Letters

Impedance-Based Stability Criterion for Grid-Connected Inverters

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***Abstract*—Grid-connected inverters are known to become unstable when the grid impedance is high. Existing approaches to analyzing such instability are based on inverter control models that account for the grid impedance and the coupling with other grid-connected inverters. A new method to determine inverter-grid system stability using only the inverter output impedance and the grid impedance is developed in this paper. It will be shown that a grid-connected inverter will remain stable if the ratio between the grid impedance and the inverter output impedance satisfies the Nyquist stability criterion. This new impedance-based stability criterion is a generalization to the existing stability criterion for voltage-source systems, and can be applied to all current-source systems. A single-phase solar inverter is studied to demonstrate the application of the proposed method.**

***Index Terms*—Current source systems, grid-connected inverters, harmonic resonance, impedance analysis, small-signal stability.**

I. Introduction

Most renewable energy sources are connected to the power grid through an inverter. In such grid-connected mode, the inverter is typically controlled as a current source [1] to inject certain amount of current into the grid. Dynamic interaction of grid-connected inverters with the power grid has been a topic of extensive study in recent years due to the rapidly increasing penetration of renewable energy and distributed generation (DG) resources. Traditional power system theory uses phasor-based models to study the effects of DG sources on grid stability below the fundamental frequency [2], [3]. Inverter-produced harmonic currents can also be a problem for power quality and have been discussed in the literature [4], [5].

An increasingly important concern for grid-connected inverter is the effects of grid impedance on inverter control performance and stability. It is known that high grid impedance can destabilize the inverter current control loop and lead to sustained harmonic resonance or other instability problems (see, e.g., [6]). Existing approaches to analyzing such instability problems require detailed inverter control models that account for the grid impedance [6], [7], as well as the coupling with other grid-connected inverters, and use root locus and other time- or frequency-domain techniques to determine inverter control stability under variable grid conditions.

Inverter instability in the presence of high grid impedance is similar to the converter-filter or the more general source-load interaction problems found in many other power electronic systems. A well-established technique to analyze such interconnected systems is by the impedance-based stability criterion: The ratio of the source output impedance to the load input impedance must satisfy the Nyquist stability criterion in order for the interconnected source-load system to be stable. The technique is widely used in the design of switching-mode power supplies with input filters [8], as well as more complex dc distributed power systems [9]. Generalization of the technique to ac power systems has also been reported [10].

Detailed inverter control models and their loop stability analysis are necessary for the design of individual grid-connected inverters. When the objective is grid system stability analysis, external behavior of an inverter is of more interest than its internal loop stability and is also easier to obtain. In such cases, the impedance-based approach is more advantageous and effective [10], as it avoids the need to remodel each inverter and repeat its loop stability analysis when the grid impedance changes, or when more inverters are connected to the same grid. It also does not require detailed design information of individual inverters, which is often not available to those performing grid system stability analysis.

There is, however, a conceptual problem when one tries to apply the existing impedance-based stability criterion to grid-connected inverters, as either the inverter or the grid could be treated as the *source* in the analysis, resulting in completely opposite stability conclusions. The purpose of this letter is to present an impedance-based stability criterion for grid-connected inverters by 1) recognizing that the existing criterion is valid only for voltage-source systems, and 2) developing a new criterion for current-source systems. Experimental results from a solar inverter operated with a DG test-bed are presented to demonstrate the application of the proposed method.

II. Impedance-Based Stability Analysis

The essence of the impedance-based stability criterion, first presented in [8], is to partition the system under study into a source and a load subsystem. The source subsystem is modeled by its Thevenin equivalent circuit consisting of an ideal voltage source (V_s) in series with an output impedance (Z_s), while the load subsystem is modeled by its input impedance (Z_l) (see Fig. 1). Since almost all power electronic circuits are

Manuscript received August 9, 2010; revised January 5, 2011; accepted March 1, 2011. Date of current version November 18, 2011. Recommended for publication by Associate Editor P. Rodriguez.

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Digital Object Identifier 10.1109/TPEL.2011.2136439

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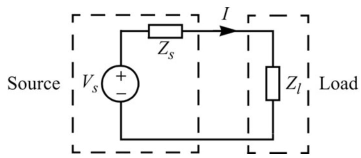

Fig. 1. Small-signal representation of a voltage source with load.

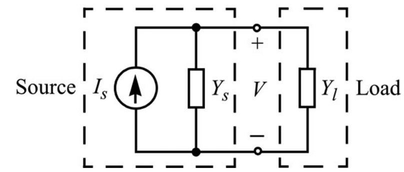

Fig. 2. Small-signal representation of a current source with load.

nonlinear, this linear representation is only valid for small-signal analysis.

With the assumed small-signal model, the current I flowing from the source to the load is

$$I(s) = \frac{V_s(s)}{Z_l(s) + Z_s(s)} \tag{1}$$

which can be rearranged in the following form:

$$I(s) = \frac{V_s(s)}{Z_l(s)} \cdot \frac{1}{1 + Z_s(s)/Z_l(s)}. \tag{2}$$

For system stability analysis, it can be assumed that the source voltage is stable when unloaded and the load current is stable when powered from an ideal source. In that case, both $V_s(s)$ and $1/Z_l(s)$ are stable, such that stability of the current depends on the stability of the second term on the right-hand side of (2)

$$H(s) = \frac{1}{1 + Z_s(s)/Z_l(s)}. \tag{3}$$

The impedance-based stability criterion is based on the observation that *H(s)* resembles the close-loop transfer function of a negative feedback control system where the forward gain is unity and the feedback gain is $Z_s(s)/Z_l(s)$; that is, the ratio of the source output impedance to the load input impedance. By linear control theory, *H(s)* is stable if and only if $Z_s(s)/Z_l(s)$ satisfies the Nyquist stability criterion [8].

When applying the impedance-based stability criterion presented above, it is important to recognize that the source is assumed to be a voltage source that is stable when unloaded. Since most practical sources are voltage sources and are stable when unloaded, this assumption tends to be forgotten. However, grid-connected inverters are usually controlled in the current-injection mode and do not behave like a voltage source. Hence, their stability cannot be analyzed by the existing impedance-based method.

To develop an impedance-based stability criterion for current-source systems, we start from a small-signal model similar to that used in the voltage-source case. However, instead of using a Thevenin equivalent circuit, we represent the current source by a Norton equivalent circuit, in the form of a current source (I_s) in parallel with an output admittance (Y_s). The load is represented by its input admittance, Y_l (see Fig. 2). With this small-signal representation, the voltage across the load is

$$V(s) = \frac{I_s(s)}{Y_l(s) + Y_s(s)}. \tag{4}$$

which can be rearranged as follows:

$$V(s) = \frac{I_s(s)}{Y_l(s)} \cdot \frac{1}{1 + Y_s(s)/Y_l(s)} \tag{5}$$

Similar to the voltage-source case, the current source can be assumed stable when unloaded (that is, when the load is a short-circuit such that its admittance is infinite), and the load is stable when powered from an ideal current source. Under these assumptions, both $I_s(s)$ and $1/Y_l(s)$ are stable, such that the stability of *V(s)* depends on the stability of the second term on the right-hand side of (5). Note again that this term resembles the close-loop transfer function of a negative feedback control system where the forward gain is unity and the feedback gain is $Y_s(s)/Y_l(s)$. Therefore, a current-source system is stable if:

1) The current source itself is stable when unloaded, i.e., when the load is a short-circuit.
2) The load is stable when supplied by an ideal current source.
3) The ratio of the source output admittance to the load input admittance satisfies the Nyquist criterion.

Note that admittances instead of impedances are used in (5) in order to highlight its duality to (2). The analysis can also be carried out based on the source output impedance (Z_s) and load input impedance (Z_l), in which case (5) becomes

$$V(s) = I_s(s) Z_l(s) \cdot \frac{1}{1 + Z_l(s)/Z_s(s)} \tag{6}$$

and stability of the interconnected system requires the ratio of the load input impedance to the source output impedance meet the Nyquist stability criterion.

Comparing (6) to (2), one can see that stability requirements for current-source systems are opposite to that for voltage-source systems: A current source should have high (ideally infinite) output impedance, while a voltage source should have low (ideally zero) output impedance in order to ensure stable operation with a wide range of loads; and a current-source system is more stable when the load impedance is low, while a voltage-source system is more stable when the load impedance is high.

Given that Thevenin and Norton equivalent circuits can be used exchangeably in circuit analysis, one might ask why it is necessary to make a distinction between current-source and voltage-source systems. Further, one may argue that a current-source system could also be represented by the Thevenin equivalent circuit shown in Fig. 1, such that, based on (3), stability of the system would require the ratio of the source output impedance to the load input impedance, not the inverse as appeared in (6), to satisfy the Nyquist stability criterion. To answer these questions, recall that two assumptions were made in order for (3) to have the same stability characteristics as the original system:

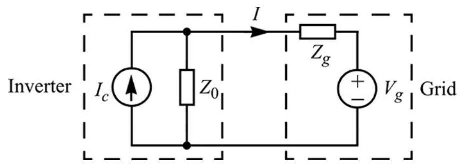

Fig. 3. Small-signal representation of an inverter-grid system.

1) The voltage source is stable when unloaded, i.e., $V_s(s)$ has no unstable poles.
2) The load is stable when powered from an ideal voltage source, i.e., its input impedance $Z_l(s)$ has no right-half plane zeros.

Since there are no native current sources, practical current sources (for power applications) are typically made of inductors with active current control. Such a current source cannot function properly when the output is an open-circuit, because there would be no external path for the current to flow in that case. Although not common in practice, an internal path could, in theory, be provided to allow the current to circulate, e.g., by having a resistor connected in parallel with the output terminals. But the impedance of such a path must be very high in order not to lower the overall output impedance of the current source, which, ideally, should be infinite. With such high impedance, the open-circuit output voltage will be extremely high unless the current is drastically reduced from its rated value by, e.g., entering a voltage-limiting mode, which is not the intended operation mode of a current source. Therefore, practical current sources cannot be assumed to function properly within their designed output current range when the output is an open-circuit. As a result, a practical current-source system in general cannot be represented using the Thevenin equivalent circuit shown in Fig. 1 for system stability analysis using (3). Instead, a current-source representation based on (5) or (6) must be used.

III. Grid-Connected Inverters

The most common grid model for system stability analysis is an ideal voltage source in series with a grid impedance, Z_g, typically consisting of an inductor in series with a resistor. With the grid-connected inverter modeled as a current source in parallel with an output impedance, the overall inverter-grid system can be represented by the small-signal equivalent circuit shown in Fig. 3. This can be viewed as a hybrid system consisting of both a voltage source and a current source. Consistent with the previous assumptions and common practice, the grid voltage can be assumed to be stable without the inverter, and the inverter to be stable when the grid impedance is zero. Based on the equivalent circuit, the inverter output current is

$$I(s) = \frac{I_c(s)Z_0(s)}{Z_0(s)+Z_g(s)} - \frac{V_g(s)}{Z_0(s)+Z_g(s)} \tag{7}$$

which can be rearranged to

$$I(s) = \left[I_c(s) - \frac{V_g(s)}{Z_0(s)}\right] \cdot \frac{1}{1+Z_g(s)/Z_0(s)}. \tag{8}$$

Based on (8) and the assumptions stated before, a grid-connected inverter will operate stably if the ratio of the grid impedance to the inverter output impedance, $Z_g(s)/Z_0(s)$, satisfies the Nyquist criterion. Stability margin of the system can also be measured from the Nyquist plot of $Z_g(s)/Z_0(s)$. Note that one arrives at the same conclusion if the system is treated as a voltage-source system powered by the grid.

The stability criterion stated above indicates that a grid-connected inverter should be designed to have as high output impedance as possible in order to operate stably under a wide range of grid conditions. The output impedance, therefore, is an important performance index for grid-connected inverters, and provides a simple means for characterizing and comparing different inverter designs. Harnefors *et al.* [11] discussed the relationship between the input impudence of a PWM rectifier and its various control parameters, which can be extended to provide guidelines for the shaping of grid-connected inverter output impedance. The output impedance of an inverter also depends on its output filter design. In particular, damping of the filter [12], whether using passive or active methods, is necessary to avoid dipping in the output impedance. Grid synchronization methods [1] also have strong effects on the inverter output impedance, which have not been given much attention so far and will be studied in a future work.

IV. Experimental Results

To demonstrate the application of the stability criterion developed in the previous section, a 3 kW commercial single-phase solar inverter has been tested on a DG test-bed, where the grid impedance can be varied. Two sets of test results are presented here. In the first case, the inverter was connected to the grid directly, and the inverter output current and voltage waveforms are shown in Fig. 4(a), which indicate stable operation. In the second case, an additional inductor $L_p = 12.8$ mH was inserted to purposely increase the grid impedance. The measured inverter output current and voltage waveforms are presented in Fig. 4(b). The grid current spectrum, shown in Fig. 5, indicates significant 5th and 7th harmonics. The inverter output power was 1700 W in both cases.

To apply the impedance-based stability criterion presented in the previous section, the output impedance of the inverter and the grid impedance was measured by using a frequency analyzer for each test condition. Fig. 6 shows the measured frequency responses of the inverter output impedance and the grid impedance for the two cases tested. Without the additional inductor L_p, the grid impedance intersects with the inverter output impedance at three different frequencies (3 kHz, 8 kHz, and 25 kHz) where the phase difference is 115°, −20°, and 110°, respectively, indicating sufficient phase margin in system stability at all points. With the additional 12.8 mH inductor, the grid impedance intersects with the inverter output impedance only once at about 420 Hz, where the phase difference is about 160°. This intersection frequency correlates closely with the dominant harmonic frequencies in the measured inverter output current and voltage, indicating that the harmonics are caused by

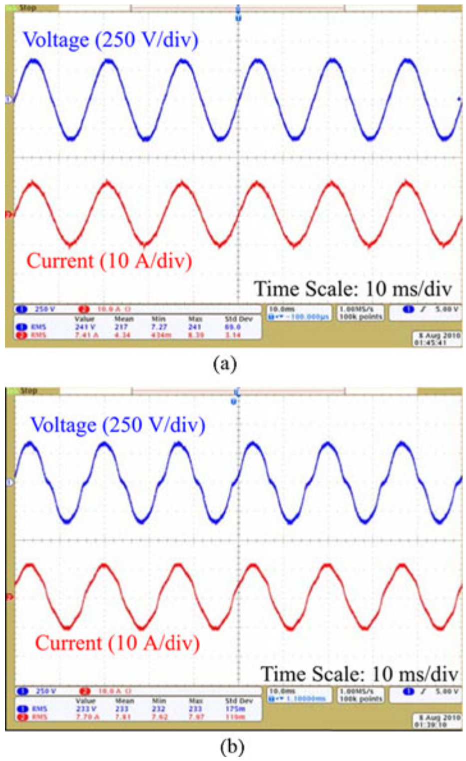

Fig. 4. Solar inverter terminal voltage and output current measurements. (a) Stable operation when connected directly to the grid. (b) Harmonic resonance when a 12.8 mH inductor is inserted in series with the grid.

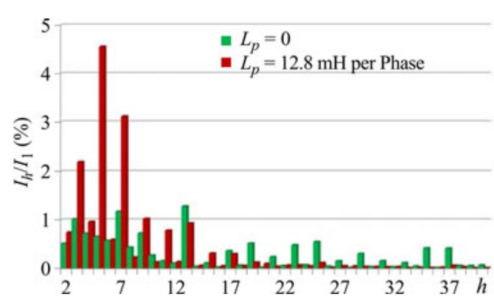

Fig. 5. Comparison of grid current harmonics with different grid impedance.

a marginally stable resonance between the inverter and the grid impedance at this frequency.

V. Summary

The effects of grid impedance on grid-connected inverter stability can be studied by using a simple small-signal model involving the grid impedance and the inverter output impedance. Under the assumption that the inverter is controlled as a current source and is stable when operating with an ideal grid, it will remain stable with a nonideal grid if the ratio of the grid impedance to the inverter output impedance satisfies the Nyquist criterion. This new impedance-based stability criterion is a generalization to a similar existing stability criterion for voltage-source systems, and can be applied to other current-source systems as well. The analysis also indicates that grid-connected inverters should be designed to have as high output impedance as

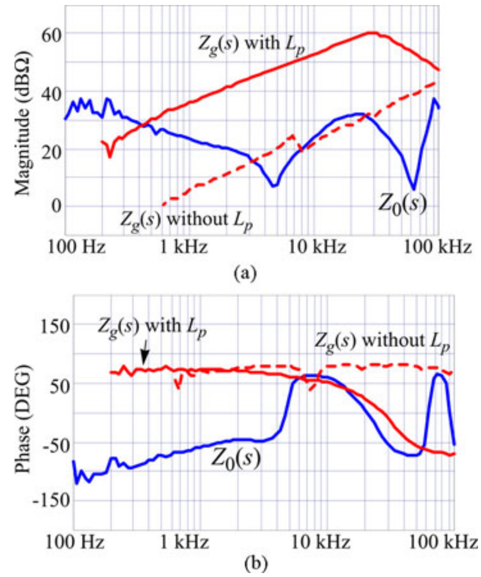

Fig. 6. Frequency response of the inverter output and the grid impedance.

possible in order to be able to operate with a wide range of grid impedance. Future work will discuss the design of grid-parallel inverter control, output filter and damping, as well as grid synchronization methods to shape and maximize the output impedance.

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