Stochastic Assessment of Harmonic Propagation and Amplification in Power Systems Under Uncertainty

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***Abstract*—The time-varying harmonics in power systems are not only caused by the stochastic harmonic emissions from different types of harmonic sources, but also the uncertain harmonic response of the system. This paper aims to reveal the impact of the latter and presents a stochastic assessment technique to study the harmonic propagation and amplifications in power systems. To achieve this, systematic indices are defined to quantify the harmonic propagation and amplification characteristics of the system. Potential system uncertainties are then categorized using aleatory and epistemic uncertainties. As a result, both the time-varying and epistemic randomness can be characterized. Finally, the stochastic behavior of the harmonic propagation and amplification is studied using the Monte Carlo simulation technique. Moreover, the impact of load modelling method on the randomness of harmonic propagation and amplification has been investigated, raising awareness to the importance of careful selection and validation of the load modelling on harmonic studies. Extensive case studies confirm the usefulness and effectiveness of the proposed method.**

***Index Terms*—Harmonic resonance, resonance amplification severity, stochastic assessment, aleatory uncertainties, epistemic uncertainties.**

I. Introduction

IN RECENT years, there have been increasing harmonic problems in power systems due to the proliferation of renewable energy systems and nonlinear devices (electronic and power electronic devices) [1]–[3]. In particular, potential harmonic resonances existing inherently in power systems with varying affected areas and the resulting amplification can enormously worsen the harmonic problems. Therefore, besides controlling the harmonic injection levels of customers, it is necessary to know the harmonic propagation and amplification characteristics in a power system.

In the literature, various efforts have been made to develop the assessment techniques for power system harmonic response, with the focus on harmonic resonance. The resonance problem can be analyzed using the frequency scan analysis (FSA) technique or the resonance mode analysis (RMA) technique. In the general FSA, the frequency response of self-impedances can reveal potential harmonic resonance frequencies existing at the harmonic source bus while the frequency response of transfer impedances can reveal potential harmonic resonance frequencies existing at non-harmonic-source buses. However, they are qualitative judgements and cannot quantitatively reveal the amplification levels of a resonance [4]. The RMA based on eigenvalue decomposition [5]–[9] or s-domain network model [10]–[13] can also identify the location where a resonance mode can be most easily excited and observed using bus participation factors, however, they are still qualitative judgements as the physical meaning of a mode that can affect a large area is still under investigation. In [14], an electrical distance-based method for predicting the non-localized harmonic resonance is presented. However, it can only give qualitative judgements of the harmonic amplification areas. In [15], the localized harmonic resonance condition and the non-localized harmonic resonance condition are distinguished and defined for the harmonic current source, and a quantitative index has been proposed to reveal the harmonic propagation area and the varying severities at the non-harmonic-source buses of a parallel resonance. It also shows that resonance amplifications not only exist at the resonance frequencies according to the results using FSA or RMA. However, quantitative index for harmonic voltage source at the non-harmonic-source bus and quantitative indices for the resonance conditions at the harmonic current source bus and harmonic voltage source bus are also needed.

On the other hand, nowadays a wide variety of uncertainties are present in different electric power systems, affecting the system resonance potentials in varying degrees and making it necessary to perform probabilistic assessment of system harmonic response. Ref. [16] studied the impacts of model abstractions and parameter uncertainties on the harmonic impedance estimation and resonance parameters. In [17], considering load and supply uncertainty, a system impedance probabilistic model is presented in detail. Similarly, the authors of [18] explored the statistical analysis of harmonic resonance frequencies due to manufacturing tolerances of capacitor and shunt filter banks in a typical medium distribution system. However, these methods can only be used to qualitatively reveal the harmonic resonance conditions at the harmonic source bus. In [19], considering the time-varying traction power consumption, the statistical study of

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the resonance location between the Steinmetz and supply system reactance is performed. However, all above studies can only reveal the stochastic behavior of the resonance frequency, which is not enough for designing an effective harmonic resonance mitigation scheme. In [20], a systematic methodology is presented to investigate the stochastic behavior of harmonic resonance due to system uncertainties using both Monte Carlo (MC) approach and RMA. Three indices, including probabilistic expressions of 1) resonance frequency band, 2) modal impedances in the resonance band, 3) sensitivity information at the bus-level and the element-level are used to represent the stochastic behaviors of harmonic resonance. However, only qualitative results can be achieved. In fact, the randomness of harmonic sources has been rather well considered in designing harmonic mitigation schemes. The stochastic harmonic response of the system is much less well considered however.

The purpose of this paper is to propose a systematic method for stochastic assessment of the harmonic propagation and amplification in power systems with two innovative features:

1) Quantitative results can be achieved by using a set of quantitative harmonic propagation and amplification indices both for harmonic current source and harmonic voltage source.
2) The time-varying characteristics and epistemic randomness of harmonic propagation and amplification are investigated respectively by categorizing potential system uncertainties using aleatory uncertainties and epistemic uncertainties.

Moreover, the randomness of harmonic propagation and amplification with different load models have been compared and analyzed to reveal the influence of load modelling method on harmonic propagation and amplification studies.

This paper is organized as follows. Firstly, quantitative resonance amplification indices are derived for harmonic current source and harmonic voltage source respectively in Section II. Then, in Section III, potential system uncertainties are distinguished into aleatory uncertainties and epistemic uncertainties. Thereafter, the framework for studying the stochastic behavior of harmonic propagation due to system uncertainties is then studied using the Monte Carlo technique is presented in Section IV. Moreover, the randomness of harmonic propagation and amplification with different load models have been compared and analyzed to reflect the truth in Section V. Finally, a brief conclusion and discussion of future works are given in Section VI.

II. Harmonic Propagation and Amplification Indices

Harmonic sources can be approximatively modelled as two types: 1) Harmonic current source, such as the magnetic ballast discharge lamp, three phase converter and three phase adjustable speed drive, etc. [21]. 2) Harmonic voltage source, such as electronic devices powered by single phase capacitor filtered diode bridge rectifier. Generally, for the parallel resonance excited by a harmonic current source, a small harmonic current injection may result in a big harmonic voltage distortion, while for the series resonance excited by a harmonic voltage source, a small harmonic voltage may result in a big harmonic current. In this paper, based on the index for harmonic current source in [15], a novel index is proposed to reveal the harmonic propagation area and varying severities at the non-harmonic-source buses of a series resonance which is excited by a harmonic voltage source. In addition, quantitative assessment indices for the resonance conditions at the harmonic current source bus and harmonic voltage source bus are also derived.

A. Harmonic Amplification Indices for Non-Harmonic-Source Bus

At the non-harmonic-source bus, for a harmonic current source, harmonic resonance condition is defined if the resulting *h*-th harmonic voltage ratio is higher than that at the harmonic source bus. For a harmonic voltage source, the resonance condition is defined if the resulting *h*-th harmonic current ratio at other branches is higher than that resulted from the harmonic voltage source [14], [15].

In the frequency scan analysis, as for the harmonic current source, at frequency *h*(pu), considering 1.0 per-unit current I_j^h injected to harmonic source bus *j*, the harmonic voltage at each bus is

$$\left[\, U_1^h \cdots U_j^h \cdots U_n^h \,\right]' = \left[\, Z_{1j}^h \cdots Z_{jj}^h \cdots Z_{nj}^h \,\right]' \quad (1)$$

where the apostrophe symbol '′' means the transpose of a matrix. Under the per-unit system, the amplification ratio of individual harmonic voltage distortion of bus *i* to bus *j* can be expressed as

$$ARV_{ij}^h = \frac{IHVD_i^h}{IHVD_j^h} = \left| \frac{U_i^h / U_i^1}{U_j^h / U_j^1} \right| = \left| \frac{U_i^h}{U_j^h} \right| \cdot \left| \frac{U_j^1}{U_i^1} \right| \quad (2)$$

where $IHVD_i^h$ and $IHVD_j^h$ are the *h*-th individual harmonic voltage distortions at bus *i* and bus *j* respectively; U_i^1 and U_j^1 are the per-unit values of the fundamental voltages at bus *i* and bus *j* respectively, and $|\cdot|$ means the absolute value. As U_i^1 and U_j^1 are both approximately equal to one, substituting with (1) in (2), the *ARV* can be further expressed as

$$ARV_{ij}^h = \left| \frac{U_i^h}{U_j^h} \right| \cdot \left| \frac{U_j^1}{U_i^1} \right| \approx \frac{|Z_{ij}^h|}{|Z_{jj}^h|} \quad (3)$$

The *ARV* in (3) is used as the parallel resonance amplification index. If $ARV_{ij}^h > 1$, the resulting *h*-th individual harmonic voltage distortion at bus *i* is higher than that at the harmonic source bus *j*. A non-localized resonance occurs if there are one or more non-harmonic-source buses satisfying the resonance condition [15]. In this paper, the quantitative index is then extended to harmonic voltage source.

As for the harmonic voltage source, such as the background harmonic voltage, rectifier with DC-link capacitor, considering 1.0 per-unit voltage applied to the harmonic voltage source bus *s*, the resulting harmonic current is

$$I_s^h = 1/Z_{ss}^h \quad (4)$$

where Z_{ss}^h is the driving point *h*-th harmonic impedance at bus *s*. Considering the branch L_{ab}, according to (1), the resulting harmonic voltage at bus *a* and bus *b* can be expressed as

$$U_a^h = Z_{sa}^h / Z_{ss}^h \quad (5)$$

$$U_b^h = Z_{sb}^h / Z_{ss}^h \tag{6}$$

Obviously, under the per-unit system, U_a^h and U_b^h are also the amplification ratio of individual harmonic voltage distortion of bus *a* and bus *b* to the harmonic voltage source bus *s*, respectively.

Thus, the resulting harmonic current at the branch L_{ab} is

$$\left|I_{ab}^h\right| = \left|\frac{U_a^h - U_b^h}{z_{ab}^h}\right| = \left|\frac{Z_{sa}^h - Z_{sb}^h}{Z_{ss}^h \cdot z_{ab}^h}\right| \tag{7}$$

where z_{ab}^h is the *h*-th harmonic impedance of the branch L_{ab}. Then, the amplification ratio of harmonic current (*ARC*) of branch L_{ab} to the harmonic current resulted from the harmonic voltage source is

$$ARC_{ab} = \left|I_{ab}^h / I_s^h\right| = \left|\frac{Z_{sa}^h - Z_{sb}^h}{z_{ab}^h}\right| \tag{8}$$

Therefore, under the per-unit system, like (2) and (3), the amplification ratio of individual harmonic current distortion of branch L_{ab} to the harmonic current resulted from the harmonic voltage source is

$$ARC_{ab} = \frac{IHCD_{ab}^h}{IHCD_s^h} = \left|\frac{Z_{sa}^h - Z_{sb}^h}{z_{ab}^h}\right| \tag{9}$$

where $IHCD_{ab}^h$ and $IHCD_s^h$ are the *h*-th individual harmonic current distortions of branch L_{ab} and harmonic voltage source branch respectively. As for the branch between bus *a* and the ground *o* with the impedance of z_{ao}^h, such as the shunt capacitors, (9) will be expressed as

$$ARC_{ao} = \frac{IHCD_{ao}^h}{IHCD_s^h} = \left|\frac{Z_{sa}^h}{z_{ao}^h}\right| \tag{10}$$

where $IHCD_{ao}^h$ is the *h*-th individual harmonic current distortions of branch L_{ao}. Therefore, the affected areas and corresponding severities of harmonic voltage source can be presented with (9) and (10).

B. Harmonic Amplification Indices for Harmonic Source Bus

At the harmonic source bus, the FSA has been used to identify the parallel and series resonance frequencies and resulting impedances. However, the FSA can only qualitatively indicate the severity of a resonance at the harmonic source bus as the damping provided by the system resistance at this resonance frequency is generally difficult to quantify. Several kinds of harmonic resonance indices have been proposed to reveal the resonance condition for shunt capacitor applications at the harmonic source bus [21]–[23]. In [21], the resonance condition at the capacitor placement bus is defined if the harmonic voltage magnitude resulting from the maximum harmonic current injection given in the IEEE Std. 519 exceeds the harmonic voltage limit, the resonance occurs and likely cause problems. In this section, two indices are derived to define the resonance conditions at the harmonic current source bus and harmonic voltage source bus, respectively.

For the harmonic current source bus *j*, the resulting *h*-th harmonic voltage distortion (%) with the maximum amplitude of the harmonic current (at harmonic *h*) injection can be calculated as

$$\begin{aligned} HRU_j^h &= \frac{U_j^h}{U_j^1} \times 100\% = \frac{Z_{jj}^h \cdot I_j^h}{\left(U_N/\sqrt{3}\right)\cdot 1000} \times 100\% \\ &= \frac{\sqrt{3}\cdot Z_{jj}^h \cdot I_j^h}{10\cdot U_N} = \frac{\sqrt{3}\cdot Z_{jj}^h \cdot I_L \cdot RI_{j-\max}^h}{10\cdot U_N} (\%) \end{aligned} \tag{11}$$

where U_j^h(V) and U_j^1(V) are the *h*-th harmonic voltage and the fundamental voltage at bus *j* respectively, $RI_{j-\max}^h$(%) is the maximum allowable individual harmonic current distortion, I_L(A) is the maximum demand load current at bus *j*, I_j^h(A) is the maximum injected *h*-th harmonic current, $Z_{jj}^h(\Omega)$ here is the system *h*-th harmonic impedance at bus *j*, U_N(kV) is the nominal line voltage at bus *j*. If HRU_j^h exceeds the harmonic voltage limits, the resonance condition at the harmonic source bus is satisfied. It can reveal the possible degree of the voltage distortions at the harmonic source bus.

For the harmonic voltage source bus *j*, according to (11), the resulting *h*-th harmonic current distortion HRI_j^h(%) with the maximum amplitude of the harmonic voltage injection can be calculated as

$$HRI_j^h = \frac{10 \cdot U_N \cdot RU_{j-\max}^h}{\sqrt{3} \cdot Z_{jj}^h \cdot I_L} (\%) \tag{12}$$

where $RU_{j-\max}^h$ is the maximum allowable individual harmonic voltage distortion (%) at the harmonic voltage source bus *j*.

In fact, for the harmonic current source bus, we can also calculate the maximum allowable value of Z_{jj}^h based on (11), as shown in (13),

$$Z_{jj-\max}^h = \frac{10 \cdot U_N \cdot RU_{j-\max}^h}{\sqrt{3} \cdot I_L \cdot RI_{j-\max}^h} \tag{13}$$

Similarly for the harmonic voltage source bus, the maximum allowable value of the driving point *h*-th harmonic admittance can be calculated based on (12), as shown in (14). This detailed process will be presented in the case studies in Section V.

$$Y_{jj-\max}^h = \frac{\sqrt{3} \cdot I_L \cdot RI_{j-\max}^h}{10 \cdot U_N \cdot RU_{j-\max}^h} \tag{14}$$

III. Power System Uncertainty Categorization and Modeling

According to the origins of power system uncertainties, we can categorize them into aleatory uncertainty and epistemic uncertainty. Aleatory uncertainty derives from inherent randomness of the power systems. Epistemic uncertainty origins from human's lack of knowledge of the power systems and lack of the ability of accurately measuring and modeling the power systems. Unlike aleatory uncertainty, given more knowledge of the system and proper methods, epistemic uncertainty can be reducible to a certain degree. Taking the harmonic analysis in a real power system, for example, the probability distribution of the harmonic amplitudes is a representation of aleatory uncertainty, reflecting an inherent randomness of the power system. We cannot reduce this type of uncertainty. On the contrary, parameters of the probability distribution imply a kind of epistemic uncertainty, constrained by the existing knowledge and corresponding model [24].

As a wide variety of uncertainties exist in power systems, appropriate categorization indeed reminds us what we should notice in representing and processing diverse uncertainty in power systems. The uncertainties in power systems can be categorized into three kinds: 1) Aleatory fluctuations, including (but not limited to) system equivalent impedance and frequency, generator output from renewable energy sources, the composition of system loads, the random behaviors of the system, such as failures and faults in the system, and equipment parameter uncertainties due to the material properties of equipment effected by temperature, etc. 2) Long-term epistemic change, such as equipment parameter uncertainties due to the material properties of equipment effected by aging, etc. 3) Component parameter uncertainties due to manufacturing tolerances and installations. In this paper, the load variations are considered as the main aleatory uncertainties to investigate the aleatory randomness of harmonic propagation and amplification in power systems, while all kinds of uncertainties are considered to investigate the epistemic randomness.

A. Loading Condition Variations

Loading variability at each system bus represents uncertainty as the composition of different types and numbers of loads connected or disconnected at any time, including passive linear loads, nonlinear loads (harmonic sources), etc. As a significant component of the system harmonic impedance, the loads can affect the system harmonic damping and even the resonance conditions. It is shown that the harmonic modeling of linear load is sensitive to the model parameters, model topology, and the load composition. In this paper, three load models including series model, parallel model, and CIGRE model, as shown in Fig. 1, have been used to demonstrate the sensitivity of harmonic propagation and amplification to the load model type. It should be noted that in the CIGRE load model, the participation K is the ratio of the induction motor demand to the total load demand or active power P [25]. Meanwhile, load power data (the active and reactive power) are modeled as normal distributions with mean values equal to nominal loading and standard deviation derived from the load uncertainty [26]–[28].

B. Power Supply System Variations

In the harmonic analysis, generators are normally modeled as a series RL circuit representing the sub-transient reactance with X/R ratio ranging between 15–50, which depends on the short-circuit power and the system frequency caused by load and generation conditions, generation dispatch, topology reconfiguration, etc. [18].

As for the transformers, the winding capacitance starts to have some effect at 10 kHz, i.e. well above the maximum harmonic frequencies present in power systems. Therefore, transformer impedance is shown to be proportional to the leakage reactance, which have a manufacturing tolerance of -10% to 10% according to manufacturing standards for transformers [29].

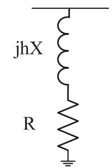
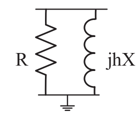
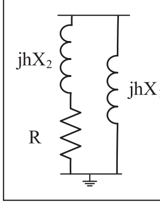
Model	Parameters
Series load model  jhX, R	$R = P \cdot \frac{U^2}{P^2+Q^2}$ $X = Q \cdot \frac{U^2}{P^2+Q^2}$
Parallel load model  R, jhX	$R = \frac{U^2}{P}$ $X = \frac{U^2}{Q}$
CIGRE model  jhX$_2$, jhX$_1$, R	$R = U^2/(1-K)\cdot P$ $X_1 = \frac{U^2}{K\cdot P\cdot(6.7\cdot\tan\phi - 0.74)}$ $X_2 = 0.073\cdot R$ $\tan\phi = Q/P$

Fig. 1. Three types of load models.

C. Component Parameter Variations

In practice, parameters of the electrical components are not exact due to manufacturing variations, the material properties effected by temperature, aging, etc. [30]–[32]. For example, capacitor units have a manufacturing tolerance ranging from -0% to 10% at 25°C uniform case, while resistance -10% to 10% and inductance -3% to 3%. Capacitance variation with temperature is typically in the range of 0.4% to 0.8% decrease per 10°C increase in temperature. Obviously, the combination of these tolerances is more significant than individual tolerances.

D. System Operation Condition Variations

The contingency system operation conditions, such as the switching of harmonic filter or other system components, reconfigurations of feeders, and system faults such as transformer outage and feeder cut-off, often have a significant impact on the system impedance and then influence the harmonic response in power systems.

IV. Stochastic Assessment Framework

A. MC Method

There are three main methods of dealing with uncertain problems such as MC simulation method [26], analytical method [18] and approximation method [27]. Though large amount of experiments is needed, the MC method can achieve the best accuracy and is used as accuracy verification standard of other methods. In this paper, the MC method is used to perform the stochastic assessment of harmonic propagation and amplification in power systems. In this work, the stopping rule (15) is used to limit the number of simulations [26]:

$$\varphi_{\bar{X}_n} = \frac{\phi^{-1}(1-\frac{\rho}{2})\sqrt{\frac{\tau^2(X_n)}{n}}}{\bar{X}_n} \tag{15}$$

Simulations are stopped if the sample mean error $\varphi_{\bar{X}_n}$ falls below a specified threshold. In (15), $\phi^{-1}(\cdot)$ is the inverse Gaussian conditional probability distribution with a mean of zero and standard deviation of one, $\tau^2(\cdot)$ represents the variance of a sample, ρ represents the desired confidence level, X_n is a sample of measured outputs consisting of *n* samples and $\bar{X}_n$ is the sample mean.

B. MC-Based Stochastic Assessment Method

With modeling the aleatory uncertainties, mainly the variations of loads, the aleatory variations of the harmonic propagation and amplification can be studied, while the whole randomness of the harmonic propagation and amplification can be revealed with considering the whole uncertainties in a power system. The proposed method can be summarized as follows:

1) Categorize and model power system uncertainties.
2) Within these probability distributions of the uncertain parameters to be investigated, simulate the two indices for the non-harmonic-source buses according to (3) and (9), respectively, while calculate the maximum allowable value of the driving point harmonic impedance or admittance according to (13) and (14), respectively. To investigate the aleatory randomness of system harmonic propagation and resonance amplification, only load uncertainties are considered in this paper, while the whole uncertainties are considered to investigate the epistemic randomness of system harmonic propagation and resonance amplification behaviors.
3) Investigate the sensitivity of stochastic harmonic propagation and resonance amplification behavior to the parameter uncertainty or modeling method using a rough global sensitivity analysis method, which is illustrated in Subsection V-D.

V. Case Studies

A. Test System Details

The proposed method has been performed on the modified IEEE 14-bus system, as shown in Fig. 2. Full system details, including basic parameters and harmonic impedance modeling methods are given in [33].

System uncertainties are presented for all generator outputs, system loads, and network elements. In this paper, for comparison, all the uncertain parameters are modeled using normal distribution with most values (99.7%) in the interval $m_n \pm \delta(\%) \cdot m_n$, i.e., $m \sim N(\mu_m, \sigma_m^2)$ with $\mu_m = m_n$ and $3 \cdot \sigma_m = (m_n \cdot \delta/100)$, where m_n represents the nominal values of the parameters, and different values of $\delta(\%)$ are adopted for different network parameters, which will be illustrated in Table I.

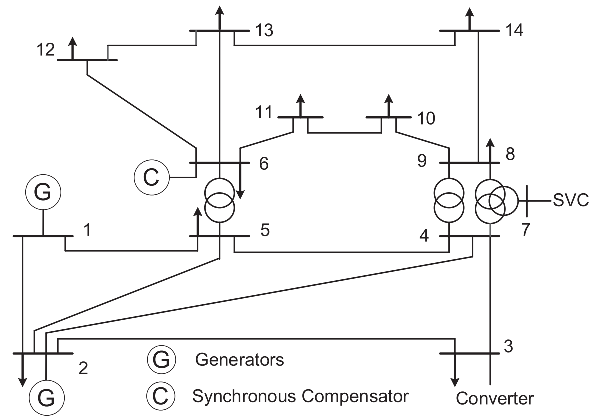


Fig. 2. Single line diagram of the IEEE-14 bus test system.

TABLE I
PARAMETERS FOR CONSIDERED UNCERTAINTIES

Uncertainties	Parameters
Reactance of Generator	$\delta = 10\%$
Resistance of Transformer	$\delta = 5\%$
Reactance of Transformer	$\delta = 10\%$
Load	$\delta = 50\%$
Line	$\delta = 10\%$
R of Filter	$\delta = 10\%$
L of Filter	$\delta = 5\%$
C of Filter	$\delta = 10\%$
Shunt Capacitor	$\delta = 10\%$

B. Epistemic Randomness of Harmonic Propagation and Amplification Due to the Whole System Uncertainties

In this paper, for each harmonic from 2 pu to 50 pu, $\rho = 0.01$ is adopted in (15) and MC procedure stopped when $\varphi_{\bar{X}_n}$ is less than a tolerance $\varphi = 0.01$ to ensure 99% confidence level of the results. With considering all kinds of potential uncertainties, the randomness of harmonic propagation and amplification due to epistemic uncertainty that origins from the lack of knowledge of the power system and lack of the ability of measuring and modeling the power system can be investigated. The CIGRE load model is used here. The box plot of the driving point impedances of bus 3 is shown in Fig. 3, where the magenta dotted line is the maximum allowable value of Z_{jj}^h based on (13). If the driving point impedance is bigger than the corresponding maximum allowable value, the harmonic voltage amplification may occur, such as the 3rd and 33rd harmonics. The box plot of the *ARVs* of harmonic source bus 3 to bus 8 is shown in Fig. 4. As the sensitivity of the index to different components vary at different frequencies, varying degrees of randomness of the *ARVs* occur at different frequencies.

With considering all kinds of potential uncertainties, the epistemic randomness of harmonic propagation and amplification in the power systems can be investigated. The box plot of the driving point admittances of bus 1 is shown in Fig. 5, where the magenta dotted line is the maximum allowable value of Y_{jj}^h

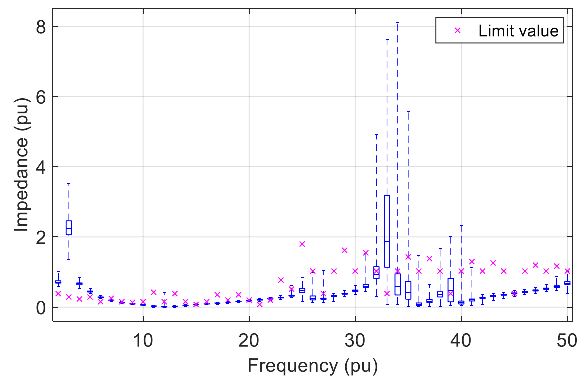


Fig. 3. Boxplot of the driving point impedances of harmonic source bus 3 considering the whole uncertainties.

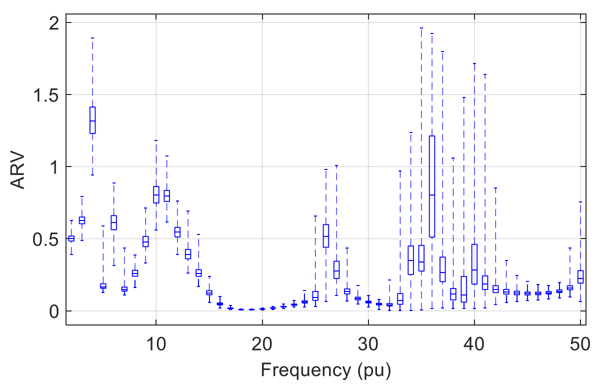


Fig. 4. Boxplot of the *ARVs* of harmonic source bus 3 to bus 8 considering the whole uncertainties.

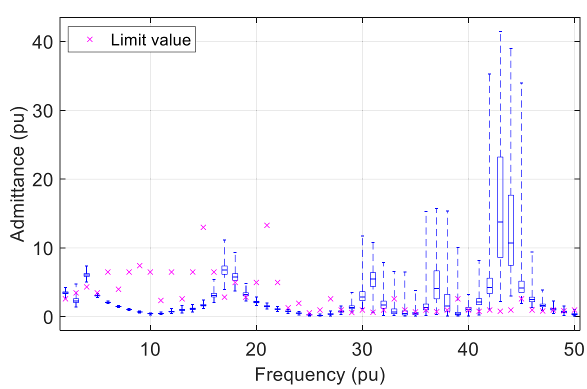


Fig. 5. Boxplot of the driving point admittances of harmonic source bus 1 considering the whole uncertainties.

based on (14). If the driving point admittance is bigger than the corresponding maximum allowable value, the harmonic current amplification may occur, such as the 31st, and 43rd harmonics. The box plot of the *ARCs* of the branch between bus 4 and bus 5 to harmonic voltage source bus 1 considering the whole uncertainties is shown in Fig. 6. Similarly, varying degrees of randomness of the *ARCs* occur at different frequencies.

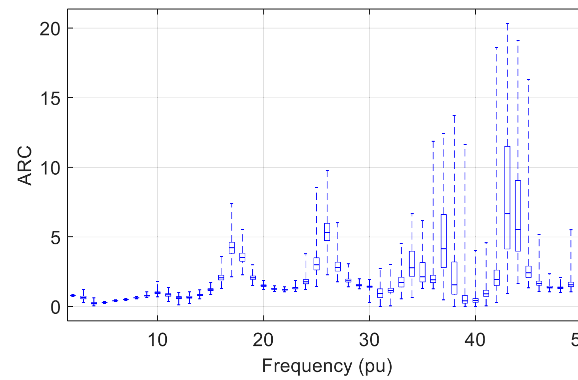


Fig. 6. Boxplot of the *ARCs* of the branch between bus 4 and bus 5 to harmonic voltage source bus 1 considering the whole uncertainties.

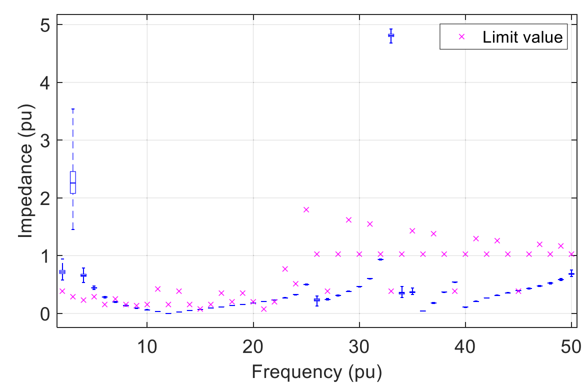


Fig. 7. Boxplot of the driving point impedances of harmonic source bus 3 only considering load uncertainties with CIGRE load model.

C. Aleatory Randomness of Harmonic Propagation and Amplification Due to Load Variations

Loads are a significant component of the system harmonic impedance. However, the loads, especially the consumer, always present the greatest uncertainties in a real-life power system, are also the main aleatory uncertainty in power systems. The load impedance is time-varying as it depends on what devices are connected. This information is not known beforehand and is a major uncertainty in harmonic load modelling. Therefore, it is necessary to study the effect of load variations on the harmonic propagation and resonance amplification. As presented in [25], load level will influence the harmonic driving point impedance of the system at a specific bus and lightly loaded systems can exhibit more pronounced resonance peaks at the harmonic source bus. Obviously, the load level will also influence the harmonic propagation and resonance amplification ratios at the non-harmonic source buses with varying degrees.

With considering the load uncertainties with CIGRE load model, the aleatory randomness of harmonic propagation and resonance amplification can be investigated. As shown in Fig. 7, the load variations have much bigger influence on the driving point impedances at low-order harmonic frequencies. However, as shown in Fig. 8, the load variations can result in rather big variations of the harmonic propagation and resonance amplification,

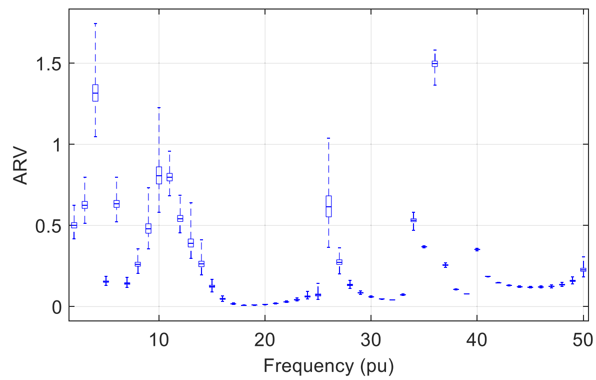


Fig. 8. Boxplot of the *ARVs* of harmonic source bus 3 to bus 8 only considering load uncertainties with CIGRE load model.

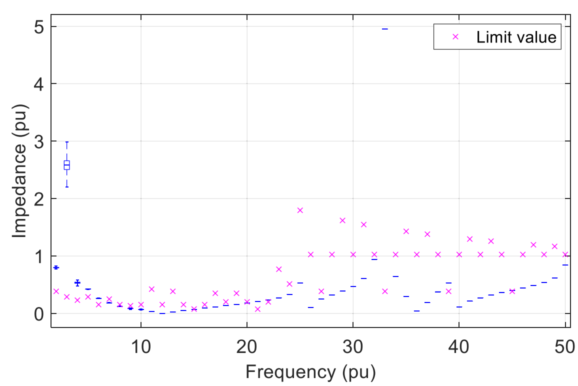


Fig. 9. Boxplot of the driving point impedances of harmonic source bus 3 only considering load uncertainties with series load model.

illustrating that the harmonic response of the power systems are rather sensitive to the load variations. In Subsection V-D, two other common load models [25] are tested to investigate whether they could reflect the influence of load variations on the power system harmonic response.

D. The Influence of Load Modelling Method

Loads have a significant impact on the frequency and amplitude of resonances, and it is necessary to choose an appropriate load model to represent the system load as accurate as possible [25], [34]. Variance-based global sensitivity analysis methods refer to ways of quantifying the contribution of each input parameter to the total variance of the output [35], [36]. Therefore, when only load uncertainties are considered in investigating the aleatory randomness of system harmonic propagation and resonance amplification, the bigger the variance of the resonance amplification index is, the more sensitive it is to the load uncertainties. In this case, this rough analysis method is used to investigate the influences of different modeling methods on the results.

Only considering load uncertainties, the box plots of the driving point impedances of harmonic source bus 3 using the series load model is shown in Fig. 9, while the parallel load model in Fig. 11, the CIGRE load model in Fig. 7. Although

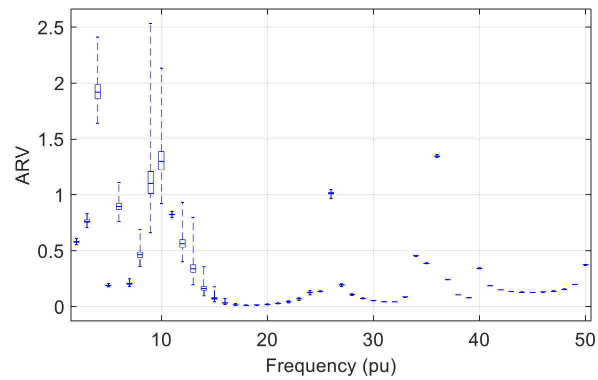


Fig. 10. Boxplot of the *ARVs* of harmonic source bus 3 to bus 8 only considering load uncertainties with series load model.

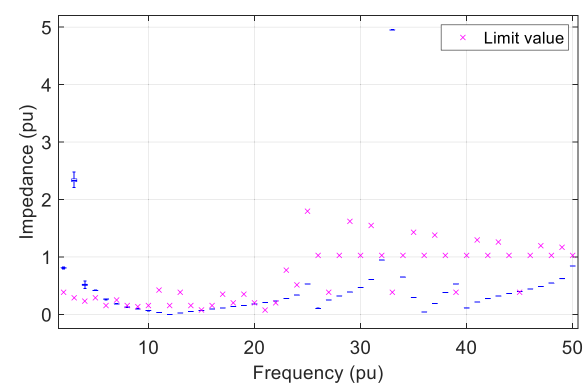


Fig. 11. Boxplot of the driving point impedances of harmonic source bus 3 only considering load uncertainties with parallel load model.

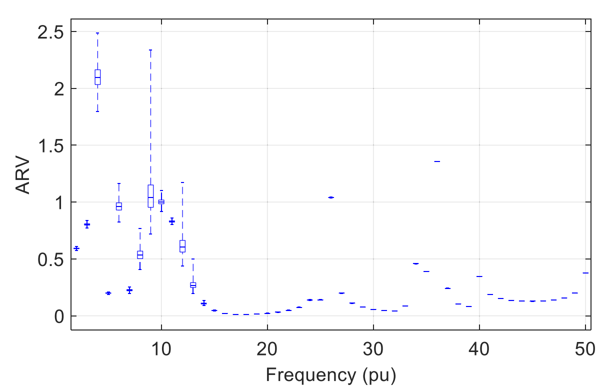


Fig. 12. Boxplot of the *ARVs* of harmonic source bus 3 to bus 8 only considering load uncertainties with parallel load model.

there are no obvious distinctions between the three types of load models in studying the driving point impedances, the CIGRE load model has the best sensitivity for revealing the influence of load variations on the driving point impedances, as bigger variances are presented in Fig. 7.

Only considering load uncertainties, the box plots of the *ARVs* of harmonic source bus 3 to bus 8 using the series load model is shown in Fig. 10, while the parallel load model in Fig. 12, the CIGRE load model in Fig. 8. The differences of

TABLE II
HARMONIC SOURCE DATA FOR THE TWELVE-PULSE HVDC TERMINAL

H-order	Mag(p.u.)	Angle(deg)
1	1.0000	-49.56
5	0	0
7	0	0
11	0.0838	-20.53
13	0.0648	85.46
17	0	0
19	0	0
23	0.0250	75.81
25	0.0266	177.95

the results due to these three different load models are obvious. At high-order harmonic frequencies, the results using CIGRE load model possess bigger variances than those using series load model or parallel load model, illustrating that the CIGRE load model can provide a better approximation of the load harmonic impedance at high-order harmonic frequencies. However, at low-order harmonic frequencies, for the variances of the results respectively using the three load models, there are varying degrees of difference at different harmonic frequencies. On the whole, the CIGRE load model can provide better approximation of the load harmonic impedance than series load model and parallel load model. Therefore, the harmonic propagation and amplification analysis results are sensitive to the load model type.

However, since loading variability at each system bus represents uncertainty as the composition of different types and numbers of loads connected or disconnected at any time, including resistive loads, motive loads, nonlinear loads (harmonic sources), etc., a comprehensive investigation of the load composition is necessary for choosing a suitable model or composition of models for the load at each system bus. Therefore, in addition to load level uncertainties, it is necessary to further consider the model uncertainties for the harmonic modeling of load at each system bus [34].

E. Application in the Placement of Passive Harmonic Filter

Passive harmonic filters are still the main devices for reducing harmonic distortion in power systems. Apart from the local filter at the harmonic source bus, additional assessment is also needed for the buses where non-localized resonance amplifications may occur or the loads are sensitive to harmonic pollution. The proposed four indices can predict whether localized or non-localized resonances will occur or not and their corresponding probabilities, which is helpful for finding effective placement of filters. It should be noted that, to determine which values of the stochastic resonance amplification severity indices one should use, harmonic source data and the harmonic requirement of the loads should be also considered.

In the IEEE 14-bus system, the twelve-pulse HVDC terminal at bus 3 is modeled as a harmonic current source [15]. The source spectra are provided in Table II. Two single-tuned 11th harmonic filters have been placed at bus 3 [33].

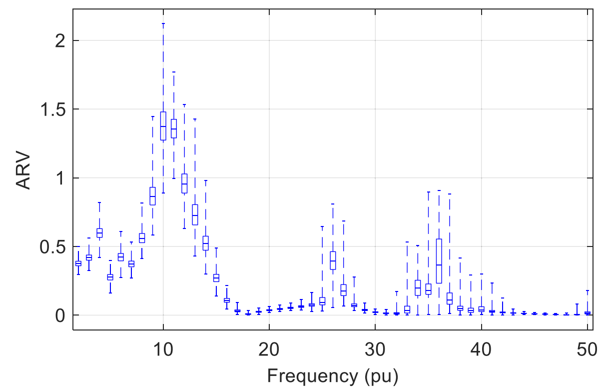


Fig. 13. Boxplot of the *ARVs* of harmonic source bus 3 to bus 10 without 11th harmonic filter at bus 10.

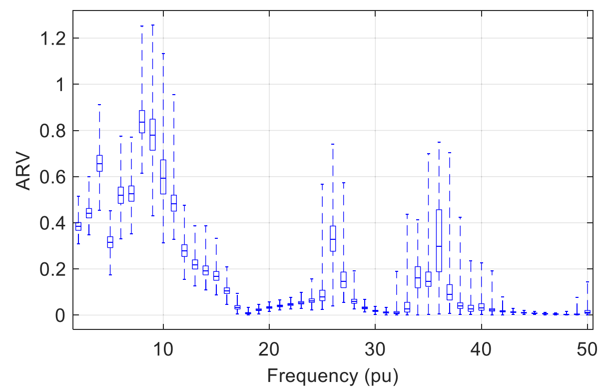


Fig. 14. Boxplot of the *ARVs* of harmonic source bus 3 to bus 10 with 11th harmonic filter at bus 10.

The ARVs of harmonic source bus 3 to bus 10 are shown in Fig. 13. It is obvious that the 10th harmonic and 11th harmonic will be amplified at bus 10, while the 12th harmonic may be amplified. However, there are almost no even harmonics in the HVDC terminal. If the loads at bus 10 are sensitive to harmonics, the 11th harmonic filter is needed at bus 10. The ARVs of harmonic source bus 3 to bus 10 with 11th harmonic filter at bus 10 are shown in Fig. 14. The 11th harmonic amplification at bus 10 has been suppressed successfully.

However, apart from reducing the harmonic distortion at the buses which are sensitive to harmonic pollution, the strategy to optimally place harmonic filters would consider optimizing the size and/or cost of a filter as well as achieving the maximum harmonic suppression in the system at the same time. Although the proposed indices can provide a preliminary placement proposal of harmonic filters, case-dependent global optimization analysis is needed for finding effective harmonic filter scheme based on the desired objectives in a power system.

VI. CONCLUSION

In this paper, a set of quantitative indices has been derived, and then a systematic stochastic assessment technique for harmonic propagation and amplifications in power systems is presented with considering different types of uncertainties. The impacts of different load models on the stochastic harmonic propagation and amplifications in power systems have been investigated. The results have revealed many interesting insights on the phenomenon. The main findings of this work are of the following.

1) For harmonic current source and harmonic voltage source respectively, the proposed quantitative indices can be used to reveal the propagation areas and the varying severities of harmonic propagation and amplification at the harmonic source bus and the non-harmonic-source bus, respectively.
2) Appropriate categorization of uncertainties existing in power systems, i.e. aleatory uncertainties and epistemic uncertainties, is very helpful for understanding the time-varying characteristic of harmonic response and reminding us what we should notice in representing and processing diverse uncertainty in power systems. Meanwhile, considering stochastic harmonic response of the system as well as the randomness of harmonic sources will be helpful for designing robust harmonic mitigation schemes.
3) The randomness of harmonic propagation and amplification with different load models have been compared and analyzed, demonstrating the high sensitivity of harmonic response to the load model type. In addition, the harmonic propagation and resonance amplification is more sensitive at low-order harmonic frequencies to the load variations.

Many significant changes are taking place in today's power systems, including high penetration of renewable energy sources, changes in load composition and changes in the grid itself, such as replacement of overhead lines by underground cables. These changes affect the harmonic propagation and amplification in power systems in varying ways and degrees, and more detailed studies need to be performed using the proposed method in the future. Meanwhile, it is hoped that this method may enable online probabilistic analysis, or optimization based upon repeated probabilistic studies, for example for harmonic suppression scheme designs for power systems including uncertainties.

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