An Improved Electromechanical Oscillation-Based Inertia Estimation Method

Bo Wang, Deyou Yang, Guowei Cai, Zhe Chen, *Fellow, IEEE*, and Jin Ma, *Member, IEEE*

***Abstract*—The existing inertia estimation based on electromechanical oscillation relies on a single mode, which is limited in complex systems. In this letter, the inertia is estimated without worrying about the number of modes in the oscillation. The inertia expression is derived based on the swing equation in the frequency domain, which considers the mode coupling in the electromechanical bandwidth using a summation calculation. The effectiveness of the proposed inertia estimation method is validated through simulation data and real measurements.**

***Index Terms*—Inertia estimation, electromechanical oscillations, frequency transform, multiple modes, power system.**

I. Introduction

When the proportion of renewable power generation increases, inertia estimation has become a basic part of power system operation [1]. For a large-scale power system, the inertia of each local subsystem is independently estimated, which can improve the accuracy and indicate to the operator where the weak inertia is [2].

Fundamentally, deriving the inertia expression based on the measurable electromechanical response is at the center of subsystem inertia estimation. The frequency response excited by the power shortage has a straightforward relationship with the inertia. On this basis, the inertia is expressed as the ratio of the frequency derivative and power shortage [3], [4] so that the inertia can be estimated by the sudden outage of the generator [5] and the abrupt change in reference power of the controller [6]. Compared with the frequency response, the electromechanical oscillation is a dynamic response near the equilibrium point caused by a small disturbance, which has a small impact on the system operation. The relationship between inertia and oscillation parameters of a single mode is established in the frequency domain to estimate the inertia by identifying the characteristic parameters of the oscillation [7]. In [8], the inertia is estimated by combining the characteristic parameters and a specific point of an equivalent oscillation generated by Fourier transform and ESPRIT. Since the existing methods rely on a single-mode oscillation with determined parameters generated from measurement, they may show low performance in some situations from the perspective of engineering applications, such as

- Noise interference in the measured oscillation signal;
- Short-term and nonstationary oscillation in a strongly damped system;
- Multiple modes in the oscillation in a large-scale system.

This letter is inspired by the gap between the above situation and the existing method to estimate inertia using the original electromechanical oscillation response. In general, the contributions of the proposed method are summarized as follows: 1) the inertia expression is derived considering all oscillation modes that occur in the measurement; 2) the inertia estimation is only related to the Fourier transformation results of the entire oscillation, without identifying the characteristic parameters of the oscillation; 3) the performances of the proposed method and other methods are compared using the simulation and real measurement.

II. Methodology

A. Swing Equation in the Frequency Domain Considering Mode Coupling

A subsystem in the power system is usually equivalent to a generator connected to a bus, whose electromechanical oscillation response near the equilibrium point is revealed by the swing equation described by a second-order oscillator [8], i.e.,

$$M\Delta\ddot{\theta} + D\Delta\dot{\theta} + K\Delta\theta = 0 \tag{1}$$

where θ is the rotor angle; ω is the rotor speed; p is the electrical power; M and D are the inertia and damping coefficient, respectively.

For a stable system, the electromechanical oscillation is finally suppressed, i.e., all variables in (1) are zero as $t \to \infty$, which implies that all terms in (1) satisfy Dirichlet condition. Subsequently, (1) can be Fourier transformed, i.e.,

$$\mathcal{F}\left\{M\Delta\ddot{\theta}\right\} + \mathcal{F}\left\{D\Delta\dot{\theta}\right\} + \mathcal{F}\left\{K\Delta\theta\right\} = 0 \tag{2}$$

where $\mathrm{F}\{x\} = \mathrm{X}\{\gamma\}$ is the Fourier transform.

The Fourier transform is the signal conversion from the time domain to the frequency domain, so $\mathrm{X}\{\gamma\}$ reflects the characteristics of signal x at frequency γ. For the i-th frequency γ_i, the relationship in (2) remains. Considering that the frequency bandwidth of the electromechanical oscillation N is composed

Manuscript received August 21, 2021; revised December 6, 2021; accepted January 30, 2022. Date of publication March 7, 2022; date of current version April 19, 2022. This work was supported in part by the National Science Foundation of China under Grants 51977031 and 51877032, and in part by the Doctoral Foundation of Northeast Electric Power University under Grant BSJXM-2021201. Paper no. PESL-00212-2021. *(Corresponding author: Deyou Yang.)*

Bo Wang, Deyou Yang, and Guowei Cai are with the School of Electrical Engineering, Northeast Electrical Power University, Jilin 132012, China (e-mail: eebowang@hotmail.com; eedyyang@hotmail.com; caigw@neepu.edu.cn).

Zhe Chen is with the Department of Energy Technology, Aalborg University, 9220 Aalborg, Denmark (e-mail: zch@et.aau.dk).

Jin Ma is with the School of Electrical and Information Engineering, University of Sydney, Sydney, NSW 2006, Australia (e-mail: jma@sydney.edu.au).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TPWRS.2022.3156441.

Digital Object Identifier 10.1109/TPWRS.2022.3156441

of $\Gamma = \{\gamma_1 \ldots, \gamma_i \ldots, \gamma_N\}$, the swing equation in the frequency domain can be obtained by the superposition of (2) in every frequency, i.e.,

$$M \sum_{\gamma=0}^{\gamma=N} \Delta\ddot{\Theta}(\gamma) + D \sum_{\gamma=0}^{\gamma=N} \Delta\dot{\Theta}(\gamma) + K \sum_{\gamma=0}^{\gamma=N} \Delta\Theta(\gamma) = 0 \quad (3)$$

Apparently, the frequency domain form of the swing equation is an algebraic equation with a fixed point, and there is no loss of any mode in the electromechanical oscillation. Compared with the swing equation in the time domain, it is convenient to derive the inertia expression using (3) because there is no need to consider the zero crossing of the oscillation.

B. Inertia Expression Based on the Electromechanical Oscillation Response

Because this letter aims to estimate the inertia, the expression to estimate the inertia should be related to the measured data, and no other parameters should be included. From a measurable perspective, the rotor speed ω and electrical power p of the equivalent generator can be determined by the bus frequency and transmission power, respectively, which are easier to obtain than the rotor angle of the equivalent generator. According to the relationships $\Delta p = K\Delta\theta$ and $\Delta\omega = \Delta\dot{\theta}$ [7], we can rewrite (3) as

$$M \sum_{\gamma=0}^{\gamma=N} \Delta\dot{\Omega}(\gamma) + D \sum_{\gamma=0}^{\gamma=N} \Delta\Omega(\gamma) + \sum_{\gamma=0}^{\gamma=N} \Delta\mathrm{P}(\gamma) = 0 \quad (4)$$

Then, the elimination of *D* is significant to express the inertia without unknown parameters. The essence of the Fourier transform is a calculation operator expressed as $\int_{-\infty}^{\infty} (\cdot) e^{-j\gamma t} dt$, i.e., the result of the Fourier transform is a complex number. Therefore, two irrelevant equations for *M* and *D* can be established from the real part, and the imaginary parts in (4) are equal so that *M* can be expressed as, (5) shown at bottom of this page.

The derivative term in the Fourier transform is represented by an algebraic term, so there is no need to perform derivative calculations. This helps improve the accuracy and efficiency of the inertia estimation, especially when the oscillation is nonstationary.

C. Considerations for Practical Estimation

Apparently, the inertia can be estimated by (5) once the electromechanical oscillation responses are captured, which makes the inertia estimation have little limitation. However, the Fourier transform result is a significant factor that can affect the estimation accuracy, i.e., the calculation of $\mathrm{X}\{\gamma_i\}$. Considering that all measurement data after sampling are discrete in actual situations, it is inconvenient to perform an integral transformation on the measurement to calculate $\mathrm{X}\{\gamma_i\}$. Therefore, the fast Fourier transform (FFT) [9], which is based on the discrete sequence, is adopted in applications to ensure the evaluation accuracy.

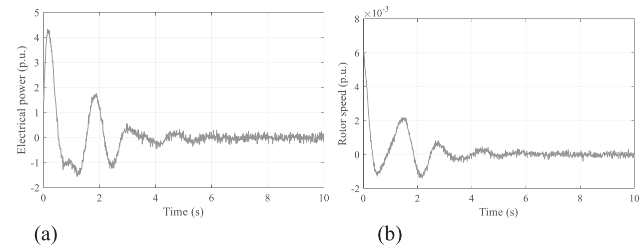

Fig. 1. Electromechanical oscillation response of (a) the electrical power and (b) rotor speed of the area.

III. Simulation

To demonstrate the performance of the proposed method, an IEEE 10-generator power system divided into three subsystems is used, where G2, G3 and G10 constitute area 1, G4-G7 constitute area 2, and G1, G8, G9 constitute area 3. The detailed division of subsystems can be found in [10]. The based power of the system is 100 MVA. The inertia of area 2 will be estimated, considering that no abrupt inertia comes from an equivalent generator to cover up the accuracy of the estimation. The speed signal of the subsystem that corresponds to the equivalent generator is represented by the average frequency of the bus [4], while the tie-line power is measured to generate the power signal [7]. Moreover, Gaussian noise is added to the simulated signals. Finally, the electromechanical oscillation response presents noise interference, multimode coupling, short-term and nonstationary, as shown in Fig. 1, all of which can occur in an actual power system.

Considering the maximum oscillation frequency in the electromechanical bandwidth, the upper bound *N* in inertia estimation is set as 2.5 Hz. Then, the real parts and imaginary parts of $\Delta\mathrm{P}(\gamma_\mathrm{i})$ and $\Delta\Omega(\gamma_\mathrm{i})$ that correspond to the filtered power and rotor speed, respectively, are calculated, as shown in Fig. 2.

Based on the FFT results for the electromechanical bandwidth, the terms $\sum_{\gamma=0}^{\gamma=N} \Delta\Omega(\gamma)$, $\sum_{\gamma=0}^{\gamma=N} \Delta\mathrm{P}(\gamma)$ and $\sum_{\gamma=0}^{\gamma=N} j\gamma\Delta\Omega(\gamma)$ can be determined by summation and yield 0.003-0.002j, -1.694+2.742j and -0.010+0.014j, respectively. Then, the inertia of the subsystem can be estimated according to (5), as shown in Table I. The real value is calculated by the aggregation of each inertia in the subsystem. To facilitate comparison, both estimated inertia and real inertia are displayed with 100 MVA as the basic power. The estimated inertia (214.7 s)

$$M = \frac{\mathrm{Re}\left[\sum_{\gamma=0}^{\gamma=N} \Delta\Omega(\gamma)\right] \mathrm{Im}\left[\sum_{\gamma=0}^{\gamma=N} \Delta\mathrm{P}(\gamma)\right] - \mathrm{Re}\left[\sum_{\gamma=0}^{\gamma=N} \Delta\mathrm{P}(\gamma)\right] \mathrm{Im}\left[\sum_{\gamma=0}^{\gamma=N} \Delta\Omega(\gamma)\right]}{\mathrm{Re}\left[\sum_{\gamma=0}^{\gamma=N} j\gamma\Delta\Omega(\gamma)\right] \mathrm{Im}\left[\sum_{\gamma=0}^{\gamma=N} \Delta\Omega(\gamma)\right] - \mathrm{Re}\left[\sum_{\gamma=0}^{\gamma=N} \Delta\Omega(\gamma)\right] \mathrm{Im}\left[\sum_{\gamma=0}^{\gamma=N} j\gamma\Delta\Omega(\gamma)\right]} \quad (5)$$

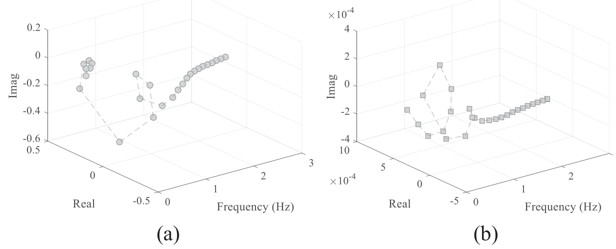

Fig. 2. FFT results of (a) the electrical power and (b) rotor speed.

TABLE I
RESULTS OF DIFFERENT METHOD (RATE POWER = 100 MVA)

Method	Estimation (s)	Error (%)	Reality (s)
Proposed	214.72	4.09	205.60
Method [6]	327.36	59.22	
Method [7]	254.65	23.86	

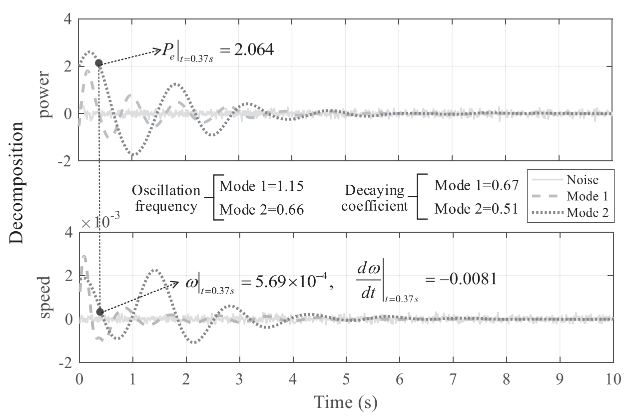

Fig. 3. Decomposition and identification for other methods.

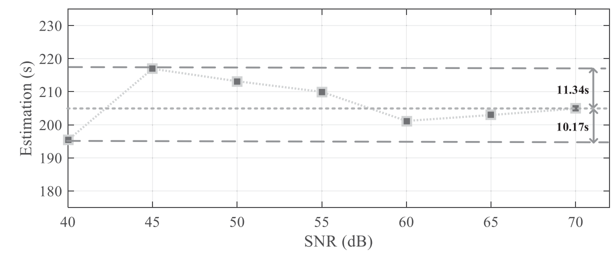

Fig. 4. Estimation with different SNR.

TABLE II
ESTIMATION BASED ON DIFFERENT SCENARIOS

Fault	Op 1	Op 2	Op 3	Op 4	Avg.
1^ϕ	209.17 s	197.16 s	211.04 s	198.31 s	203.92 s
2^ϕ	212.69 s	217.78 s	199.01 s	206.45 s	208.98 s
3^ϕ	204.66 s	206.39 s	196.64 s	201.05 s	202.04 s
Avg.	208.84 s	207.11 s	202.03 s	201.94 s	205.29 s

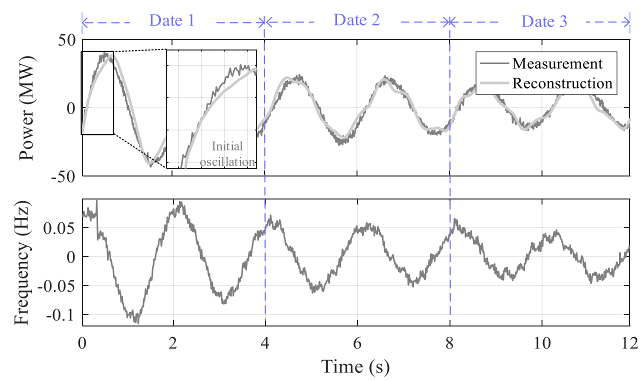

Fig. 5. Electromechanical oscillation response of the actual power system.

has a small deviation from the real inertia (205.6 s). In addition, the relative errors are calculated based on the estimation and reality, which reflects the high accuracy of the estimation.

To illustrate the advantage of the proposed method, the inertia is estimated by the methods proposed in [7] and [8] using the corresponding parameters in Fig. 3. As shown in the comparison in Table I, the other methods have much higher relative errors than the proposed method mainly because other methods have more sources of error than the proposed method, such as decomposition error and identification error. The error of the proposed method mainly comes from the FFT calculation of the oscillation containing the noise. To articulate the effect of the level of noise on the performance of the proposed method, the signal-noise ratio (SNR), which can characterize the noise in the measurement from a PMU, is utilized [11]. As shown in Fig. 4, the deviation from the estimation to reality has small differences when the SNR changes between 40-70 dB, which exhibits a small impact on the estimation results. The reasons are both the small proportion of noise information in the electromechanical bandwidth and the reduced noise interference on the inertia estimation due to the subtraction in (5).

A further test is performed for different scenarios constructed by different system operations and the typical faults, where the operation is adjusted based on the normal load from -10% to 10% with steps of 5%. The estimation result of different scenarios is listed in Table II, where the estimation with maximum deviation to reality is 12.18 s. Based on the estimation results, three types of average values are calculated: avg. of different faults (as shown in the column), avg. of different operations (as shown in the row) and avg. of all estimation results. The effectiveness of the method is demonstrated by the small deviation between statistics and reality.

IV. REAL MEASUREMENT

To further verify the feasibility of the proposed inertia estimation method in an actual power system, the HN grid system is employed, which connects to the main grid in South China through 500-kV tie-lines. The deviations of the post-disturbance frequency of the HN grid and tie-line power are shown in Fig. 5, which are recorded by a wide-area measurement system with a sampling frequency of 100 Hz. Due to the weak damping of the system, the oscillation has not been suppressed at 12 s. Considering the verification of accuracy and robustness, three sets of data with different windows are generated: the first 4 s of data (data 1), the first 8 s of data (data 2) and the entire data (data 3).

TABLE III
ESTIMATION BASED ON DIFFERENT DATA (RATE POWER = 100 MVA)

Method	Data 1	Data 2	Data 3
Proposed	64.50 s	67.14 s	66.31 s
Method [7]	100.69 s	87.69 s	60.17 s
Method [8]	132.17 s	99.69 s	71.66 s

For the accuracy verification, the inertia is estimated by the entire dataset, which is 66.31 s with 100 MVA as the basic power. However, since detailed information about the test system is rarely available, the real inertia cannot be exactly known. Thus, the accuracy of the proposed inertia estimation method is verified by comparing the measured power and reconstructed power during the initial oscillation. The high coincidence between measurement and reconstruction supports the accuracy of the estimation.

To verify the robustness of the oscillation duration, the inertia is estimated based on the data with different lengths. As shown in Table III, the estimation results have no significant difference. The proposed method is strongly robust because the results of FFT are little impacted by the oscillation duration once the signal exhibits the basic characteristics of oscillation.

The inertia is also estimated by method [7] and method [8] for performance comparison, as shown in Table III. Taking the entire data as input, the inertia estimation results based on the other methods are slightly different from the estimation results of the proposed method. However, the other methods are less robust than the proposed method because shorter data lead to worse decomposition performance.

V. CONCLUSION

In this letter, an inertia estimation method that is slightly selective about electromechanical oscillation is proposed. The test results show that as long as the electromechanical oscillation trajectory is captured, the inertia can be accurately estimated, even if the oscillation has noise interference, strong damping and coupled multimode. This advantage makes the proposed method convenient for practical engineering applications.

REFERENCE

[1] F. Milano, F. Dörfler, G. Hug, D. J. Hill, and G. Verbič, "Foundations and challenges of low-inertia systems," in *Proc. Power Syst. Computation Conf.*, 2018, pp. 1–25.

[2] P. M. Ashton, C. S. Saunders, G. A. Taylor, A. M. Carter, and M. E. Bradley, "Inertia estimation of the GB power system using synchrophasor measurements," *IEEE Trans. Power Syst.*, vol. 30, no. 2, pp. 701–709, Mar. 2015.

[3] D. P. Chassin, Z. Huang, M. K. Donnelly, C. Hassler, E. Ramirez, and C. Ray, "Estimation of WECC system inertia using observed frequency transients," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 1190–1192, May 2005.

[4] J. Schiffer, P. Aristidou, and R. Ortega, "Online estimation of power system inertia using dynamic regressor extension and mixing," *IEEE Trans. Power Syst.*, vol. 34, no. 6, pp. 4993–5001, Nov. 2019.

[5] Y. Bian, H. Wyman-Pain, F. Li, R. Bhakar, S. Mishra, and N. P. Padhy, "Demand side contributions for system inertia in the GB power system," *IEEE Trans. Power Syst.*, vol. 33, no. 4, pp. 3521–3530, Jul. 2018.

[6] R. J. Best, P. V. Brogan, and D. J. Morrow, "Power system inertia estimation using HVDC power perturbations," *IEEE Trans. Power Syst.*, vol. 36, no. 3, pp. 1890–1899, May 2021.

[7] G. Cai, B. Wang, D. Yang, Z. Sun, and L. Wang, "Inertia estimation based on observed electromechanical oscillation response for power systems," *IEEE Trans. Power Syst.*, vol. 34, no. 6, pp. 4291–4299, Nov. 2019.

[8] R. K. Panda, A. Mohapatra, and S. C. Srivastava, "Online estimation of system inertia in a power network utilizing synchrophasor measurements," *IEEE Trans. Power Syst.*, vol. 35, no. 4, pp. 3122–3132, Jul. 2020.

[9] K. R. Rao, D. N. Kim, and J. J. Hwang, *Fast Fourier transform—Algorithms and Applications*. Amsterdam, Netherlands: Springer-Verlag, 2010.

[10] K. Emami, T. Fernando, H. H. Iu, B. D. Nener, and K. P. Wong, "Application of unscented transform in frequency control of a complex power system using noisy PMU data," *IEEE Trans. Ind. Inform.*, vol. 12, no. 2, pp. 853–863, Apr. 2016.

[11] M. Brown, M. Biswal, S. Brahma, S. J. Ranade, and H. Cao, "Characterizing and quantifying noise in PMU data," in *Proc IEEE Power Energy Soc. Gen. Meeting*, 2016, pp. 1–5.