probabilistic method proposed in this paper were recorded during one of those 27-hour sessions beginning at 11:00 in the morning and ending at 13:40 the next afternoon.

B. The VUF Calculated from Recorded Active and Reactive Power Data

Fig. 3 shows the recorded active powers P_a, P_b and P_c, and reactive powers Q_a, Q_b and Q_c. All the power data are expressed in per-unit of the megavoltamperes per phase. The base value of the megavoltamperes per phase equals 5 MVA in this example, being one third of the rated three-phase megavoltamperes of the transformer. It is noted that in Fig. 3(b), there are a sudden increase and a sudden decrease in reactive power at about the 1400th and 2600th time steps, respectively. The sudden changes in reactive power were caused by switching of capacitor banks in the substation.

Fig. 4 shows the transformer low-tension side line-to-line voltages V_{ab}, V_{bc} and V_{ca} in per-unit (on the line-to-neutral voltage base) which have been obtained using a three-phase load flow program in Fortran. In the load flow program the 69 kV bus has been taken as the slack bus, and the power data shown in Fig. 3 as injected power at the load buses. The star-delta (Y-Δ) three-phase model of the transformer proposed by Arrillaga *et al.* [15] has been used in the program, and the iterative method described by (9) and (10) has been coded to find the voltages at the 11 kV buses. The corresponding VUF has been calculated using (11)–(13) and depicted in Fig. 5(a).

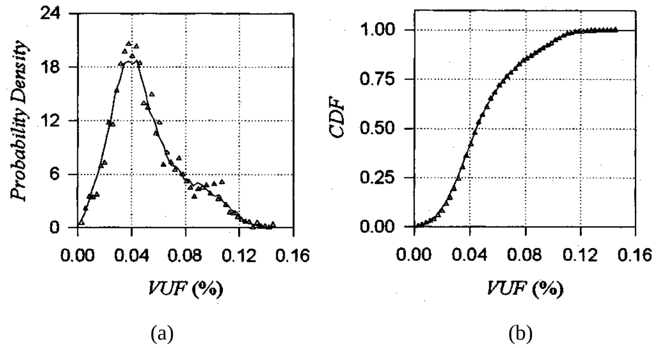


Fig. 6. Comparison of (a) the PDFs and, (b) the CDFs of the VUF, calculated using field recorded data (triangular dots) and using Monte Carlo simulation method (solid lines).

C. Monte Carlo Simulation

To simulate the random variation of the VUF shown in Fig. 3, we need to model the active powers P_a, P_b and P_c and the reactive powers Q_a, Q_b and Q_c using correlated Gaussian random variables. It is noted that neither is the power as a function of time purely deterministic, nor purely probabilistic, but a combination of both. The power can be resolved into a deterministic component and a probabilistic component. The former accounts for the average (expected) daily power curve, and the later for the random deviation from the expected quantity. Therefore, we can write

$$P_i(t) = \overline{P}_i(t) + \xi_i \tag{14}$$

$$Q_i(t) = \overline{Q}_i(t) + \delta_i \tag{15}$$

where

$i = a$, b or c refers to the phase specified,
$\overline{P}_i(t)$ and $\overline{Q}_i(t)$ to the deterministic components of active and reactive powers, and
ξ_i and δ_i to their probabilistic components.

The deterministic components $\overline{P}_i(t)$ and $\overline{Q}_i(t)$ can be obtained using a low-pass filter to filter out the random variations of $P_i(t)$ and $Q_i(t)$, and the probabilistic components ξ_i and δ_i can be modeled by Gaussian random variables. The method proposed by [10] then allows six correlated Gaussian random variables ξ_a, δ_a, ξ_b, δ_b, ξ_c and δ_c to be generated from a random number generator. The foregoing modeling of $P_i(t)$ and $Q_i(t)$ is indeed an application of random processes. $P_i(t)$ and $Q_i(t)$ are normal random processes, and ξ_a, δ_a, ξ_b, δ_b, ξ_c and δ_c are strict-sense stationary random processes.

The active and reactive powers P_a, P_b, P_c, Q_a, Q_b and Q_c simulated according to (14) and (15) have been taken as the inputs to the three-phase load flow calculation program to compute the low-tension side voltages of the distribution transformer. And the corresponding VUF have also been computed and depicted in Fig. 5(b). Comparison of Fig. 5(a) and Fig. 5(b) shows that the proposed method is capable of closely simulating both deterministic and probabilistic variation of the unbalance in the three-phase voltage. While Fig. 5 provides the reader with a qualitative evaluation of the proposed method, Fig. 6, which compares the PDFs and the CDFs of the VUF obtained from the measured and from the simulated power data, may offer a quantitative validation of it.

IV. CONCLUSION

A method that integrates Monte Carlo technique, load flow study and network reduction theorem, has been presented in this paper for simulating random variations of three-phase loads, voltages and the VUF. In this paper, the three-phase active and reactive powers were separated into the deterministic and the stochastic components. The deterministic components were modeled by simple time functions while the stochastic components were modeled by correlated Gaussian random variables. This modeling approach has allowed both the time variation and the probability property of the powers to be taken into account and simulated accurately.

Field recorded three-phase power data of a 69/11 kV distribution substation have been used as an example to illustrate how the proposed Monte Carlo method can be incorporated into the load flow calculations. Comparison of the VUF calculated using measured power data with that obtained from the proposed Monte Carlo simulation, has showed satisfactory results. Both the variation with time and the PDF and the CDF of the VUF are close to those calculated using recorded data.

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Yaw-Juen Wang was born in Taiwan in 1962. He received his diploma in electrical engineering from the National Kaohsiung Institute of Technology, Taiwan, his Master's degree in energy technology from the Asian Institute of Technology, Bangkok/Thailand, the Degree of Engineer from the National Electrical Engineering School of Grenoble (ENSIEG), Grenoble/France and the Doctor of Engineering from the National Polytechnique Institute of Grenoble (INPG) in 1982, 1987, 1989 and 1993, respectively. He is at present an Associate Professor at the Department of Electrical Engineering, National Yun-Lin University of Science and Technology, Yunlin/Taiwan. His research interests include electric power quality, probabilistic modeling of power systems disturbances, numerical methods for electromagnetics and renewable energy resources. Dr. Wang is a Member of the IEEE, a permanent member of the Taiwanese Solar Energy Association and the Chinese Electrical Engineers Association.

Lambert Pierrat was born in France in 1939. He received his diploma in electrical, electronic and automatic control engineering from University of Nancy, France, and his Degree of Engineer from the National Electrical Engineering School (ENSIEG), Grenoble, France, in 1970 and 1972, respectively. From 1973 to 1983, he was a Research Engineer with French Electricity Board (EDF) and from 1984 to 1989, Head of Electrical Branch in EDF. He was a Scientific Attaché at EDF, a research advisor with the French National Council of Scientific Research (CNRS) and a Visiting Professor at the ENSIEG from 1989 to 1998. He is at present a Senior Researcher at the Laboratory of Geophysical and Industrial Flows (LEGI), Grenoble/France. He has published more than 200 technical papers in the areas of electric machinery, power electronics and power systems. His current research interests are stochastic approaches to electric power quality, reliability and renewable energy. Professor Pierrat is a corresponding member of Science Contact of the French Academy of Science and an international expert on the Committee 22G, International Electrotechnical Commission (IEC).