A New Hybrid Method for Signal Estimation Based on Haar Transform and Prony Analysis

Nedim Aktan Yalcin and Fahri Vatansever

Abstract— **The signal estimation is very important in electrical and electronic engineering. In this study, it is shown that signal parameters' (frequency, amplitude, and phase) estimation can be realized with the implementation of Prony method on Haar transform coefficients. In order to accomplish this, mathematical relationship between roots of Prony polynomial which are found with original signal values and roots which are calculated with Haar approximation/detail coefficients is constructed. Frequency components of signal are estimated with this relationship. Next, the second part of Prony algorithm which constructs the matrix equation between roots and signal values in order to find the amplitude and phase values is implemented with Haar coefficients. In other words, a new matrix equation is derived for finding amplitudes and phases with the found roots in the first step and Haar coefficients. Thus, implementations of the first and second steps give signal parameters. Derived equations are valid for all degrees of Haar coefficients not just the first one. The use of Haar coefficients decreases the data size and increases the speed and accuracy. The proposed method is also more robust of selection of different Prony polynomial coefficient sizes.**

Index Terms— **Haar transform, harmonic estimation, interharmonic estimation, Prony method, signal estimation.**

I. Introduction

PRONY analysis is a powerful signal estimation method which is created by the French mathematician and engineer Gaspard Clair François Marie Riche de Prony [1]. This method was used in many fields. Besides, many hybrid algorithms were developed based on this method. Reason of using Prony analysis and many hybrid methods which are variation of Prony method is their capability of revealing not only harmonics but also interharmonics which are present at noninteger multiples of the main harmonic. Many studies were realized in numerous fields using Prony-based methods including communication systems, medical image processing, and power electronics [2]–[9].

Harmonic analysis is an important topic which is widely studied in signal processing and many methods were derived in this context. The most common methods in this field are discrete Fourier transform (DFT) and fast Fourier transform (FFT) [10]. These methods analyze the harmonics of signal based on integer multiples of the main harmonic. Thus, the found frequencies consist of integer multiples of the main harmonic. However, if signal contains noninteger multiples of the main harmonic, it could not be detected with these methods. Basic solution for this context may be short time DFT (STDFT). However, character of interharmonics is nonintegral multiples of the main/fundamental frequency. Therefore, wide windows are needed for revealing them in order to reach necessary resolution. However, many signals have nonstationary structure and this is the main problem. For this reason, various methods are developed for revealing interharmonics [11].

Especially when considering power electronics, harmonics and interharmonics are essential issue in terms of power quality. Reasons of interharmonics in power systems are nonlinear loads (arc furnace and daily electronic devices like computers, TVs, and so on), frequency converter circuits (cycloconverters, inverters, matrix converters, etc.), nonlinear mechanical characteristics of generators, power plants, etc. [12]. In general manner, the nonlinear structure of systems causes interharmonics.

In this article, it is intended to derive a Prony-based method for revealing signal parameters. It is useful to investigate other Prony-based methods in the literature. There are many studies which are extended forms of Prony method. In [8], a new method based on digital filtering and Prony method is defined. This method performs more accurate and fast determination of the main frequency of gridline than DFT. In [13], a new algorithm which modifies the classical least-square Prony method is proposed. In this work, the main frequency is selected first like Fourier transform. Advantage of this work is remarkable reduction of computational complexity and gives opportunity for real-time implementation. On the other hand, this method assumes that the damping factor is zero. This assumption removes one of the important benefits of Prony method. In [14], finite difference time-domain approach is presented for the analysis of wave propagation. In this study, fast inverse Laplace transform is used for the transformation of dispersion expressed in the frequency domain to time domain and Prony method is used for the estimation of signal parameters in the time domain. In [15], a new time-domain algorithm based on a modified Prony method is proposed for the estimation of the frequencies and amplitudes of broken rotor bar faults. This method is implemented with

Manuscript received August 3, 2020; accepted September 5, 2020. Date of publication September 16, 2020; date of current version December 1, 2020. The Associate Editor coordinating the review process was Dr. Shoaib Amin.
(Corresponding author: Nedim Aktan Yalcin).

The authors are with the Electrical-Electronics Engineering Department, Bursa Uludağ University, 16059 Bursa, Turkey (e-mail: aktanyalcin@uludag.edu.tr; fahriv@uludag.edu.tr).

Digital Object Identifier 10.1109/TIM.2020.3024358

dividing stator current into short overlapped time windows and analyzing them with least-squares Prony method. It is based on using short time windows for frequency/amplitude estimation in order to reduce the data size. It is reported that this technique can analyze frequencies and amplitudes near sideband of fundamental frequency accurately.

Besides Prony-based papers, there are studies which compare Prony method with other estimation techniques. In [6], the proposed methods which are based on "subspace" methods are compared with Prony method. Min-norm methods are selected for representing "subspace" methods. In this article, many test signals are created for testing these methods such as industrial frequency converter, out-of-step operation of synchronous generator, and dc arc furnace. In this work, it is reported that "subspace" methods are successful for high-resolution estimation and independent of SNR value. It is stated that Prony method was also successful for finding signal components. In [16], dynamic mode decomposition (DMD) and Prony method are investigated. It is showed that Prony and DMD methods give acceptable results. However, DMD method showed higher performance if signal has close distinct eigenvalues. In [17], modal information which is extracted from dynamic response of power systems is considered. It is showed that Matrix Pencil is more successful for extracting the modal information of signals than Prony. It is also stated that Matrix Pencil is more immune to noise inherently.

After considering the aforementioned articles, it can be said that Prony analysis is a powerful method in signal estimation, but it has some disadvantages. One of them is high calculation time when data size is large. The other disadvantage is uncertainty about selection criteria of Prony polynomial coefficients' size for large number of data. High number of coefficients' size does not guarantee more accurate results but causes more memory consumption and calculation time for solution. It is also seen that Prony method is not immune to noise unlike investigated methods.

In this study, a new hybrid method which finds signal parameters (frequencies, phases, and amplitudes) using Haar approximation/detail coefficients and Prony method is presented. Derived equations for this purpose can calculate signal parameters for the nth-degree Haar coefficients not only the first degree. The proposed method decreases the calculation time, has robust characteristic for different number of Prony polynomial coefficients, performs more stable under white noise, and produces more accurate results compared to original Prony method.

This article constitutes of five sections. In Section I, it is intended to summarize previously realized works. In Section II, Prony method and discrete Haar wavelet transform analysis are explained. In Section III, the proposed method is explained. Then simulations are realized in Section IV. Finally, general evaluation of this study is demonstrated in Section V.

II. Materials and Methods

This section is divided in two subsections. First, Prony method is explained. Next, discrete Haar transform is described.

A. Prony Analysis

Prony analysis is an extended form of Fourier analysis. It uncovers frequency, amplitude phase, and damping factor of the analyzed signals. Due to consideration of damping factor, it has advantages for finding interharmonics. In Prony analysis, a signal can be represented as the following equation [18], [19]:

$$\hat{y}(t) = \sum_{i=1}^{\infty} A_i e^{\sigma_i t} \cos(2\pi f_i t + \varphi_i). \tag{1}$$

In (1), $\hat{y}(t)$ is the approximation of $y(t)$ signal, f is the frequency, σ is the damping factor, and real, φ is the phase. If $\cos(2\pi f_i t + \varphi_i) = 0.5\left(e^{j(2\pi f_i t+\varphi_i)} + e^{-j(2\pi f_i t+\varphi_i)}\right)$ is applied to (1), the following equation can be written:

$$\hat{y}(t) = \sum_{i=1}^{\infty} \left(B_i e^{\lambda_i t}\right). \tag{2}$$

In (2), $B_i = 0.5 A_i e^{\pm j\varphi_i}$ are the amplitude coefficients and $\lambda_i = \sigma_i \pm j2\pi f_i$ are the eigenvalues. The $y(t)$ signal comprises $y(t_k)$ discrete values and t_k values are equally spaced time vector, $k = 0, 1, \ldots, N-1$. Calculation of Prony analysis is abbreviated as follows [18].

Step 1: Create a linear prediction model (LPM) from the obtained $y(t_k)$ values.

Step 2: Based on the created LPM, find characteristic polynomial.

Step 3: Using roots which can be found in step 2, calculate frequency, amplitude, and phase values.

If the sampling time is t_k, (2) can be expressed as follows:

$$\hat{y}(k) = \sum_{i=1}^{\infty} B_i z_i^k, \quad z_i = e^{\lambda_i t_k}. \tag{3}$$

After construction of these equations, B_i and z_i values, which satisfy $y(k) = \hat{y}(k)$ equation, $k = 0, 1, \ldots, N-1$, should be determined. In order to achieve this, the following equation can be written that implements (3) for each t_k [18]:

$$\begin{bmatrix} B_1 z_1^0 + \cdots + B_n z_n^0 \\ \vdots \\ B_1 z_1^{N-1} + \cdots + B_n z_n^{N-1} \end{bmatrix} = \begin{bmatrix} z_1^0 & \cdots & z_n^0 \\ \vdots & \ddots & \vdots \\ z_1^{N-1} & \cdots & z_n^{N-1} \end{bmatrix} \begin{bmatrix} B_1 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} y(0) \\ \vdots \\ y(N-1) \end{bmatrix}. \tag{4}$$

Equation (4) can be written in a compact form as follows:

$$\mathbf{ZB} = \mathbf{Y}. \tag{5}$$

If all z_i values are found, λ_i values can be found from (3). z_i values must be roots of the nth-degree polynomial with coefficients a_i. Thus, the following equation can be written:

$$z^n - \left(a_1 z^{n-1} + a_2 z^{n-2} + \cdots + a_n z^0\right) = 0. \tag{6}$$

Based on (6), $N \times 1$ vector which is described in the following equation can be created:

$$\bar{\mathbf{A}} = \begin{bmatrix} -a_n & \cdots & -a_1 & 1 & 0 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} -\mathbf{a} & 1 & \mathbf{0} \end{bmatrix}. \tag{7}$$

The following equation can be written after the implementation of (7) to (4):

$$\bar{\mathbf{A}}\mathbf{Y} = y(n) - [a_1 y(n-1) + \cdots + a_n y(0)] = \bar{\mathbf{A}}\mathbf{Z}\mathbf{B} \\ = B_1\left[z_1^n - \left(a_1 z_1^{n-1} + a_2 z_1^{n-2} + \cdots + a_n z_1^0\right) + \cdots\right] = 0. \quad (8)$$

The last step in (8) is written due to the achieved equality of (6) by each z_i. It is seen that (8) is an extended form of (6) [18]. If the initial time is selected arbitrarily, (8) can be written as follows:

$$\begin{bmatrix} y(n-1) & \cdots & y(0) \\ \vdots & \ddots & \vdots \\ y(N-2) & \cdots & y(N-n-1) \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} y(n+0) \\ \vdots \\ y(N-1) \end{bmatrix}. \quad (9)$$

Solution of (9) gives polynomial coefficients in (6). After finding roots z_i of polynomial, eigenvalues λ_i in (3) are detected. These operations create the first and second steps of Prony method. The last step is the construction of matrix in (4) and finding complex amplitude coefficients B_i [18].

B. Discrete Haar Transform

Although it has a simple structure, the Haar basis is important in harmonic analysis due to its capability of uncovering time and frequency information of signals simultaneously [20]. Basically, Haar transform is an operation which calculates sum and difference of signal with grouping its elements' binary. Implementation of this process consecutively gives higher order Haar coefficients. Basis function of Haar is given in the following equations:

$$\varphi_{2k}[n] = \begin{cases} 1/\sqrt{2}, \ n = 2k, & n = 2k+1 \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

$$\varphi_{2k+1}[n] = \begin{cases} 1/\sqrt{2}, & n = 2k \\ -1/\sqrt{2}, & n = 2k+1 \\ 0, & \text{otherwise.} \end{cases} \quad (11)$$

Each Haar basis is applied on signal values consecutively so discrete Haar Transform is realized. The following equations show this process:

$$\varphi_{2k}[n] = \varphi_0[n-2k] \quad (12)$$

$$\varphi_{2k+1}[n] = \varphi_1[n-2k]. \quad (13)$$

Discrete Haar transform is given in (14) and (15) and reconstruction of transformed signal is given in (16). Equation (14) is called as approximation coefficients and (15) is called as detail coefficients:

$$X[2k] = \langle \varphi_{2k}, x \rangle = (x[2k] + x[2k+1])/\sqrt{2} \quad (14)$$

$$X[2k+1] = \langle \varphi_{2k+1}, x \rangle = (x[2k] - x[2k+1])\Big/\sqrt{2} \quad (15)$$

$$x[n] = \sum_{k \epsilon Z} X[k]\varphi_k[n]. \quad (16)$$

III. Proposed Method

In this section, it must be emphasized that the solution of (9) may not be always the same. This means that Prony polynomial coefficients may be changed if its roots converge exponential term in (2). This situation can be observed with artificial neural network-based solution of Prony [5]. In these types of studies, neurons which represent polynomial coefficients are obtained differently for each run of program but almost the same eigenvalues. Using this observation, it is intended to derive lower order polynomial which provides a way to reach roots of original polynomial in order to decrease the computation time of roots. This can be achieved selecting some polynomial coefficients as zero. The other benefits of decreasing the polynomial degree is decreasing the matrix dimension in (9) and (43), so reducing their computational complexity. In this section, it is described that how discrete Haar transform can be used for this purpose.

This section consists of three parts. First, derivations of the proposed formulas for finding frequency parameters are presented. Second, computation of complex amplitude parameters based on discrete Haar transform and Prony method is presented. Finally, computational complexity of the proposed method is discussed. These formulas find signal parameters using either approximation coefficients or detail coefficients.

A. Frequency Estimation

The first-order approximation coefficients of discrete y signal with N elements are given for $k+1$ elements as follows:

$$\langle \varphi_{2k,y} \rangle = \left[\frac{y[0]+y[1]}{\sqrt{2}}, \frac{y[2]+y[3]}{\sqrt{2}}, \ldots, \frac{y[k-1]+y[k]}{\sqrt{2}}\right]. \quad (17)$$

The first-order detail coefficients of the analyzed y signal are given in the following equation:

$$\langle \varphi_{2k,y} \rangle = \left[\frac{y[0]-y[1]}{\sqrt{2}}, \frac{y[2]-y[3]}{\sqrt{2}}, \ldots, \frac{y[k-1]-y[k]}{\sqrt{2}}\right]. \quad (18)$$

Based on (17), the following equations can be written:

$$y[k-1] = a_{k-1}y[0] + a_{k-2}y[1] + \cdots + a_1 y[k-2] \quad (19)$$

$$y[k] = a_{k-1}y[1] + a_{k-2}y[2] + \cdots + a_1 y[k-1] \quad (20)$$

$$y[k-1] + y[k] = a_{k-1}(y[0] + y[1]) + a_{k-2}(y[1] + y[2]) \\ + \cdots + a_1(y[k-2] + y[k-1]). \quad (21)$$

In (19), the $(k-1)$th element of the y signal is considered and the first element is represented with $y[0]$. Therefore, the number of polynomial coefficient size is selected as $k-1$. In the first row of (9), the nth element of the y signal is taken into account and the first element is also showed with $y[0]$. Hence, number of coefficient size is n.

If there is enough number of Prony coefficients, $y[k-1] + y[k]$ can also be found with arbitrarily setting a_i coefficients to zero for $i = 2m+1, m \in N^+$. After setting odd indexed coefficients to zero, the following equation can be written and it is noted that k is selected as odd number:

$$y[k-1]+y[k]=a_{k-1}(y[0]+y[1])+a_{k-3}(y[2]+y[3]) +\cdots+a_2(y[k-3]+y[k-2]) \quad (22)$$

$$y[k-1]+y[k]=a'_{(k-1)/2}(y[0]+y[1]) +a'_{(k-3)/2}(y[2]+y[3]) +\cdots+a'_1(y[k-2]+y[k-1]). \quad (23)$$

Coefficient vector $\mathbf{a}'=[a'_1,a'_2,\ldots,a'_{(k-1)/2}]$ which is used in (23) is a reconstructed form of the other coefficient vector $\mathbf{a}=[0,a_2,0,a_4,\ldots,a_{(k-3)},0,a_{(k-1)}]$ which is implemented in (22) and $\mathbf{a}'$ vector is obtained by removing the zero coefficients from $\mathbf{a}$.

If (9) is implemented on the first-order approximation coefficients, $\mathbf{a}'$ vector is obtained. Relationship between polynomials which are constructed from $\mathbf{a}'$ and $\mathbf{a}$ vectors is given in (24) and (25). More clearly, solution of (9) based on the matrix which is created by using values from (23) is $\mathbf{a}'$. In this step, $\mathbf{a}$ vector can be created by placing zeros in appropriate index from $\mathbf{a}'$. Equation (24) is polynomial with coefficient vector $\mathbf{a}$. Equation (25) is a rearranged form of the following equation:

$$a_{k-1}x^{k-2}+0x^{k-3}+\cdots+a_4x^3+0x^2+a_2x^1+0x^0=0 \quad (24)$$

$$x\left(a_{k-1}x^{k-3}+\ldots a_8x^6+a_6x^4+a_4x^2+a_2x^0\right)=0. \quad (25)$$

If $y=x^2$ is applied in (25), (26) can be written as

$$\sqrt{y}\left(a_{k-1}y^{(k-3)/2}+\cdots+a_8y^3+a_6y^2+a_4y+a_2\right)=0. \quad (26)$$

It is easily seen that coefficients in (26) are $\mathbf{a}'$ vector. Therefore, roots of polynomial which are directly found from $\mathbf{a}'$ are related with roots of essential polynomial with coefficient $\mathbf{a}$ vector (at this point, essential polynomial refers to the obtained polynomial from original signal values by using (9) and essential coefficients refer to its coefficients). This relation is expressed with the following equation:

$$x=\sqrt{y}. \quad (27)$$

In order to analyze the second-order approximation coefficients for finding roots of essential polynomial, the following equations can be written similar to (19)–(21):

$$E_1=y[k-1]=a_{k-1}y[0]+a_{k-2}y[1]+\cdots+a_1y[k-2] \quad (28)$$

$$E_2=y[k]=a_{k-1}y[1]+a_{k-2}y[2]+\cdots+a_1y[k-1] \quad (29)$$

$$E_3=y[k+1]=a_{k-1}y[2]+a_{k-2}y[3]+\cdots+a_1y[k] \quad (30)$$

$$E_4=y[k+2]=a_{k-1}y[3]+a_{k-2}y[4]+\cdots+a_1y[k+1] \quad (31)$$

$$E_1+E_2+E_3+E_4=a_{k-1}(y[0]+y[1]+y[2]+y[3]) +\cdots+a_1(y[k-2]+y[k-1]+y[k]+y[k+1]) \quad (32)$$

$$E_1+E_2+E_3+E_4=a'_{(k-1)/4}(y[0]+y[1]+y[2]+y[3]) +\cdots+a'_1(y[k-2]+y[k-1]+y[k]+y[k+1]). \quad (33)$$

In (32), all coefficients arbitrarily set to zero except $a_i, i=4m, m\in N^+$. The last index a_{k-1} in (32) is selected according to $k-1=4n, n\in N^+$. In this way, (9) can be implemented for just only the second-order approximation coefficients. Coefficient vector $\mathbf{a}''=[a'_1,a'_2,\ldots,a'_{(k-5)/4},a'_{(k-1)/4}]$ which is used in (33) is a reconstructed form of essential coefficient vector $\mathbf{a}=[0,0,0,a_4,0,0,0,a_8,\ldots,a_{k-5},0,0,0,a_{k-1}]$ which is implemented in (32) and $\mathbf{a}''$ vector is obtained by removing the zero coefficients from $\mathbf{a}$.

If (9) is implemented on the second-order approximation coefficients, $\mathbf{a}''$ vector is obtained. Relationship between polynomials which are constructed from $\mathbf{a}''$ and $\mathbf{a}$ vectors is given in (34) and (35). More clearly, solution of (9) based on the matrix which is created by using values from (33) is $\mathbf{a}''$. In this step, $\mathbf{a}$ vector can be created by placing zeros in appropriate index from $\mathbf{a}''$. Equation (34) is the essential polynomial with coefficient vector $\mathbf{a}$ and (35) is a rearranged form of (34):

$$a_{k-1}x^{k-2}+0x^{k-3}+\cdots+a_4x^3+0x^2+0x^1+0x^0=0 \quad (34)$$

$$x^3\left(a_{k-1}x^{k-5}+\ldots a_{16}x^{12}+a_{12}x^8+a_8x^4+a_4\right)=0. \quad (35)$$

If $y=x^4$ is applied in (35), the following equation can be written:

$$\sqrt[4]{y^3}\left(a_{k-1}y^{(k-5)/4}+\ldots a_{16}y^3+a_{12}y^2+a_8y+a_4\right)=0. \quad (36)$$

It is easily seen that coefficients in (36) are $\mathbf{a}''$ vector. Therefore, roots of polynomial which are directly created from $\mathbf{a}''$ are related with roots of essential polynomial with coefficient $\mathbf{a}$ vector. This relation is expressed with the following equation:

$$x=\sqrt[4]{y}. \quad (37)$$

If (22), (26), (32), and (36) are investigated, general formula can be constructed with using the nth-order approximation coefficients for finding roots of essential polynomial. Derived equations are given as follows:

$$\sqrt[\upsilon]{y^{\upsilon-1}}\left(a_{(k-1)\upsilon}y^{(k-\upsilon-1)/\upsilon}+\cdots+a_{3\upsilon}y^2+a_{2\upsilon}y+a_\upsilon\right)=0 \quad (38)$$

$$x=\sqrt[\upsilon]{y} \quad (39)$$

$$\upsilon=2^n,\quad k,\ n\in N^+. \quad (40)$$

After finding roots of essential polynomial, frequency parameter of signal can be calculated. It is important to say that (38)–(40) is valid for not only approximation coefficients but also detail coefficients. It is easily proved with substituting detail coefficients for approximation coefficients in (9).

Finding frequencies with the Haar-based Prony method can be abbreviated as follows.

1) Find the nth-order approximation coefficients of the analyzed signal.
2) Implement the approximation coefficients in (9) and calculate the $\mathbf{a^n}$ Prony coefficient vector.
3) Construct the Prony polynomial with the $\mathbf{a^n}$ coefficient vector.
4) Find the roots of Prony polynomial.
5) Find the roots of essential polynomial with (39).
6) Find the frequencies from roots of essential polynomials.

Implementation of root calculation of derived method in computer program has an important point. In the fifth stage, the υth-degree roots are calculated. In many computer languages, there is no function which directly computes the υth degree roots of numbers. For example, if υ is 4 and we want to calculate $\sqrt[4]{1}$, computer program may return (-1 and 1). However, four different roots are available $\sqrt[4]{1} = e^{2\pi j/4}, i \in N$. Solution is $(-1, 1, -j, j)$. It is crucial to use or write appropriate method in computer program for finding all roots.

B. Complex Amplitude Estimation

Complex amplitudes can be calculated using estimated z_i values. For this purpose, (4) must be rearranged. Before finding general formula for amplitude detection, it is convenient to start with the first-order approximation coefficients to reveal the complex amplitude values. At this point, $1/\sqrt{2}$ coefficients are ignored, just focused on $y[k]+y[k+1]$ sums for the sake of simplicity. Because $y[k]+y[k+1]$ values are known, the rows of (4) can be summed in pairs. Therefore, the following equation can be written:

$$\begin{bmatrix} (1+z_1^1) & \cdots & (1+z_n^1) \\ \vdots & \ddots & \vdots \\ z_1^{N-2}(1+z_1^1) & \cdots & z_n^{N-2}(1+z_n^1) \end{bmatrix} \begin{bmatrix} B_1 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} y[0]+y[1] \\ \vdots \\ y[N-2]+y[N-1] \end{bmatrix}. \quad (41)$$

The same operation can be realized on (41). If the rows of (41) are summed in pairs, the right side of the obtained equation consists of the second-order approximation coefficients. Matrix equation for calculation of complex amplitudes with the second-order approximation coefficients is written in the following equation:

$$\begin{bmatrix} (1+z_1^1)(1+z_1^2) & \cdots & (1+z_n^1)(1+z_n^2) \\ \vdots & \ddots & \cdots \\ z_1^{(N-4)}(1+z_1^1)(1+z_1^2) & \cdots & z_n^{(N-4)}(1+z_n^1)(1+z_n^2) \end{bmatrix} \begin{bmatrix} B_1 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} y[0]+y[1]+y[2]+y[3] \\ \vdots \\ y[N-4]+y[N-3]+y[N-2]+y[N-1] \end{bmatrix}. \quad (42)$$

With consideration of (41) and (42), general formula which reveals complex amplitudes can be written as (43) for the υth-order approximation coefficients. In (43), $k+1$ is the row number and z_i' is defined in (44) as follows:

$$\begin{bmatrix} z_1^{0\times(2^\upsilon)} z_1' & \cdots & z_n^{0\times(2^\upsilon)} z_n' \\ \vdots & \ddots & \\ z_1^{k\times(2^\upsilon)} z_1' & \cdots & z_n^{k\times(2^\upsilon)} z_n' \end{bmatrix} \begin{bmatrix} B_1 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} \sum_{j=0}^{2^\upsilon-1} y[j] \\ \vdots \\ \sum_{j=k\times 2^\upsilon}^{(k+1)\times 2^\upsilon-1} y[j] \end{bmatrix} \quad (43)$$

$$z_i' = \prod_{p=1}^{\upsilon}\left(1+z_i^{(2^{p-1})}\right), \quad i = 1, 2, \ldots, n. \quad (44)$$

It is also possible to calculate the complex amplitude values by detail coefficients. Related equality for the first-order detail coefficients is obtained with subtraction of rows in pairs using (4). However, construction of the matrix equation with higher order detail coefficients is a little different. Before finding higher order detail coefficients, approximation coefficients should be calculated. For example, the fifth-order detail coefficients are calculated after finding fourth-order approximation coefficients. Therefore, the right side of (43) is rewritten as in (45) and the left side of (43) is modified by changing z_i' values as in (46) as follows:

$$\begin{bmatrix} z_1^{0\times(2^\upsilon)} z_1' & \cdots & z_n^{0\times(2^\upsilon)} z_n' \\ \vdots & \ddots & \\ z_1^{k\times(2^\upsilon)} z_1' & \cdots & z_n^{k\times(2^\upsilon)} z_n' \end{bmatrix} \begin{bmatrix} B_1 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} \sum_{j=0}^{2^{\upsilon-1}-1} y[j] - \sum_{j=2^{\upsilon-1}}^{2^\upsilon-1} y[j] \\ \vdots \\ \sum_{j=k\times 2^\upsilon}^{2^{\upsilon-1}(2\times k+1)-1} y[j] - \sum_{j=2^{\upsilon-1}(2\times k+1)}^{(k+1)2^\upsilon-1} y[j] \end{bmatrix} \quad (45)$$

$$z_i' = \left(1 - z_{\mathrm{i}}^{(2^\upsilon)}\right)\prod_{p=1}^{\upsilon-1}\left(1+z_{\mathrm{i}}^{(2^{p-1})}\right), \quad i = 1, 2, \ldots, n. \quad (46)$$

As stated before, $1/\sqrt{2}$ multiples are ignored while derivation steps of (43) and (45). In practice, each sum of the right-hand side in (43) and (45) has $(1/\sqrt{2})^\upsilon$ multiples. υ is the degree of approximation or detail coefficients and $\mathbf{B_{haar}}$ is the obtained complex amplitude vector when considering $(1/\sqrt{2})^\upsilon$ multiplier. Therefore, in order to find the complex amplitude values, the last operation which is stated in (47) must be realized. Thus, complex amplitudes (amplitude and phase information) are obtained as follows:

$$\mathbf{B} = \left(\sqrt{2}\right)^\upsilon \mathbf{B_{haar}}. \quad (47)$$

C. Computational Complexity

Prony method consists of two major steps: frequency estimation and complex amplitude estimation. Frequency estimation step also implements two stages. In the first stage, solution of (9) ($N \times N$ matrix equation) is realized. After solution of the first stage, Prony polynomial coefficients are obtained. Calculation of roots which are zeros of the polynomial constitutes the second stage.

The proposed method consists of three major steps: calculation of Haar transform, solution of reduced form of (9), and finding roots of (26). It is assumed that the $y[k]$ signal has N discrete values and Prony polynomial coefficient size is also N. Solution of (9) has computational complexity of $O(N^3)$ if Gauss–Jordan elimination method is used [21]. The proposed method uses Haar transform coefficients instead of original signal values. The nth-degree Haar transform reduces the data size $N/2^n$ so that computational cost of (9) can be reduced to $O(N^3/2^{3n})$, if coefficients' size is also selected as $N/2^n$. Computing all complex zeros of the Nth-degree polynomial is $O(N^3\log^2(N))$ [22]. In the proposed method,

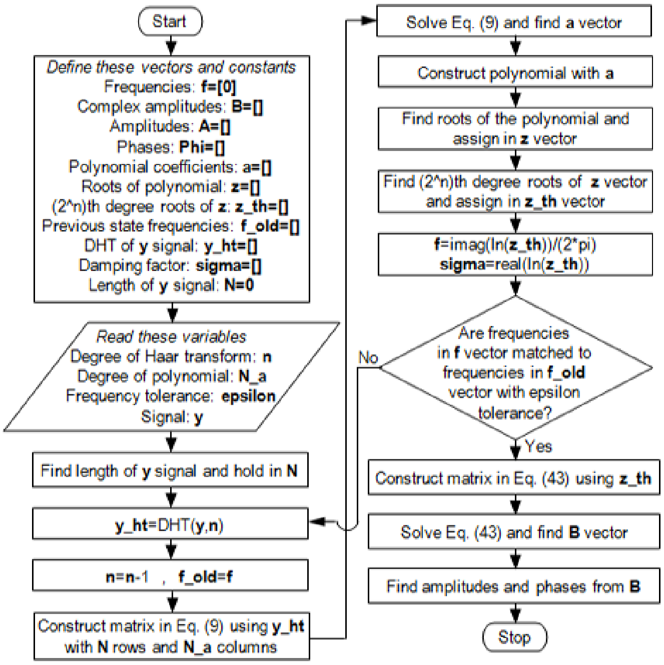

Fig. 1. Flowchart of the proposed algorithm.

due to reduction of data size to $N/2^n$, computational complexity is $O\left(\left(N^3/2^{3n}\right)(\log N - 3n\log 2)^2\right)$. In the second stage, it is assumed that all zeros of polynomial are complex. This is reasonable because almost all values of $e^{\lambda_i t}$ are expected imaginary in (2). Computational cost of revealing complex amplitudes depends on solutions of $N \times N$ and $N/2^n \times N/2^n$ matrices for Prony and proposed methods, respectively. Therefore, computational complexity is $O\left(N^3\right)$ and $O\left(N^3/2^{3n}\right)$ for Prony and proposed methods. Total computational complexity of Prony and proposed methods is shown, respectively, in the following equations:

$$\mathrm{Cost}_{\mathrm{Prony}} = 2\times O\left(N^3\right) + O\left(N^3\log^2(N)\right) \tag{48}$$

$$\mathrm{Cost}_{\mathrm{Proposed}} = 2\times O\left(N^3/2^{3n}\right) + O\left(\frac{N^3}{2^{3n}}(\log N - 3n\log 2)^2\right). \tag{49}$$

Flowchart of the proposed method is shown in Fig. 1. In this algorithm, it is needed to use a loop. Reason of this is to detect the convergence step of frequencies to constant values. Convergence step of algorithm represents the discrete Haar transform degree which is used for calculating frequencies. Therefore, (49) should be updated as follows:

$$\mathrm{Cost}_{\mathrm{Proposed}} = \sum_{i=1}^{k}\left(O\left(\frac{N^3}{2^{3n}}(\log N - 3n\log 2)^2\right)\right) + 2\times O\left(N^3/2^{3n}\right). \tag{50}$$

In (50), $N = 2^p, k, p \in N^+, k < p$. It is clearly seen that the proposed method has less computational complexity than Prony method for all k and p.

IV. Simulations and Evaluations

In this section, the simulations are realized with MATLAB [23]. In these simulations, it is intended to compare the proposed method and Prony method according to their relative error rates and calculation speeds. Therefore, their performance is investigated on signals which have different sampling frequencies but the same harmonics. Distinct sampling frequencies affect the number of data so that the speeds of algorithms can be examined. Besides, their accuracy performances are analyzed on data model which has interharmonic components and diverse frequency spectrum.

The used data set in this article is obtained from the mathematical model of perturb and observe maximum power point tracking (P&O MPPT) algorithm which is used in photovoltaic systems [24], [25]. In this model, two important parameters specify the behavior of harmonics and interharmonics: step voltage and MPPT frequency. Step voltage is the source of harmonics/interharmonics' amplitudes and MPPT frequency assigns closeness of frequencies. It is expected that low amplitudes and close frequencies are harder to be estimated than the others. Flexibility of frequency distribution and amplitude value arrangement give opportunity to test algorithms in various conditions. The used mathematical model is given in (51). Equations (52)–(54) clarify the used parameters mathematically. In these equations, n is the harmonic number, A_n is the harmonic amplitude, φ_n is the harmonic phase, V_{step} is the step voltage, f_{MPPT} is the MPPT frequency, and f_g is the grid frequency (the main frequency):

$$i_g(t) = \sum_{n=1}^{\infty}\frac{A_n}{2}\left[\cos\left(2\pi t\left(f_g - f_n\right) + \varphi_n\right) - \cos\left(2\pi t\left(f_g + f_n\right) + \varphi_n\right)\right] \tag{51}$$

$$a_n = \frac{2V_{\mathrm{step}}}{\pi n}\sin\left(\frac{\pi n}{2}\right), \quad b_n = \frac{V_{\mathrm{step}}}{\pi n}\cos(\pi(n-1)) \tag{52}$$

$$A_n = \sqrt{a_n^2 + b_n^2}, \quad \emptyset_n = \tan^{-1}\left(\frac{b_n}{a_n}\right) \tag{53}$$

$$f_n = \frac{(2n-1)f_{\mathrm{MPPT}}}{4}. \tag{54}$$

In this section, two comparative simulations are realized with Prony and proposed methods. Besides these simulations, the proposed method is tested under white Gaussian noise with SNR = 10, SNR = 20, and SNR = 30 values. The simulation results are produced from the MATLAB 2014 software and an Intel Core i7-2670QM 2.20-GHz CPU, 6.00-GB RAM, 64-Bit Windows 10 computer [23]. Steps of algorithms are practically explained in Table I. Signal parameters which are used for the simulations are given in the first two columns of Tables II and III. The comparative simulation results are also given in Tables II and III. Prony and proposed algorithms' speeds are given in Table IV. Finally, the test results of the proposed method under white Gaussian noise are shown in Table V.

The simulation parameters are clarified in related tables. In Table II, the step voltage is 12 V and the MPPT frequency is 5 Hz; and in Table III, the step voltage is 24 V and the MPPT frequency is 20 Hz. Frequencies in Table II are closer than Table III and amplitudes in Table II are lower

TABLE I
ALGORITHM STEPS ($f_{\text{MPPT}} = 5 - \text{Hz}, V_{\text{step}} = 12 - \text{V}$, and $F_{s2} = 100 - \text{kHz}$)

Steps	i	FREQUENCY VALUES FOR CORRESPONDED STEP	Duration (s)	PD
1	0	[1.1137]	0.4931	2
2	1	[1.2753]	0.4826	3
3	2	[1.1726 2.4049 4.7643 6.1035]	0.6695	7
4	3	[1.17562.43685.16647.467710.057012.207012.2070]	1.0359	13
5	4	[0.0492 1.17192.42202.57824.92195.07817.42197.5781]	3.2670	31
6	5	[36.406338.750038.906241.250041.406243.750043.9062]	7.9727	61
7	6	[38.750041.250043.750046.250048.750050.0000 51.250053.7500 56.2500 58.750061.2500]	21.6484	127
8	7	[38.750041.250043.750046.250048.750050.0000 51.250053.7500 56.2500 58.750061.2500]	52.6259	251

TABLE II
COMPARISON OF PRONY AND PROPOSED METHODS ($f_{\text{MPPT}} = 5 - \text{Hz}$ and $V_{\text{step}} = 12 - \text{V}$)

Prony Method $F_{s1} = 8\ kHz$			Prony Method $F_{s2} = 100\ kHz$			Proposed Method $F_{s1} = 8\ kHz$			Proposed Method $F_{s2} = 100\ kHz$		
f (Hz)	A (V)	Phi (DEGREE)	f (Hz)	A (V)	Phi (DEGREE)	f (Hz)	A (V)	Phi (DEGREE)	f (Hz)	A (V)	Phi (DEGREE)
38.7500	0.8541	-26.5650	39.3719	0.3704	-40.4619	38.7500	0.8541	-26.5651	38.7500	0.8541	-26.5651
41.2500	0.4775	89.9999	~	~	~	41.2500	0.4775	90.0000	41.2500	0.4775	90.0000
43.7500	1.4235	26.5651	~	~	~	43.7500	1.4235	26.5651	43.7500	1.4235	26.5651
46.2500	0.9549	-89.9999	46.7824	1.3476	-5.3820	46.2500	0.9549	-90.0000	46.2500	0.9549	90.0000
48.7500	4.2706	-26.5650	~	~	~	48.7500	4.2706	-26.5651	48.7500	4.2706	-26.5651
50.0000	12.0000	-0.0001	49.7197	9.4540	-70.5839	50.0000	12.0000	0.0000	50.0000	12.0000	0.0000
51.2500	4.2706	-26.5650	~	~	~	51.2500	4.2706	-26.5651	51.2500	4.2706	-26.5651
53.7500	0.9549	-89.9999	~	~	~	53.7500	0.9549	90.0000	53.7500	0.9549	90.0000
56.2500	1.4235	26.5650	55.7052	0.2914	45.3914	56.2500	1.4235	26.5651	56.2500	1.4235	26.5651
58.7500	0.4775	89.9999	~	~	~	58.7500	0.4775	90.0000	58.7500	0.4775	90.0000
61.2500	0.8541	-26.5650	61.1106	0.5246	-57.8912	61.2500	0.8541	-26.5651	61.2500	0.8541	-26.5651

TABLE III
COMPARISON OF PRONY AND PROPOSED METHODS ($f_{\text{MPPT}} = 20 - \text{Hz}$ and $V_{\text{step}} = 24 - \text{V}$)

Prony Method $F_{s1} = 8\ kHz$			Prony Method $F_{s2} = 100\ kHz$			Proposed Method $F_{s1} = 8\ kHz$			Proposed Method $F_{s2} = 100\ kHz$		
f (Hz)	A (V)	Phi (DEGREE)	f (Hz)	A (V)	Phi (DEGREE)	f (Hz)	A (V)	Phi (DEGREE)	f (Hz)	A (V)	Phi (DEGREE)
5.0000	1.7082	-26.5651	~	~	~	5.0000	1.7082	-26.5651	5.0000	1.7082	-26.5651
15.0000	0.9549	-90.0000	10.9464	~	~	15.0000	0.9549	90.0000	15.0000	0.9549	90.0000
25.0000	2.8471	26.5651	~	~	~	25.0000	2.8471	26.5651	25.0000	2.8471	26.5651
35.0000	1.9099	90.0000	30.2691	~	~	35.0000	1.9099	-90.0000	35.0000	1.9099	-90.0000
45.0000	8.5412	-26.5651	47.9690	~	~	45.0000	8.5412	-26.5651	45.0000	8.5412	-26.5651
50.0000	12.0000	0.0000	~	~	~	50.0000	12.0000	0.0000	50.0000	12.0000	0.0000
55.0000	8.5412	-26.5651	55.2592	~	~	55.0000	8.5412	-26.5651	55.0000	8.5412	-26.5651
65.0000	1.9099	-90.0000	~	~	~	65.0000	1.9099	90.0000	65.0000	1.9099	90.0000
75.0000	2.8471	26.5651	74.3062	~	~	75.0000	2.8471	26.5651	75.0000	2.8471	26.5651
85.0000	0.9549	90.0000	84.5187	~	~	85.0000	0.9549	90.0000	85.0000	0.9549	-90.0000
95.0000	1.7082	-26.5651	94.9880	~	~	95.0000	1.7082	-26.5651	95.0000	1.7082	-26.5651

TABLE IV
CONVERGENCE SPEEDS OF ALGORITHMS (S)

Algorithm	Table II		Table III	
	F_{s1}	F_{s2}	F_{s1}	F_{s2}
Prony	20.9090	561.8876	8.9365	81.9546
Proposed algorithm	0.6661	88.1951	0.2500	3.2102

than Table III. In both tables, the effect of data sizes is also investigated. Therefore, different sampling frequencies F_s are used and showed in two different columns. F_{s1} is 8 kHz (2^{11} data samples) and F_{s2} is 100 kHz (2^{15} data samples). In this way, algorithms' speeds are also compared.

Prony polynomial coefficient degree is specified as the nearest prime number smaller than half of data size. Data size corresponds to size of approximation/detail coefficients for Haar transform. For large data size, the increase of Prony coefficient number causes high calculation time of solution of (4) and (9). However, it does not guarantee more accurate results for higher number of coefficients. There is no clue how to select the coefficient size for Prony method. In the proposed method, coefficient size does not dramatically affect the accuracy of solution due to low number of data size. For Prony method, the coefficient size is selected as 1021 for Tables II and III. This value satisfies the first condition (because data size is 2^{11}) but not the second condition. Because considering Prony algorithm speed makes it compulsory. Polynomial degree for

TABLE V
RESULTS OF THE PROPOSED METHOD FOR WHITE GAUSSIAN NOISE

Signal parameters		SNR=10dB		SNR=20dB		SNR=30dB	
f (Hz)	A (V)	Relative Error (f)	Relative Error (A)	Relative Error (f)	Relative Error (A)	Relative Error (f)	Relative Error (A)
5	1.7082	0.0036	0.0225	0.0042	0.0136	0.9800e-3	0.0027
15	0.9549	0.0067	0.0008	0.0074	0.0080	0.4733e-3	0.0264
25	2.8471	0.0008	0.0161	0.0001	0.0423	0.1160e-3	0.0049
35	1.9099	0.0019	0.0446	0.0007	0.0124	0.0971e-3	0.0030
45	8.5412	0.0004	0.0069	0.0002	0.0026	0.0022e-3	0.0002
50	12.0000	0.0001	0.0170	0.0000	0.0059	0.0240e-3	0.0003
55	8.5412	0.0001	0.0346	0.0001	0.0001	0.0145e-3	0.0005
65	1.9099	0.0015	0.0891	0.0005	0.0637	0.0338e-3	0.0047
75	2.8471	0.0005	0.0564	0.0001	0.0076	0.1307e-3	0.0081
85	0.9549	0.0010	0.1189	0.0002	0.0379	0.0588e-3	0.0157
95	1.7082	0.0020	0.0020	~0.0000	0.0395	0.0021e-3	0.0052

derived method is shown in the last column of Table I (PD column) for each step.

In Table II, the proposed method gives high accuracy for frequency and amplitude values. On the other hand, Prony method could not estimate 41.25, 43.75, 48.75, 51.25, 53.75, and 58.75 Hz and their corresponding amplitude and phase values. Besides, the frequency results for Prony have a relative error rate between 0.0023 and 0.0160 and the amplitude results have a relative error rate between 0.2122 and 0.7953. For the proposed algorithm, the relative error rates are under $1e-14$ for frequency estimation and $1e-4$ for amplitude estimation.

In Table III, Prony method and the proposed method give high accuracy for low sampling frequency (8 kHz). However, if sampling frequency is increased to 100 kHz, Prony method could not find $5-$, $25-$, $50-$, and $65-$Hz frequency components. The found frequencies by Prony for $100-$kHz sampling frequency have a relative error rate between $1.2632e-4$ and 0.2702. Prony algorithm could not find any amplitude components in this step. On the other hand, the proposed algorithm for $100-$kHz sampling frequency in Table III has a relative error rate under $1e-14$ for all frequencies. The amplitude results by the proposed algorithm have relative error rates under $1e-4$.

It is important to note that considering Tables II and III for $100-$kHz sampling frequency, the calculation time for Prony method and the proposed algorithm is higher in Table II according to Table III. For Prony method, the size of the constructed matrix from (9) is equal in both cases (in Tables II and III). However, it takes more calculation time for frequency components close to each other. Similarly, the proposed method also needs more time for calculation of frequency components in Table II than Table III, although, in both cases, the same degrees (eighth) of Haar approximation coefficients are used.

It should be noted that the proposed method finds frequencies which are not available at frequencies of original signal like Prony and Prony-based methods. The underlying reason for this is the degree of Prony polynomial. Higher order polynomials have more roots. Thus, more roots cause finding more frequencies. On the other hand, the found frequencies which are not available in original signal have smaller amplitudes than the others. In Tables II and III, the corresponding amplitudes to unavailable frequencies in signals are both lower than $1e-11$.

In Table IV, the total time for finding signal parameters (including Haar coefficients' calculation time) is 88.1951 s. Prony algorithm calculation time for finding the signal parameters with the same data is 561.8876 s as seen in Table IV.

Another important result of this study is that frequency estimation can be realized with Haar detail coefficients as stated in (38). The relative error rate for finding frequencies with Haar detail coefficients is under $1e-14$ like approximation coefficients. However, in this study, amplitudes cannot be calculated with detail coefficients using (45).

V. CONCLUSION

In this study, a new hybrid method which combines Prony and Haar approximation/detail coefficients for revealing signal parameters is proposed. It is shown that Haar approximation/detail coefficients can be used as input for Prony method instead of original signal for finding frequencies, amplitudes, and phases of signal if derived formulas are used. Basic idea of this algorithm is to find the lower degree of polynomial which holds a way to reach Prony polynomial. Inspiration of this method is based on parametric estimation of Prony polynomial's roots (such as ADALINE method). These types of algorithms produce different polynomial coefficients for each run. In the proposed method, some coefficients are forced to remain zero so that lower order polynomial is created. Roots of the derived polynomial depend on Prony polynomial with mathematical equations if Haar approximation/detail coefficients are used. Advantages of this method are estimation of signal parameters with lower number of data, higher speed, and higher accuracy rate than original Prony method. The proposed method also gives more stable solution for different number of Prony polynomial coefficient size due to lower number of data. Although the proposed algorithm is quite fast compared to original Prony method, it can be modified in future works in order to increase the approximation speed (for example, starting point of Haar decomposition can be optimized, algorithm steps' range can be changed, and parallel computing of Haar decomposition can be realized for very large number of data). The proposed method is also a parametric method like Prony method, so previously realized studies for solving Prony parameters (such as ADALINE) can be carried out by using equations in this study. As a result, a new hybrid method which has advantages compared to original estimation technique (Prony method) for signal estimation is presented in this article.

REFERENCES

[1] G. R. B. Prony, "Essai experimantal et analytique, etc," *J. L'Ecole Polytech.*, vol. 1, no. 1, pp. 24–76, 1795.

[2] C. Spillard and G. J. R. Povey, "Application of the Prony algorithm to a predictive RAKE receiver," in *Proc. Int. Symp. Spread Spectr. Techn. Appl. (ISSSTA)*, 1996, pp. 1039–1042.

[3] S. Dencks and G. Schmitz, "Estimation of multipath transmission parameters for quantitative ultrasound measurements of bone," *IEEE Trans. Ultrason., Ferroelectr., Freq. Control*, vol. 60, no. 9, pp. 1884–1895, Sep. 2013.

[4] C.-I. Chen and G. W. Chang, "An efficient prony-based solution procedure for tracking of power system voltage variations," *IEEE Trans. Ind. Electron.*, vol. 60, no. 7, pp. 2681–2688, Jul. 2013.

[5] G. W. Chang, C.-I. Chen, and Q.-W. Liang, "A two-stage ADALINE for harmonics and interharmonics measurement," *IEEE Trans. Ind. Electron.*, vol. 56, no. 6, pp. 2220–2228, Jun. 2009.

[6] T. Lobos, Z. Leonowicz, J. Rezmer, and P. Schegner, "High-resolution spectrum-estimation methods for signal analysis in power systems," *IEEE Trans. Instrum. Meas.*, vol. 55, no. 1, pp. 219–225, Feb. 2006.

[7] M. D. Kusljevic, "On LS-based power frequency estimation algorithms," *IEEE Trans. Instrum. Meas.*, vol. 62, no. 7, pp. 2020–2028, Jul. 2013.

[8] T. Lobos and J. Rezmer, "Real-time determination of power system frequency," *IEEE Trans. Instrum. Meas.*, vol. 46, no. 4, pp. 877–881, Aug. 1997.

[9] A. Bracale, G. Carpinelli, Z. Leonowicz, T. Lobos, and J. Rezmer, "Measurement of IEC groups and subgroups using advanced spectrum estimation methods," *IEEE Trans. Instrum. Meas.*, vol. 57, no. 4, pp. 672–681, Apr. 2008.

[10] J. W. Cooley and J. W. Tukey, "An algorithm for the machine calculation of complex Fourier series," *Math. Comput.*, vol. 19, no. 90, p. 297, Apr. 1965.

[11] C. Li, W. Xu, and T. Tayjasanant, "Interharmonics: Basic concepts and techniques for their detection and measurement," *Electric Power Syst. Res.*, vol. 66, no. 1, pp. 39–48, Jul. 2003.

[12] M. B. Marz, "Interharmonics: What they are, Where they come from and What they do," in *Proc. Minnesota Power Syst. Conf. Pap.*, Nov. 2016, pp. 1–3.

[13] J. Zygarlicki, M. Zygarlicka, J. Mroczka, and K. J. Latawiec, "A reduced Prony's method in power-quality analysis—Parameters selection," *IEEE Trans. Power Del.*, vol. 25, no. 2, pp. 979–986, Apr. 2010.

[14] J. Chakarothai, "Novel FDTD scheme for analysis of frequency-dependent medium using fast inverse laplace transform and Prony's method," *IEEE Trans. Antennas Propag.*, vol. 67, no. 9, pp. 6076–6089, Sep. 2019.

[15] M. Sahraoui, A. J. M. Cardoso, and A. Ghoggal, "The use of a modified Prony's method to track the broken rotor bars characteristic frequencies and amplitudes, in three-phase induction motors," in *Proc. Int. Symp. Power Electron., Electr. Drives, Autom. Motion*, Jun. 2014, pp. 210–215.

[16] Z. S. Machado, Jr., and G. de Vasconcelos Eng, "Comparison between DMD and Prony methodologies applied to small signals angular stability," in *Proc. 4th Brazilian Technol. Symp. (BTSym)*, 2019, pp. 213–222.

[17] L. L. Grant and M. L. Crow, "Comparison of matrix pencil and Prony methods for power system modal analysis of noisy signals," in *Proc. North Amer. Power Symp.*, Aug. 2011, pp. 1–7.

[18] J. F. Hauer, C. J. Demeure, and L. L. Scharf, "Initial results in Prony analysis of power system response signals," *IEEE Trans. Power Syst.*, vol. 5, no. 1, pp. 80–89, Feb. 1990.

[19] J. Xiong, B. Wang, and S. Zhang, "Interharmonics analysis based on windowed interpolation and Prony algorithm," in *Proc. 2nd Int. Asia Conf. Informat. Control, Autom. Robot. (CAR)*, Mar. 2010, pp. 430–433.

[20] M. Vetterli and J. Kovacevic, *Wavelets and subband coding*. Upper Saddle River, NJ, USA: Prentice-Hall, 1995.

[21] M. Mucha and P. Sankowski, "Maximum matchings via Gaussian elimination," in *Proc. 45th Annu. IEEE Symp. Found. Comput. Sci.*, Oct. 2004, pp. 248–255.

[22] V. Pan, "Algebraic complexity of computing polynomial zeros," *Comput. Math. with Appl.*, vol. 14, no. 4, pp. 285–304, 1987.

[23] Mathworks.com. *Matlab*. The MathWorks. Accessed: 2019. [Online]. Available: http://mathworks.com

[24] A. Sangwongwanich, Y. Yang, D. Sera, H. Soltani, and F. Blaabjerg, "Analysis and modeling of interharmonics from grid-connected photovoltaic systems," *IEEE Trans. Power Electron.*, vol. 33, no. 10, pp. 8353–8364, Oct. 2018.

[25] N. A. Yalçin and F. Vatansever, "Comparison of Prony and ADALINE method in inter-harmonic estimation," *Uludağ Univ. J, Fac. Eng.*, vol. 25, no. 1, pp. 405–418, 2020.

Nedim Aktan Yalcin received the B.Sc. degree from the Electrical-Electronics Engineering Department, Zonguldak Karaelmas University, Zonguldak, Turkey, in 2010, and the M.Sc. degree in electronics engineering from Uludağ University, Bursa, Turkey, in 2014.

He has been with the Engineering Faculty, Electrical-Electronics Engineering Department, Bursa Uludağ University, Bursa, as a Research Assistant. He performs research in the areas of circuits and systems, engineering software development, and engineering education.

Fahri Vatansever received the B.Sc., M.Sc., and Ph.D. degrees in electrical-electronics engineering from Sakarya University, Sakarya, Turkey, in 1997, 1999, and 2005, respectively.

He has been with the Engineering Faculty, Electrical-Electronics Engineering Department, Bursa Uludağ University, Bursa, Turkey, as a Professor. He performs research in the areas of circuits and systems, computational and artificial intelligence, and engineering education.