Criteria to Estimate the Voltage Unbalances due to High-Speed Railway Demands

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Abstract - This paper has presented simple criteria to estimate the voltage unbalances due to high-speed railway demands that are generally single-phase loads. Feeding traction loads from the public power system may lead to some voltage unbalance on the latter and consequently affect the operation of its energy-supply system and other equipments connected with it. Three transformer connection schemes that are commonly used in power-supply systems for the high-speed railway are discussed and compared. The estimating criteria have been derived and represented by simple formula that can be easily applied to evaluate this voltage unbalance. The results are of value to related engineers and consultants especially during periods of planning and design.

I. INTRODUCTION

The electric locomotive is a strong contender to provide the most efficient and economical motive power for heavy traffic density. The steam railway has been converted to an electric railway in Taiwan during the last decades of the twentieth century. A high-speed electric railway between Taipei and Kaohsiung is currently planned to solve the problem of deteriorating highway. The required traction power of this West Taiwan High-Speed Rail Link will be supplied by electrical energy from seven feeder stations situated along the railroad route and each feeder station will be supplied by two independent incoming feeds from the 161-kV high-voltage grid of the Taiwan Power Company (Taipower). The significant railroad electrification confronts Taipower engineers with the problem of supplying the railroad's electric energy, and with the engineering problems associated with erecting and operating the catenary system.

The most economical electrification method is customarily based upon single-phase 25-kV or 50-kV ac traction with thyristor-controlled locomotives. The three-phase power-supply system will therefore be called upon to supply a large single-phase load, which will cause distortion of the supply voltage and current waveforms due to harmonics and be subject to sudden large load changes during the transition of trains through neutral sections and when trains change from a driving to a braking mode or vice versa[1,2]. These problems which may have detrimental effects on the traction energy-supply system, its devices and other consumers demand great attention of related engineers and consultants. The voltage unbalance and fluctuation, harmonic, and power factor compensation problems that have been identified require detailed analyses to realize their effects.

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To estimate the generated disturbances the complete set of related data and many detailed studies in normal and abnormal system conditions are necessary. Here we devote attention to only the voltage unbalance problem. The rigorous study of voltage unbalance effects needs advanced system analysis capabilities that are not available with the existing packages. It is therefore necessary to develop a three-phase power-flow analysis program with rigorous component models to satisfy the requirements of the rigorous system analysis[3-5]. On the basis of this powerful analysis tool, complete unbalance background information and parameters of energy-supply system, the more rigorous studies of the effects of voltage unbalance due to the traction load of the high-speed railway can be undertaken. Furthermore, ways to reduce the voltage unbalance can be considered.

However, it should take much time to develop a three-phase power-flow program that can analyze a multivoltage transmission system. Besides, some component models are lacking. Much effort should be expended before rigorous evaluations of the effects of voltage unbalance can be done. Therefore, some simple criteria have been applied by consultants to estimate the voltage unbalances due to high-speed railway demands supplied by ac single and two-phase schemes. The criteria of three substation connection schemes are derived and compared in this paper. It is of value to the engineers who are concerned with the unbalance problems cause by the high-speed railway demands.

II. POWER-SUPPLY SCHEMES

Many factors affect the selection of power-supply schemes. However, they are beyond the scope of this paper, and are not discussed here. The most commonly used power-supply schemes are 1 × 25 kV and 2 × 25 kV[6]. The maximum length of a sector of the latter is greater than that of the former. The minimum required number of substations for these two power-supply schemes also differ. Generally, the required number of substations of the latter is less than that of the former. However, neither affects the degree of unbalance on the supply high-voltage network. The most important factor that affects the degree of voltage unbalance is the connection method of the traction substation. The three most commonly used connection schemes are single-phase, V- and Scott-connections shown in Figs. 1-3. These have different characteristics, and therefore different effects on the energy-supply system, and also have different costs in installation and maintenance. For these three connection schemes, formulas are derived to evaluate easily the voltage unbalances due to high-speed railway demands. Comparison of characteristics and effects of these three schemes are also drawn. The S_{L1} and S_{L2} in Figs. 1-3 are the power demands on respectively the first and second side of the railway substation.

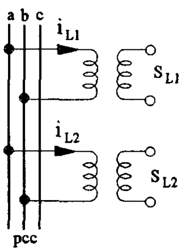

Fig. 1 Single-phase connection

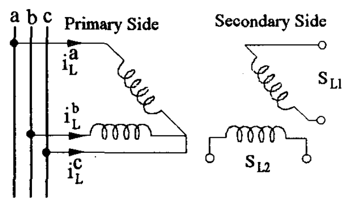

Fig. 2 V-connection

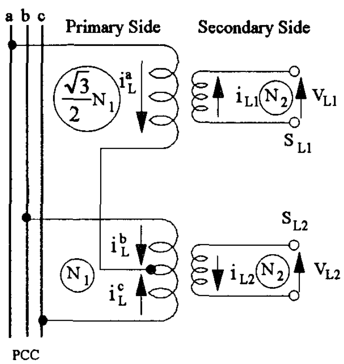

Fig. 3 Scott-connection

III. DERIVATIONS OF VOLTAGE UNBALANCES DUE TO HIGH-SPEED RAILWAY DEMANDS

To simplify the derivations, the transformers were assumed to be ideal. Hence no impedance exists in the transformers and therefore no voltage drop and loss is incurred. According to this assumption, the traction load and transformers in the railway substation are simply represented by their equivalent loads and the voltage unbalances of the primary side of the transformer that is the point of common coupling (PCC) due to high-speed railway demands can be derived. The three most commonly used transformer connections in the high-speed railway power-supply systems are discussed and compared in the following sections. The results are simple and can be easily applied to estimate roughly the degrees of voltage unbalance due to high-speed railway demands.

A. Single-phase connection

In general, the connection of transformers in the traction substations is worked out between two phases of the high-voltage network when the single-phase connection scheme is adopted. Feeding traction substations between two phases may lead to some voltage unbalances on the high-voltage network and consequently affect the operation of the energy-supply system and other equipment connected with it. These unbalances are as strong as when three-phase short circuit powers are weak. In addition, the two transformers of a traction substation are fed on the same phases shown in Fig. 1, but substations in succession are generally fed on different phases alternatively as follows: A-B, B-C, C-A. Thus the traction load is almost equally distributed on the three phases. This arrangement allows a small voltage unbalance in the high-voltage network. The circuit diagram of the traction power-supply system with a single-phase connection scheme can therefore be simplified as shown in Fig. 4. The public high-voltage network is represented by its Thevenin equivalent circuit by using the short-circuit power at the point of common coupling. The corresponding equivalent impedance diagram of Fig. 4 is shown in Fig. 5.

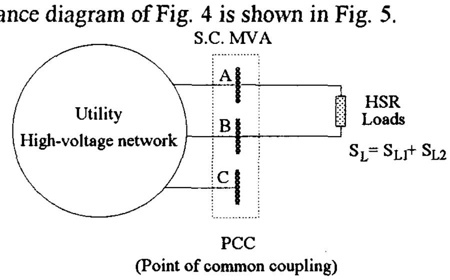

Fig. 4 Simplified circuit diagram for single-phase connection

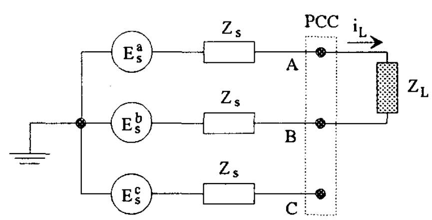

Fig. 5 Equivalent impedance diagram for single-phase connection

For the per unit system

$$Z_L \propto \frac{1}{S_L^*};\ \text{and}\ Z_S \propto \frac{1}{S_S^*} \tag{1}$$

Furthermore, for the same bases

$$Z_L = \lambda \frac{1}{S_L^*};\ \text{and}\ Z_S = \lambda \frac{1}{S_S^*} \tag{2}$$

in which λ is an arbitrary value depending on the base values selected.

From Fig. 5,

$$V_{PCC}^a = E_S^a - Z_s i_L$$
$$V_{PCC}^b = E_S^b + Z_s i_L$$
$$V_{PCC}^C = E_s^c \tag{3}$$

and

$$i_L = \frac{1}{Z_L}(V_{Pcc}^a - V_{Pcc}^b) \tag{4}$$

Substituting eqn. 4 into eqn. 3 yields

$$\begin{bmatrix} 1+\frac{Z_s}{Z_L} & \frac{-Z_s}{Z_L} & 0 \\ \frac{-Z_s}{Z_L} & 1+\frac{Z_s}{Z_L} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{Pcc}^a \\ V_{Pcc}^b \\ V_{Pcc}^c \end{bmatrix} = \begin{bmatrix} E_s^a \\ E_s^b \\ E_s^c \end{bmatrix} = \begin{bmatrix} E_s^a \\ \alpha^2 E_s^a \\ \alpha E_s^a \end{bmatrix} \tag{5}$$

Thus,

$$\begin{bmatrix} V_{Pcc}^a \\ V_{Pcc}^b \\ V_{Pcc}^c \end{bmatrix} = \frac{E_s^a}{1+\frac{2Z_s}{Z_L}} \begin{bmatrix} \left(1+\frac{Z_s}{Z_L}\right)+\alpha^2\frac{Z_s}{Z_L} \\ \alpha^2\left(1+\frac{Z_s}{Z_L}\right)+\frac{Z_s}{Z_L} \\ \alpha\left(1+\frac{2Z_s}{Z_L}\right) \end{bmatrix} \tag{6}$$

By definition,

$$V_{PCC}^{012} = T_S^{-1} V_{PCC}^{abc} \tag{7}$$

in which

$$V_{PCC}^{012} \equiv \begin{bmatrix} V_{PCC}^0 & V_{PCC}^1 & V_{PCC}^2 \end{bmatrix}^T$$
$$V_{PCC}^{abc} \equiv \begin{bmatrix} V_{PCC}^a & V_{PCC}^b & V_{PCC}^c \end{bmatrix}^T \tag{8}$$

and

$$T_s^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \tag{9}$$

in which $\alpha = e^{j\frac{2\pi}{3}};\ \ and\ \ \alpha^2 = e^{-j\frac{2\pi}{3}}$ (10)

Eqns. 6-10 give

$$V_{PCC}^{012} = \begin{bmatrix} 0 \\ \frac{Z_s + Z_L}{Z_L} \\ \frac{\alpha^2 Z_s}{Z_L} \end{bmatrix} \tag{11}$$

Eqn. 11 shows that no zero-sequence voltage occurs and a value of $\frac{Z_S}{Z_L}$ of negative-sequence voltage exists for the single-phase connection scheme. By definition, the amount of voltage unbalance is expressed in the ratio of the negative sequence voltage to the positive sequence voltage:

$$Voltage-unbalance\ factor(d_2) = \frac{Negative-sequence\ voltage(|V_2|)}{Positive-sequence\ voltage(|V_1|)} \tag{12}$$

Therefore,

$$d_2 = \frac{|V_{PCC}^{(2)}|}{|V_{PCC}^{(1)}|} = \frac{|Z_S|}{|Z_S + Z_L|} = \frac{|Z_S^*|}{|Z_S^* + Z_L^*|} \tag{13}$$

Substituting eqn. 2 into eqn. 13 yields

$$d_2 = \frac{S_L}{S_S + S_L} \tag{14}$$

in which

S_L = apparent power of the traction load

S_S = short circuit power of the PCC

Eqn. 14 indicates the voltage unbalance due to traction demands supplied by an ac single-phase scheme and shows that the single-phase connection scheme is unsuitable if the short-circuit capacity is small for a large traction load.

If $s_L << s_S$, then d_2 can be approximated by d_2' as

$$d_2 \cong \frac{S_L}{S_S} \equiv d_2' \tag{15}$$

Eqn. 15 shows that the approximate value of the voltage unbalance is estimated from the ratio of the traction power demand between two phases to the high-voltage network short-circuit power at the PCC, that is s_L/s_S. Because of the simplicity the eqn. 15 is the widely used criterion to estimate an unbalance of this kind.

The tolerance of d_2' is

$$\varepsilon\% = \frac{d_2 - d_2'}{d_2} \times 100\% \tag{16}$$

The relationships of d_2, short circuit power and traction demands are shown in Fig. 6. The error of d_2' is indicated in Fig. 7. The larger is the S_L/S_S ratio, the smaller the error that occurs.

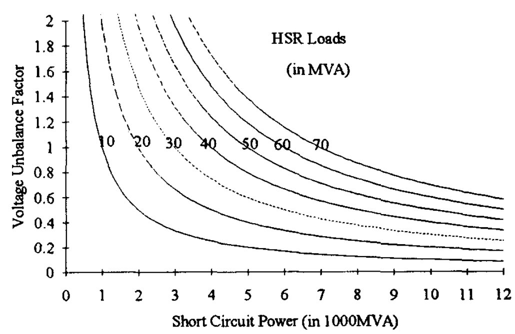

Fig. 6 Voltage unbalance factor for single-phase connection

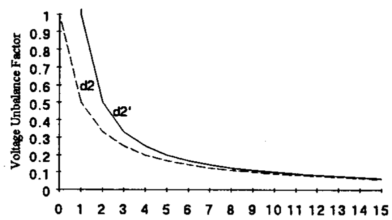

Fig. 7 Comparison of d_2 and d_2'

B. V-connection

The major disadvantage of the single-phase connection is the great extent of voltage unbalance if the power-supply system is small relative to the traction loads. Therefore, the V-connection scheme has been used to reduce the unbalance caused by the single-phase traction loads. However, this connection scheme requires three high-voltage conductors, and requires a neutral section at a substation. The simplified circuit diagram for the V-connection scheme appears in Fig. 8. Its corresponding impedance diagram appears in Fig. 9.

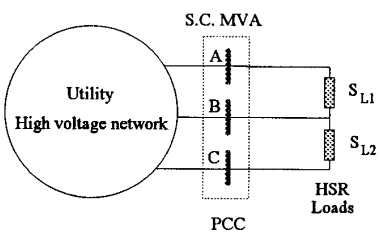

Fig. 8 Simplified circuit diagram for V-connection

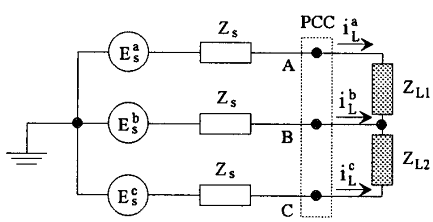

Fig. 9 Equivalent impedance diagram for V-connection

For the source side of the PCC in Fig. 9,

$$\begin{bmatrix} E_s^a \\ E_s^b \\ E_s^c \end{bmatrix} - \begin{bmatrix} Z_s & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & Z_s \end{bmatrix} \begin{bmatrix} i_L^a \\ i_L^b \\ i_L^c \end{bmatrix} = \begin{bmatrix} V_{pcc}^a \\ V_{pcc}^b \\ V_{pcc}^c \end{bmatrix} \qquad (17)$$

Transforming into symmetrical components, that is

$$\begin{bmatrix} E_s^{(0)} \\ E_s^{(1)} \\ E_s^{(2)} \end{bmatrix} - \begin{bmatrix} Z_s & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & Z_s \end{bmatrix} \begin{bmatrix} i_L^{(0)} \\ i_L^{(1)} \\ i_L^{(2)} \end{bmatrix} = \begin{bmatrix} V_{pcc}^{(0)} \\ V_{pcc}^{(1)} \\ V_{pcc}^{(2)} \end{bmatrix} \qquad (18)$$

in which

$$E_s^{(0)} = 0; \quad and \quad E_s^{(2)} = 0 \qquad (19)$$

Combining eqns. 18 and 19 yields

$$V_{pcc}^{(0)} = -Z_s i_L^{(0)} \qquad (20)$$

$$V_{pcc}^{(1)} = E_s^{(1)} - Z_s i_L^{(1)} \qquad (21)$$

$$V_{pcc}^{(2)} = -Z_s i_L^{(2)} \qquad (22)$$

Furthermore, for the load side of the PCC in Fig. 9,

$$i_L^a + i_L^b + i_L^c = 0 \qquad (23)$$

Therefore,

$$i_L^{(0)} = 0 \qquad (24)$$

Also from Fig. 9

$$V_{pcc}^a - Z_{L1} i_L^a = V_{pcc}^n$$

$$V_{pcc}^b = V_{pcc}^n$$

$$V_{pcc}^c - Z_{L2} i_L^c = V_{pcc}^n \qquad (25)$$

The above equations are written in matrix form as follows:

$$\begin{bmatrix} V_{pcc}^a \\ V_{pcc}^b \\ V_{pcc}^c \end{bmatrix} - \begin{bmatrix} Z_{L1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Z_{L2} \end{bmatrix} \begin{bmatrix} i_L^a \\ i_L^b \\ i_L^c \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} V_{pcc}^n \qquad (26)$$

Transforming into symmetrical components, that is

$$\begin{bmatrix} V_{pcc}^{(0)} \\ V_{pcc}^{(1)} \\ V_{pcc}^{(2)} \end{bmatrix} - \frac{1}{3} \begin{bmatrix} Z_{L1} + Z_{L2} & Z_{L1} + \alpha Z_{L2} & Z_{L1} + \alpha^2 Z_{L2} \\ Z_{L1} + \alpha^2 Z_{L2} & Z_{L1} + Z_{L2} & Z_{L1} + \alpha Z_{L2} \\ Z_{L1} + \alpha Z_{L2} & Z_{L1} + \alpha^2 Z_{L2} & Z_{L1} + Z_{L2} \end{bmatrix} \begin{bmatrix} i_L^{(0)} \\ i_L^{(1)} \\ i_L^{(2)} \end{bmatrix} = \begin{bmatrix} V_{pcc}^n \\ 0 \\ 0 \end{bmatrix} \qquad (27)$$

From eqn. 27

$$V_{pcc}^{(1)} = \frac{1}{3}(Z_{L1} + Z_{L2}) i_L^{(1)} + \frac{1}{3}(Z_{L1} + \alpha Z_{L2}) i_L^{(2)} \qquad (28)$$

$$V_{pcc}^{(2)} = \frac{1}{3}(Z_{L1} + \alpha^2 Z_{L2}) i_L^{(1)} + \frac{1}{3}(Z_{L1} + Z_{L2}) i_L^{(2)} \qquad (29)$$

Substituting eqn. 22 into eqn. 29 yields

$$i_L^{(1)} = -\frac{Z_{L1} + Z_{L2} + 3Z_s}{Z_{L1} + \alpha^2 Z_{L2}} i_L^{(2)} \qquad (30)$$

and substituting eqn. 30 into eqn. 28 gives

$$V_{pcc}^{(1)} = \frac{-Z_{L1} Z_{L2} - (Z_{L1} + Z_{L2}) Z_s}{Z_{L1} + \alpha^2 Z_{L2}} i_L^{(2)} \qquad (31)$$

Dividing eqn. 22 by eqn. 31 provides

$$\frac{V_{pcc}^{(2)}}{V_{pcc}^{(1)}} = \frac{Z_s (Z_{L1} + \alpha^2 Z_{L2})}{Z_{L1} Z_{L2} + (Z_{L1} + Z_{L2}) Z_s} \qquad (32)$$

Eqn. 32 is rewritten

$$\frac{V_{pcc}^{(2)}}{V_{pcc}^{(1)}} = \frac{Z_s \left(\frac{1}{Z_{L2}} + \alpha^2 \frac{1}{Z_{L1}} \right)}{1 + \left(\frac{1}{Z_{L1}} + \frac{1}{Z_{L2}} \right) Z_s} \qquad (33)$$

Assume $S_{L1} = kS_L$ (34)

Therefore,

$$S_{L2} = S_L - S_{L1} = (1-k) S_L \qquad (35)$$

For the per unit system with the same bases

$$Z_L \propto \frac{1}{S_L^*}; \quad and \quad Z_S \propto \frac{1}{S_S^*}$$

therefore,

$$\frac{1}{Z_{L1}}=\frac{k}{Z_L};\quad and\quad \frac{1}{Z_{L2}}=\frac{1-k}{Z_L} \tag{36}$$

Substituting eqn. 36 into eqn. 33 yields

$$\frac{V_{pcc}^{(2)}}{V_{pcc}^{(1)}}=\frac{\frac{Z_S}{Z_L}(1-k+\alpha^2 k)}{1+\frac{Z_S}{Z_L}} \tag{37}$$

Hence,

$$d_2=\left|\frac{V_{pcc}^{(2)}}{V_{pcc}^{(1)}}\right|=\frac{|Z_S|}{|Z_L+Z_S|}\sqrt{3k^2-3k+1} \tag{38}$$

That is

$$d_2=\left|\frac{V_{pcc}^{(2)}}{V_{pcc}^{(1)}}\right|=\frac{S_L}{S_L+S_S}\sqrt{3k^2-3k+1} \tag{39}$$

The voltage unbalance caused by traction loads in the V-connection scheme depends not only on the apparent traction power and the short-circuit power of the tapping point but also on the coefficient k. The definition of k is shown in eqn. 34. The minimum value of d_2 in eqn. 39 is

$$Min\ d_2=\frac{1}{2}\frac{S_L}{S_S+S_L}$$

when k equals 0.5. Hence the unbalance of the V-connection scheme is only half that of the single-phase connection scheme if two transformer loads are balanced. However, it is the same when k equals 0 and 1.

If $S_L << S_S$ then

$$d_2=\left|\frac{V_{PCC}^{(2)}}{V_{PCC}^{(1)}}\right|\cong\frac{S_L}{S_S}\sqrt{3k^2-3k+1}\equiv d_2' \tag{40}$$

C. Scott-connection

Although the V-connection is generally better than the single-phase connection scheme from the unbalance point of view, the V-connection scheme is also unsuitable if the short-circuit capacity is insufficient. The Scott-connection scheme is therefore adopted for its better performance. The connection diagram of windings in the Scott transformer appears in Fig. 3.

From Fig. 3

$$\begin{cases} i_L^a=\frac{-2}{\sqrt{3}}\frac{N_2}{N_1}i_{L1} \\ i_L^b=-\frac{i_L^a}{2}-\frac{N_2}{N_1}i_{L2}=\frac{N_2}{N_1}\left(\frac{1}{\sqrt{3}}i_{L1}-i_{L2}\right) \\ i_L^c=-\frac{i_L^a}{2}+\frac{N_2}{N_1}i_{L2}=\frac{N_2}{N_1}\left(\frac{1}{\sqrt{3}}i_{L1}+i_{L2}\right) \end{cases} \tag{41}$$

$$\begin{cases} V_{L1}=-\frac{N_2}{N_1}V_{pcc}^{ab}\angle-30^\circ \\ V_{L2}=j\frac{N_2}{N_1}V_{pcc}^{ab}\angle-30^\circ \end{cases} \tag{42}$$

and

$$\begin{cases} V_{pcc}^a=\frac{V_{pcc}^{ab}\angle-30^\circ}{\sqrt{3}} \\ V_{pcc}^b=\frac{V_{pcc}^{ab}\angle-30^\circ}{\sqrt{3}}\alpha^2 \\ V_{pcc}^c=\frac{V_{pcc}^{ab}\angle-30^\circ}{\sqrt{3}}\alpha \end{cases} \tag{43}$$

Therefore,

$$\begin{cases} V_{L1}=-\sqrt{3}\frac{N_2}{N_1}V_{pcc}^a \\ V_{L2}=j\sqrt{3}\frac{N_2}{N_1}V_{pcc}^a \end{cases} \tag{44}$$

The relationship of complex powers, currents, and voltages of the secondary are

$$\begin{cases} i_{L1}^*=\frac{S_{L1}}{V_{L1}}=\frac{-S_{L1}}{\sqrt{3}\frac{N_2}{N_1}V_{pcc}^a} \\ i_{L2}^*=\frac{S_{L2}}{V_{L2}}=\frac{-jS_{L2}}{\sqrt{3}\frac{N_2}{N_1}V_{pcc}^a} \end{cases} \tag{45}$$

Substitute eqn.45 to eqn. 41

$$\begin{cases} i_L^{a*}=\frac{2}{3}\frac{S_{L1}}{V_{pcc}^a} \\ i_L^{b*}=-\frac{1}{3}\frac{S_{L1}}{V_{pcc}^a}+j\frac{S_{L2}}{\sqrt{3}V_{pcc}^a} \\ i_L^{c*}=-\frac{1}{3}\frac{S_{L1}}{V_{pcc}^a}-j\frac{S_{L2}}{\sqrt{3}V_{pcc}^a} \end{cases} \tag{46}$$

Therefore,

$$\begin{cases} S_L^a=V_{pcc}^a i_L^{a*}=\frac{2}{3}S_{L1} \\ S_L^b=V_{pcc}^b i_L^{b*}=-\frac{\alpha^2}{3}S_{L1}+j\frac{\alpha^2}{\sqrt{3}}S_{L2} \\ S_L^c=V_{pcc}^c i_L^{c*}=-\frac{\alpha}{3}S_{L1}-j\frac{\alpha}{\sqrt{3}}S_{L2} \end{cases} \tag{47}$$

Eqn. 47 shows the equivalent three-phase load of the two single-phase traction loads S_{L1} and S_{L2}. The corresponding equivalent impedances are

$$\begin{cases} Z_L^a=\frac{1}{3}\left(\frac{3}{2}Z_{L1}\right)=\frac{1}{2}Z_{L1} \\ Z_L^b=\frac{1}{3}\left(\frac{1}{-\frac{\alpha}{3Z_{L1}}-j\frac{\alpha}{\sqrt{3}Z_{L2}}}\right)=\frac{\alpha^2 Z_{L1}Z_{L2}}{-Z_{L2}-j\sqrt{3}Z_{L1}} \\ Z_L^b=\frac{1}{3}\left(\frac{1}{-\frac{\alpha^2}{3Z_{L1}}+j\frac{\alpha^2}{\sqrt{3}Z_{L2}}}\right)=\frac{\alpha Z_{L1}Z_{L2}}{-Z_{L2}+j\sqrt{3}Z_{L1}} \end{cases} \tag{48}$$

The simplified circuit diagram of the Scott-connection is therefore represented by Fig. 10. Its corresponding equivalent impedance diagram appears in Fig. 11.

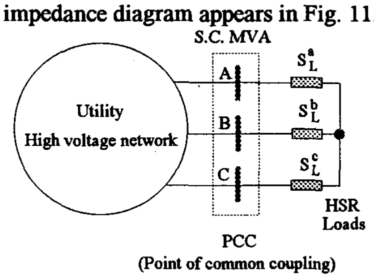

Fig. 10 Simplified circuit diagram for the Scott-connection

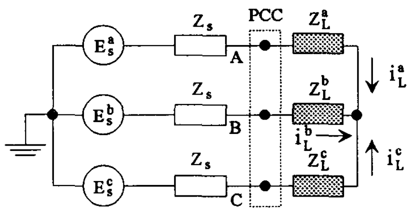

Fig. 11 Equivalent impedance diagram for the Scott-connection

For the load side of the PCC in Fig. 11,

$$i_L^a + i_L^b + i_L^c = 0$$

Therefore,

$$i_L^{(0)} = 0 \tag{49}$$

Also from Fig. 11

$$\begin{bmatrix} V_{pcc}^a \\ V_{pcc}^b \\ V_{pcc}^c \end{bmatrix} - \begin{bmatrix} Z_L^a & 0 & 0 \\ 0 & Z_L^b & 0 \\ 0 & 0 & Z_L^c \end{bmatrix} \begin{bmatrix} i_L^a \\ i_L^b \\ i_L^c \end{bmatrix} = \begin{bmatrix} V^n \\ V^n \\ V^n \end{bmatrix} \tag{50}$$

Transforming to symmetrical components, that is

$$\begin{bmatrix} V_{pcc}^{(0)} \\ V_{pcc}^{(1)} \\ V_{pcc}^{(2)} \end{bmatrix} - \begin{bmatrix} Z_X & Z_Y & Z_Z \\ Z_Z & Z_X & Z_Y \\ Z_Y & Z_Z & Z_X \end{bmatrix} \begin{bmatrix} i_L^{(0)} \\ i_L^{(1)} \\ i_L^{(2)} \end{bmatrix} = \begin{bmatrix} V^n \\ 0 \\ 0 \end{bmatrix} \tag{51}$$

in which

$$\begin{cases} Z_X = \frac{1}{3}(Z_L^a + Z_L^b + Z_L^c) \\ Z_Y = \frac{1}{3}(Z_L^a + \alpha^2 Z_L^b + \alpha Z_L^c) \\ Z_Z = \frac{1}{3}(Z_L^a + \alpha Z_L^b + \alpha^2 Z_L^c) \end{cases} \tag{52}$$

From eqns. 49 and 51

$$V_{pcc}^{(1)} = Z_X i_L^{(1)} + Z_Y i_L^{(2)} \tag{53}$$

$$V_{pcc}^{(2)} = Z_Z i_L^{(1)} + Z_X i_L^{(2)} \tag{54}$$

For the source side of the PCC in Fig. 11

$$V_{pcc}^{(0)} = -Z_s i_L^{(0)} \tag{55}$$

$$V_{pcc}^{(1)} = E_s^{(1)} - Z_s i_L^{(1)} \tag{56}$$

$$V_{pcc}^{(2)} = -Z_s i_L^{(2)} \tag{57}$$

Substituting eqn. 57 into eqn. 54 yields

$$i_L^{(1)} = -\frac{Z_X + Z_S}{Z_Z} i_L^{(2)} \tag{58}$$

Substituting eqn. 58 into eqn. 53 gives

$$V_{pcc}^{(1)} = \frac{-Z_X(Z_X + Z_S) + Z_Y Z_Z}{Z_Z} i_L^{(2)} \tag{59}$$

Dividing eqn. 57 by eqn. 59 provides

$$\frac{V_{pcc}^{(2)}}{V_{pcc}^{(1)}} = \frac{Z_S Z_Z}{Z_X^2 + Z_X Z_S - Z_Y Z_Z} \tag{60}$$

Substituting eqn. 52 into eqn. 60 yields

$$\frac{V_{pcc}^{(2)}}{V_{pcc}^{(1)}} = \frac{(Z_L^a + \alpha Z_L^b + \alpha^2 Z_L^c) Z_S}{(Z_L^a Z_L^b + Z_L^b Z_L^c + Z_L^c Z_L^a) + (Z_L^a + Z_L^b + Z_L^c) Z_S} \tag{61}$$

in which

$$Z_L^a Z_L^b + Z_L^b Z_L^c + Z_L^c Z_L^a = \frac{3}{2} \frac{Z_{L1}^2 Z_{L2}(Z_{L1} + Z_{L2})}{3Z_{L1}^2 + Z_{L2}^2}$$

$$Z_L^a + \alpha Z_L^b + \alpha^2 Z_L^c = \frac{3}{2} \frac{(Z_{L1}^2 - Z_{L2}^2) Z_{L1}}{3Z_{L1}^2 + Z_{L2}^2}$$

$$Z_L^a + Z_L^b + Z_L^c = \frac{3}{2} \frac{(Z_{L1} + Z_{L2})^2 Z_{L1}}{3Z_{L1}^2 + Z_{L2}^2} \tag{62}$$

Therefore, eqn. 61 becomes

$$\frac{V_{pcc}^{(2)}}{V_{pcc}^{(1)}} = \frac{Z_S(Z_{L1} - Z_{L2})}{Z_S(Z_{L1} + Z_{L2}) + Z_{L1} Z_{L2}} \tag{63}$$

Using the same definitions of the V-connection scheme shown in eqns. 34-36

$$\frac{V_{pcc}^{(2)}}{V_{pcc}^{(1)}} = \frac{-Z_S(2k-1)}{Z_S + Z_L} \tag{64}$$

and

$$\frac{V_{pcc}^{(2)}}{V_{pcc}^{(1)}} = \frac{-S_L(2k-1)}{S_S + S_L} \tag{65}$$

So

$$d_2 = \left| \frac{V_{pcc}^{(2)}}{V_{pcc}^{(1)}} \right| = \frac{S_L |2k-1|}{S_S + S_L} \tag{66}$$

Eqn. 66 shows that the unbalance is eliminated when the two transformer loads are equal, when k equals 0.5.

If $S_L << S_S$ eqn. 66 becomes

$$d_2 = \left| \frac{V_{PCC}^{(2)}}{V_{PCC}^{(1)}} \right| \cong \frac{S_L}{S_S}(2k-1)^2 \equiv d_2' \tag{67}$$

D. Comparison of voltage unbalance effects for three power supply schemes

Comparison of the voltage unbalances for single-phase, V-and Scott-connection schemes appears in Fig. 12. Fig. 12 shows that the unbalance is significantly reduced if the two transformer loads are closer. However, the two traction loads depend on the normal scheduling and the need to maintain an adequate distance between trains. The inherent diversity of the traction loads also makes this condition impossible. The advantage of the Scott-connection may be therefore not useful.

Fig. 12 also shows that the Scott-connection is better than the V-connection scheme, and the latter is better than the single-phase connection scheme from the unbalance point of view. However, the great cost of the Scott transformers and the small probability in time to use the efficiency of the system are major disadvantages of the Scott-connection scheme. The extra neutral sections in the substations and one or two more high-voltage conductor lines required are the drawbacks of the V-connection compared with the single-phase connection scheme. Because the selection of the substation connection scheme is a complicated and difficult problem a detailed analysis requires a comprehensive examination of the selection which is beyond the scope of this paper.

The adoption of the V- and Scott-connection schemes also requires a suitable operating schedule for the trains to use the advantages of these systems during the entire operating period.

A specific power system instead of the public power system can be used to supply the energy of the traction loads. The disturbances on the public power system would thereby be eliminated, but the cost of the specific high-voltage network is great. The criteria developed in this paper can also be applied to estimate the unbalance on the specific power system. As expected, the unbalance will be relatively high because the short-circuit capacity is generally smaller than the general public power system.

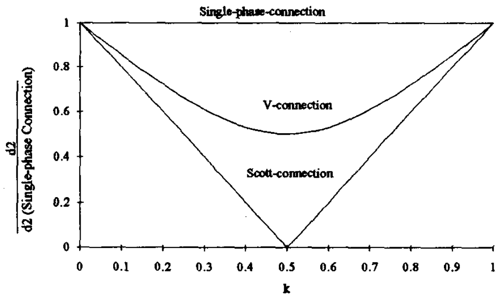

Fig. 12 Comparison of voltage unbalances for three substation connection schemes

IV. Conclusion

Simple criteria to estimate the voltage unbalance due to a high-speed railway traction load supplied by ac single- and two-phase schemes have been presented in this paper. Three substation connection schemes, that is single-phase, V- and Scott-connection, have been discussed and compared. Although much detailed study of the unbalance effects of the traction load on the power-supply system is necessary to guarantee the quality of the power-supply system, this paper has introduced a simple criterion to estimate this unbalance. The results are of value to related engineers and consultants especially during periods of planning and design.

However, for a rigorous analysis a three-phase power-flow program with some advance functions is required. A three-phase power-flow program that is suitable to analyze the transmission system unbalance problem is under development by the author and his research team. More rigorous results will be obtained in the near future.

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