OPTIMAL INTERGOVERNMENTAL TRANSFER: EQUITY AND EFFICIENCY [*][†]

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Abstract

We develop a principal-agent model to characterize optimal place-based intergovernmental transfer policies, incorporating both asymmetric information and a general form of externalities. The optimal nonlinear transfer scheme addresses the dual objectives of equity and efficiency. Using county-level fiscal data from China, we conduct numerical simulations to explore potential transfer reforms. Our results show that the optimal marginal transfer is lower than the current level, and the presence of externalities reverses its sign. Transitioning from the existing transfer system to the optimal one leads to welfare gains equivalent to an increase in per capita consumption ranging from 4.02% to 4.25%.

Keywords. Optimal Transfer; Externality; Information Asymmetry; Flypaper Effect

JEL. H22; H41; H77; D62

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1 Introduction

Empirical evidence has shown that transfers from the central government to sub-national governments play important roles in the fiscal operation of sub-national governments. The sub-national governments depend heavily on intergovernmental transfers or tax sharing systems, especially for developing countries[1]. When it comes to intergovernmental transfers, previous literature often prioritizes fiscal equity and always treats transfers as a pure equalization tool (Gross, 2021; Boadway, 2004; Fajgelbaum et al., 2019; Albouy, 2012). This consideration still echoes due to widening economic gaps in both developing and developed countries (Gal and Egeland, 2018; Ehrlich and Overman, 2020; Rao, 2017). However, the efficiency aspect of transfers has not received sufficient attention in the extant literature. Since intergovernmental transfer policies contingent on local economic statistics (instead of customary place-blind transfers) inevitably distort local government fiscal behavior, inefficient transfer design would endogenously influence interregional fiscal disparities and regional welfare.

Inefficiency primarily stems from two key dimensions, both of which have a significant impact on fiscal equity and efficiency in reality. First, central governments always face challenges of information asymmetry between different tiers of government. Apart from the asymmetric information on the local preference for public goods, which is widely discussed in the fiscal decentralization literature, policymakers in some developing countries are also confronted with challenges in effectively monitoring the economic data of local governments due to the limited effectiveness of oversight mechanisms and shortcomings in auditing systems[2]. This situation creates incentives for local officials to utilize their private information and manipulate economic indicators to secure additional subsidies from the central governmentAs a result, policymakers are hindered from making accurate assessments of the suitable levels of local public expenditure, and people witness the high dependency on

[1]In China, transfers from the central to local governments accounted for up to 39% of national tax revenues in 2019 and remained at approximately 33% for the five years before 2019. In India, the aggregate transfers to states as a proportion of the gross state domestic product (GSDP) of all states was set to be 7.3% by the 14th Financial Commission. Unconditional transfers in Mexico in 2007 accounted for 42% of state-level governments' tax revenues Martinez-Vazquez and Sepulveda (2011). We also observe similar situations in a few developed countries. In Germany in 2010, transfers including intergovernmental tax redistributions, accounted for 12.4% of total tax revenues, or approximately 65.0 billion euro, according to Henkel et al. (2021). In 2019, The Ministry of Housing, Communities & Local Government in the U.K. spent 15.2 billion pounds in grants to local authorities, social care funds, and other local affairs, and total governmental grants reached 118 billion pounds, accounting for 13% of government expenditures, and 5 billion pounds more than in 2018 (Source: U.K. government official statistics. https://www.gov.uk/government/statistics).

[2]For instance, local governments in China are considered to conceal their true economic statistics from the central government, resulting in the bias of data reported by local governments (Chen et al., 2019).

transfers observed in various countries, or the so-called "flypaper effect"[3]. Although some countries try to mitigate the distortion induced by asymmetric information in transfer policies (see next section), we still witness that in China, Argentina, Peru, Bolivia, and Brazil, the correlation between transfers and GDP per capita turns out to be positive, which is entirely contrary to the widely-discussed equalization role of transfers(Martinez-Vazquez and Sepulveda, 2011).

The second efficiency issue stems from the externalities of local public spending since intergovernmental transfer policies would no doubt influence the provision of regional public goods[4]. Externalities can take various forms, including spillover effects of local public goods, population mobility incurred by regional fiscal policies, and interregional strategic interactions occurring when localities engage in fiscal competition. Although some studies point out that these externalities may play important roles in economy (Tombe and Zhu, 2019; Li et al., 2019; Alloza et al., 2019), extant studies or policies on transfer design often fail to take them into account. Ignoring externalities can introduce an additional dimension of inefficiency into a transfer system, while the interplay between externalities and the asymmetric information structure further complicates the efficiency problem. Thus it is necessary to gauge the extent to which externalities influence the effectiveness of current transfer policies and explore how to mitigate efficiency losses induced by externalities[5].

Unfortunately, few studies address the efficiency challenges above in designing place-based transfer policies, much less establish a comprehensive framework that integrates both efficiency and equity concerns. Hence, previous studies are unable to answer critical questions: are the current transfer policies efficiently allocated to regions with various levels of economic development, and which concern should be primarily focused on by policymakers in designing transfer policies? Without a study that incorporates both crucial efficiency and equity considerations, we cannot fundamentally reexamine the prevailing transfer policies to identify necessary reforms or understand the potential welfare gains associated with these reforms.

This paper uses a principal-agent model with two levels of government – the central and heterogeneous local governments to develop a comprehensive framework of intergovern-

[3]Many developing countries have witnessed a growing dependence on transfers instead of local taxation and increasing local expenditures, such as India(Abiad et al., 2020) or Mexico(Sour, 2013). Figure 1 in the next section displays the status of transfers in countries like China, exhibiting the reliance of local governments on transfers.

[4]See Lockwood (1999); Ercolani and Valle e Azevedo (2014).

[5]Gaubert et al. (2021) examine place-blind transfer policies in the context of mobile households as a kind of externalities. Contrary to their work, our work focuses on place-based transfer policies, extending the discussion to a broader range of externalities within an infinite-region model.

mental transfers. The model highlights three main issues that transfer policies must address. The first is the information asymmetry between central and local governments. Local governments are distributed along a continuum and have private information on the level of local economic endowments, preference for public spending, or both. Non-linear transfer policies in our model can capture the distortions in public spending behavior and the issue of incentive incompatibility arising from this information asymmetry. The second issue is public goods externalities, and we discuss a general form of externalities capturing various scenarios of externalities. The last one, fiscal equity, is reflected by the welfare weights of heterogeneous regions. Furthermore, the framework of our model aligns with the existing policies in plenty of countries, ensuring the practical relevance of our research findings.

Via the variation (dual) approach, we derive the optimal transfer formula. The introduction of externalities integrates general equilibrium effects into the derivation, explaining up to 7% of the overall local public spending perturbation under marginal transfer reforms in the current policy. We find that the optimal transfer policy depends on the elasticities of public spending, the impact of externalities, the hazard rate of public goods expenditures, and the redistribution term. These four factors exactly correspond to the aforementioned dependency, equity, and externality issues of transfers, thereby enabling quantitative analysis of current policies. We use elasticities of public spending and externalities as sufficient statistics introduced by Chetty (2009) to make it feasible for policymakers to design transfers using empirical estimation.

We then analyze how externalities influence transfer design in scenarios that closely align with real-world settings, specifically considering the mobility of residents and intergovernmental competition. Since transfer policies can create externalities by affecting residents' relocation decisions and strategic interactions between local governments, these effects can be incorporated into a unified general framework extended from the benchmark model. As a result, the optimal transfer structures in these scenarios remain similar to those in the benchmark case.

Considering the unobservable preferences of public goods are widely discussed in the literature on fiscal federalism, we extend our analysis to the bi-dimensional heterogeneity scenario where regional governments possess private information not only about their economic endowments but also about residents' preferences for public goods. We explore the optimal transfer design under this environment by extending the variation approach to the multidimensional heterogeneity case with externalilites.

Utilizing the dataset of Chinese county-level governments, we conduct a numerical simulation to analyze the optimal transfer policy in China. Our estimation involves parameters related to public spending and externalities within the utility function. Employing these parameters and fiscal data, our numerical model yields the following outcomes:

First, the optimal marginal transfer is significantly lower than the marginal transfer currently in effect in China, even when considering a utilitarian social preference that is not particularly redistributive. In contrast to the optimal transfer policy, the existing marginal transfer system in China allocates excessive transfers to regions with high levels of public expenditure while inadequately funding regions with low expenditure. This results in a situation where the majority of Chinese regions have both low public spending and low transfers.

Second, we find the optimal marginal transfer in simulation is U-shaped, which is mainly determined by the shape of the redistribution term. This finding is close to those of recent studies such as Heathcote and Tsujiyama (2021a), which also emphasizes the importance of the distribution term in deciding the shape of the optimal tax policy.

Thirdly, externalities play key roles in determining the sign of optimal marginal transfer policy. The transfers undertake the function of a kind of Pigouvian correction subsidies for positive externalities, which would result in a much higher marginal transfer for all regions. Thus, we observe that externalities cause the marginal transfer for regions at both tails of the public spending distribution to deviate from zero. The optimal transfer schedule initially ascends but then decreases after reaching a certain threshold, and eventually reverses to increase again. This indicates that the optimal transfer policy is far from a pure redistributive fiscal tool as a monotonic function of regional fiscal expenditures, but should combine both incentive and redistributive functions.

We further gauge the welfare improvement due to the reform of the transfer policy and show how externalities amplify the behavioral change of local public spending in the numerical simulation. The result also shows that the optimal transfer policy in our setup improves social welfare by 4.02–4.25% of the average consumption per capita derived by the model, which is a substantial amount. Moreover, we discuss the feasibility of the optimal transfer by partially using sufficient statistics obtained via empirical methods. It is more straightforward for policymakers to use the empirical elasticities of public spending to public goods price than the structural model to design optimal transfer policies. Utilizing the panel fiscal data, we estimate the elasticities of public spending and compare the simulation outcomes based on empirically derived elasticities with those derived from a fully structural model. Our analysis suggests that these two methods yield similar optimal transfer schedules, indicating the feasibility of using sufficient statistics in guiding policymakers in reality when implementing transfer policy reforms.

This paper contributes in three dimensions. First, this work constructs a general framework to analyze intergovernmental transfers. Compared to recent literature on fiscal decentralization such as Janeba and Wilson (2011); Bellofatto and Besfamille (2018, 2021); Fajgelbaum and Gaubert (2020), we revisit the information asymmetry which is the crucial characteristic of fiscal federalism. Our work aligns more closely with that of Lehmann et al. (2014); Berriel et al. (2024), who highlight information frictions and externalities in interregional fiscal policy design. However, this work extends these discussions by modeling infinite heterogeneous regions, providing a more general framework to derive an optimal transfer schedule under various scenarios. Second, we extend the Mirrleesian principal-agent model by applying it to the context of fiscal decentralization. Unlike earlier studies employing similar approaches (Lockwood, 1999; Persson and Tabellini, 1996; Boadway et al., 1999), this paper enriches the transfer framework by incorporating household mobility and multidimensional heterogeneities. Finally, compared to extant studies, this paper is, to our limited knowledge, the first work to provide a numerical analysis of optimal transfer design, which is the final contribution of this paper. Through the numerical simulation, we offer a quantitative scheme for reforming existing transfer policies in typical countries like China. We also emphasize the significance of the redistribution component in determining optimal transfers and conduct a welfare analysis of potential transfer reforms.

Related Literature. Our work is connected to three branches of the literature. First, our work is related to the theory of transfers. As Gross (2021) comments, transfers in previous research always take the form of either revenue equalization or tax-base equalization. Few studies have focused on the compound problems of transfer policies in practice, such as issues of information frictions and externalities. Some studies discuss the role of transfer policies when there are information asymmetry issues between central and local governments, such as the moral hazard (Persson and Tabellini, 1996), adverse selection (Lockwood, 1999), or both of the two inefficiencies (Boadway et al., 1999) induced by the transfer system. These studies discuss the inter-regional heterogeneities in terms of the cost of providing public goods (Cornes and Silva, 2000), local income levels (Lockwood, 1999; Cremer et al., 1996, 2001) or public goods preference (Lockwood, 1999; Cremer et al., 1996). Gaubert et al. (2021) examine the optimal design of place-based income taxation alongside place-blind transfers in the presence of mobile populations. Contrary to their work, our study focuses on place-based transfer policies across a large number of regions, incorporating multiple types of externalities, including migration. Ours is the first work to provide a comprehensive framework of transfers with a clear structure of the optimal policy, providing intuitive economic insights. Based on Chinese county-level data, we are able to offer implementable advice for designing the optimal non-linear transfer in countries with huge inter-regional fiscal disparities and relatively limited national financial resources for redistribution.

The second branch of literature connected with our work is about externalities induced by public goods. Koethenbuerger (2008) and Bjorvatn and Schjelderup (2002) are important works on the provision of public goods in the face of externalities and fiscal decentralization with tax sharing or competition. More recent papers mainly focus on externalities such as the agglomeration or crowding-out effects arising from population migration or the choice of occupation (Fajgelbaum and Gaubert, 2020). This strand of articles either tends to use models with limited discrete areas (Bloch and Zenginobuz, 2006; Kline, 2010) or is primarily concerned with tax policies for individuals or firms rather than for sub-national governments. Instead, we focus on the different kinds of externalities of public spending and show how they act with the nonlinear transfer system in a principal-agent model with infinite local governments. By showing a Mirrleesian expression for optimal transfers, our study fills the gap in the literature by providing insights about the importance of introducing externalities into the design of transfer policies. Using the data from China, we also shed light on the important impact of externalities on deciding the monotonicity of transfer policies.

The last related strand of the literature is on optimal nonlinear taxation. As Oates (1999) mentions, inter-regional grants can be considered as a form of tax to some extent. Different from Mirrlees (1971), Saez (2001); Gerritsen (2023) propose the variation approach to analyze tax incidences under tax perturbation with clearer economic insights. Sachs et al. (2020) analyzes the optimal nonlinear income tax in the case of endogenous wages, which is a kind of price externality. Kaplow (2012) and Jacobs and de Mooij (2015) discuss optimal income and commodity taxation with the externalities from goods consumption. Compared with the literature above, our work applies the variation approach to the case of a nonlinear inter-regional transfer system. Notably, we extend the variation approach to address a multidimensional heterogeneity model that incorporates general equilibrium effects induced by externalities. Moreover, scenarios such as interregional transfers design in the context of the mobile population also represent novel applications of optimal taxation theories[6].

[6] Lehmann et al. (2014) and Huggett and Luo (2023) also investigate the Mirrleesian principal-agent problem in scenarios where households can migrate between regions. While they concentrate on optimal individual taxes, this paper focuses on optimal intergovernmental transfer policies and the information asymmetry that exists between two tiers of government, rather than between individuals and a government.

The remainder of the paper is organized as follows: Section 2 overviews the transfer policies in a typical country, China, and builds the benchmark model based on the common points of these policies. Section 3 exhibits the optimal transfer design of the benchmark model. Section 4 applies the framework to more general cases with various types of externalities. Section 5 introduces the extension of the model in terms of regional heterogeneities. Section 6 introduces the fiscal data and presents the outcomes of numerical simulation based on the data. Section 7 concludes.

2 Transfer Policies: Background and Model

As we emphasized in Section 1, the transfer policy faces three important issues: fiscal equity, information asymmetry, and externalities. In developing countries, these challenges are more severe due to the insufficient governance capacities and limited fiscal resources to address interregional fiscal disparities. In this section, we will introduce the transfer policy in a typical country, China, as a background to explore the practical importance of these issues and conclude the main concerns of transfer policies in countries where transfers play crucial roles in fiscal practice. These contexts will help us understand how to build models to optimize intergovernmental transfer policies. Based on the policy review and facts, we will build a theoretical framework to accommodate the efficiency and fiscal equity problems of intergovernmental transfers.

2.1 Intergovernmental Transfer Policy: China as a Typical Case Analysis

China can serve as a typical analytical object due to its three characteristics. First, China is a typical country with numerous local governments in a decentralized fiscal system. Intergovernmental transfers play key roles in this system because the tax-sharing arrangement between the central and local governments remains quite stable (only adjusted in 2002 and 2016 after the establishment of the decentralized system in 1994), making transfer policies one of the few fiscal tools that policymakers can frequently adjust to influence local fiscal operations. Second, as detailed later in this section, Chinese policymakers employ a formula-based transfer system. This approach allows us to uncover their primary considerations and provides a basis for comparing the existing policies with the optimal policies proposed in this study. Lastly, China's transfer policies face dual challenges of efficiency and equity (as discussed in Section E.2), aligning closely with the central focuses discussed in Section 1.

In China, the intergovernmental transfer system, called "**transfer payment**", plays an important role in determining local governments' finances. Henceforth we use the terms "transfer" and "transfer payment" interchangeably. The total amount of transfer funds accounts for approximately one-third of the general budget revenue according to the data.

China's transfer policy can be divided into two main categories. The first is "the general transfer" (GT), which accounts for more than half of all transfer funds. This category contains the balance transfer, which mainly includes unconditional transfers to less-developed regions and a series of subsidies to frontier or ethnic minority areas. Another category is the special transfer (ST), in which all the items are earmarked for particular uses, such as transfers dedicated to education or infrastructure investments. In 2018, the Chinese Ministry of Finance implemented a transfer policy reform. Some items of the ST funds that require the collaborative provision of public goods by both the central and local governments were allocated to GT, and this portion is referred to as "transfer funds for shared fiscal powers."

GT is the most important type of transfer payment, not only because it has the highest proportion in the total amount of transfer payments, but also because its design best matches the discussion of the functions of intergovernmental transfers in fiscal decentralization, as its main function is to "narrow the financial gap between region". Specifically, GT is more concerned about the gap in "basic public services" between regions than the productivity gap of local governments, as announced by the Chinese central government[7]. Basic public services mainly encapsulate expenditures that are closely associated with the primary needs of residents, e.g. public expenditure for local government operation, education, medical care, and other related expenditures for local residents[8].

As GT is designed to alleviate the financial pressure of local governments in providing basic public services, it has two important functions: to narrow the economic disparities between regions and to restrict the overspending behavior of local governments. GT is distributed based on some formulas, known as the "transfer payment formula method." We will briefly conclude the major concern in this formula method, and leave the introduction of the formula detail in Appendix E.2.

The core concerns and characteristics of Chinese transfer payment policies are as follows:

(i) **Information Asymmetry**. The central government is unsure about the income level of local governments, or at least the revenue-collecting capacity of local governments. There-

[7]A document issued by China's State Council in 2014 called *The State Council's opinion on reforming and improving the central-to-local transfer policy system* pointed out that the main goal of Chinese transfer policies is to "promote the equalization of basic public services among regions" and to "improve the growth mechanism for general transfer payments", which means the focus should be on general transfer payments.

[8]In Appendix E, we specifically show the descriptions of basic public services in different policy documents in China, as well as the selection of basic public services in this paper from a data perspective.

fore it uses the formula method to deduce local revenues.

(ii) **Dependency on Transfer Payments and Flypaper Effect**. The central government has recognized the potential local expenditure distortion induced by transfer payments under the asymmetric information structure. It constrains excessive spending and soft budgetary constraints of local governments by computing pseudo public expenditures. However, local financial statistics still affect the amount of transfer payments. Besides, as noted in the relative document, some items in the transfer formula could also be adjusted based on the reported local statistics. Hence the issue of the high dependency on transfers still exists, which will be shown later.

(iii) **Inter-regional Equity**. This is the main target of the transfer payment policy. Notably, transfer payments focus on the equity of basic public services rather than the overall level of local economic development. The central government recognizes that it is challenging to narrow the economic development gap between regions solely through transfer payments.

(iv) **Awareness of Externalities**. The transfer formula takes externalities such as the inter-regional population mobility and free-rider problems of public goods into consideration.

In short, China has developed a complex formula-based interregional transfer system that utilizes objective indices to mitigate the adverse selection problem and externalities. However, this transfer policy falls short of optimal design.

First, the central policymaker lacks sufficient confidence in relying solely on these formulas for distributing transfers, leading to frequent adjustments in transfer policies and reliance on reported statistics from local governments. As a result, the observed transfers increasing with public spending levels indirectly encourage the rising dependency on transfer funds, leaving the room for local governments to widen their fiscal gaps via expanding public expenditures, as shown in Figure 1. The figure indicates that there are huge discrepancies in local public expenditures and transfer payments per capita between regions, although narrowing the fiscal gap is the main target of the Chinese transfer payment policy. Most counties are clustered in an area featuring low levels of public spending and transfer payments, while a few are scattered in areas with extremely high public spending or extremely high transfers. The positive correlation between transfers and local expenditures creates an incentive for local governments to expand their fiscal gaps by increasing public spending[9].

Second, this transfer policy also undermines fiscal equity, as most regions are characterized by both low public spending and low transfer payments. This indicates a right-skewed distribution of public service provision, which is contrary to the intended goals of the trans-

[9] We provide more evidence about the transfer dependency in Appendix E.3.

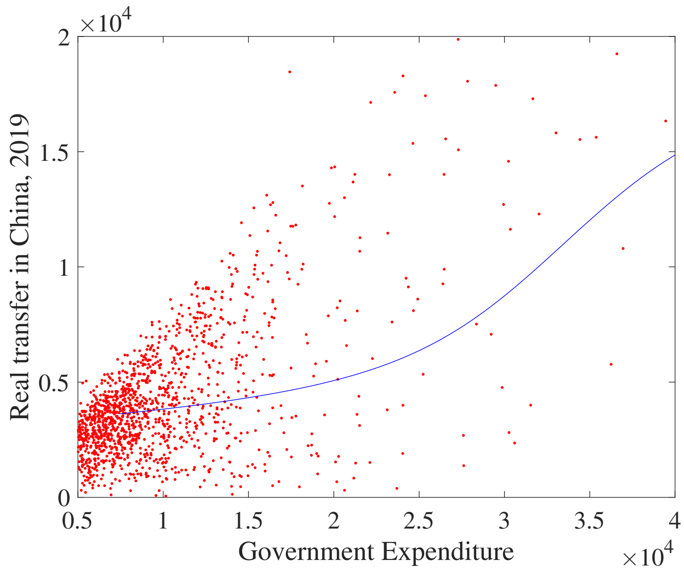

Figure 1: The relationship between per capita government expenditures in the general budget (horizontal axis) and per capita transfer payments (vertical axis) across county-level regions in China, 2019.

Notes: Each region is depicted as a red dot located at the intersection of its transfer and spending levels. The blue solid line represents the curve fitted to the relationship between the two variables using the Gaussian kernel function.

fer design.

Finally, greater attention should be given to externalities, as their influence is far from negligible. For instance, regional infrastructure investment accounts for 14%, a considerable proportion of Chinese economic growth (Dinlersoz and Fu, 2022). These infrastructures such as highways and railways, then generate significant nationwide externalities, including spillover effects and large-scale population migration with profound influence (Shi and Huang, 2014; Baum-Snow et al., 2017; Tombe and Zhu, 2019)[10]. Nevertheless, current transfer policies lack effective coping strategies to address these various types of externalities.

A more striking fact is that these situations are not unique to China. We observe similar policy targets and considerations in transfer design in other countries such as India, Mexico and other Latin American countries, which also grapple with comparable dilemmas, as detailed in Martinez-Vazquez and Sepulveda (2011); Abiad et al. (2020); Rao (2017); Sour (2013); Ahmad and Brosio (2007). The background of transfer policies in various countries motivates us to question how to reform the prevailing transfer policies. The optimal transfer

[10]According to the seventh national population census of China, in 2020, approximately 124.84 million individuals migrated across provincial borders, while an additional 367.93 million relocated within their respective provinces.

design should simultaneously accommodate information frictions between different tiers of governments, fiscal equity and externalities of public spending. Next, we will introduce the settings of the model as the baseline of our study.

2.2 Benchmark Model Environment

In the rest of this article, we will use a theoretical framework to accommodate the information asymmetry, fiscal equity, and externality problems of transfer design, focusing on the asymmetric information regarding local economic data.

In this part, we will exhibit a simple model illustrating the basic idea of our framework: how transfer design influences the public goods allocation and therefore affecting efficiency. Consider a country with two tiers of governments: one central government (the policy-maker), and a continuum of local government, called provinces, each of which has a heterogeneous income level $n \in \mathcal{N} \subset \mathbb{R}_+$. Let the lowest and highest levels of $\mathcal{N}$ be $\underline{n}$ and $\overline{n}$.The reality underlying the setup of an infinite number of regions is that the number of lowest-tier governments tends to be large in countries such as China[11]. In this framework, we simplify the relationship between the central government and various tiers of government as the direct central-local fiscal system.

Residents in each region consume both private and public goods. The utility function of the representative resident from a region with $n \in \mathbb{R}_+$ is defined as:

$$U(n) = U\left(g, c, e, n\right), \tag{1}$$

where $c(n)$ is the level of private consumption and $g(n)$ is the level of public goods provided in the region, mainly referring to the non-productive public goods, consistent with the background introduced in Section 2.1. e represents the externalities in any region induced by the public service in other regions. In the benchmark model, we only consider the classical spillover effect of public goods, which will be extended in Section 4.1. g is observable for both central and local governments.

Through this paper, we set preferences to be separable and additive between private consumption and public goods, which makes the analysis easier. The utility function is

$$U = u(g, e) + v(c), \tag{2}$$

where u is strictly concave and twice continuously differentiable for both g and e, which

[11]That the number of regions is infinite does not fundamentally influence the results.

means that $u_g > 0$ and $u_{gg} < 0$. The positive externality from the public goods means that $u_e > 0$, and we assume that the cross derivative $u_{ge} \geq 0$. In the case of negative externalities, such as environmental pollution induced by public spending, $u_e < 0$, and the discussion and derivation below remain unchanged. Through this paper, we only consider the positive spillover effect. The utility function in terms of c can be risk-neutral or risk-averse.

The budget constraint for the residents in a region is

$$c = n - r. \tag{3}$$

n is the exogenous income of a representative resident in this region and represents regional economic development.

n is private information for local governments and unobservable for the central government. This corresponds to the fact of information asymmetry introduced in Section 2.1. Particularly, local Chinese governments have an incentive to misreport their GDP to the central government, a fact which has been verified by Chen et al. (2019) and Song and Xiong (2023)[12].

r is the lump-sum tax on residents levied by the local government. We keep the tax structure in a simplified way because the focus of this study is to analyze the impact of transfers on governmental behavior, and income or commodity taxation is always connected with the behavior of residents or consumers. The tax policy is therefore treated herein in a simplified manner[13].

The local government provides public goods $g(n)$ only for the local residents and imposes a lump-sum tax $r(n)$. Since we do not consider the difference between the private and public goods markets, the price of public goods is assumed to be equal to that of private goods[14]. Local governments receive a nonlinear transfer $\tau(\cdot) \in C^2(\mathbb{R}, \mathbb{R}_+)$ from the central government while levying the tax from the local residents. In reviewing the policy of transfer payments in Section 2.1, the central government determines the amount of transfer payments using a formula approach, where the amount of transfers can be viewed as a func-

[12] The central government may even ask the National Bureau of Statistics (NBS) of China to compile separate GDP data for each province instead of directly using the data reported by the localities themselves. Even the NBS statistics often exhibit inaccuracies due to statistical problems (Feenstra et al., 2013) and political requirements and have been doubted in the previous literature (Rawski, 2001).

[13] Gross (2021) uses a model including taxation, debt, and transfers with consumers, producers, and investors in two regions in a union. In contrast with their paper, we focus mainly on the optimal design of a nonlinear transfer policy and observe how different elements influence behavior and welfare along a continuum of regions.

[14] However, introducing this heterogeneity in the cost of public goods into our model is not complicated.

tion of public expenditures in countries like China, India, or South Africa. Therefore, the design of this model should be consistent with the existing policy framework. Here the non-linear transfer schedule τ is the function of g. The budget constraint for a local government with income level n is

$$\tau(n) + r(n) = g(n). \tag{4}$$

The transfer policy can be understood as a contract $\{\tau, g\}$ to local governments. The rules are announced by the central government simultaneously with the decision process for local governments. Once the central government chooses $\tau(n)$ for region n, local governments also determine their consumption of public and private goods $g(n)$ and $c(n)$.

We follow the customary assumption of the optimal taxation theory:

Assumption 1 *$g(n) \in \mathbb{R}_+$ and $c(n) \in \mathbb{R}_+$ are differentiable and strictly increasing with n, i.e., monotonicity*[15].

This enables us to take $\tau \in C^2(\mathbb{R}_+, \mathbb{R})$ as $\tau(g(n))$. This also coincides with the reality in present transfer policies that the central government formulates the transfer payment policy mainly based on levels of basic public services, as shown in Figure 1.

The local government chooses $g(n)$ to maximize utility $U(n)$ subject to the budget constraint. From the first-order condition for this optimization problem, we can obtain the marginal rate of substitution between the public and private goods as

$$-\frac{u_g}{v_c} = \tau'(g) - 1. \tag{5}$$

From the equation (5) we can see that the marginal transfer directly influences local fiscal decisions regarding public goods provision. The increasing transfer schedule in China depicted in Figure 1, indicating that $1 - \tau'(g) > 0$ is relatively small, thus encourages local governments to pursue high levels of g since $u_{gg} < 0$. This first-order condition aligns with the observed expansion of fiscal gaps since 1993, as shown in Figure E.2.

Another assumption serves a technical requirement:

Assumption 2 *For any $n \in \mathcal{N}$ and $(g(n), c(n)) \in \mathbb{R}_+ \times \mathbb{R}_+$, the function $MRS \triangleq -\frac{u_g}{v_c}$ is differentiable with respect to n, g and c.*

The externality is one of the focal point of our study. The general form of the externality

[15] Although we do not use this constraint during mathematical derivation process, we check whether Assumption 1 is satisfied during numerical simulation.

in the transfer design can be defined as:

$$e(n) = \int_{\underline{n}}^{\overline{n}} \Theta(n, n') \mathcal{E}(g(n'); n', T) dn'. \tag{6}$$

The first element of $\mathcal{E}$, g, captures how regional public choices influence the broader economy. Additionally, we denote T as the direct effect of transfer design on externalities. For instance, when households are mobile, transfers directly shape migration decisions through their income effects on residents, which we will discuss in Section 4.1. The weight function, $\Theta(n, n')$, reflects the potential asymmetry in nationwide effects of regional expenditures. In Section 4, we will see how this setup encapsulates various scenarios of externalities prevailing in China.

The central government optimizes the social welfare function $W = \int_{\underline{n}}^{\overline{n}} \gamma(n) U(n) f(n) dn$, where $\gamma(n)$ is the weight that reflects the value that the central government places on the region with income level n[16]. The budget constraint for the central government is:

$$\int_{\underline{n}}^{\overline{n}} \tau(n) f(n) = \bar{R}, \tag{7}$$

where $\bar{R}$ is the fiscal revenue of the central government, which is assumed to be constant.

2.3 Elasticities

Our primary focus is on how policymakers design and implement the transfer schedule accounting for the potential behavioral responses of local governments to such policy reforms. It is necessary to introduce several elasticities as sufficient statistics during our analysis.

First, under Assumptions 1 and 2, we take the total differential of each side of (5); thereafter, we define the income and Marshallian price elasticities as

$$(\tau'(g) - 1) \frac{\partial g}{\partial n} \triangleq \tilde{\eta}_n = -\frac{v''(\tau'(g) - 1)^2}{u_{gg} + v''(\tau' - 1)^2 + v'\tau''}, \tag{8}$$

$$\frac{\partial g}{\partial (\tau' - 1)} \frac{\tau' - 1}{g} \triangleq \tilde{\epsilon}_n^u = -\frac{v' + gv''(\tau'(g) - 1)}{u_{gg} + v''(\tau' - 1)^2 + v'\tau''} \cdot \frac{\tau'(g) - 1}{g}, \tag{9}$$

[16] Saez and Stantcheva (2016) proposes a more general definition of welfare weights that embody concern for the consumption of agents. They argue that generalized welfare weights can characterize those weights based on utility functions. This paper does not include some of the special cases proposed in that article, so the use of traditional welfare weights is still appropriate.

and derive the elasticity of compensated demand (Hicksian elasticity) as:

$$\widetilde{\epsilon}_n^c = -\frac{v'}{u_{gg} + v''(\tau'-1)^2 + v'\tau''} \cdot \frac{\tau'(g)-1}{g}. \tag{10}$$

From the price and income elasticities, we can illustrate the flypaper effect. They reflect not only the unilateral impact of the price of public spending $(1-\tau')$ on expenditures g but also the reverse impact of expenditures on the price. If Assumption 1 holds, it is not hard to obtain $\widetilde{\epsilon}_n^c < 0$, which means that the decrease of the price of public spending $1-\tau'$ (equalling the increase of $\tau'-1$) will lead to the increase of public spending. Many previous studies show that the flypaper effect is due to the decrease in the cost of providing public service (Hamilton, 1986; Bailey and Connolly, 1998). Hence the price elasticity $\widetilde{\epsilon}_n^u$ shows the flypaper effect of transfers, which is one of the core concerns of the central government.

Another concern with public goods is externalities. The elasticities presented above indirectly contain the influence of e because public spending is influenced by e, but the change in e induced by a change in transfer policies remains to be explained. To show this, it is necessary to introduce the variation approach, which will be discussed in detail in the following section. Due to a perturbation of the transfer policy, the public spending choice g changes, further altering the externalities. This process can be reflected in the elasticity of externalities, which reflects the total influence of a change in e on the regional choice of g, as:

$$\widetilde{\epsilon}^{g,e} \triangleq \frac{\partial g}{\partial e}\frac{e}{g} = -\left(\frac{u_{ge}}{u_{gg} + v''(\tau'-1)^2 + v'\tau''}\right)\frac{e}{g}. \tag{11}$$

(11) reveals the relationship between changes in public spending and externalities at the local government level. We can decompose the way in which externalities moderate the changes in local government behavior into two parts. First, a change in public spending across all regions alters externalities. These changes in the externalities further change public spending in each region. This elasticity $\widetilde{\epsilon}^{g,e}$ reflects the result of these two channels of interaction between externalities and public goods spending. A more detailed analysis can be found in Section 3.

By introducing elasticities into the analysis for optimal transfers, unlike the analytical result in Lockwood (1999) providing opaque insights about the influence of transfer policies, it is possible to derive the optimal transfer rule based on observable statistics reflecting the behavioral responses of localities, as we establish in Section 3.2. We show how the flypaper effect and externalities affect the design of the optimal transfer policy.

3 Transfer Reform and Optimal Transfer

In this section, we choose the variation approach used by Gerritsen (2023) and Sachs et al. (2020) to solve the optimal transfer schedule in the benchmark model with the sole heterogeneity in n. The advantage of this method is that one can examine the behavior of each region and of the central government as well as the welfare change due to a shift in the transfer policy. We also observe that the optimal transfer rule can be expressed in terms of sufficient statistics, which provide richer economic insights compared to the results derived under the primal approach.

3.1 Perturbation

Consider τ as the optimal transfer policy schedule g[17]. The perturbation $\kappa\hat{T}$ is a minimal perturbation to the schedule τ, where $\hat{T} \in L^2(\mathbb{R}_+, \mathbb{R})$ is a nonlinear, first-order differentiable transfer reform function and $\kappa \in \mathbb{R}$ parameterizes the scale of the reform[18]. By adding the perturbation to the transfer schedule, we can analyze the first-order effect on the local governmental choices and its welfare impact on each region. The most important choice is the public goods expenditure:

$$g = g(e(\tau), \tau; n),$$

where the first independent variable, e, indicates the influence of externalities on the expenditure on public goods, and the second variable, τ, indicates the effect of the transfer policy schedule on the choice of g due to the reform of the policy. The perturbation of public goods is defined as:

Definition 1 *The perturbation of public spending under the transfer perturbation $\kappa\hat{T}(n; \cdot)$ is*

$$\hat{g}(n) \triangleq \lim_{\kappa \to 0} \left[g(e(\tau + \kappa\hat{T}), \tau + \kappa\hat{T}; n) - g(e(\tau), \tau; n) \right] / \kappa.$$

The term $\hat{g}(n)$ represents the change in public goods in a region with income level n given the perturbation of the transfer policy schedule $\kappa\hat{T}$, considering the interaction between the change in the externalities $e(\tau + \kappa\hat{T})$ and the induced change in public goods spending due to the change in externalities when $\kappa \to 0$.

[17] Here, the second-best allocation is defined as the solution of the Mirrlees approach. In Appendix B.4, we show the equivalence between the Mirrlees approach and variation approach, or the dual approach.

[18] Although the reform $\hat{T}$ is a function over the whole interval along the distribution of public spending g, there is a particular case in which the transfer changes only the specific level of g^*, called the "elementary reform" (see Saez (2001)).

After the perturbation happens, we can show the efficiency impact of the transfer reform via the perturbation of public goods provision as follows:

Proposition 1 *The perturbation of a reform of the transfer policy schedule $\hat{T}$ on the public goods in a region with income level n is given as follows: for all $n \in [\underline{n}, \overline{n}]$, we solve the Fredholm integral equation problem as:*

$$\frac{\hat{g}(n)}{g(n)} = \underbrace{\frac{(\widetilde{\epsilon}_n^c \hat{T}' + \widetilde{\eta}_n \hat{T}/g)}{\tau'(g) - 1}}_{\text{direct effect}} + \underbrace{\widetilde{\epsilon}^{g,e} \cdot \frac{\hat{e}(\hat{T})}{e}}_{\text{indirect effect}}, \tag{12}$$

The introduction of e makes (12) become a Fredholm integral equation similar to Sachs et al. (2020). The percentage change of the price of local governments $\frac{\hat{T}'(g)}{\tau(g)-1}$ (the retention rate in their article) first causes the direct percentage change of public spending, $\frac{\hat{g}(n)}{g(n)}$. The price elasticity $\widetilde{\epsilon}_n^c$ determines the magnitude of this change. Besides, since we consider the income effect in this paper, the change in transfers for regions with public spending g, in percentage terms as $\frac{\hat{T}}{g(\tau'(g)-1)}$, will also directly influence the local governments' expenditures. The income elasticity $\widetilde{\eta}_n$ determines how large the influence is. However, these are all partial-equilibrium impacts of the perturbation on the transfer schedule, which ignores the change of inter-regional externalities due to public spending changes in all regions.

The perturbation of the transfer schedule not only leads to the **direct** changes in spending on public goods. These partial-equilibrium changes, in turn, lead to a change in the externalities of the public goods. This change further alters the public spending choice of local governments, the scale of which depends on the percentage change of externalities $\frac{\hat{e}}{e}$, and the elasticity of externalities, which is the **indirect** impact of the transfer perturbation.

$\hat{g}$ is jointly determined by the direct and indirect impacts of transfer perturbation, as shown in equation (12). Since the change of externalities is also the result of the perturbation of public spending nationwide, the final perturbation of public goods spending is the solution to a fixed-point equation. However, the general definition in (6) results in a complex formulation involving series, as shown by Sachs et al. (2020), making the roles of externalities less understandable. Thus in the benchmark model, we define the externalities as the weighted sum of public goods spending overall regions, consistent with Lockwood (1999) and Kaplow (2012):

$$e \triangleq \int_{\underline{n}}^{\overline{n}} g(n) f(n) dn, \tag{13}$$

where $f(n)$ is the PDF of regions with available resources n. e can be viewed as a constant for a single region. We also discuss more general case in Section 4.1, introducing mobile households, and in Appendix C we discuss the interregional competition which is a particular case in China, as revealed by Song and Xiong (2023); Li et al. (2019).

Based on the form of externalities above, we simulate $\hat{g}$ by fitting China's current transfer policy using an extended HSV function (discussed in Appendix D.3) in order to examine how transfer reforms impact local public spending levels. To better highlight the role of externalities, we adopt a specific transfer reform function, $\hat{T}(g) = g - g^*$, rather than the elementary reform employed by Sachs et al. (2020), where the substitution effect would dominate other influences. The setup of utility function and parameter calibration is introduced in Section 6. Figure 2 exhibits the impact of the transfer perturbation on public spending levels at g^* and the overall externalities.

As the starting point of the transfer reform increases, the response of local public spending at g^* becomes more pronounced. Since $\hat{T} = g - g^*$, there is no income effect at the point $g = g^*$. Therefore, the upward trend of both the black solid and red dashed lines suggests that substitution effects are more significant in regions with higher income levels, which means a greater distortion induced by the reform. We can also observe that the externality effect is relatively small compared to the substitution effect, considering the scale of elasticity of externalities (0.027). However, the impact of externalities on transfer design remains significant, as it still accounts for up to 7% of the overall changes in local public spending. Besides, the perturbation of e also directly alters the utility for all localities. As shown in Figure 2, the absolute value of the perturbation of externalities is comparable to $\hat{g}$, and becomes more substantial when the transfer reform affects a wider range of regions. In Section 6, we will show that the presence of externalities has a significant impact on the optimal transfer design, and even reverses the sign of the marginal transfer.

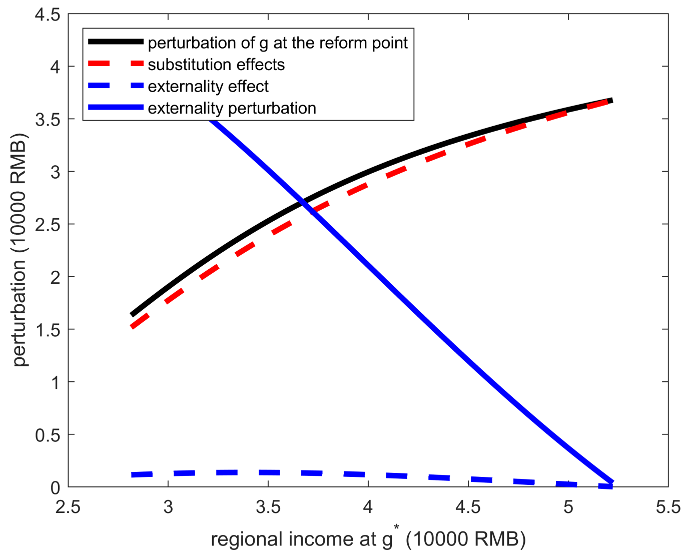

Figure 2: Perturbation induced by the marginal transfer reform $\hat{T} = g - g^*$ on the current transfer policy in China

Next, we shed light on the impact of transfer perturbation on the welfare of the local governments and society. The perturbation of regional utility is given by the following corollary:

Corollary 1 *The perturbation of the reform $\hat{T}$ on the welfare $\hat{U}(n)$ of a region with income level n is:*

$$\hat{U}(n) = u_e(n)\hat{e} + v'(n)\hat{T}. \tag{14}$$

The interpretation of equation (14) is straightforward: the reform of the transfer policy schedule first directly relaxes the budget constraint of every region and therefore improves the welfare of regions according to the envelope theorem, which is shown through the latter part of the equation (14). Furthermore, the former portion of (14) indicates that the transfer policy reform also changes the externalities, influencing the utility of the region.

From this equation, the welfare change across all regional governments can be expressed as:

$$\hat{W}_{region} = \int_{\underline{n}}^{\overline{n}} \gamma(n)\hat{U}(n)f(n)dn. \tag{15}$$

$\gamma(n)$ is the marginal social weight.

The reform also leads to a change in the government's revenue R, and the perturbation on government revenue $\hat{R}$ can be derived as:

Corollary 2 *The transfer perturbation on the revenue of central government $\hat{R}$ is given by:*

$$\hat{R}(\hat{T}) = \int_{\underline{n}}^{\overline{n}} \hat{T} f(n) dn + \int_{\underline{n}}^{\overline{n}} \tau'(n) \hat{g} f(n) dn. \tag{16}$$

This equation can also be interpreted as two channels that impact government revenue. The first channel is the direct change of the national transfer budget caused by $\kappa\hat{T}$, reflected by the former part of the equation (16). This is the "mechanical effect" of transfer policy reform. The change in each region's provision of public goods at the margin, $\hat{g}$, further changes the revenue of the central government by $\tau'\hat{g}$, which is the "behavioral effect" of transfer policy reform. This effect is embodied in the latter part of the equation (16).

The perturbation of social welfare is the sum of the changes in each region's welfare and the government revenue, accounting for the marginal social value of the central government's expenditure, λ[19]. λ can be derived as the function of consumption and welfare weights:

Lemma 1 *The marginal social value of the central government's budget is given by:*

$$\lambda = -\frac{\int_{\underline{n}}^{\overline{n}} \gamma(n') f(n') dn'}{\int_{\underline{n}}^{\overline{n}} \frac{1}{v'(n')} f(n') dn'}$$

.

Proof. See Appendix B.2. ■

We combine (15) and (16) to represent the change in social welfare $\hat{W}$ as:

$$\hat{W} = \int_{\underline{n}}^{\overline{n}} \gamma(n) \left[u_e(n)\hat{e} + v'(n)\hat{T} \right] f(n) dn + \lambda \int_{\underline{n}}^{\overline{n}} \hat{T} f(n) dn + \lambda \int_{\underline{n}}^{\overline{n}} \tau'(n) \hat{g} f(n) dn, \tag{17}$$

where the term $\frac{\gamma}{\lambda}$ can be interpreted as the marginal social welfare function for the regions compared with the marginal value of the central government's funds.

3.2 Optimal Policy

Based on some computations (see Appendix A.3), we can now give a clear solution for the optimal transfer design.

[19] We assume that the central government does not have public expenditures, which means all transfers are distributed to local governments.

Proposition 2 *The Diamond-Saez form of the optimal marginal transfer can be expressed as follows:*

$$\frac{\tau'(g)}{\tau'(g)-1} = A(n)B(n)C(n) + D(n), \tag{18}$$

where

$$A(n) = \frac{1}{\tilde{\epsilon}^c_n}, \tag{19}$$

$$B(n) = \frac{1-H(g)}{gh(g)}, \tag{20}$$

$$C(n) = \int_g^{\overline{g}} \frac{h(g')}{1-H(g)} (\beta(g') - 1) e^{\int_{g'}^{g} \frac{v_{cc}}{v_c}(\tau'-1)dg''} dg', \tag{21}$$

$$D(n) = -\frac{\int_{\underline{g}}^{\overline{g}} \left(\frac{\tilde{\gamma}}{\lambda} u_e + \tilde{\epsilon}^{g,e} \frac{g'}{e} \tau' \right) h(g') dg'}{(\tau'(g) - 1) \left(1 - \int_{\underline{g}}^{\overline{g}} \tilde{\epsilon}^{g,e} \frac{g'}{e} h(g') dg' \right)}, \tag{22}$$

$$\beta(g) = -\frac{\tilde{\gamma}(g) v_c(g)}{\lambda}. \tag{23}$$

The left-hand side of (18) is a decreasing function of τ'. Similar to Saez (2001), (18) can be decomposed into a few elements determining the optimal transfer policy. First, the social value $\beta(g)$ determines the term $C(n)$, which represents the social preference for inter-regional redistribution. Suppose that the central government pays more attention to regions with public goods expenditure levels above g. In that case, $C(n)$ becomes larger (recall that $\beta \geq 0$). Despite the change in $D(n)$, since $A(n) < 0$, then a higher $\beta(g)$ means a higher marginal transfer for this region[20]. We will show in Section 6.4 that the redistribution term is the main determinant of the shape of the optimal marginal transfer curve.

The second element is the distribution of public goods, indicated by $B(n)$. As Saez (2001) emphasizes, this item can be understood by investigating the cost-benefit analysis of policy perturbation. Considering a perturbation to the marginal transfer $\tau'(g)$ in regions whose public goods levels are near g, the gain from the marginal change in the transfer policy is proportional to that of regions whose public goods level is above g, expressed as $(1 - H(g))$. The utilities of these regions increase as a result of receiving lump-sum transfers, whereas central government revenues decrease as they are used to fund these lump-sum transfers. The cost associated with the marginal rate change resulting from the distortion

[20]If the monotonic condition for g is satisfied, $A(n) < 0$ always holds.

in consumption choices is linked to the number of regions close to g and their consumption of public goods, denoted as $gh(g)$. Consequently, the distortion is more significant when the proportion of regions where g' exceeds g is low, while the proportion of regions near g is high. Therefore, a higher value of $1 - H(g)$ relative to density indicates greater social welfare, and as a result, higher marginal transfers should be implemented if $A(n)C(n) < 0$.

The third determinant of the optimal marginal rate in the transfer policy is the Hicksian elasticity. The term $A(n)$ is negatively connected with the marginal transfer, as a higher elasticity of compensated demand means that the distortion is higher for a change in the marginal rate $\tau'(g)$. Most notably, a high elasticity of substitution implies that an increase in marginal transfers leads to a more pronounced flypaper effect in the region, causing excessive public overspending and leading these regions to become more dependent on transfers. Therefore, it is advisable to decrease the marginal transfer level to mitigate the impact of the flypaper effect, thus enhancing overall social efficiency.

The key element differing from the results of Saez (2001) are the functions of the externalities. Equation (22) directly shows the mechanisms by which externalities influence the optimal transfer policy, which requires the transfer policies to internalize the externalities as a kind of Pigouvian tax. The influence of the externalities can be divided into following two channels:

$$D(n) = \underbrace{-\frac{\int_{\underline{g}}^{\overline{g}} \frac{\widetilde{\gamma}}{\lambda} u_e h(g')dg'}{(\tau'(g)-1)\left(1-\int_{\underline{g}}^{\overline{g}} \widetilde{\epsilon}^{g,e}\frac{g'}{e}h(g')dg'\right)}}_{\text{Equity Channel}} \underbrace{-\frac{\int_{\underline{g}}^{\overline{g}} \widetilde{\epsilon}^{g,e}\frac{g'}{e}\tau' h(g')dg'}{(\tau'(g)-1)\left(1-\int_{\underline{g}}^{\overline{g}} \widetilde{\epsilon}^{g,e}\frac{g'}{e}h(g')dg'\right)}}_{\text{Efficiency Channel}},$$

Firstly, externalities change social equality through the direct change of the utility function, $\frac{\tilde{\gamma}}{\lambda}u_e$, according to the envelope theorem. In addition, changes in e induce behavioral adjustments among local governments. They reallocate expenditures between public and private goods not only based on their own preferences but also in response to externalities generated by other regions. Hence externalities also play a crucial role in the efficiency considerations of transfer design. These two channels highlight the trade-off between efficiency and equity in the presence of externalities. Given that $\lambda < 0$ and $\widetilde{\epsilon}^{g,e} > 0$ when the absolute value of τ'' is sufficiently small, the first term of $D(n)$ is positive while the second term is negative. This suggests that externalities affect the optimal transfer design through opposing forces: the efficiency channel and the equity channel. It is also remarkable that the two channels above can be directly derived via the variation approach, while the primal approach requires some transformation (which is not obvious) to obtain the same expression, as shown in Appendix

B.4.

Besides, the variation approach also makes the expression of the externality term $D(n)$ more practical for policymakers in designing transfers by incorporating elasticities as sufficient statistics, compared to the result of Lockwood (1999) derived via the primal approach[21]. Although policymakers still need to compute the welfare effect u_e determined by parameters in a structural model, one can also employ a "partial" sufficient statistics method by solely estimating those elasticities and incorporating them into the structural model.

It is also notable that while the definition of externality e does not affect the derivation of welfare perturbation, it is directly related to the expression of the externality term D. The optimal transfer schedule in (18) just serves as a fundamental framework for balancing efficiency and equity considerations in the design of place-based intergovernmental policies.

3.3 Sign of Marginal Transfer

Externalities also make the marginal rate for the top-tail (or bottom) regions no longer zero. First we can demonstrate that the sign of $D(n)$ is given by the following proposition:

Proposition 3 *In the case of positive externalites, which means $u_e > 0$, $D(n)(\tau' - 1) > 0$ under the optimal transfer system, which means $\lim_{n \to \overline{n}} \tau' > 0$.*

Proof. See Appendix B.4. ■

Hence under the existence of externalities, even for the regions with the highest economic resources, the central government should provide transfers increasing with the public spending to encourage the positive spillover effect as a Pigouvian tax.

The variation approach clearly shows the different channels through which externalities affect the optimal transfer policy. We also illustrate the roles played by the elasticities, the hazard rate, and redistribution for social equity in determining social welfare, as well as how they can be derived from the perturbation in the transfer policy.

4 Scenarios of Externalities

Equation (18) highlights the key determinants of transfer policies in addressing information asymmetry, externalities, and redistribution challenges. However, we set externalities as

[21] Both results are undoubtedly equivalent, as demonstrated in Appendix B.4. However, the optimal transfer derived using the primal approach depends solely on the derivatives of the utility function, which are less feasible to be estimated in practice.

atmospheric and symmetric among regions, simplified from the general case in the equation (6). From Section 1 and Section E.2, we argue that externalities arising from interregional transfers can manifest through household migration or interregional fiscal competition. In this section, we demonstrate how the general formulation of externalities in (6) can be adapted to encapsulate various externality scenarios. Through this way can we validate the tractability of the optimal transfer rule structure in the baseline formula (18). This section presents model extensions in two directions: interregional migration and fiscal competition. Although applying the model to completely different scenarios, we can still demonstrate that the benchmark model is heuristic, as the extensions in this section do not change the structure and basic mechanisms of the optimal transfer rule outlined in Proposition 2.

4.1 Mobile Households

As introduced in Section 1, one of the externalities for local public expenditures is the migration of residents. Tombe and Zhu (2019) has pointed out that migration in China played an important role in explaining Chinese economic growth. The locality choice is also a main factor in the literature of fiscal federalism theories. We argue that the mobile households will lead to a similar result of the optimal transfer as (18).

The underlying intuition is that individuals make choices based on location preferences, leading to shifts in population distribution. This relocation process, often called "voting with the feet," is influenced by the availability of local public goods. Thus migration induced by transfer policies can also be summarized as a type of externalities. In a sorting model, the externality term e can be expressed as:

$$e(n) = \int \Theta(v(n))\mathcal{E}(v(n'), T)dn'.$$

In this case, $\Theta(v(n))$ and $\mathcal{E}(v(n))$ capture how regional public spending behavior affects the indirect utility $v(n)$ of residing in region n, thereby influencing residents' location choices.

We build a model with discrete regions with mobile individuals in a continuum to formally discuss the externalities and the optimal transfers under this scenario. Suppose there are finite numbers $N \in \mathbb{Z}$ of sub-national regions in a country. A region $i \in \mathbb{N} = \{1, 2, \cdots, N\}$ has an endogenous population f_i since individuals are mobile. Each representative individual in the region i consumes consumption goods c_i. A representative producer in the region i combines labor and public goods (or amenities) provided by the local government to produce the final goods. The production function is:

$$Y_i = n_i \cdot (\alpha \cdot G_i),$$

where Y_i represents the aggregate quantity of provided consumption goods within region i. n_i represents the heterogeneous productivity specific to the region, and G_i denotes the aggregate quantity of public goods provided by the local government. α is the proportion of productive public spending in relation to G_i. We denote the corresponding variables at the per capita level as $y_i = \frac{Y_i}{f_i}$ and $g_i = \frac{G_i}{f_i^{\chi}}$. The production function above can be rewritten as:

$$y_i = n_i \cdot \alpha g_i \cdot f_i^{\chi-1}, \tag{24}$$

where χ represents the congestion effect of public goods (e.g. see Fajgelbaum et al. (2019)). For the sake of simplification, we set $\chi = 1$, indicating the absence of any congestion effect.

Same with the benchmark model, the local government i levies a lump-sum tax r on the producer. The individual consumption level c_i equals the after-tax quantity of final goods $y_i - r$. However, in this case, the central government distributes the transfer funds according to the aggregate levels of output Y_i instead of g_i. Hence the budget constraint for the local government i is:

$$rf_i + T(Y_i) = G_i, \tag{25}$$

and the overall budget constraint for the region i:

$$c_i f_i + T(Y_i) = y_i f_i + G_i. \tag{26}$$

The proportion α of G_i is allocated as the factor for production and $1 - \alpha$ of G_i for non-productive public goods[22]. The utility function for each individual in the region i can be written as:

$$U_i = u((1-\alpha) \cdot g_i) + \psi(c_i). \tag{27}$$

Following classical settings of inter-regional migration of households such as Li et al. (2019); Fajgelbaum et al. (2019), we let v_i the indirect utility of households in the region i, and the ex-ante utility V_i for households in the region i is the indirect utility multiplied by a random variable ε_i:

$$V_i = v_i \cdot \varepsilon_i. \tag{28}$$

[22] Here we simplify the endogenous selection of productive and non-productive spending of local governments.

The variable ε_i follows a Fréchet distribution with CDF $\Pr(\varepsilon_i < x) = \exp(-x^{-k})$ where $k > 1$. Using the property of the distribution and law of large numbers, the fraction of individuals living in a region i is

$$f_i = \frac{v_i^k}{\sum_{j=1}^{N} v_j^k}. \tag{29}$$

Therefore the distribution f_i is also endogenous for public goods spending and transfer policies since it is the function of v_i.

Since the information asymmetry is between the central and local governments, IC conditions should be considered for local governments rather than directly for residents. We assume that local governments care about the total amounts of public goods and private consumption but share the same utility structures with residential utilities[23]. This setup is consistent with the bureaucracy problem discussed in Niskanen (1971). Denote the utility for the local government i as:

$$U_i^L = U^L((1-\alpha)G_i, C_i) = u((1-\alpha)G_i) + \psi(C_i), \tag{30}$$

where $C_i = c_i f_i$ is the total amount of private consumption in the region i. Hence we can rewrite the utility function of local residents as:

$$U_i = u((1-\alpha)\frac{G_i}{f_i}) + \psi(\frac{C_i}{f_i}) = U(G_i, C_i, e_i^{-1}), \tag{31}$$

and the elasticity term here is consistent with the equation (6):

$$e_i = \sum_j \Theta(n_i)\mathcal{E}(g(n_j); n_j, T)$$

where $\Theta(n_i) = (v_i^k)^{-1}$ and $\mathcal{E}(g(n_j); n_j, T) = v_j^k$.

The IC constraints for local governments can be divided into downward and upward constraints (see Heathcote and Tsujiyama (2021b)). The necessary condition for incentive compatibility requires the downward IC constraint binding for all $i \in \mathbb{N}$, which implies:

$$U^L(n_i, n_i) = U^L(n_i, n_{i-1}). \tag{32}$$

The first n_i in U^L refers to the productivity of the region i and the second one refers to the

[23]The setup that local governments share the same utility function structure with residents is not necessary for obtaining the structure of our optimal transfer scheme.

allocation $Y(n_i)$, $C(n_i)$ and $G(n_i)$.

The central government uses the transfers to optimize social welfare, which is the weighted sum of local households' utilities:

$$W = \sum_{i=1}^{N} \omega_i f_i v_i = \sum_{i=1}^{N} \omega_i f_i v(C_i/f_i, G_i/f_i), \tag{33}$$

where ω_i is the social welfare weight for households in the region i. The central government has a budget constraint similar to the main text:

$$\sum_{i=1}^{N} T(Y_i) = 0. \tag{34}$$

The central government allocates the production y_i and consumption goods c_i to maximize the social welfare, subject to the budget constraint and downward IC constraint. The public goods spending g_i can be replaced by (24). Hence the optimization problem for the central government is:

$$\begin{aligned}
&\max_{\{Y_i, C_i\}} W \\
\textbf{s.t.} \quad &\sum_{i=1}^{N} (C_i - Y_i - G_i) = 0 \\
&u(\frac{Y_i}{n_i}\frac{1-\alpha}{\alpha}) + \psi(C_i) = u(\frac{Y_{i-1}}{n_i}\frac{1-\alpha}{\alpha}) + \psi(\widetilde{C}_{i-1}).
\end{aligned} \tag{35}$$

where $\widetilde{C}_i = C_{i-1} + G_{i-1} - \frac{Y_{i-1}}{\alpha n_i}$. Since f_i is the function of v_i for $i = 1, 2, \cdots, N$, we can view the population distribution f_i as the function of the allocation $\{Y_i, C_i\}$.

Following Heathcote and Tsujiyama (2021b), we further let $U_i = (\frac{1-\alpha}{\alpha}\frac{y_i}{n_i})^{\sigma+1}\frac{1}{\sigma+1} + \log(c_i)$. Then the optimal transfer in the optimization problem (35) can also be decomposed into an ABC-D form similar to (18) in the following proposition:

Proposition 4 *When households are mobile in discrete finite regions denoted as $i = 1, 2, \ldots, N$, the optimal marginal transfer for a region i is*

$$\frac{T_i'}{(\frac{1}{\alpha n_i} + 1) - T_i'} = A \cdot B(n_i) \cdot C + D_1 + D_2,$$

where the term D represents the externality induced by the migration of households, α the proportion *of productive public goods to the total public spending in local governments, n_i the local heterogeneous endowment and σ the parameter of the utility function.*

See Appendix C.1 for the proof.

We can observe that, as discussed in Heathcote and Tsujiyama (2021b,a), the three terms exactly correspond to those in (18), where

$$A(n_i, n_{i+1}) = -\left((\frac{n_i}{n_{i+1}})^{\sigma+1} - \frac{C_i}{\widetilde{C}_i} - \frac{1}{\widetilde{C}_i u_{Yi}} \frac{n_{i+1} - n_i}{\alpha n_i n_{i+1}} \right)$$

is the elasticity term and

$$B(n_i) = \frac{1}{f_i} = \frac{v_i^k}{\sum_{j=1}^N v_j^k}$$

is the distribution term. As the number of regions approaches infinity, the term A multiplied by B approaches the elasticity and hazard rate terms in the optimal marginal transfer (18) of the benchmark model.

$$C(n_i) = \sum_{s=i}^{N} (\frac{\omega_s v_{cs}}{\lambda \psi_{Cs} c_i} + \frac{1}{\psi_{Cs} c_i}) \prod_{j=i}^{s-1} \frac{\psi_C(\widetilde{C}_j)}{\psi_{Cj}}$$

is the redistribution term, and the two terms corresponding to the externality term (22) are:

$$D_1(n_i) = \omega_i \Psi_i (v_{Yi} \frac{\psi_{Ci}}{u_{Yi}} - v_{Ci}) / \lambda$$

and

$$D_2(n_i) = A(n_i) B(n_i) \sum_{s=i+1}^{N} \left(\omega_s \frac{v_{cs}}{\lambda \psi_{Cs} c_i} \left[\underbrace{(1 + \varepsilon_{vs}^f)(1 + \varepsilon_{Cs}^f (1 - \frac{u_{yi}}{\psi_{ci}}))}_{\text{Efficiency Channel}} - \underbrace{1}_{\text{Equity Channel}} \right] \right) \prod_{j=i}^{s-1} \frac{\psi_C(\widetilde{C}_j)}{\psi_{Cj}}$$

where $\varepsilon_v^f = \frac{\partial f_i}{\partial v_i} \frac{v_i}{f_i} = \frac{\Psi_i}{f_i} - 1$ is the elasticity of migration and

$$\Psi_i \triangleq \frac{(k+1) v_i^k (\sum_{i=1}^N v_i^k) - k v_i^{2k}}{(\sum_{i=1}^N v_i^k)^2} = f_i + v_i \frac{\partial f_i}{\partial v_i}.$$

The first externality term D_1 arises from the disparity between the utility functions of the central and local governments. As the central government only cares about residents' welfare, any variation in transfers will impact population distribution and residents' utilities, which depend on per capita consumption and public spending. However, for local governments, this migration of residents only impacts their utilities through the total amounts of consumption and public spending. $v_{Yi}\frac{\psi_{Ci}}{u_{Yi}} - v_{Ci}$ captures such difference in utility changes between the central and local governments, while Ψ_i represents the marginal welfare change induced by a marginal change in the residential utility v_i.

The second externality term D_2 reflects how the mobility of households influences the optimal design of transfer. A marginal change in the transfer schedule will alter the local governmental trade-off between the total consumption and total public spending levels. The fiscal behavior changes in each region then affect the social welfare in two channels.

First, comparable to the externality term (22) in the benchmark case, the total consumption change will directly alter residential utilities in each region, as represented by the term v_{cs}, which influences equity among regions. Next, this utility change will lead to subsequent household migration, which can be captured by two elasticities ε^f_{vs} and ε^f_{Cs}. The former elasticity reflects how variation in residential utilities affects the population of a region s. The later elasticity ε^f_{Cs} reflects the multiplier effect of migration. Given the total amounts of C in one region, it measures how the change in local population affects the per capita consumption and per capita public goods, which will further alter the residential utility once more. In a new equilibrium under the new transfer policy, residents settle down again, and per capita consumption and public spending support such allocation of population, as in (29). Thus these two elasticities capture the total effects of the demographic transition in the region s from the original equilibrium to the new one on the social welfare.

The two terms D_1 and D_2 represent mechanisms of externalities that differ from the baseline model in Section 3. D_1 captures the externalities associated with bureaucratism, where local governments focus on the total amounts of public budgets and consumption while neglecting the effects of incurred population flow on the welfare of local residents. D_2 highlights the general equilibrium effect of local public spending through the migration behavior of local governments. However, the two terms influence the optimal marginal transfer by added up with the canonical ABC form, sharing a similar expression structure with the result of the benchmark model.

4.2 Interregional Fiscal Competition

Another noteworthy phenomenon in China is the competition among officials in sub-national governments to meet the GDP growth targets set by higher-level government authorities (see Song and Xiong (2023); Li et al. (2019)). Since the officials in higher-level governments can not accurately gauge the ability of local officials, the local governments have incentives to overspend on productive public goods and pursue higher GDP growth rates. This strategic choice enhances their prospects of achieving victory in competitive tournaments. In this case, the expenditures in one region will also influence the probability of winning the tournaments of the other regions, and cause fiscal externalities. Particularly, we can define externalities in this scenario as:

$$e(y) = \int_n \Theta(y)\mathcal{E}(n,y)dn = \frac{\int \tilde{n}h(y \mid \tilde{n}, g)f^N(\tilde{n})d\tilde{n}}{f^Y(y)}, \tag{36}$$

where $y = A(n+\varepsilon) \cdot g$ is the ex-post local final production, determined by productivity A, local endowment n, and idiosyncratic shocks ε, the last two terms of which are local private information. g denotes local productive public expenses. $h(y|n)$ is the conditional probability density function (PDF) of ex-post output. f^N and f^Y are marginal distribution of n and y. We will introduce the detail of this heuristic model in Appendix C.

In this scenario, the optimal transfer serves the purpose of alleviating the distortions arising from tournaments, akin to the role of Pigouvian taxes in the foundational model. Notably, in this case, the transfer also reflects the game structure between the central and local governments. Since the central government cannot observe the private information and behavior of local governors, and local governors can only decide their own ex-ante public spending, both central and local governments formulate strategies based on their respective beliefs of the actions of the other governments.

5 Extension of Information Structure

We discuss the heterogeneity of local endowments in the benchmark model. However, the heterogeneity of local preference is always emphasized by previous literature (Cremer et al., 1996; Lockwood, 1999). It is reasonable to assume that the central government cannot accurately observe these preferences.

In this section, we build a bi-dimensional heterogeneity model to encompass the unobservable preferences instead of solely discussing the heterogeneous preferences. The reason is that the correlation between the distribution of preference and income heterogeneities might be intricate. Thus neglecting either of the two dimensions would result in a dramatically different transfer design. For instance, residents in a region with low income may prefer a high level of public goods for public subsidies and more public infrastructure investment. Conversely, previous literature has also pointed out that the demand for governmental public goods may increase with local income due to the escalation of public goods' costs (Blackley and DeBoer, 1987). Therefore the uni-dimensional heterogeneity model may yield dramatically different numerical results of the optimal transfer design, contingent on which type of heterogeneity it prioritizes, since the marginal distribution function of the heterogeneity it captures may diverge significantly with another one's.

Suppose that a continuum of regions differs from each other in two dimensions: the income level $n \in \mathcal{N} \subset \mathbb{R}_+$, and the preference for public goods $\theta \in \Theta \subset \mathbb{R}_+$. The two heterogeneities are subject to a joint distribution $\Phi(n,\theta)$ with the conditional distribution of n denoted as $f(n \mid \theta)$ and the marginal distribution of θ denoted as $\pi(\theta)$. Both the income level n and preference θ are the private information of local governments. Although in reality, preference for public goods can be partially revealed by behavioral outcomes of local governments, the precise estimation of preference is still challenging. Incorporating income level heterogeneity further complicates the identification process. Hence the central government designs the optimal transfer rules only based on the observable local public expenditures.

The utility function of local governments takes the form as follows:

$$U = u(g,e,\theta) + v(c). \tag{37}$$

The budget constraint is:

$$n + \tau = g + c. \tag{38}$$

Hence the MRS still satisfies (5). The externality e is defined as:

$$e \triangleq \iint_{\{n,\theta\}\in\mathcal{N}\times\Theta} g f(n \mid \theta) dn d\pi(\theta). \tag{39}$$

The introduction of externalities distinguishes this model from other studies with multi-dimensional heterogeneities, such as Jacquet and Lehmann (2021); Alves et al. (2024); Kapička (2023); Ferey et al. (2024) , while also creating a new challenge in solving for optimal transfers with both externalities and two dimensions of heterogeneities. We will introduce the approach to address this obstacle later in this section.

Here we make the following assumption by categorizing regions into different groups based on their preference θ:

Assumption 3 *$\forall \theta \in \Theta$, given θ we have:*

$$\frac{\partial MRS(n,\theta)}{\partial n} \leq 0, \forall n \in [\underline{n}, \overline{n}],$$

which is the within-group single-crossing condition.

Assumption 4 *$\forall \theta \in \Theta$, given θ, $g(n,\theta)$ satisfies $\frac{\partial g}{\partial n} \geq 0$, which is the with-group monotonicity condition.*

According to Jacquet and Lehmann (2021), the two assumptions above ensure the interchangeability of the income level n and the public spending g, given the preference θ, which can be expressed as:

$$h(g(n,\theta) \mid \theta) = \frac{f(n \mid \theta)}{\dot{g}(n,\theta)}, \tag{40}$$

We further define the marginal distribution of g as:

$$\bar{h}(g) \triangleq \int_{\theta \in \Theta} h(g \mid \theta) d\pi(\theta), \tag{41}$$

with its CDF as $\bar{H}(g)$, where $g \in \mathcal{G}$.

We derive the optimal transfer rules via the extended variation approach with double perturbations (see Appendix C.3 for detail). Before we obtain the optimal transfer rule, it is useful to define the **total** elasticities where Jacquet and Lehmann (2021) discuss the composition effect of two heterogeneities. The elasticities $\widetilde{\epsilon}^c_n$ and $\widetilde{\eta}_n$ depict the response of public spending by a specific local government to changes in income or MRS, belonging to so-called "local" elasticities. The total elasticities denote the weighted mean of local elasticities of regions with the same public spending level. Define:

$$\bar{\epsilon}(g) = \int_{\theta \in \Theta} \epsilon(n(g,\theta),\theta) \frac{h(g \mid \theta)}{\bar{h}(g)} d\pi(\theta), \tag{42}$$

and

$$\bar{\eta}(g) = \int_{\theta \in \Theta} \eta(n(g,\theta),\theta) \frac{h(g \mid \theta)}{\bar{h}(g)} d\pi(\theta) \tag{43}$$

as the total substitution and income elasticities of regions with public spending level g.

For the computational simplicity, we further define:

$$\bar{\zeta}(g) \triangleq \int\limits_{\theta\in\Theta} \widetilde{\gamma}(g,\theta)u_e \frac{h(g \mid \theta)}{\bar{h}(g)} d\pi(\theta), \tag{44}$$

reflecting the average impact of externalities on the social welfare of regions with public spending g, and

$$\bar{\phi}(g) \triangleq \int\limits_{\theta\in\Theta} \widetilde{\gamma}(g,\theta)v' \frac{h(g \mid \theta)}{\bar{h}(g)} d\pi(\theta), \tag{45}$$

as the average social welfare weights for regions sharing the same g. Therefore we can finally derive the optimal transfer schedule shown in the following proposition via the computation of the variation approach:

Proposition 5 *The optimal marginal transfer in the scenario of bi-dimensional heterogeneities takes the following ABC-D form:*

$$\frac{\tau'}{\tau'-1} = A(g(n,\theta))B(g(n,\theta))C(g(n,\theta)) + D(g(n,\theta)), \tag{46}$$

$$A(g(n,\theta)) = \frac{1}{\bar{\epsilon}}, \tag{47}$$

$$B(g(n,\theta)) = \frac{1-\bar{H}(g)}{\bar{h}(g)g}, \tag{48}$$

$$C(g(n,\theta)) = \int_g (\bar{\beta}-1)\frac{\bar{h}(g')}{1-\bar{H}(g)} \exp\left(\int_{g'}^{g} \frac{\bar{\eta}}{\bar{\epsilon}g''}dg''\right) dg', \tag{49}$$

$$D(g(n,\theta)) = \frac{\int\limits_{g\in\mathcal{G}} (\frac{\bar{\phi}}{\lambda}+1)\left[\int_{\underline{g}}^{g'} \bar{\epsilon}_e \frac{\tau'(g'')-1}{\bar{\epsilon}g''} \exp\left(\int_{g'}^{g''} \frac{\bar{\eta}}{\bar{\epsilon}g'''}dg'''\right) dg''\right] \bar{h}(g')dg' - \int\limits_{g\in\mathcal{G}} \frac{\bar{\zeta}(g)}{\lambda}\bar{h}(g')dg'}{\tau'(g)-1}, \tag{50}$$

where

$$\bar{\beta}(g) = -\frac{\bar{\phi}(g)}{\lambda} \tag{51}$$

is the average welfare weight.

For the proof, see Appendix C.3[24].

The optimal marginal transfer given by Proposition 5 has a similar structure to that in the

[24] We also display the solution for the shadow price λ in the appendix.

benchmark model. The optimal transfer formula consists of four terms: the public spending elasticity $A(g)$, the hazard rate of public spending $B(g)$, the redistribution term $C(g)$, and the externality term $D(g)$. However, since the two dimensions of heterogeneities simultaneously determine the spending choice of local governments, the optimal transfer contingent on local public spending needs to consider the composition of regions bundled with the same public spending level. Therefore some of these terms are different from their counterparts in (18). Firstly, the elasticity $\bar{\epsilon}$, as the sufficient statistics, is the mean of elasticities of regions with the same g but different preference θ. As the transfer policies change, the composition of regions with the same g also endogenously changes, which is the composition effect. Henceforth it is a caveat to directly implement the sufficient statistics estimated with the real data in determining the optimal transfer policy under the multidimensional heterogeneities, as pointed out by Jacquet and Lehmann (2021).

The second difference between the bi-dimensional heterogeneity case and the baseline arises in the redistribution term. Both the welfare weight $\bar{\beta}$ and the income effect $\bar{\eta}$ represent the average levels across bundles of regions. Given the fixed level g, the composition effect causes these bundles to vary with the transfer policies, making the welfare weight also endogenously change. Besides, in the context of multidimensional heterogeneities, the relationship between $\bar{\eta}$ and $\bar{\epsilon}$ can no longer be simplified into a closed-form expression as in the uni-dimensional case, and the exponential term remains in $C(g)$.

The externality term $D(g)$ can be clearly divided into two parts exactly corresponding to (22), the mechanical welfare effect, represented by the term $\frac{\bar{\zeta}(g)}{\lambda}$, and the behavioral effect, represented by the first term in the numerator of $D(g)$. In the multidimensional case, we observe that although the two effects of externalities can be represented by a few elasticities and welfare terms such as $\bar{\phi}$ and $\bar{\zeta}$, estimating these welfare terms remains challenging. Thus a structural method is recommended when designing the optimal transfer policy with multidimensional characteristics.

Section 3 discusses the key elements of the optimal transfer policy, highlighting the comprehensive consideration of local dependency on transfers, externalities and interregional equity. Section 4 applies the framework to various scenarios of externalities. This section extends the benchmark model by exploring the case of bi-dimensional heterogeneity. Nonetheless, we need to further exhibit how much the optimal transfer policy differs from the transfer system used in reality, as discussed in Section 1. In the next section, we show the numerical results of the optimal transfer policy in the Chinese context under the setup of the benchmark model and quantitatively answer the questions raised in Section 1.

6 Numerical Simulation

6.1 Data Introduction

We use Chinese county-level fiscal data to show how we can improve Chinese transfer payment policies in reality using our theoretical model. First, we explain the variables used in the simulation. After that, we introduce the data that we use to implement the numerical simulation.

China's governmental budget structure consists of four distinct sets of budgets: the general budget, the central government-managed fund budget, the national state capital operations budget, and the national social insurance fund budget. Among these, the general budget is the most frequently discussed and encompasses expenditures related to the governmental operation. Notably, the primary component of transfer payments, GT, is closely associated with the revenue and expenditure of the general budget[25]. Conversely, the other three budgets have limited relevance to China's transfer system. Consequently, we employ revenue and expenditure data from the general budget to gauge governmental financial resources and the demand for public goods (this will be revisited later).

Public goods can be divided into two types according to previous literature: one directly enters the utility function which represents the basic public services in our model and also the main concern of the present formula method for BT, while the other type enters the production function and corresponds to expenditure such as infrastructure and technology investment(Keen and Marchand, 1997). These two types are noted as productive and nonproductive public goods in Barro (1990). In our model, public goods mainly refer to basic public services, which are closely connected with nonproductive public spending. One challenge is to differentiate between purely productive and nonproductive public goods expenditures since many expenditure items in official fiscal reports exhibit attributes of both. Compared to the central government-managed fund budget and the national state capital operations budget, expenditures within the general budget align more closely with the definition of non-productive public goods. Thus we mainly use the total expenditures of the general budget as the proxy of public spending.

Besides, we provide a more detailed classification of non-productive public expenditures as the robustness check. Specifically, we select the sum of nine subcategories within the general budget as a proxy variable for public goods expenditure g: general public service

[25] For the definition of GT and BT, see Section 2.1.

expenditures (primarily including operational and facility funding for local governments), national defense expenditures, public safety expenditures, education expenditures, cultural, sports, and media expenditures, social security and employment expenditures, medical and health care expenditures, urban and rural community affairs expenditures, and housing security expenditures. The expenditures on these nine items basically correspond to the scope of basic public services[26].

We collected county-level public financial settlement data during the year 2016-2019 in 26 Chinese provinces. We also merge with the county-level population data from the China Population and Employment Statistics Yearbook, which enables us to calculate the per capita variables since g and c in the model represent per capita public goods and consumption. Finally, we use the social consumption data (total retail sales of social consumer goods) from China's National Bureau of Statistics to calibrate some parameters. The raw data contains 5,667 observations in total, and 3,763 after merged with the consumption data. Finally, 2,671 observations remain after further excluding observations without the data of nine subcategories of the general budget expenditures (as we defined above) during 2016–2019.

6.2 Numerical Methods

We follow the Mankiw et al. (2009) (MWY) fixed-point algorithm [27]. We initially use the realistic public goods expenditure of regional governments g_0 and China's transfer payment schedule τ_r, approximated by a function in an extended Heathcote-Storesletten-Violante (HSV) form to obtain the distribution of n (see Appendix D.3 for the detail of approximation of the transfer schedule).

Next, we set an initial transfer schedule using the current transfer payment policy τ_r. Then we reorganize the equation and utilize the fixed-point algorithm to obtain τ'. Notably, although the monotonicity condition is not considered in this model, it is a sufficient condition for IC. In this paper, the monotonicity of g is verified in the data (see Figure D.5).

Our study continues with a comparison between the existing policy framework in China and the optimally derived one through numerical simulations. We aim to identify the key factor that predominantly influences the shape of the optimal transfer curve. Additionally, we illustrate the impact of transfer policy reform by examining how various regions respond

[26] See Appendix E for our criteria and details of categorization.

[27] MWY neglects the second-order derivative of income tax, and we adopt the same approach in this study. It is worth noting that the numerical distinction between MWY's approach and the marginal tax curves when considering τ'' is negligible, as shown in previous research (Jacquet and Lehmann, 2021). This suggests that the MWY method can be implemented to obtain an approximation of the optimal tax.

to these policy changes. The simulation allows us to assess the welfare improvements associated with transitioning from the existing policy to the theoretically optimal one. We also investigate the feasibility of utilizing empirically estimated elasticities as sufficient statistics in the formulation of transfer policies, thereby enhancing practicality for policymakers.

6.3 Parameters

The utility function for the regional government is defined as $U(g, c, e) = \beta g^{\alpha} e^{\theta} + \frac{c^{1-\sigma}}{1-\sigma}$ in this section, where β is the weight on the public goods and α, θ are the elasticities of public goods and externalities, respectively. σ is the relative risk aversion factor, as discussed in Friend and Blume (1975) and Mehra and Prescott (1985). They argue that σ should be larger than 1 but less than 2. Szpiro and Outreville (1988) and Hanna and Lindamood (2004) identify this parameter for different countries, genders, and surveys. Meyer and Meyer (2005) discusses this parameter for consumption instead of income or portfolios. In summary, σ can be 0.8–1.6 or approximately 2, and in some studies' settings, it can be more than 4. Mankiw et al. (2009) sets σ to 1.5. Therefore, we set σ to be 2 in our numerical simulation.

Table 1: Parameter Calibration

Parameter	Symbol	Value	Estimation Method
relative risk aversion factor	σ	2	from literature
elasticity of public goods	α	0.74	panel data regression
elasticity of externalities	θ	0.027	calibration
weight of utility	β	0.12	calibration
Pareto weight on regions	$\widetilde{\gamma}$	1	utilitarian preference

We use the empirical method to estimate the parameters α, β, and θ. Given the utility function above, we can rewrite the first order condition (5) as:

$$\alpha \beta g^{\alpha-1} e^{\theta} = (1 - \tau') c^{-\sigma}. \tag{52}$$

The logarithmic form of the equation is:

$$\ln \alpha\beta + \theta \ln e + (\alpha - 1) \ln g = -\sigma \ln c + \ln 1 - \tau'. \tag{53}$$

This enables us to use the empirical method to estimate the coefficient $\alpha - 1$. The basic idea of the estimation is: First, we fit the marginal transfer payments $\tau' - 1$ using a Gaussian kernel function by using the fiscal data in 2019. Next, since σ is always estimated by using micro data while this paper mainly focuses on the macro-level issue, we substitute $\sigma = 2$ directly into the right-hand side of (53) and use $\sigma \ln c - \ln 1 - \tau'$ as the dependent variable. Finally, we use the following panel-data regression to obtain the estimation:

$$y_{it} = (1 - \alpha) \ln g_{it} + \epsilon_{it} + \chi_i + \psi_t, \tag{54}$$

where $y_{it} = \sigma \ln c - \ln 1 - \tau'$, ϵ_{it}, χ_i and ψ_t are the residuals and the county and year fixed effects. To implement the regression, we merge the county-level fiscal data with the county-level total retail sales of consumer goods per capita (as the proxy for c) using the China County Statistical Yearbook (CCSY) for 2016-2019. This panel data is an unbalanced panel with 2,619 observations from 2016-2019. The number of observations decreases due to the exclusion of observations with estimated τ' larger than one.

We choose two pairs of variables as proxies for public expenditure and transfer payments in the regression[28]. The first one is GT as τ and total expenditures in the general budget as g, while the other chooses BT as τ and the basic public service expenditures including nine items of expenditures as g. As introduced in Section 6.1, GT is mainly connected with the general budget, therefore the first pair of variables reflect the comprehensive interaction between the central and local governments. Besides, BT is the main part of GT which undertakes the responsibility of inter-regional redistribution and is also based on the formula method. In Section 1, we show that BT mainly focuses on basic public services. Hence the basic public services selected in Section 6.1 should be the corresponding g to BT. The results of the regression are shown in Table 2.

Columns (1) and (3) in Table 2 control the time and county fixed effects, while columns (2) and (4) represents the results of the random effect model. We can find that either the fixed effect model or random effect model shows a coefficient $0 < 1 - \alpha < 1$ for $\ln g$. The coefficients estimated under the fixed effect model are much lower than those under the random effect model. The results of the revised Hausman test recommend using the fixed effect model, while the R^2 also shows the fixed effect model fits the data better.

It could be argued that our static model does not account for inter-temporal investment, and therefore, the estimation may not fully capture residents' economic behavior. In our approach, we treat household savings as consumption in the future, which is then discounted to its present value. We use the indicator "per capita balance of savings deposits for urban and rural residents" in CCSY as a proxy for savings (denoted as K) and treat the

[28]The county-level fiscal data we collected only includes BT for the year 2016-2018, not for 2019.

Table 2: The Estimation of α

VARIABLES	(1) y(GT)	(2) y(GT)	(3) y(BT)	(4) y(BT)	(5) y(GT& K)
ln(g)	0.268***	0.655***			0.235***
	(0.0880)	(0.0615)			(0.0648)
ln(basic)			0.256***	0.589***	
			(0.0569)	(0.0547)	
Observation	2,619	2,619	1,492	1,492	2,276
Number of county	1,185	1,185	803	803	1,043
R-squared	0.308	0.117	0.444	0.170	0.535
Random effect	No	Yes	No	Yes	No
County fixed effect	Yes	No	Yes	No	Yes
Year fixed effect	Yes	No	Yes	No	Yes

Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

sum of this variable and c introduced above as a broad measure of consumption per capita. The regression result is displayed in column (5) in Table 2. The new c fits the data better and provides a coefficient that is close to other estimated results. Finally we assume $\alpha = 0.74 \in [0.732, 0.744]$.

The parameters β and θ are calibrated by fitting the mean of county-level total retail sales of consumer goods per capita in 2016-2019 and per capita GDP. The units of the chosen c, g, and per capita GDP are 10,000 RMB per capita. The GMM estimation method reports that $\beta = 0.1055$ and $\theta = 0.417$.

Finally, in terms of central governments, we let the equity weight $\widetilde{\gamma} = 1$ for all regions, which means that the preference for social welfare is utilitarian. Even under utilitarian preferences, which are less redistributive than reality (as the central government tends to allocate more resources to underdeveloped areas), our numerical results show a lower level of progressivity compared to reality. We ensure that the total transfer payments generated by the simulation, which correspond to per capita transfers in China due to population standardization, match the average GT in China during the simulation process.

6.4 Numerical Optimal Transfer

6.4.1 Optimal Marginal Transfer and Externalities

First, we use the MWY method to obtain the optimal marginal transfer policy τ' with GT in 2019 as τ and total expenditures of the general budget in 2019 as g. We also conduct the simulation using the BT as τ and the basic public service expenditures as g (see Figure D.1)[29]. The results are depicted in Figure 3. Graph (b) in Figure 3 deserves the most attention: the optimal marginal transfer with externalities is the solid blue line. Compared with the actual marginal transfer for China in 2019, the theoretically optimal τ' is much lower. The results suggest that the optimal transfer policy should feature a considerably flatter schedule compared to the current policy in China (see Figure D.6). This implies that subsidies for less-developed regions with low per capita public goods expenditure should be increased, while transfers to regions with high levels of public goods expenditure should be reduced. The results of the numerical simulation offer insights into the advisability of increasing transfers to regions with low public spending in a typical country like China, which grapples with both efficiency and equity issues.

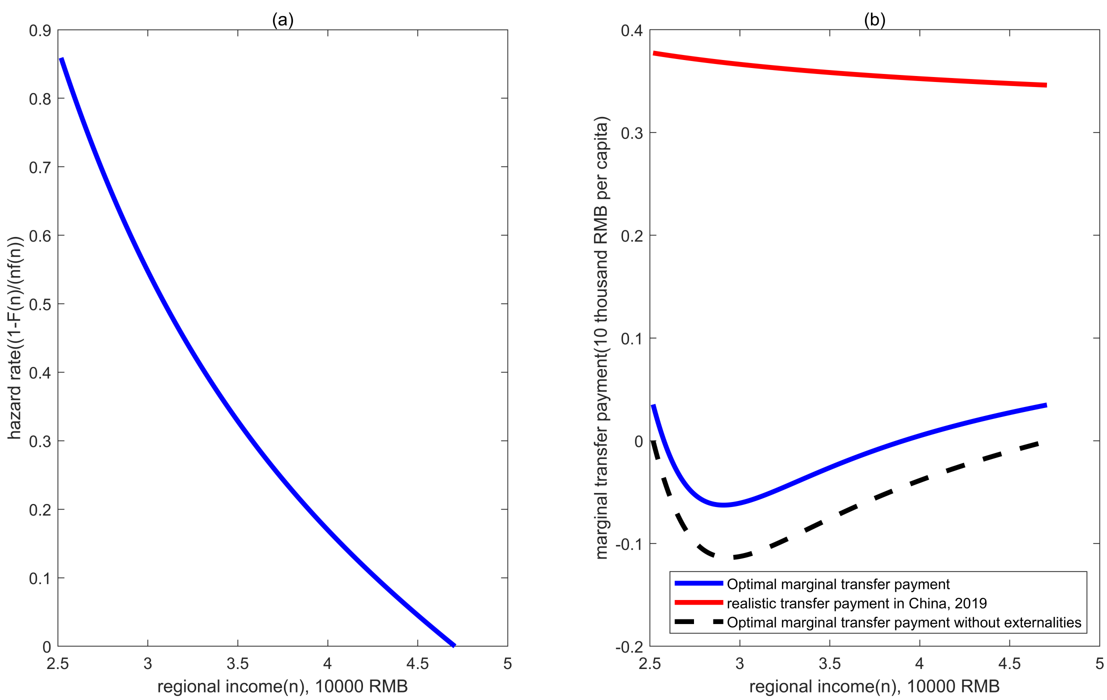

Figure 3: Graph (a): The hazard rate for public goods expenditure in China, 2019. Graph (b): The optimal marginal transfer τ' with and without externalities compared with the actual data from China, 2019.

The results also highlight the significance of externalities in influencing the size and progressivity of the optimal transfer policy, as theorized in the preceding sections. Specifically,

[29] As mentioned before in the footnote, we use the basic public service expenditures and the BT during the period of 2016-2018. The number of observations is 1,810 after including the BT variable.

the optimal transfer policy functions as a Pigouvian subsidy aimed at addressing positive externalities. Without externalities, the marginal transfer is negative. Thus the optimal τ without externalities suggests a completely redistributive plan where the transfer decreases as regional income levels increase. However, when externalities are considered, the transfer scheme becomes non-monotonic. The marginal transfer curve in this case is greater than that without externalities and remains above zero within certain intervals, as clearly illustrated in subgraph (b).

Particularly, externalities encourage higher transfers for regions with the highest and lowest levels of g (indicating the highest and lowest levels of n) at the two ends of the distribution, thereby fostering greater positive externalities for society. As discussed in Section 3.2, the approximation of τ' among the regions with the highest (or lowest) levels of g depends only on the effect of externalities. The Pigouvian subsidies intended to correct these externalities thus overwhelm the redistributive requirements of transfers in these cases, suggesting greater rewards for regions with relatively high levels of public expenditure. Therefore, failing to take into account externalities induced by governmental expenditure may lead to the misguided design of inter-regional transfer payments, overlooking the importance of incentivizing public spending in certain regions.

6.4.2 Determinants of Optimal Transfer

From Graph (a) in Figure 3, we can see that the hazard rate for public goods expenditure follows a U-shaped pattern with a fat tail. The hazard rate has long been considered the primary determinant of the optimal curve's shape. Although Graph (b) in Figure 3 presents a gently U-shaped pattern that is consistent with that for the hazard rate, the shape of marginal transfer is not really solely decided by the shape of $B(n)$. The situation in this model is different because the transfer τ is the opposite of taxation of local government spending. Ignoring the impact of externalities and letting $t(g) = -\tau(g)$, t', which can be denoted as the optimal marginal "tax", is positive and shows an inverted-U shape pattern (imagining inverting the curve of the black dashed curve in Graph (b)). This contradicts the seminal numerical results of optimal tax theory (Saez, 2001)).

As the efficiency term $A(g)$, the hazard rate $B(g)$, and the redistribution term $C(g)$ jointly determine the shape of transfer policies, it would be better to decompose τ', as done in (18), to reveal the most influential factor dominating the transfer policies. The shape of the curve τ' is determined by τ'', and the derivative of the left-hand side of (18) in terms of

g is directly related to τ''[30]. Therefore, taking the logarithm of each side of the equation (18) and neglecting the externality term, it is obvious that if $C(n) > 0$, $A(n) < 0$, $\tau'' \propto (\frac{dlog(-A)}{dg} + \frac{dlog(B)}{dg} + \frac{dlog(C)}{dg})\frac{1}{1-\frac{\phi}{\lambda}}$. If $\frac{\phi}{\lambda} < 1$, then τ'' is positively related to the growth rates for $-A(n)$, $B(n)$, and $C(n)$[31]. Therefore, an examination of the growth rate for each term is the most effective method for distinguishing their influence on the shape of the transfer policy schedule, as illustrated in Figure 4.

Figure 4 presents the different influences of the terms $-A(n)$ (represented by A in the figure), $B(n)$, and $C(n)$ in (18). Graph (a) shows that the absolute value of the elasticity term $-A(n)$, represented by the solid blue line, exhibits a consistent downward trajectory. In contrast, the hazard rate $B(n)$ follows a U-shaped pattern, and $C(n)$ is as large as $B(n)$ in terms of the absolute value. However, Graph (b) also reveals that despite the higher absolute values of $-A(n)$ and $B(n)$, the slope of $C(n)$ surpasses that of both $-A(n)$ and $B(n)$ across most of the entire interval. In Graph (c), we visualize the growth rate of the three terms, and it becomes evident that the growth rate of $C(n)$ has the most significant influence at the left end of τ'' and the U-shaped pattern is mainly dominated by the redistribution effect of transfers for regions with low levels of g.

[30] Externalities do not significantly influence the slope of τ'.

[31] In Appendix B.4 we prove that $\frac{\phi}{\lambda} < 0$.

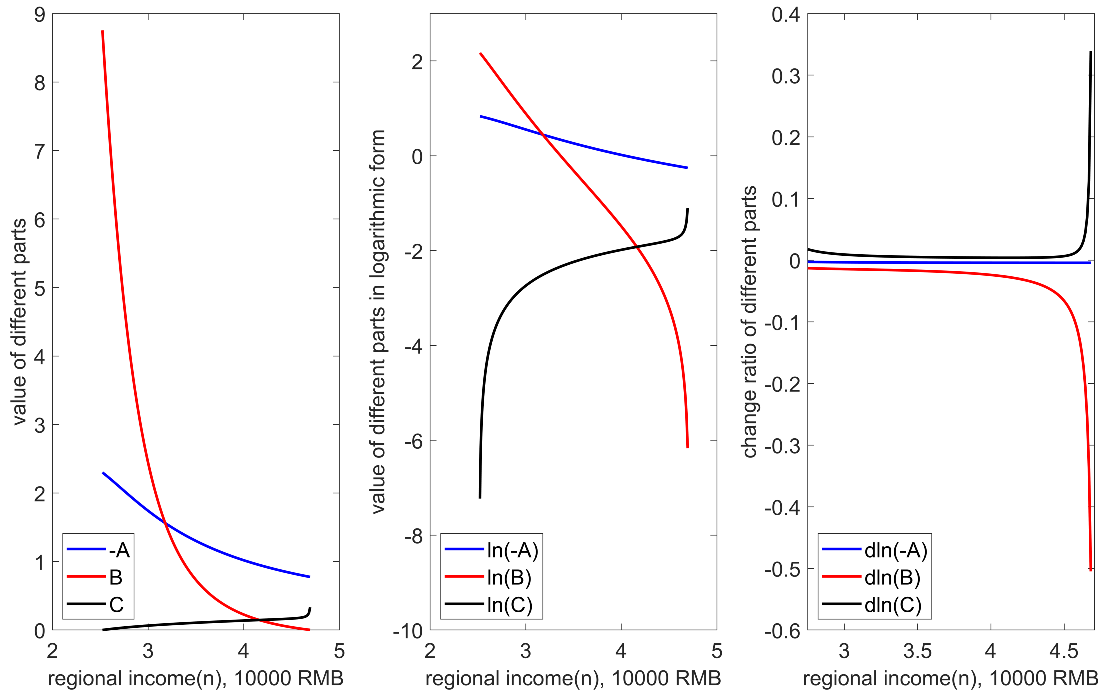

Figure 4: The decomposition of the effects of the efficiency term, distribution term, and equity term on the shape of the transfer policy.

The slope of the redistribution and inter-regional insurance term $C(n)$ is determined by $\frac{1}{v'(g)}$ and the integral lower bound g. With an increasing g, the number of regions influenced by a marginal elementary change in $\tau'(g)$ decreases, which leads to a change in social equity. At the same time, the exponential term $e^{\int_{g'}^{g} \frac{v_{cc}}{v_c}(\tau'-1)dg''} = \frac{v'(g')}{v'(g)}$, representing the ratio of income and Hicksian elasticities, increases due to the concavity of $v(\cdot)$[32]. When g is extremely low, an increase in g primarily influences the scale of $C(n)$ via the substantial behavioral changes across the majority of regions with spending levels above g. When g is extremely high, the impact on the scale of the social equity term is largely driven by the surge in $\frac{1}{1-H(g)}$. These two effects can explain the extremely rapid growth of the redistribution term at the two ends of the g distribution.

We also investigate the impact of the term $\frac{1}{1-H(g)}$ on the optimal τ'. Canceling out the influence of $\frac{1}{1-H(g)}$ in both $B(n)$ and $C(n)$ does not alter the dominance of $C(n)$ in determining the shape of τ' (see Figure D.2). However, without the influence of $\frac{1}{1-H(g)}$, the rapid reduction of regions affected by changes in the marginal transfer at the point g now primarily influence the scale of the redistribution term, and the term $C(1-H(g))$ decreases at the

[32] Note that $\frac{v_{cc}}{v_c}(\tau'-1) = \frac{\widetilde{\eta}_n}{\widetilde{\epsilon}_n^c g}$.

right tail of the distribution.

6.5 Welfare Analysis

Next, we analyze the welfare change due to the transfer policy reform and evaluate the welfare improvement when shifting from the actual transfer system to the optimal system.

In Table 3, we compare the optimal transfer policy τ^* with the current transfer policy in China (denoted as τ_r) and estimate the welfare improvement of transfer payment reform. The welfare gain is measured by the compensating variation (CV):

$$W(\tau^*, c^* - CV) = W(\tau_r, c_0) \tag{55}$$

where $W(\cdot)$ is the indirect social utility function and c^* is the vector of all regional consumption levels under the optimal transfer τ^*. The term CV indicates the extent to which consumption should be deducted evenly in each region to equalize the social welfare values when transitioning from τ_r to τ^*. The second column of Table 3 displays the CV under different parameter combinations. The third column is the ratio of $\frac{CV}{E(c_0)}$, where c_0 is the consumption level generated by the model and $E(c_0)$ is the mean value of c_0.

In the first row of Table 3, the shift from the actual transfer system to the theoretical optimal system generates welfare gain equaling at most 1530 RMB per capita increase of private consumption, accounting for approximately 4.11% of consumption per capita ec_0, which is a considerably large ratio. We also choose various values of the parameter σ and α to see the impact of risk aversion on the degree of welfare improvement. Neither of them shows a significant impact on the level of the results. The inclusion of externalities always amplifies the welfare gain except when σ is lower than the benchmark. This is understandable since the current transfer policy fails to address both the information asymmetry issue and externalities, leading to greater efficiency losses compared to the optimal policy. The welfare improvement equals about 4.02-4.25% of the average consumption in the model.

Table 3: Welfare improvement relative to the current transfer policy schedule

Scenario	Welfare improvement (RMB per capita)	Percentage of $E(c_0)$ (%)
$\alpha = 0.74$, with externalities (benchmark)	1.35×10^3	4.19
$\alpha = 0.74$, without externalities	1.32×10^3	4.10
$\alpha = 0.7$, with externalities	1.35×10^3	4.10
$\alpha = 0.7$, without externalities	1.33×10^3	4.03
$\sigma = 1.75$, with externalities	1.48×10^3	4.04
$\sigma = 1.75$, without externalities	1.53×10^3	4.11
$\sigma = 2.25$, with externalities	1.23×10^3	4.25
$\sigma = 2.25$, without externalities	1.15×10^3	4.02

6.6 Sufficient Statistics

Since the optimal transfer encompasses the elasticities of public spending as sufficient statistics, it is necessary to discuss the practicability of formulating transfer payments using these sufficient statistics. As shown in Figure 4, the elasticity term $A(n)$ ranges from about -2.5 $\sim$ -1 (since the term A in Figure 4 equals $-|A|$). This is estimated via the structural model rather than the empirical method. Nevertheless, it is worth noting that the optimal transfer policy might differ if the central government were to employ empirically estimated sufficient statistics. First, we use the following regression to estimate the price elasticity $\widetilde{\epsilon}_n^c$:

$$\begin{aligned}\ln g_{basic,i,t} =& \beta_1 \ln\left(1-\tau'_{BT,i,t}\right) + \beta_2 \ln \tau_{ST,i,t} + \beta_3 \ln\left(1-\tau'_{BT,i,t-1}\right) + \beta_4 \ln \tau_{ST,i,t-1} \\ &+ \vec{\beta}\vec{X}_{i,t-1} + \chi_i + \psi_t + \epsilon_{i,t},\end{aligned} \tag{56}$$

where $\ln g_{basic,i,t}$ is the basic public service expenditure in a region i for the year t. $(1-\tau'_{BT,i,t})$ is the price of basic public service expenditure in a region i for the year t using BT as the proxy to transfer payment. $\tau_{ST,i,t}$ is the amount of ST which also influences the local public spending. We control the lagged terms of GDP, savings, and consumption per capita (these variables are consistent with those used in Section 6.3) in order to avoid the endogeneity of these macro variables in the same years. Using the fixed effect model, the price elasticity is around -1.4$\sim$-5.9, shown in Table D.1 in Appendix D.2, which almost includes the elasticity interval obtained from the structural model. Considering the estimation bias and the issue of low degrees of freedom in some of our regressions, we choose three different levels of $\widetilde{\epsilon}_n^c$: -1.5, -3, and -4.5 to check the robustness.

Figure D.3 shows that even if the central government uses the elasticity estimated through empirical methods to design the optimal transfer, the marginal transfer still exhibits a U-shaped pattern, which reaffirms that the shape of the optimal policy curve is dominated by the hazard rate term $B(n)$ and redistribution term $C(n)$ rather than the elasticity. Hence using sufficient statistics estimated through empirical approaches in designing transfer policies could yield the transfer scheme close to the optimal one. However, the structural method is still recommended, as the outcomes derived from the empirical method might alter the monotonicity of transfers over certain intervals. When the absolute value of elasticity equals 3 or 4.5, the marginal transfer will become more progressive, and regions with lower levels of public spending will receive relatively fewer transfer funds than the optimal one. Once the central government chooses a lower elasticity, such as -1.5, it results in a decrease in marginal transfers in certain intervals while an increase in others. Besides, some elasticities, such as the elasticities related to externalities, may be challenging to estimate empirically in reality, depending on the specific scenarios of externalities.

7 Discussion and Conclusion

This study investigates the optimal nonlinear transfer policy schedule, focusing on inter-regional equity and efficiency concerning local public spending behavior. These issues are crucial for addressing significant fiscal disparities between regions and the information asymmetry between central and local governments, particularly in developing countries with numerous localities but relatively limited fiscal resources for redistribution, such as China. We develop a principal-agent model to capture the challenges for transfer policymakers in such contexts. This model offers a comprehensive framework for analyzing the impacts of externalities, information asymmetry (including incurred flypaper effects), and inter-regional equity considerations on the formulation of optimal transfer policies.

First, building on the methods of Gerritsen (2023) and Sachs et al. (2020), we explore how local governmental behavioral distortions and variations in externalities impact social welfare as a result of changes in transfer policy. To illustrate the behavioral effects of transfer perturbations, we introduce the elasticities of public spending as sufficient statistics to clarify the interaction between transfers and public expenditures, and numerically exhibit the behavioral results of transfer reforms. We derive the optimal transfer policy and decompose the determinants of the optimal transfer policy into the hazard rate, the elasticities for regional governments corresponding to the flypaper effect, the social welfare weight reflecting the consideration of societal equity, and the total influence of the externalities on social welfare. Externalities are crucial in this context, as they not only induce general equilibrium effects that influence the fiscal behavior of local governments but also ensure that the optimal marginal transfer for regions at both ends of the public spending distribution is no longer zero.

To demonstrate the universality of our framework, we first discuss the general form of externalities under various scenarios. We highlight the impact of migration or competitive strategies on the optimal design of transfers, whose results share similar structures to the optimal transfer rule in the benchmark model. We also explore the optimal transfers when regions differ in both income levels and preferences for public spending. By extending the double perturbation methodology to a multidimensional heterogeneity case with externalities, we derive the optimal marginal transfer, which is determined by the total elasticities and the average levels of variables at the given levels of public spending.

Subsequently, we perform numerical simulations using Chinese fiscal and consumption data spanning the years 2016 to 2019. Our target is to provide quantitative insights into feasible reforms for current policies in typical countries such as China. The optimal marginal transfer policy is much lower than the marginal curve of the actual policy. This indicates the necessity for the Chinese central government to allocate more resources to regions with low public goods expenditures through inter-regional redistribution. Externalities play vital roles in designing optimal transfers as their inclusion inverts the sign of the marginal transfer, revealing the function of transfers as a kind of Pigouvian subsidy. Additionally, the optimal transfer schedule displays a U-shaped pattern, primarily influenced by the growth rate of the inter-regional public redistribution term. In the welfare analysis section, we show that the transition from the current system in China to the optimal one generates welfare gains equalling about 1.15×10^3–1.53×10^3 RMB per capita private consumption, equivalent to approximately 4.02%–4.25% of per capita consumption, representing a substantial improvement.

Appendix A Proof about Transfer Perturbation

A.1 Perturbation of g

Here, we provide more details regarding the variation approach. Considering the perturbation $\kappa\hat{T}$ to the transfer policy schedule $\tau(g)$, first-order condition (7) can be rewritten as

$$u_g(\widetilde{g},\widetilde{e}) + v'(n+\tau(\widetilde{g})+\kappa\hat{T}-\widetilde{g})\cdot(\tau'(\widetilde{g})-1+\kappa\hat{T}'(\widetilde{g}))=0, \tag{A1}$$

where $\widetilde{e}=e+\kappa\hat{e}$.

We can obtain the equation for $\hat{g}$ and $\hat{e}$:

$$\begin{aligned}&\left[u_{gg}+v''(\tau'-1)^2+v'\tau''\right]\hat{g}+\\&u_{ge}\hat{e}+v'\hat{T}'+v''(\tau'-1)\hat{T}=0.\end{aligned} \tag{A2}$$

Here, $\hat{g}$ represents the whole perturbation of the public goods expenditures given the change in the transfer schedule and the perturbation of e which equals the integral of $\hat{g}$ among all regions.

From (A2), we can reorganize the equation to obtain the expression for $\hat{g}_1$:

$$\hat{g}=-\hat{e}\left(\frac{u_{ge}}{u_{gg}+v''(\tau'-1)^2+v'\tau''}\right)-\frac{v'\hat{T}'+v''(\tau'-1)\hat{T}}{u_{gg}+v''(\tau'-1)^2+v'\tau''}. \tag{A3}$$

A.2 Welfare Analysis

Next, we provide the proof of Corollary 1. We define the welfare of a region after the reform of the schedule as:

$$\widetilde{U}\triangleq u(\widetilde{g},\widetilde{e})+v(n+\tau(\widetilde{g})+\kappa\hat{T}-\widetilde{g}). \tag{A4}$$

Similar to the definition of $\hat{g}$, we define as follows: $\hat{U}=\lim_{\kappa\to 0}\frac{\widetilde{U}-U}{\kappa}$. We combine the definition of U from (1) with the first-order condition (7) and use the Taylor expansion of (B5) to derive the marginal effect on welfare $\hat{U}$ in (14).

Regarding the impact of the reform on central government revenue, total government revenue after the reform of the transfer policy can be shown as follows:

$$\hat{R}=\int_{\underline{n}}^{\overline{n}}(\tau(\widetilde{g})+\kappa\hat{T})f(n)dn. \tag{A5}$$

Equation (16) is based on the Taylor expansion of (A5) at the point $\mu = 0$.

A.3 Proof of Proposition 2

To show the properties of and clearly express the optimal marginal transfer policy, it is harmless to specify the form of the reform $\hat{T}$. Following Saez (2001) and Sachs et al. (2020), we can assume an "elementary" reform among regions whose public goods expenditures are beyond g^* as follows[33]:

$$\hat{T}(g) = (1 - H(g^*))^{-1}\mathbb{I}_{\{g \geq g^*\}}, \tag{A6}$$

where $H(g^*)$ is the cumulative distribution function for the public goods level g^*. The marginal reform of the transfer policy schedule is a Dirac function:

$$\hat{T}' = (1 - H(g))^{-1}\delta(g, g^*). \tag{A7}$$

$\delta(g, g*)$ is the Dirac function, which is infinite for $g = g^*$ and zero for any $g \neq g^*$. This special function contains a good property to derive the optimal transfer policy:

Lemma 2 *For a well-behaved (continuous and with compact support) function $\Omega(\cdot)$,*

$$\Omega(g^*) = \int_{\underline{g}}^{\overline{g}} \delta(g', g^*)\Omega(g')dg'$$

always holds.

The proof of this lemma can be found in the appendix to Sachs et al. (2020).

Next, we should transfer each PDF for the integrals in (17), $f(n)dn$, into the PDF for g, which is $h(g)dg$. Considering that it is impossible for the central government to observe the true income levels of the regional governments in our setting, the real workable weights for the central government are those derived according to the provision of public goods in each region, namely, $\widetilde{\gamma}(g)$. The Assumption 1 implies $f(n)dn = h(g)dg$. Then, (15) can be rewritten into the integral in g. The perturbation of social welfare (17) as a function of g is

[33] Some studies may assume that $g(\kappa, \tau; n)$ is Lipschitz continuous, which would require $\hat{g}(\hat{T}, \tau; n)$ to be finite. Gomes et al. (2017) and Sachs et al. (2020) show that the Dirac function can be approached by any continuous derivable function.

given by:

$$\hat{W} = \int_{\underline{g}}^{\overline{g}} \widetilde{\gamma}(g')u_e(g')\hat{e}h(g')dg' + \int_{g^*}^{\overline{g}} \frac{\widetilde{\gamma}(g')v'(g')}{1-H(g^*)}h(g')dg' + \lambda\left(1 + \int_{\underline{g}}^{\overline{g}} \tau'(g')\hat{g}'h(g')dg'\right). \tag{A8}$$

The optimal transfer policy requires that $\hat{W} = 0$ around $\tau(\cdot)$ for an arbitrary perturbation $\kappa\hat{T}$ when $\kappa \to 0$. Therefore, let $\hat{W} = 0$ in (A8), and this equation is an integral equation with $\hat{g}$ and $\hat{e}$. The optimal transfer policy τ is contained in $\hat{g}$ and $\hat{e}$ so that (A8) is an implicit function for τ. To obtain a clear form of τ, one can solve the Fredholm integral equation (12) and replace $\hat{e}$ in (A8) with τ, the elasticities, and the distribution function of g, which is is given by:

$$\begin{aligned}\hat{e} &= \int_{\underline{g}}^{\overline{g}} \hat{g}h(g)dg \\ &= \left[\widetilde{\epsilon}_n^c(g^*)\frac{g^*h(g^*)}{\tau'(g^*)-1} + \int_{g^*} \widetilde{\eta}_n \frac{h(g')}{\tau'(g')-1}dg'\right]\left(\frac{1}{1-\int_{\underline{g}}^{\overline{g}} \widetilde{\epsilon}^{g,e}\frac{g'}{e}h(g')dg'}\right).\end{aligned} \tag{A9}$$

From (A9), the perturbation of the externalities includes two components: the first component is how much g changes due to the transfer perturbation, which is captured by the Hicksian and income elasticities. The second component, captured by the term $\widetilde{\epsilon}^{g,e}$, is the overall result of the mutual influence between g and e.

Replacing (A9) into (A8) and utilizing the properties of Dirac δ function, we can easily derive the expression of (18).

Appendix B Mirrlees Approach

B.1 Basic Setting for the Mirrlees Approach

The Mirrlees method requires that the transfer policy curve and the public goods contracts meet the Mirrlees-Spence condition. Suppose that both the assumptions about the properties of the utility function and Assumption 2 are satisfied; we can deduce that[34]:

$$MRS_g = -\frac{u_{gg}}{v_c} > 0. \tag{B1}$$

[34]We use equation MRS_g to denote the partial derivative of the MRS for the public goods rather than the full derivative $\frac{dMRS}{dg}$. This notation is used from this point forward in this paper.

$MRS_g > 0$ gives us a foundation on which to apply the method of mechanism design in our analysis. Before delving into the analysis in detail, we should give a proof for the satisfaction of this condition in our setting by using (B1).

Lemma 3 *When the assumptions about the utility function's basic properties (concavity) and Assumptions 1 and 2 are satisfied, and if the resident is risk-averse, then the Mirrlees–Spence condition holds.*

Proof. Using (5) under Assumption 1 and 2, we can induce that

$$-MRS(n) = \frac{d\tau}{dg} - 1.$$

Then, we can derive $\frac{d\tau}{dg}$ with respect to income n and simplify the equation. The envelope theorem implies the following:

$$\left.\frac{d\frac{d\tau}{dg}}{dn}\right|_{\text{constant } U} = \frac{u_g.v_{cc}}{v_c^2}. \tag{B2}$$

If the resident is risk averse, then $v_{cc} \leq 0$, which implies that (9) is nonpositive. Therefore, the Mirrlees–Spence condition holds. ■

Now, we should give the expression for IC. When the Mirrlees–Spence condition is met under the normal assumptions regarding the properties of utility, the derivative of U with respect to n is:

$$U'(n) = u_g \cdot g'(n) + v_c \cdot [(\tau' - 1)g'(n) + 1]. \tag{B3}$$

(B3) represents how the change in a region's type influences the utility of that region if it does not hide its real economic income level. This condition coincides with the first-order condition (5) since this first-order condition is the necessary condition for the optimal choice for region n. Therefore, using (B3) and (5), the necessary condition for IC is as follows:

$$\text{IC} : U'(n) = v_c = U_n. \tag{B4}$$

(B4) is the direct result of the envelope theorem and the so-called first-order truth-telling constraint. When $\frac{\widetilde{\eta}_n}{\tau' - 1} \geq 0$, equation (B3) becomes a necessary and sufficient condition. Moreover, the RC is:

$$\text{RC} : \int_{\underline{n}}^{\overline{n}} \tau(n) f(n) dn = 0, \tag{B5}$$

where $\overline{n}$ and $\underline{n}$ are the upper and lower bounds of the regional income range, respectively.

Now, we present the optimization problem for the central government below. The central government chooses the public goods allocation $\{g\}$, the level of the utility function U and the externalities e to maximize social welfare for all regions[35]:

$$\max_{\{g(n),e,U(n)\}} \int_{\underline{n}}^{\overline{n}} \gamma(n)U(n)f(n)dn, \tag{B6}$$

subject to the IC constraint and RC in (B4) and (B5) :

$$U'(n) = v_c,$$

$$\int_{\underline{n}}^{\overline{n}} \tau(n)f(n)dn = 0,$$

and the definition equation for e as the weighted average of public spending over all regions in (13)

$$\int_{\underline{n}}^{\overline{n}} g(n)f(n)dn = e,$$

combined with the budget constraint for the representative residents:

$$c(n) = \tau(n) - g(n) + n, \tag{B7}$$

where $\gamma(\cdot)$ is the welfare weight placed by the central government on a region exogenously.

Then we can deduce the optimal marginal transfer:

Proposition 6 *The optimal transfer policy among regions, subject to the IC and RC constraints, can be expressed as follows:*

$$\frac{\tau'(g) + \frac{\phi}{\lambda}}{\tau'(g) - 1} = \frac{1}{\widetilde{\epsilon}_n^c} \cdot \frac{1 - H(g)}{gh(g)} \cdot \int_g^{\overline{g}} \frac{h(g')}{1 - H(g)} (\beta(g') - 1) e^{\int_{g'}^{g} \frac{v_{cc}}{v_c}(\tau' - 1)dg''} dg', \tag{B8}$$

where $\frac{\phi}{\lambda} = -\int_{\underline{n}}^{\overline{n}} \frac{u_e}{v_c} f(n')dn' - \int_{\underline{n}}^{\overline{n}} u_e \frac{v_{cc}}{v_c} dn' \int_{n'}^{\overline{n}} (1 - \beta(m)) f(m) \frac{1}{v'(g(m))} dm$ *is the marginal social value of the effect on social welfare,* $H(g)$ *is the CDF of regional consumption of public goods, and* $h(g)$ *is the corresponding PDF.* $\beta(g) = -\frac{\widetilde{\gamma}(g)v_c(g)}{\lambda} \geq 0$ *is the social marginal value of a resident possessing public goods* g*. The term* $\widetilde{\gamma}(g) = \gamma(n(g))$ *is the welfare weight on the region with public goods provision* g*.*

[35] g, e and U can determine c, given the utility function.

B.2 Proof of Proposition 6 and Lemma 1

To solve (B6), we construct the following Lagrange function:

$$L = \int_{\underline{n}}^{\overline{n}} \gamma(n')U(n')f(n')dn' + \phi(\int_{\underline{n}}^{\overline{n}} g(n')f(n')dn' - e) + \lambda \int_{\underline{n}}^{\overline{n}} (c(n') + g(n') - n')f(n')dn' - \int_{\underline{n}}^{\overline{n}} \mu(n')(U_n(n') - v_c(c(n'))dn', \tag{B6$'$}$$

where τ is replaced by the combination of g, c and n. Alternatively, we can use the properties of the conjugate variables $\mu(n)$ and integration by parts:

$$L = \int_{\underline{n}}^{\overline{n}} \gamma(n')U(n')f(n')dn' + \phi(\int_{\underline{n}}^{\overline{n}} g(n')f(n')dn' - e) + \lambda \int_{\underline{n}}^{\overline{n}} (c(n') + g(n') - n')f(n')dn' + \int_{\underline{n}}^{\overline{n}} \mu(n')v_c(c(n')) + \dot{\mu}(n')U(n')dn'.$$

By totally differentiating the utility function U, we obtain:

$$\frac{dc}{dU} = \frac{1}{v_c}, \ \frac{dc}{dg} = -\frac{u_g}{v_c}, \ \frac{dc}{de} = -\frac{u_e}{v_c}. \tag{B9}$$

The state variable is $U(n)$, while the public goods allocation $g(n)$ and the sum of public goods expenditure e can be set as control variables in the problem[36]. Using (B9), the FOC for (B6$'$) can be derived as:

$$L_U = -\dot{\mu} = \gamma(n)f(n) + \mu(n)\frac{v_{cc}}{v_c} + \frac{\lambda f(n)}{v_c}, \tag{B10}$$

$$L_e = -\phi + \int_{\underline{n}}^{\overline{n}} \mu(n')v_{cc} \cdot (-\frac{u_e}{v_c})dn' - \int_{\underline{n}}^{\overline{n}} \frac{\lambda u_e f(n')}{v_c}dn' = 0, \tag{B11}$$

$$L_g = \phi f(n) + \mu(n)v_{cc} \cdot (-\frac{u_g}{v_c}) + \lambda(1 - \frac{u_g}{v_c})f(n) = 0. \tag{B12}$$

[36] This leaves room for a discussion for the treatment of e. Some may argue that e should not be a control variable, as it does not change with n. However, if we treat it as a function of the distribution of g, the same conclusions can be reached. We can think of the variation in $e(n)$ in the Lagrangian problem as a perturbation to the distribution of g, and we can define this perturbation as a Dirac function at the point n^*, which means that $de = \int \hat{g}h(g)dg$ and $\hat{g} = \delta(n, n^*)$. $\delta(n, n^*)$ is the Dirac function that is equivalent to zero except at point n. Choosing g as the only control variable in the Lagrange problem and using the variation method results in the perturbation of e for L being the same as L_e, the original first-order condition.

From (B10), note that $\mu(\overline{n}) = \mu(\underline{n}) = 0$; we can deduce that:

$$\frac{\mu v_c(n)}{\lambda} = \int_n^{\overline{n}} f(n')\left(\frac{v_c(n)}{v_c(n')} + \frac{\gamma(n')v_c(n)}{\lambda}\right)e^{\int_n^{n'} \frac{v_{cc}}{v_c} dm} dn'. \tag{B13}$$

By letting $n = \underline{n}$ in (B13), and using $\mu(\underline{n}) = 0$, we can deduce Lemma 1.

Let $\beta(n) = -\frac{\gamma(n)v_c(n)}{\lambda}$, and combine (5), (B7) to rewrite (A5) as:

$$\frac{\mu v_c(n)}{\lambda} = \int_n^{\overline{n}} f(n')(1 - \beta(n'))e^{\int_{g(n')}^{g(n)} \frac{v_{cc}}{v_c}(\tau' - 1)dg} dn'. \tag{B14}$$

From (B11):

$$\begin{aligned}\frac{\phi}{\lambda} &= -\int_{\underline{n}}^{\overline{n}} \frac{u_e}{v_c} f(n')dn' - \int_{\underline{n}}^{\overline{n}} \frac{\mu(n')}{\lambda} v_{cc} \frac{u_e}{v_c} dn' \\ &= -\int_{\underline{n}}^{\overline{n}} \frac{u_e}{v_c} f(n')dn' - \int_{\underline{n}}^{\overline{n}} u_e \frac{v_{cc}}{v_c} \int_{n'}^{\overline{n}} (1 - \beta(m))f(m)\frac{1}{v'(g(m))} dm dn'.\end{aligned} \tag{B15}$$

(B14) represents the externalities of public goods. We also have:

Corollary 3 *If the utility functions of local governments satisfy the setup in Section 2.2, then:*

$$\int_{\underline{n}}^{\overline{n}} u_e \frac{v_{cc}}{v_c} dn' \int_{n'}^{\overline{n}} (1 - \beta(m))f(m)\frac{1}{v'(g(m))} dm = 0.$$

From (B12):

$$\phi f(n) + \mu(n) MRS \cdot v_{cc} + \lambda(MRS + 1)f(n) = 0. \tag{B16}$$

Recall the definition of MRS from (5), which gives:

$$MRS_c = -MRS \cdot \frac{v_{cc}}{v_c}. \tag{B17}$$

Combining (B15) and (B16) yields:

$$\frac{\mu(n)v_c}{f(n)\lambda} \cdot \frac{MRS_c}{MRS} = \frac{MRS + 1 + \frac{\phi}{\lambda}}{MRS}. \tag{B18}$$

Then, we can use (5) and (10) to derive the relationship between the marginal transfer policy and the elasticity of compensated demand as presented below:

$$\frac{\tau'(g)+\frac{\phi}{\lambda}}{\tau'(g)-1} = -\frac{\mu(n)v_c}{\lambda}\cdot\frac{1}{\widetilde{\epsilon}_n^c(g)gh(g)}. \tag{B19}$$

Note that $F(n)$, the CDF of n, can be converted into the CDF of g since the IC condition guarantees that the number of public goods increases along with the income level n, which means that:

$$F(n) = F(n^{-1}(g)) = H(g). \tag{B20}$$

We differentiate (B20) with respect to n so that:

$$f(n) = h(g)g'(n). \tag{B21}$$

Then, we can derive (B18) using the PDF of g, which is exactly what we have done. Finally, combining (B18) and (B13) and substituting $h(g)$ for $f(n)$ in (B13), we can obtain the optimal transfer policy equation:

$$\frac{\tau'(g)+\frac{\phi}{\lambda}}{\tau'(g)-1} = A(n)B(n)C(n), \tag{B22}$$

$A(n)B(n)C(n)$ is the same as that shown in Section 2. Note that $\frac{\phi}{\lambda}$ is the integral of $\frac{\mu}{\lambda}$. Part of the integration function in $\frac{\phi}{\lambda}$ can be transformed from a function in terms of n into a function in terms of g by using the following equation:

$$\frac{v_{cc}/v_c}{g'(n)} = -\frac{MRS_c}{MRS}\cdot\frac{dg}{dn} = \frac{1}{\widetilde{\epsilon}_n^c g}, \tag{B23}$$

which can be obtained by using (B17) and (5) simultaneously.

B.3 Proof of Corollary 3

To prove Corollary 3, note that the state variable U under the Mirrlees approach acts similarly to e because they do not directly change resource allocation, but indirectly change private consumption c and affect IC and budget constraints. Multiplying u_e on both sides of the (B10) equation, it yields:

$$-\dot{\mu}u_e = \gamma u_e f(n) + \mu(n)\frac{v_{cc}}{v_c}u_e + \frac{\lambda u_e}{v_c}f(n). \tag{B24}$$

Since the equation above holds for all different income levels n, it can be integrated over both sides :

$$-\int_{\underline{n}}^{\overline{n}} \dot{\mu} u_e dn = \int_{\underline{n}}^{\overline{n}} \gamma u_e f(n) dn + \int_{\underline{n}}^{\overline{n}} \mu(n) \frac{v_{cc}}{v_c} u_e + \int_{\underline{n}}^{\overline{n}} \frac{\lambda u_e}{v_c} f(n) dn. \tag{B25}$$

Notice that $\mu(\overline{n}) = \mu(\underline{n}) = 0$, we have:

$$\mu/\lambda u_e \Big|_{\underline{n}}^{\overline{n}} + \int_{\underline{n}}^{\overline{n}} \frac{\mu}{\lambda} \frac{du_e}{dn} dn = \int_{\underline{n}}^{\overline{n}} \frac{\gamma}{\lambda} u_e f(n) dn + \int_{\underline{n}}^{\overline{n}} \frac{\mu}{\lambda} \frac{v_{cc}}{v_c} u_e dn + \int_{\underline{n}}^{\overline{n}} \frac{u_e}{v_c} f(n) dn. \tag{B26}$$

Since $\frac{du_e}{dn} = u_{ge} \frac{dg}{dn}$, it leads to:

$$\int_{\underline{n}}^{\overline{n}} \frac{\mu}{\lambda} u_{ge} \frac{dg}{dn} dn = \int_{\underline{n}}^{\overline{n}} \frac{\gamma}{\lambda} u_e f(n) dn + \int_{\underline{n}}^{\overline{n}} \frac{\mu}{\lambda} \frac{v_{cc}}{v_c} u_e dn + \int_{\underline{n}}^{\overline{n}} \frac{u_e}{v_c} f(n) dn. \tag{B27}$$

This equation implies:

$$\frac{\mu}{\lambda} u_e \Big|_{\underline{g}}^{\overline{g}} - \int_{\underline{g}}^{\overline{g}} \frac{u_e}{\lambda} \frac{d\mu}{dg} dg = \int_{\underline{g}}^{\overline{g}} (\frac{\widetilde{\gamma}(g)}{\lambda} + \frac{1}{v_c}) u_e h(g) dg + \int_{\underline{n}}^{\overline{n}} \frac{\mu}{\lambda} \frac{v_{cc}}{v_c} u_e dn. \tag{B28}$$

Notice that in (B15), $e^{\int_{g(n')}^{g(n)} \frac{v_{cc}}{v_c}(\tau'-1)dg} = e^{\int_{g(n')}^{g(n)} d\ln(v_c(g))dg} = \frac{v_c(g(n'))}{v_c(g(n))}$, therefore:

$$\frac{\mu'(g)}{\lambda} = -(\frac{\widetilde{\gamma}(g)}{\lambda} + \frac{1}{v_c}) h(g) dg. \tag{B29}$$

Substitute the equation (B29) into (B27), and it exactly means:

$$\int_{\underline{n}}^{\overline{n}} \frac{\mu}{\lambda} \frac{v_{cc}}{v_c} u_e dn = 0. \tag{B30}$$

B.4 Equivalence between Variation and Mirrlees Approaches

Here we prove the equivalence between the optimal policies derived by the variation and Mirrlees approaches. Since the other parts of (18) and (B8) are the same, what remains is to prove that

$$\frac{\phi}{\lambda} = \frac{\int_{\underline{g}}^{\overline{g}} \left(\frac{\widetilde{\gamma}}{\lambda} u_e + \widetilde{\epsilon}^{g,e} \frac{g'}{e} \tau' \right) h(g') dg'}{1 - \int_{\underline{g}}^{\overline{g}} \widetilde{\epsilon}^{g,e} \frac{g'}{e} h(g') dg'}.$$

Noticing that the marginal transfer τ' is present in the numerator on the left-hand side of (18), we assume that $\frac{\tau'+X}{\tau'-1} =$ *Right-Hand Side of (18)*, where X equals $\frac{\int_{\underline{g}}^{\overline{g}}\left(\frac{\widetilde{\gamma}}{\lambda}u_e+\widetilde{\epsilon}^{g,e}\frac{g'}{e}\tau'\right)h(g')dg'}{1-\int_{\underline{g}}^{\overline{g}}\widetilde{\epsilon}^{g,e}\frac{g'}{e}h(g')dg'}$.
Combining this with the definition of $\widetilde{\epsilon}^c_n$, we have:

$$X\left(1-\int_{\underline{g}}^{\overline{g}}(\widetilde{\epsilon}^{g,e}\frac{g'}{e})h(g')dg'\right)=\int_{\underline{g}}^{\overline{g}}-\frac{\widetilde{\gamma}}{\lambda}u_e h(g)dg+$$
$$\int_{\underline{g}}^{\overline{g}}\left(\widetilde{\epsilon}^{g,e}\frac{g'}{e}h(g)\right)\left(-X-(u_{gg}+v_{cc}(\tau'-1)^2+v_c\tau'')(h(g))^{-1}\int_{g'}^{\overline{g}}(-\frac{\widetilde{\gamma}}{\lambda}-\frac{1}{v_c(g')})h(g')dg'\right)h(g)dg. \tag{B31}$$

Since X is a constant, we use the definition of $\tilde{\epsilon}^{g,e}$ to obtain:

$$X=\int_{\underline{g}}^{\overline{g}}\frac{\widetilde{\gamma}}{\lambda}u_e h(g)dg+\int_{\underline{g}}^{\overline{g}}u_{ge}\int_{g'}^{\overline{g}}(-\frac{\widetilde{\gamma}}{\lambda}-\frac{1}{v_c(g')})h(g')dg'dg \tag{B32}$$

By changing the order of integration for the second and third terms on the right-hand side, we can exactly obtain:

$$\begin{aligned} X &\triangleq \frac{\int_{\underline{g}}^{\overline{g}}\left(\frac{\widetilde{\gamma}}{\lambda}u_e+(\widetilde{\epsilon}^{g,e}\frac{g'}{e})\tau'\right)h(g')dg'}{1-\int_{\underline{g}}^{\overline{g}}(\widetilde{\epsilon}^{g,e}\frac{g'}{e})h(g')dg'} \\ &= -\int_{\underline{n}}^{\overline{n}}\frac{u_e}{v_c}f(n')dn'. \end{aligned} \tag{B33}$$

Hence, the equivalence is proven. Besides, since $u_e>0$ and $v_c>0$, therefore $\frac{\phi}{\lambda}<0$. Then $D(n)(\tau'-1)=-\frac{\phi}{\lambda}>0$, Proposition 3 is proved.

Appendix C Optimal transfer in Extension

C.1 Proof of Proposition 4

For the sake of computational simplicity, we denote

$$\Psi_i \triangleq \frac{(k+1)v_i^k(\sum_{i=1}^N v_i^k)-kv_i^{2k}}{(\sum_{i=1}^N v_i^k)^2}=f_i+v_i\frac{\partial f_i}{\partial v_i}.$$

The first-order conditions for the problem (35) are[37]:

$$\text{For} \quad Y_i\text{:} \quad \omega_i \Psi_i v_{Y_i} - \lambda(1+\frac{1}{\alpha n_i}) + \mu_i u_{Yi}(Y_i, n_i) - \mu_{i+1}\left(u_Y(Y_i, n_{i+1}) - \psi_c(\widetilde{C}_i)(\frac{1}{\alpha n_i} - \frac{1}{\alpha n_{i+1}})\right) = 0, \tag{D1}$$

$$\text{For} \quad C_i\text{:} \quad \omega_i \Psi_i v_{Ci} + \lambda + \mu_i \psi_{Ci} - \mu_{i+1}\psi_C(\widetilde{C}_i) = 0. \tag{D2}$$

From (D2) we have:

$$\omega_i \Psi_i = \mu_{i+1}\frac{\psi_C(\widetilde{C}_i)}{v_{Ci}} - \mu_i \frac{\psi_{Ci}}{v_{Ci}} - \lambda\frac{1}{v_{Ci}}. \tag{D3}$$

Summing up the equation above and denoting $\psi_C(\widetilde{C}_0)/\psi_{C1} = 1$, use $\mu_{N+1} = \mu_1 = 0$ and we derive λ:

$$\lambda = -\frac{\sum_{i=1}^N \omega_i \Psi_i v_{Ci} \prod_{s=0}^{i-1} \frac{\psi_C(\widetilde{C}_s)}{\psi_{C,s+1}}}{\sum_{i=1}^N \prod_{s=0}^{i-1} \frac{\psi_C(\widetilde{C}_s)}{\psi_{C,s+1}}}. \tag{D4}$$

We can also rewrite (D2) as:

$$\frac{\psi_C(\widetilde{C}_i)}{\psi_{Ci}}\mu_{i+1} = \mu_i + \Psi_i \omega_i \frac{v_{Ci}}{\psi_{Ci}} + \lambda\frac{1}{\psi_{Ci}}, \tag{D5}$$

which equals:

$$\mu_i = -\sum_{s=i}^{N}\left(\Psi_s \omega_s \frac{v_{Cs}}{\psi_{Cs}} + \lambda\frac{1}{\psi_{Cs}}\right)\prod_{j=i}^{s-1}\frac{\psi_C(\widetilde{C}_j)}{\psi_{Cj}}, \tag{D6}$$

and when $s-1 < i$, we let $\prod_{j=i}^{s-1}\frac{\psi_C(\widetilde{C}_j)}{\psi_{Cj}} = 1$.

Now we turn to a decentralized economy, where the local government in one region i solve the following optimization problem:

$$\begin{aligned} &\max_{y_i, c_i} u((1-\alpha)g_i f_i) + \psi(c_i f_i) \\ \text{s.t.} \quad & y_i f_i + g_i f_i = T(Y_i) + c_i f_i. \end{aligned} \tag{D7}$$

The first-order conditions with respect of y_i and c_i are:

$$u_{Yi} f_i (1+\varepsilon_{y,i}^f) + \psi_{Ci}\frac{\partial f_i}{\partial y_i}c_i + \gamma\left((1+\frac{1}{\alpha n_i})f_i(1+\varepsilon_{y,i}^f) - T_i' f_i(1+\varepsilon_{y,i}^f) - c_i\frac{\partial f_i}{\partial y_i}\right) = 0, \tag{D8}$$

$$u_{Yi} y_i \frac{\partial f_i}{\partial c_i} + \psi_{Ci} f_i(1+\varepsilon_{ci}^f) + \gamma\left((1+\frac{1}{\alpha n_i})(y_i\frac{\partial f_i}{\partial c_i}) - T_i' \cdot y_i\frac{\partial f_i}{\partial c_i} - f_i(1+\varepsilon_c^f(f_i, c_i))\right) = 0, \tag{D9}$$

[37] Here we use u, ψ to represent the utility of local governments $u(Y_i, n_i) + \psi(C_i)$ and v to represent the utility of local residents $v(y_i, c_i)$.

where γ is the Lagrange multiplier for the budget constraint, while $\varepsilon^f_y(f_i, y_i) = \frac{\partial f_i}{\partial y_i}\frac{y_i}{f_i}$ and $\varepsilon^f_c(f_i, c_i) = \frac{\partial f_i}{\partial c_i}\frac{c_i}{f_i}$ are the elasticities of population with respect to per capita output and per capita private consumption. Combining the two equations above we obtain:

$$T'_i = \frac{u_{Yi}}{\psi_{Ci}} + 1 + \frac{1}{\alpha n_i}. \tag{D10}$$

Back to (D1) and (D2), multiply (D2) by $\frac{v_{Yi}}{v_{Ci}}$ and subtract it from (D1):

$$\omega_i \Psi_i \left(v_{Yi} - v_{Ci}\frac{u_{Yi}}{\psi_{Ci}} \right) - \lambda \left(1 + \frac{1}{\alpha n_i} + \frac{u_{Yi}}{\psi_{Ci}} \right) = \mu_{i+1} \left(u_Y(Y_i, n_{i+1}) - \psi_C(\widetilde{C}_i)(\frac{u_{Yi}}{\psi_{Ci}} + \frac{n_{i+1} - n_i}{\alpha n_i n_{i+1}}) \right). \tag{D11}$$

We further use the setup $U^L_i = (\frac{1-\alpha}{\alpha}\frac{Y_i}{n_i})^{\sigma+1}\frac{1}{\sigma+1} + \log(C_i)$ and $v_i = (\frac{1-\alpha}{\alpha}\frac{Y_i}{n_i f_i})^{\sigma+1}\frac{1}{\sigma+1} + \log(C_i/f_i)$. Therefore (D11) equals

$$1 + \frac{1}{\alpha n_i} + \frac{u_{Yi}}{\psi_{Ci}} = \omega_i \Psi_i \left(v_{Yi} - v_{Ci}\frac{u_{Yi}}{\psi_{Ci}} \right)/\lambda - \mu_{i+1}(\frac{1-\alpha}{\alpha})^{\sigma+1} \cdot \left(\frac{Y_i^\sigma}{n_{i+1}^{\sigma+1}} - \frac{Y_i^\sigma}{n_i^{\sigma+1}}\frac{C_i}{\widetilde{C}_i} - \frac{1}{\widetilde{C}_i}\frac{n_{i+1} - n_i}{\alpha n_i n_{i+1}} \right)/\lambda. \tag{D12}$$

Substituting (D12) into (D10) we have:

$$\frac{T'_i}{(\frac{1}{\alpha n_i} + 1) - T'_i} = \frac{\mu_{i+1}\left((\frac{n_i}{n_{i+1}})^{\sigma+1} - \frac{C_i}{\widetilde{C}_i} - \frac{1}{\widetilde{C}_i u_{Y_i}}\frac{n_{i+1}-n_i}{\alpha n_i n_{i+1}} \right)/\lambda}{C_i} + \omega_i \Psi_i (v_{Yi}\frac{\psi_{Ci}}{v_{Yi}} - v_{Ci})/\lambda, \tag{D13}$$

which leads to the classical Diamond-Saez formula of the optimal marginal transfer as:

$$\begin{aligned} \frac{T'_i}{(\frac{1}{\alpha n_i} + 1) - T'_i} = &- \left((\frac{n_i}{n_{i+1}})^{\sigma+1} - \frac{C_i}{\widetilde{C}_i} - \frac{1}{\widetilde{C}_i u_{Yi}}\frac{n_{i+1} - n_i}{\alpha n_i n_{i+1}} \right) \frac{1}{C_i \lambda} \sum_{s=i+1}^{N} \left(\Psi_s \omega_s \frac{v_{Cs}}{\psi_{Cs}} + \lambda \frac{1}{\psi_{Ci}} \right) \prod_{j=i}^{s-1} \frac{\psi_C(\widetilde{C}_j)}{\psi_{Cj}} \\ &+ \omega_i \Psi_i (v_{Yi}\frac{\psi_{Ci}}{v_{Yi}} - v_{Ci})/\lambda, \end{aligned} \tag{D14}$$

Compared with the classical Diamond-Saez form of optimal taxation, the distinctions emerge in four aspects: the ratio $\frac{n_{i+1}}{n_i}$, the region-selection effect represented by Ψ_i, the shadow price of budget constraint λ, and the correction term $\omega_i \Psi_i (v_{Yi}\frac{\psi_{Ci}}{v_{Yi}} - v_{Ci})/\lambda$ of local governmental utilities by transfers due to the difference between the central and local governments' target.

Eliminating the mobile factors, the first-order conditions in (D1) and (D2) can be rewritten as:

$$\omega_i f_i v_{yi}\frac{1-\alpha}{\alpha n_i f_i} - \lambda\left(1+\frac{1}{\alpha n_i}\right) + \mu_i u_Y(Y_i, n_i) - \mu_{i+1}\left(u_Y(Y_i, n_{i+1}) - \psi_c(\widetilde{C}_i)\left(\frac{1}{\alpha n_i} - \frac{1}{\alpha n_{i+1}}\right)\right) = 0 \tag{D15}$$

$$\omega_i f_i v_{ci}\frac{1}{f_i} + \lambda + \mu_i \psi_{Ci} - \mu_{i+1}\psi_C(\widetilde{C}_i) = 0 \tag{D16}$$

the optimal marginal transfer $T_i^{*\prime}$ is:

$$\begin{aligned}\frac{T_i^{*\prime}}{(\frac{1}{\alpha n_i}+1) - T_i^{*\prime}} = &-\left(\left(\frac{n_i}{n_{i+1}}\right)^{\sigma+1} - \frac{C_i}{\widetilde{C}_i} - \frac{1}{\widetilde{C}_i u_{Yi}}\frac{n_{i+1}-n_i}{\alpha n_i n_{i+1}}\right)\frac{1}{C_i\lambda^*}\sum_{s=i+1}^{N}\left(\omega_s\frac{v_{cs}}{\psi_{Cs}} + \lambda^*\frac{1}{\psi_{Cs}}\right)\prod_{j=i}^{s-1}\frac{\psi_C(\widetilde{C}_j)}{\psi_{Cj}} \\ &+ \omega_i\left(v_{yi}\frac{1-\alpha}{\alpha n_i}\frac{\psi_{Ci}}{v_{Yi}} - v_{ci}\right)/\lambda^*,\end{aligned} \tag{D17}$$

where $\lambda^* = -\frac{\sum_{i=1}^{N}\omega_i v_{ci}\prod_{s=0}^{i-1}\frac{\psi_C(\widetilde{C}_s)}{\psi_{C,s+1}}}{\sum_{i=1}^{N}\prod_{s=0}^{i-1}\frac{\psi_C(\widetilde{C}_s)}{\psi_{C,s+1}}}$. We should remark that: $v_{Yi} = v_{yi}\left(1+\varepsilon^f_{Yi}(1-\frac{\psi_{ci}}{u_{yi}})\right)/f_i$ and $v_{Ci} = v_{ci}\left(1+\varepsilon^f_{Ci}(1-\frac{u_{yi}}{\psi_{ci}})\right)/f_i$, where $\varepsilon^f_{Yi} = \frac{\partial f_i}{\partial Y_i}\frac{Y_i}{f_i}$ and $\varepsilon^f_{Ci} = \frac{\partial f_i}{\partial c_i}\frac{c_i}{f_i}$.

Comparing the two optimal transfer equations in two cases, and using the definition of Ψ_i, we can rewrite (D17) as a comparable form with (18):

$$\frac{T_i^{\prime}}{(\frac{1}{\alpha n_i}+1) - T_i^{\prime}} = A\cdot B(n_i)\cdot C + D_1 + D_2, \tag{D18}$$

C.2 Governmental Tournament: a Heuristic Model

In this part we introduce another scenario where officials in local governments join the competition for GDP growth. Song and Xiong (2023) gives a framework to illustrate the inter-regional economic tournaments among the local governors in China, using the career concern model. However, Song and Xiong (2023) fails to discuss the inter-regional fiscal policies to correct the distortion induced by tournaments. We argue that the tournaments among local governors will also cause the externality of public goods.

Here we provide a heuristic model to illustrate the tournament. For the rigorous analysis, see Wang et al. (2024). We assume the private information of local governments, $n \in \mathbb{N}$, represents the ability of local governors, which serves as the evaluation criteria for the promotion of these officials. The local final goods production function is $y = A(n+\varepsilon)\cdot g$, where g is the productive public goods in the region, and ε is a random shock which is independent of n. The distribution of both n and ε are common knowledge, but the separate n, ε, and g are unobservable to the central government. The local governments make the decision of ex-ante public expenditures $g(n)$ before observing the shock ε. The central government can use the ex-post production y to update the expectation of local governors' abilities n using the following equation:

$$E(n \mid y) = \int \tilde{n} f(\tilde{n} \mid y) d\tilde{n}, \tag{D19}$$

where $f(n \mid y)$ is the conditional probability distribution function of abilities n. The inter-governmental transfers $T(\cdot)$ are based on the final production y. The evaluation of the local ability is based on the production y, which means local governments can use public spending strategies to change the final production and have a higher probability to be promoted.

The ex-post budget constraint for a local government with ability level n is:

$$T(y) + y = g(n) + c(n, y). \tag{D20}$$

The local governments care about the consumption of local residents $c(n, y)$ and the expectation of n from the central governments, $E(n \mid y)$. Hence there is a trade-off between local consumption and governors' promotion. We argue that this causes the externality induced by public spending. To see this, using the Bayesian rule and we rewrite the (D19) as:

$$E(n \mid y) = \frac{\int \tilde{n} h(y \mid \tilde{n}, g) f^N(\tilde{n}) d\tilde{n}}{f^Y(y)}, \tag{D21}$$

where f^Y and f^N are the marginal distribution functions of y and n. It is obvious that the function $g(\cdot)$ influences the expectation, and therefore the infinitesimal change of the public spending in one region will influence the curve of public spending and thus the utility of all regions. Let the ex-post utility function of local governments as:

$$U(n, y) = u(c(n, y)) - v(g(n)) + \chi E(n \mid y), \tag{D22}$$

where the parameter χ captures the influence of inter-regional tournaments. Here, we assign a negative utility to g, indicating that investment in local output is costly due to distortions or administration costs in tax collection behavior. The central government only cares about residential consumption, and its welfare function is:

$$W = \int U(c(n, y)) dH(n, y). \tag{D23}$$

H is the joint distribution function of n and y. We assume the central government is utilitarian. Since the central government does not take the tournaments into welfare function, the transfer policy also undertakes the function of correcting the competition among local governments. Therefore the optimization problem of the central government is:

$$\begin{aligned} &\max W \\ \text{s.t.}\quad & U(n, g(n), c(n,y), y) \geq U(n, g(n'), c(n', y), y) \quad \forall n, n' \in \mathbb{N} \quad \text{and} \quad y \in [\underline{y}, \overline{y}] \\ & \int_{\underline{y}}^{\overline{y}} T(y) f^Y(y) dy = 0 \end{aligned} \tag{D24}$$

Here we assume the upper and lower bounds of y only depend on the bound of n.

We argue that the inter-regional tournament factor, represented by the parameter χ, enters the optimal transfer in two channels. First, due to the existence of tournaments, the transfer policy will no doubt change the local public spending behavior through the channel of tournaments. Secondly, there is a Bayesian Nash equilibrium between the central and local governments. Given the behavior of local governments, the central government should design the transfer based on the strategies of local governments. In equilibrium, the actual local behavior under the optimal transfers and the given strategies are the same. Therefore the parameter χ will directly influence the transfer design. The logic is similar to the Fredholm equation in the main text.

To solve the problem (D24), first consider the decentralized economy where local governments select public spend levels g to maximize their expectation of utility $E(U \mid n) = \int (u(c(n,y)) + \chi E(n \mid y, g)) \, h(y \mid n, g) dy$. The first-order condition is

$$\int (u + \chi E(n \mid y, g)) h_g(y \mid n, g) dy - \int (u_c + v_g) h(y \mid n, g) dy = 0. \tag{D25}$$

Here we should introduce the game structure between central and local governments. The central government holds a belief about the spending behavior of local governments denoted as $g^*(\cdot)$, and it updates its assessment of n based on this belief using Bayesian rules. Given this belief and the strategy adopted by the central government, local governments decide their ex-ante public spending denoted as $g(\cdot)$. In a Nash equilibrium, the public spending choice of local governments aligns with the central government's belief. In the decentralized economy, the function $g(\cdot)$ in the expectation term $E(n \mid y, g)$ is unaffected by changes in the public spending behavior of local governments. However, when solving the central government's planning problem, the change of allocation function $g(\cdot)$ will also influence the expectation term. We can set the Langrage equation for the problem (D24) as follow:

$$
\begin{aligned}
L = & \int_n \int_f \left(u(c(n,y)) - v(g(n))\right) h(y \mid n,g) dy f^N(n) dn + \\
& \lambda(\int_n g(n) f^N(n) dn + \int_y \int_n c(n,y) h(y \mid n,g) dy f^N(n) dn - \int_y y f^Y(y) dy) + \\
& (\int_n \mu(n) f^N(n) \left(\int_y (u + \chi E(n \mid y,g)) h_g(y \mid n,g) - (u_c + v_g) h(y \mid n,g)) dy \right) dn),
\end{aligned} \tag{D26}
$$

where λ is the multiplier of budget constraint and $\mu(n)$ is the costate variable of IC constraint. We can use the variation approach to obtain the first-order conditions:

$$
\text{For } c\text{:} \quad u_c h(y \mid n,g) f^N(n)(1+\lambda) + \mu(n) f^N(n)(u_c h_g(y \mid n,g) - u_{cc} h(y \mid n,g)) = 0, \tag{D27}
$$

$$
\begin{aligned}
& \text{For } g\text{:} \quad \int_y (u - v) h_g(y \mid n,g) f^N(n) dy - \int_y v_g h(y \mid n,g) f^N(n) dy + \lambda f^N(n)(1 + \int_y c h_g(y \mid n,g) dy) \\
& + \mu(n) f^N(n) \left(\int_y (u + \chi E(n \mid y,g)) h_{gg}(y \mid n,g) - (u_c + v_g) h_g(y \mid n,g)) dy - \int_y v_{gg} h(y \mid n,g) dy \right) \\
& + \chi(\int_y n h_g(y \mid n,g) / f^Y(y) \int_n \mu(n) h_g(y \mid n,g) f^N(n) dn dy) = 0
\end{aligned} \tag{D28}
$$

Rewrite (D27) as:

$$
(1+\lambda) u_c f^N(n) + \mu(n) f^N(n)(u_c \phi(n,y,g) - u_{cc}) = 0, \tag{D29}
$$

where $\phi = \frac{h_g}{h}$ is the likelihood ratio of the distribution of public spending. Integrating the equation on n and differentiating it with respect to y, we obtain:

$$
\int_n u_{cc}(1+\lambda) c_y f^N(n) dn + \int_n \mu(n)(u_{cc}\phi - u_{ccc}) c_y f^N(n) dn + \int_n \mu(n) u_c \phi_y f^N(n) dn = 0. \tag{D30}
$$

Noticing that $T'(y) = c_y - 1$ using the budget constraint of local governments, we can derive the relationship between μ and marginal transfer T':

$$
T'(y) + 1 = E_n\left(\frac{\mu(n)\phi_y / \mathcal{R}}{-(1+\lambda) - \mu(n)\phi + \mu(n)\mathcal{P}}\right). \tag{D31}
$$

Here, $\mathcal{R} = -\frac{u_{cc}}{u_c}$ is the coefficient of risk aversion, and $\mathcal{P} = -\frac{u_{ccc}}{u_{cc}}$ is the prudence of local residents. From this equation, we can find that the transfer connects with the rent of IC, μ, the likelihood ratio, and the risk aversion level of local governments. Back to the condition

(D28), it indicates that μ is impacted by the parameter representing the tournament effect χ in two ways: the direct effect of tournaments on the behavior of local government, where μ is outside the integral sign, and the indirect effect on the central government's evaluation of n, where μ is inside the integral. Hence (D28) is also an integral equation and shares a similar structure to (12) in the main text. Since the optimal transfer is the function of μ, the optimal marginal transfer is the solution to (D28) and can be decomposed into the "ABC-D" form to some extent.

C.3 Proof for Multidimensional Heterogeneity Case

In this part, we provide the relative proof of the content in Section 5. We begin by introducing the extended double perturbation method to derive the perturbation of g. Subsequently, we solve for the transfer perturbation. Finally, by substituting the transfer perturbation into the welfare function, we arrive at the optimal transfer formula.

C.3.1 Extension of Double Perturbation Method

Consider a perturbation $\kappa\hat{T}$ on the prevailing transfer schedule $T(g)$, where $\hat{T}$ is Lipschitz continuous. Define the perturbation of g as $\hat{g} = \lim_{\kappa\to 0} \frac{\tilde{g}(\tau+\kappa\hat{T},e+\kappa\hat{e};n,\theta)-g(\tau,e;n,\theta)}{\kappa}$, where $\hat{e}$ is the perturbation of externality:

$$\hat{e} = \iint_{\{n,\theta\}\in\mathcal{N}\times\Theta} \hat{g} f(n \mid \theta) dn d\pi(\theta). \tag{D32}$$

The perturbation on g is given by (12):

$$\frac{\hat{g}(n)}{g(n)} = \frac{(\widetilde{\epsilon}_n^c \hat{T}' + \widetilde{\eta}_n \hat{T}/g)}{\tau'(g) - 1} + \widetilde{\epsilon}^{g,e} \cdot \frac{\hat{e}}{e}$$

This equation is the integral equation with multiple integrals. Similarly to Section 3, we can obtain the perturbation on the social welfare as:

$$\begin{aligned}\hat{W} = &\iint_{\{g,\theta\}\in\mathcal{G}\times\Theta} \widetilde{\gamma}(g,\theta) u_e \hat{e} h(g \mid \theta) dg d\pi(\theta) + \iint_{\{g,\theta\}\in\mathcal{G}\times\Theta} \widetilde{\gamma}(g,\theta) v' \hat{T} h(g \mid \theta) dg d\pi(\theta) \\ &+ \lambda \iint_{\{g,\theta\}\in\mathcal{G}\times\Theta} \hat{T} h(g \mid \theta) dg d\pi(\theta) + \lambda \iint_{\{g,\theta\}\in\mathcal{G}\times\Theta} \tau' \hat{g} h(g \mid \theta) dg d\pi(\theta) = 0,\end{aligned} \tag{D33}$$

where $\widetilde{\gamma}(g,\theta)$ is the welfare weight for the region $\{g,\theta\}$.

To solve this equation, we follow the approach of Sachs et al. (2020) with double perturbations and extend it to the multi-dimensional heterogeneity case. We choose a specific combination of two perturbations on T:

$$\hat{T} = \omega_1 + \omega_2,$$

where the first perturbation $\omega_1(g)$ is a elementary reform:

$$\omega_1(g) = (1 - \bar{H}(g^*))^{-1} \mathbb{I}_{\{g \geq g^*\}}. \tag{D34}$$

The second perturbation $\omega_2(g)$ should cancel out the effect of externalities. However, different from Sachs et al. (2020), it is almost impossible to use the uni-dimensional transfer perturbation to satisfy $\hat{g}(n,\theta) = \hat{g}_{PE}(n,\theta), \quad \forall \{n,g\} \in \mathcal{N} \times \Theta$, where $\hat{g}_{PE}(n,\theta) = -\frac{v_c(g,\theta)\omega_1'(g) + v_{cc}(g,\theta)(\tau'(g)-1)\omega_1}{u_{gg}(g,\theta) + v_{cc}(g,\theta)(\tau'(g)-1)^2 + v_c(g,\theta)}$ is the perturbation of public spending under partial equilibrium without externalities. Instead, we assume the second perturbation cancels out the effects of externalities on the average level of expenditures of all regions with the same preference θ:

$$E(\hat{g} \mid \theta) = E(\hat{g}_{PE} \mid \theta). \tag{D35}$$

Defining $E(\hat{g}_{PE} \mid \theta) = \bar{\hat{g}}_{PE}$, we can rewrite the perturbation on externalities $\hat{e}$ as:

$$\hat{e} = \iint\limits_{\{g,\theta\} \in \mathcal{G} \times \Theta} \hat{g} h(g \mid \theta) dg d\pi(\theta) = \int\limits_{g \in \mathcal{G}_+} \bar{\hat{g}}_{PE} \bar{h}(g) dg. \tag{D36}$$

Using (D36) we can obtain the second perturbation ω_2 satisfying (D35):

Lemma 4 *The second transfer perturbation exactly canceling out the externality effect is:*

$$\begin{aligned} \omega_2(g) = &-\frac{1}{1 - \bar{H}(g^*)} \left(\frac{\bar{\epsilon}(g^*)\bar{h}(g^*)g^*}{\tau'(g^*) - 1} + \int_{g^*} \frac{\bar{\eta}}{\tau' - 1} \bar{h}(g) dg \right) \\ &\int_{\underline{g}}^{g} \frac{\tau'(g') - 1}{\bar{\epsilon} g'} \bar{\epsilon}_e \cdot \exp\left(\int_{g}^{g'} \frac{\bar{\eta}}{\bar{\epsilon} g} dg'' \right) dg'. \end{aligned} \tag{D37}$$

where

$$\bar{\epsilon}_e \triangleq \int\limits_{\theta} \frac{u_{ge}}{u_{gg} + v_{cc}(\tau' - 1)^2 + v_c \tau''} \frac{h(g \mid \theta)}{\bar{h}(g)} d\pi(\theta) \tag{D38}$$

represents the weighted average of the elasticity of public spending to externalities across regions with spending level g.

See Appendix C.3.2 for the proof. One can find that it is the total elasticities that determine the perturbation function rather than the local elasticities of any particular local government since the marginal transfers remain constant across all regions with the same public spending g, regardless of preference level.

C.3.2 Proof of Lemma 4

Combining (D35) and (D36) we have:

$$\begin{aligned}\int_{\theta\in\Theta}\hat{g}h(g\mid\theta)d\pi(\theta)=&\int_{\theta\in\Theta}\left(-\frac{u_{ge}}{u_{gg}+v''(\tau'-1)^2+v'\tau''}\right)\hat{e}h(g\mid\theta)d\pi(\theta)\\&-\int_{\theta\in\Theta}\frac{v'\hat{T}'+v''(\tau'-1)\hat{T}}{u_{gg}+v''(\tau'-1)^2+v'\tau''}h(g\mid\Theta)d\pi(\theta).\end{aligned}\tag{D39}$$

Substituting the equation above into the perturbation of g in (12), the second perturbation ω_2 is given by a first-order linear differential equation as follows:

$$\begin{aligned}0=&\omega_2'\int\limits_{\theta\in\Theta}\frac{\epsilon g}{\tau'(g)-1}\frac{h(g\mid\theta)}{\bar{h}(g)}d\pi(\theta)+\omega_2\int\limits_{\theta\in\Theta}\frac{\eta}{\tau'-1}\frac{h(g\mid\theta)}{\bar{h}(g)}d\pi(\theta)\\&-\int\limits_{\theta\in\Theta}\left(\frac{u_{ge}}{u_{gg}+v_{cc}(\tau'-1)^2+v_c\tau''}\right)\frac{h(g\mid\theta)}{\bar{h}(g)}d\pi(\theta)\cdot\\&\iint\limits_{\{g,\theta\}\in\mathcal{G}\times\Theta}\frac{v_c\omega_1'+v_{cc}(\tau'-1)\omega_1}{u_{gg}+v_{cc}(\tau'-1)^2+v_c\tau''}h(g\mid\theta)dgd\pi(\theta),\end{aligned}\tag{D40}$$

Introducing the total elasticities in (42) and (43), the equation can be rewritten as:

$$\begin{aligned}&\iint\limits_{\{g,\theta\}\in\mathcal{G}\times\Theta}\frac{v_c\omega_1'+v_{cc}(\tau'-1)\omega_1}{u_{gg}+v_{cc}(\tau'-1)^2+v_c\tau''}h(g\mid\theta)dgd\pi(\theta)\\=&\iint\limits_{\{g,\theta\}\in\mathcal{G}\times\Theta}\left(-\frac{\epsilon g}{\tau'-1}\omega_1'-\frac{\eta}{\tau'-1}\omega_1\right)h(g\mid\theta)dgd\pi(\theta)\\=&-\int\limits_{g\in\mathcal{G}}\left(\frac{\omega_1'}{\tau'-1}\bar{\epsilon}\bar{h}(g)g+\frac{\omega_1}{\tau'-1}\bar{\eta}\bar{h}(g)\right)dg.\end{aligned}\tag{D41}$$

Since ω_1 is the indicator function, its first-order derivative $\omega_1'(g)$ is the Dirac delta function. By using the properties of the Dirac delta function we can derive:

$$0 = \omega_2' \frac{\bar{\epsilon} g}{\tau'(g) - 1} + \omega_2 \frac{\bar{\eta}}{\tau'(g) - 1} + \int\limits_{\theta} \left(\frac{u_{ge}}{u_{gg} + v_{cc}(\tau' - 1)^2 + v_c \tau''} \right) \frac{h(g \mid \theta)}{\bar{h}(g)} d\pi(\theta) \cdot \\ \frac{1}{1 - \bar{H}} \left[\frac{\bar{\epsilon}(g^*)\bar{h}(g^*)g^*}{\tau'(g^*) - 1} + \int_{g^*}^{\bar{g}} \frac{\bar{\eta}(g)}{\tau'(g) - 1} \bar{h}(g) dg \right]. \tag{D42}$$

Lemma 4 is derived by solving the equation above.

C.3.3 Proof of Proposition 5

Substituting the two perturbations ω_1 and ω_2 given by (D34) and (D37) into the welfare perturbation equation (D33), it leads to:

$$\hat{W}(\omega_1, \omega_2) = \int\limits_{g \in \mathcal{G}} (\bar{\phi} + \lambda)(\omega_1 + \omega_2)\bar{h}(g) dg + \int\limits_{g \in \mathcal{G}} \bar{\zeta} \bar{h}(g) dg \cdot \int\limits_{g \in \mathcal{G}} \bar{\tilde{g}}_{PE} \bar{h}(g) dg + \lambda \int\limits_{g \in \mathcal{G}} \tau'(g) \bar{\tilde{g}}_{PE} \bar{h}(g) dg. \tag{D43}$$

If the transfer schedule $\tau(g)$ is optimal, then the perturbation of welfare should be zero. Using the properties of the elementary reform we obtain:

$$- \int_{g^*} (\bar{\phi}(g) + \lambda) \frac{\bar{h}(g)}{1 - \bar{H}(g^*)} dg = \bar{\epsilon}(g^*) \frac{\bar{h}(g^*) g^*}{1 - \bar{H}(g^*)} \cdot \\ \frac{\lambda \tau' + \int\limits_{g \in \mathcal{G}} \bar{\zeta}(g) \bar{h}(g) dg - \int\limits_{g \in \mathcal{G}} (\bar{\phi}(g) + \lambda) \left[\int_{\underline{g}}^{g} \bar{\epsilon}_e \frac{\tau'(g') - 1}{\bar{\epsilon} g'} \exp\left(\int_g^{g'} \frac{\bar{\eta}}{\bar{\epsilon} g} dg'' \right) dg' \right] \bar{h}(g) dg}{\tau' - 1} + \\ \int_{g^*} \bar{\eta} \frac{\lambda \tau' + \int\limits_{g' \in \mathcal{G}} \bar{\zeta}(g') \bar{h}(g') dg' - \int\limits_{g' \in \mathcal{G}} (\bar{\phi}(g') + \lambda) \left[\int_{\underline{g}}^{g'} \bar{\epsilon}_e(g'') \exp\left(\int_g^{g'} \frac{\bar{\eta}}{\bar{\epsilon} g'''} dg''' \right) dg'' \right] \bar{h}(g') dg'}{\tau' - 1} \bar{h}(g) dg. \tag{D44}$$

The left-hand side of the equation above illustrates the mechanical welfare effect by uniformly transferring a unit of fiscal funds to all local governments while disregarding local behavioral responses. The right-hand side represents the behavioral effects of local governments, including the substitution and income effects.

Then we let

$$y(g^*) \triangleq \frac{\bar{h}(g^*)g^*\bar{\epsilon}(g^*)}{1-\bar{H}(g^*)}\cdot$$

$$\frac{\lambda\tau' + \int\limits_{g\in\mathcal{G}} \bar{\zeta}(g)\bar{h}(g)dg - \int\limits_{g\in\mathcal{G}} (\bar{\phi}(g)+\lambda)\left[\int_{\underline{g}}^{g} \bar{\epsilon}_e \frac{\tau'(g')-1}{\bar{\epsilon}g'} \exp\left(\int_{g}^{g'} \frac{\bar{\eta}}{\bar{\epsilon}g} dg''\right) dg'\right] \bar{h}(g)dg}{\tau'-1},$$

Hence we can transform the equation (D44) into a first-order linear differential equation as:

$$-\int_{g^*} (\bar{\phi}(g)+\lambda)\frac{\bar{h}(g)}{1-\bar{H}(g^*)}dg = y(g^*) + \int_{g^*} \frac{\bar{\eta}}{\bar{\epsilon}g} y(g)dg, \tag{D45}$$

from which we can derive the optimal transfer as:

$$\begin{aligned}
&\frac{\tau' + \int\limits_{g\in\mathcal{G}} \frac{\bar{\zeta}(g)}{\lambda}\bar{h}(g)dg - \int\limits_{g\in\mathcal{G}} (\frac{\bar{\phi}}{\lambda}+1)\left[\int_{\underline{g}}^{g} \bar{\epsilon}_e \frac{\tau'(g')-1}{\bar{\epsilon}g} \exp\left(\int_{g}^{g'} \frac{\bar{\eta}}{\bar{\epsilon}g} dg''\right) dg'\right] \bar{h}(g)dg}{\tau'-1} \\
&= -\frac{1-\bar{H}(g)}{\bar{h}(g)g}\frac{1}{\bar{\epsilon}}\int_{g}(\frac{\bar{\phi}}{\lambda}+1)\frac{\bar{h}(g')}{1-\bar{H}(g)}\exp\left(\int_{g'}^{g} \frac{\bar{\eta}}{\bar{\epsilon}g''}dg''\right)dg'.
\end{aligned} \tag{D46}$$

C.3.4 Shadow Price λ

Here we give the details about the determinant of the shadow price λ. To obtain λ, we now keep ω_1 constant when applying the double-perturbation approach, while still requiring ω_2 to ensure $\bar{\bar{g}} = \bar{\bar{g}}_{PE}$. Hence ω_2 should satisfy:

$$\begin{aligned}
0 =&\omega_2' \int\limits_{\theta\in\Theta} \frac{\epsilon g}{\tau'(g)-1}\frac{h(g\mid\theta)}{\bar{h}(g)}d\pi(\theta) + \omega_2 \int\limits_{\theta\in\Theta} \frac{\eta}{\tau'-1}\frac{h(g\mid\theta)}{\bar{h}(g)}d\pi(\theta) \\
&- \int\limits_{\theta\in\Theta} (\frac{u_{ge}}{u_{gg}+v_{cc}(\tau'-1)^2+v_c\tau''})\frac{h(g\mid\theta)}{\bar{h}(g)}d\pi(\theta)\cdot \\
&\iint\limits_{\{g,\theta\}\in\mathcal{G}\times\Theta} \frac{v_{cc}(\tau'-1)\omega_1}{u_{gg}+v_{cc}(\tau'-1)^2+v_c\tau''}h(g\mid\theta)dgd\pi(\theta),
\end{aligned} \tag{D47}$$

Then ω_2 can be solved as:

$$\omega_2(g) = -\int\limits_{g\in\mathcal{G}} \frac{\bar{\eta}}{\tau'-1}\bar{h}(g)dg \cdot \\ \int_{\underline{g}}^{g} \frac{\tau'(g')-1}{\bar{\epsilon}g'} \left[\int\limits_{\theta\in\Theta} \left(\frac{u_{ge}}{u_{gg}+v_{cc}(\tau'-1)^2+v_c\tau''}\right) \frac{h(g\mid\theta)}{\bar{h}(g)} d\pi(\theta)\right] \cdot \exp\left(\int_{g}^{g'} \frac{\bar{\eta}}{\bar{\epsilon}g} dg''\right) dg'. \tag{D48}$$

Replacing two perturbations into (D43) we have:

$$\int\limits_{g\in\mathcal{G}} \frac{\bar{\eta}}{\tau'-1}\bar{h}(g) \cdot \left(\lambda\tau + \int\limits_{g\in\mathcal{G}} \bar{\zeta}(g)\bar{h}(g)dg - \int\limits_{g\in\mathcal{G}} (\bar{\phi}(g)+\lambda) \int_{\underline{g}}^{g} \bar{\epsilon}_e \frac{\tau'(g')-1}{\bar{\epsilon}g'} \exp\left(\int_{g}^{g'} \frac{\bar{\eta}}{\epsilon g''} dg''\right) dg'\right) \\ = -\int\limits_{g\in\mathcal{G}} (\bar{\phi}(g)+\lambda)\bar{h}(g)dg. \tag{D49}$$

By using the optimal transfer (46), λ can be expressed as:

$$\lambda = -\frac{\int\limits_{g\in\mathcal{G}} \bar{\phi}(g)\bar{h}(g)dg - \int\limits_{g\in\mathcal{G}} \frac{\bar{\eta}}{\bar{\epsilon}g} \int_{g}^{\overline{g}} \bar{\phi}(g')\bar{h}(g') \exp\left(\int_{g}^{g'} \frac{\bar{\eta}(g'')}{\bar{\epsilon}(g'')g''} dg''\right) dg' dg}{1 - \int\limits_{g\in\mathcal{G}} \frac{\bar{\eta}}{\bar{\epsilon}g} \int_{g}^{\overline{g}} \bar{h}(g') \exp\left(\int_{g}^{g'} \frac{\bar{\eta}(g'')}{\bar{\epsilon}(g'')g''} dg''\right) dg' dg}. \tag{D50}$$

Appendix D Supplement to the Numerical Simulation

D.1 Robustness Check for the Numerical Simulation

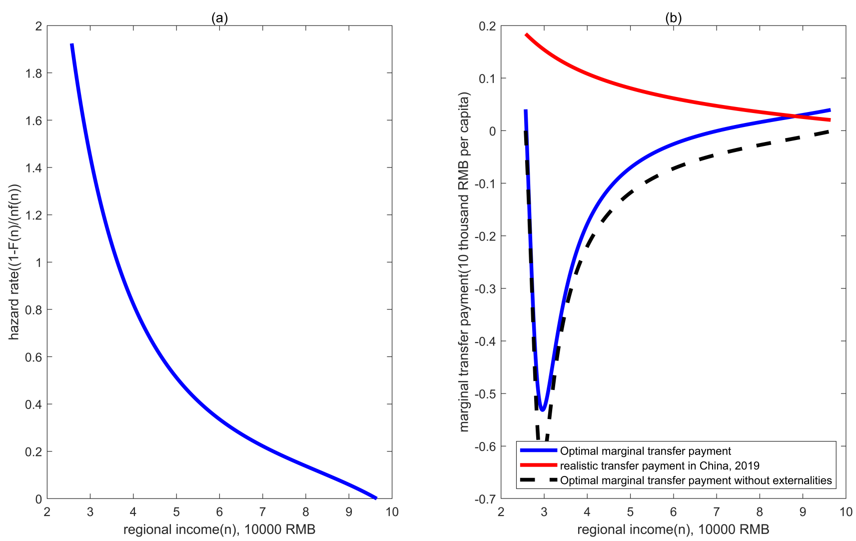

Figure D.1: The optimal marginal transfer using the BT and basic public service expenditures as τ and g

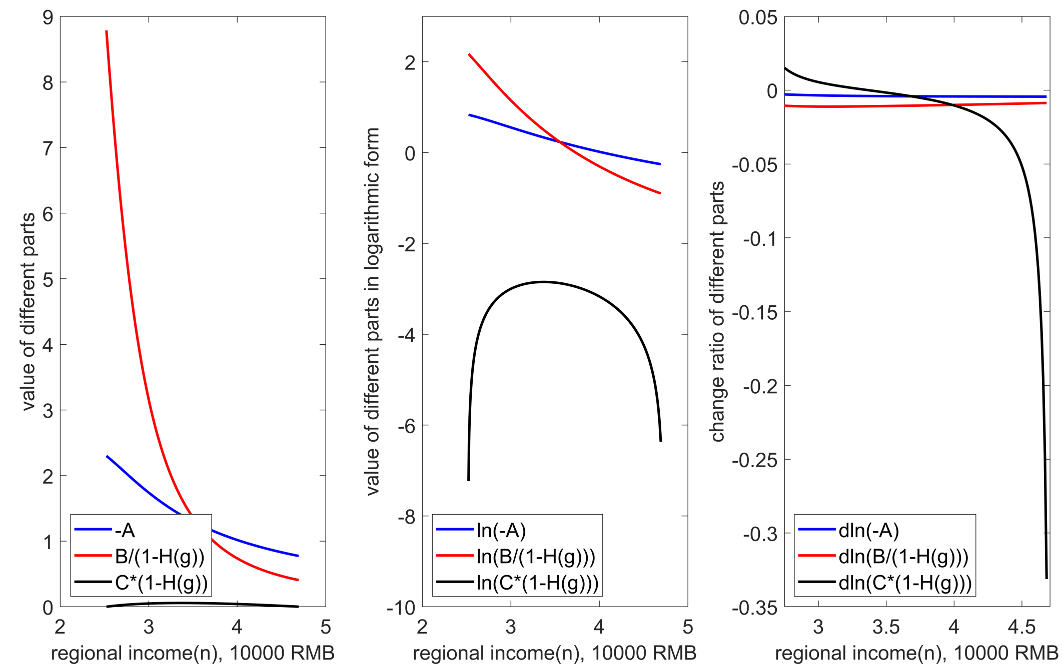

Figure D.2: The decomposition of optimal transfer without adding $1 - H(g)$ term

D.2 Sufficient Statistics

Table D.1: The estimation results for the price elasticity of public spending

	(1)	(2)	(3)	(4)
VARIABLES	$\ln g_{basic}$	$\ln g_{basic}$	$\ln g_{basic}$	$\ln g_{basic}$
L.lnc			-0.106	-0.0944
			-0.21	-0.202
L.lnk			0.117	0.18
			-0.231	-0.248
L.lny			0.837*	0.842*
			-0.439	-0.449
ln(1-τ'_{BT})	-1.411***	-3.258***	-4.134***	-5.850***
	-0.154	-0.463	-0.443	-0.566
ln(t_{ST})	0.143***	0.0897**	0.0459	0.144**
	-0.0185	-0.035	-0.0356	-0.0629
L.ln(1-τ'_{BT})		-0.286		1.486***
		-0.324		-0.374
L.ln(t_{ST})		0.0283		0.113*
		-0.0414		-0.0622
Constant	7.439***	7.571***	-0.601	-2.984
	-0.137	-0.47	-2.067	-2.252
Observations	1,471	647	375	296
R-squared	0.181	0.23	0.607	0.723
Number of county	788	453	288	228

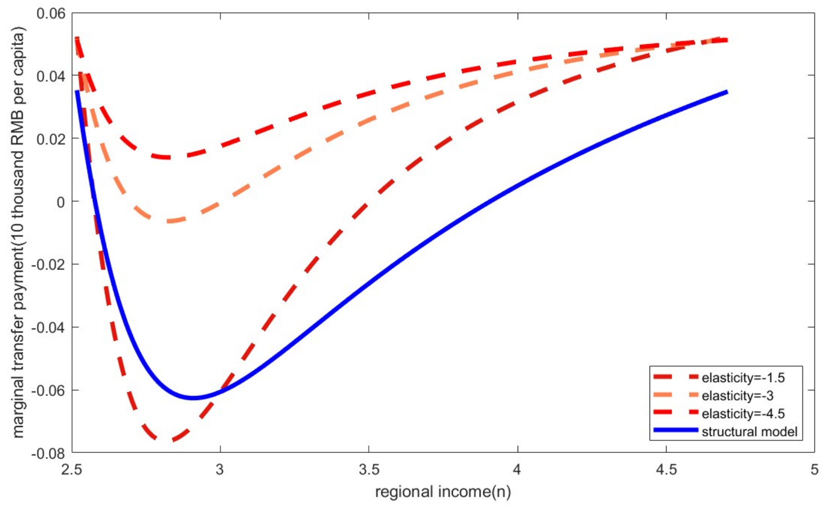

Figure D.3: The comparison between the structural model and the sufficient statistic method

D.3 Material for the Numerical Simulation

D.3.1 The Extended HSV Form of Transfer Schedule in China

The per capita government expenditure and per capita GDP exhibit a linear relationship in logarithmic form:

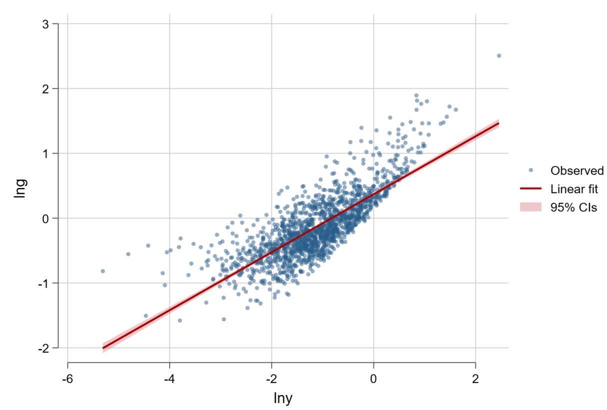

Figure D.4: The Relationship between per capita government expenditure and GDP

Therefore we can use the following function to fit the data, which is a kind of extended HSV form:

$$T = y_0 + g - \varphi_1 g^{1-\varphi_2}, \tag{F3}$$

We use the panel data of China ranging from 2016 to 2019 to estimate the coefficients in (F3).

The raw data is divided by 10000 to match the unit of measurement in Section 6.3. We also conduct the estimation when the unit of the data is 1000 RMB, and the estimation using the BTs and basic public service expenditures. The results are displayed in the Table D.2.

Table D.2: The estimation results for the parameters in (F3)

VARIABLES	unit: 10000 RMB	unit: 1000 RMB	BT and basic public service (1000 RMB)
y_0	-0.0240**	-0.240**	-0.0224***
	(0.0104)	(0.104)	(0.00869)
φ_1	0.589***	0.536***	0.846***
	(0.0117)	(0.0240)	(0.0106)
φ_2	-0.0412***	-0.0412***	-0.0534***
	(0.0111)	(0.0111)	(0.00912)
Observations	5,587	5,587	1,810
R-squared	0.598	0.598	0.254

Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

In the numerical section, we use the estimated parameters to approximate the prevailing transfer payment schedule in China.

D.3.2 More about Optimal transfer

The monotonicity condition $\frac{dg}{dn} > 0$ is met since we can obtain the relationship in the data as shown below:

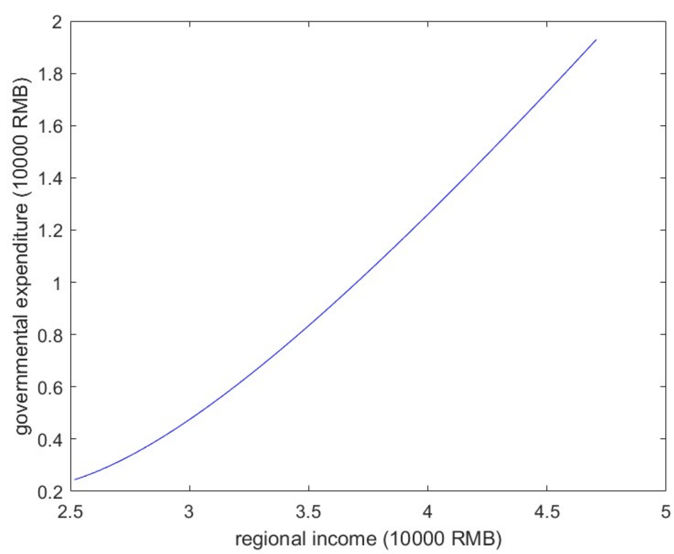

Figure D.5: The relationship between g and n in the data.

1 Keeping the budget constraint of the central government, we can derive the optimal
2 transfer policy (rather than marginal transfer) as follows:

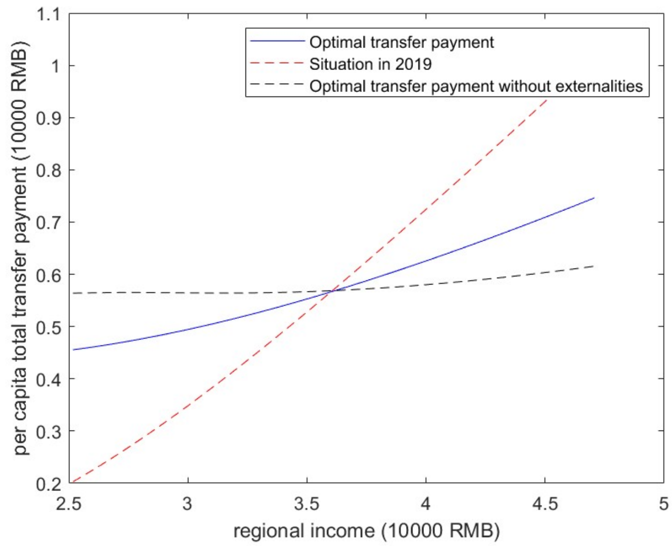

Figure D.6: Optimal transfer in China, 2019

Appendix E Supplement for Transfer Policy in China

E.1 More Facts about China

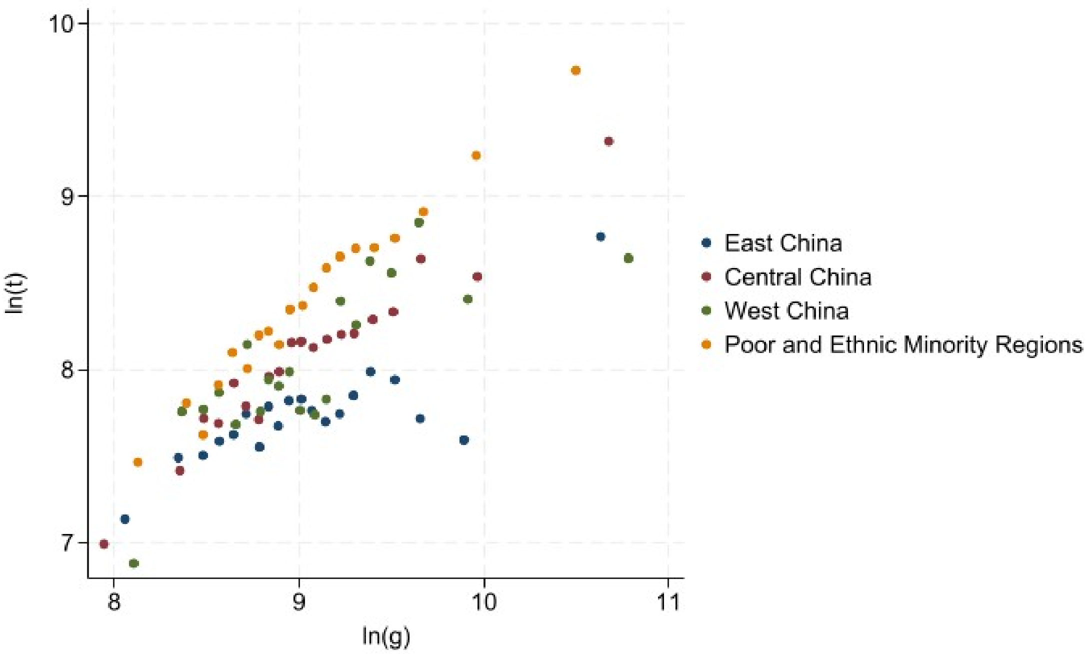

Figure E.1: Per capita transfer payments and public spending in logarithmic form: four categories of regions in 2019. The poor and ethnic minority regions are excluded from the other three regional categories.

E.2 Formula Method

After undergoing significant fiscal adjustments through a widely-discussed reform of the tax-sharing system in 1994, the Chinese central government established “the balanced transfer payment"(BT) program in 1995, aiming to regulate the financial gap between regions through balanced transfer payments[38]. BT constituted the earliest form of GT and has always been the most important component of GT since its establishment. It is allocated according to the formula method. At the beginning of its establishment, BT established the allocation method of transfer payments through the formula method, and this fundamen-

[38]Montinola et al. (1995); Qian and Roland (1998) and their following articles give detailed introduction and analysis about Chinese tax-sharing system reform in 1994. This reform not only completely transformed the tax-sharing method, but also significantly weakened the tax autonomy of local governments, leading to an increasing reliance on the transfer system.

tal framework persisted in subsequent BT policies until 2022. The main formula of BT is as follows:

$$BT_i = (SE_i - SR_i) \times TC_i,$$

where BT_i is the amount of BT for a region i, SE_i the standard financial expenditure of a region i, and SR_i the standard financial revenue of a region. Both the SE_i and SR_i are not the actual values of local government expenditures and revenues, but rather the estimated values. TC_i is a transfer payment coefficient.

The computation of SR_i reflects the concern about information asymmetry between the central and local governments. The formula for SR_i can be concluded as follows:

$$SR_i = B\hat{a}se_i \times \overline{Rate},$$

The general idea is to calculate the regional tax base $B\hat{a}se_i$ based on economic indicators that the central government can observe. For instance, value-added tax requires the calculation of value-added amount, while business tax necessitates the calculation of sales revenue and income. If the tax collection and management standards for a particular industry are unclear or if it is difficult to obtain tax base data, the standard tax revenue can be temporarily determined based on the public financial settlement of tax collections reported by local governments. Next, the SR_i is determined by multiplying $B\hat{a}se_i$ by the national average tax rate $\overline{Rate}$, instead of using the economic income data reported directly by local governments. This indicates that the central government has a deep understanding of the problem where the hidden information makes it difficult to obtain their true and reasonable income levels, or at least the real revenue-raising capacity of local governments.

The formula to determine SE_i reflects the concern for the high dependency on transfer payments and restricts public expenditure to the "basic public services" emphasized by the central government. Only a subset of the expenditure items is included in the standard financial expenditure, which encompasses the expenditures closely associated with the basic needs of local government and residents, such as education, medical care, culture, agriculture, housing security, energy conservation, environmental protection, etc. Some expenditures directly connected with local production, which are prone to overspending and always with low efficiency, are not taken into account by BT, such as technology expenditures, financial and service industry expenditures, etc. This clearly shows that China's transfer payment strategy aims to guarantee the welfare of local residents and narrow the inter-regional gap in public goods provided for residents.

Moreover, the central government is aware of a caveat when directly using the level of local government's public expenditure to calculate transfer payments. This is because local governments can potentially overspend to artificially increase fiscal gaps, thereby securing more subsidies. This can lead to soft budget constraints and the emergence of flypaper effect issues. Therefore, policymakers adopt a method for calculating the required expenditures for regions based on certain exogenous observable indicators. These exogenous parameters encompass objective indicators such as population size, area, temperature, altitude, transportation distance, and road conditions, as well as statistical data such as national average expenditure levels. The central government computes the standard values for various expenditure categories by combining these exogenous parameters with the actual expenditure levels of local governments. For example, the SE of general public service, which is one of the expenditure items, for a region i is based on the formula below:

$$SE_{general\ public\ service,i} = Pop_i \times \overline{Exp/Pop} \times Cost_i(\vec{W}_i, \vec{Exp}_i),$$

where $\overline{Exp/Pop}$ is the national standard level of expenditure of the general public service per capita set by the central government. Pop_i is the population in the region i. Exp_i is the actual expenditure of the general public service in the region i.

The cost coefficient, denoted as $Cost_i$, reflects the regional characteristics that affect the cost of providing transfer payments. The formula for the cost coefficient determining the SE of the general public service is as follows:

$$\begin{aligned}
Cost_i = &Pop_i \times \overline{Exp/Pop} \times (Weight_{pop,i} \times 0.85 + Weight_{area,i} \times 0.15) \\
&\times \{Weight_{remote\ and\ underdeveloped\ area,i} \times \frac{Exp_{faculty,i}}{Exp_i} + Weight_{tempreture,i} \times \frac{Exp_{heating,i}}{Exp_i} \\
&+ Weight_{altitude,i} \times Weight_{distance,i} \times \frac{Exp_{fuel,i}}{Exp_i} + Weight_{road\ condition,i} \times \frac{Exp_{vehicle\ maintenance,i}}{Exp_i} \\
&+ [1 - (\frac{Exp_{faculty,i}}{Exp_i} + \frac{Exp_{heating,i}}{Exp_i} + \frac{Exp_{fuel,i}}{Exp_i} + \frac{Exp_{vehicle\ maintenance,i}}{Exp_i})]\}... \\
&\times Weight_{working\ age\ population,i} \times Weight_{minority\ ethnic\ region,i} \\
&\times Weight_{municipality\ directly\ under\ the\ Central\ Government,i}
\end{aligned}$$

The terms $EXP_{\cdot,i}$ are the actual expenditures for various sub-items of the general public service in the region i. The coefficients $Weight_{\cdot,i}$ represent different factors about the region i. Although the formula may be complex, we can summarize it as a function of local basic public service expenditures and exogenous parameters.

The consideration of externalities in Chinese transfer policies is reflected in two aspects. First, the computation of SE_i takes the inter-regional population mobility into consideration. Next, in 2016, the document of BE required to treat expenditures with strong externalities and policy-driven expenditures separately from other expenditures.

The coefficient TC_i reflects how the central government cares about fiscal equity between regions. TC_i is determined by two factors, one is the national total funds of BE and the other is the financial pressure of the province where the region is located. The financial pressure is mainly determined based on the proportion of local basic public service expenditures to the total local public expenditures, as well as the local government fiscal deficit. Therefore the coefficient TC_i can also be viewed as a function of local basic public services. During computation, the central government will assign weights to reflect its prioritization of local government financial pressures.

E.3 Transfer Dependency

In this part, we present a more comprehensive analysis and a more detailed explanation of how the transfer system in China has led to local governments' dependency on transfer funds. Figure E.2 illustrates the relationship between fiscal gaps before the reform in 1994 when the intergovernmental transfer system in China was not established and those during 2016–2019.

We can observe two important facts from Figure E.2: first, the linear fitting curve increased with fiscal gaps in 1993, indicating that transfer policies in China have not succeeded in mitigating interregional fiscal disparities. A region with a large-scale fiscal gap before the establishment of the transfer system is also likely to experience relatively high fiscal pressure during the period from 2016 to 2019. The second fact is that most data points are clustered above the 45-degree line. This result verifies our intuition that the prevailing transfer schedule in China encourages the expansion of fiscal gaps (the fly-paper effect) in sub-national governments. Although the observed expansion can also be attributed to other political or economic factors, such as interregional expenditure competition, we can assert that the transfer policy in China has not succeeded in reversing the trend of deteriorating financial conditions among local governments[39].

[39] We also discuss the fiscal competition and migration in our theoretical framework in Section 4.

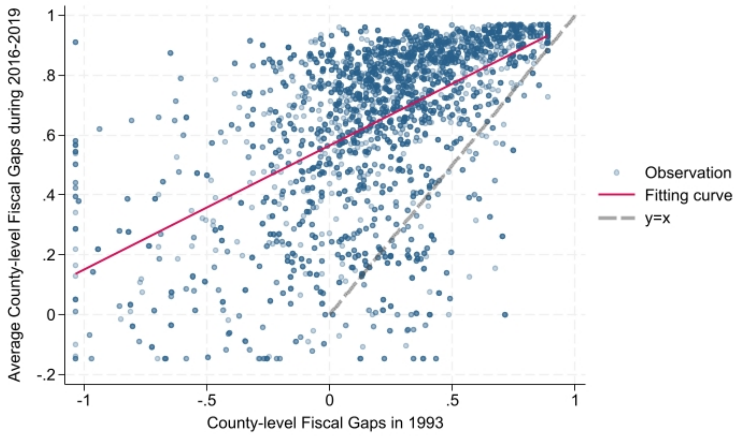

Figure E.2: The longstanding local fiscal gaps in China after the critical fiscal system reform in 1993

Notes: Fiscal gaps are defined as the difference between public expenditures and tax and non-tax revenues divided by public expenditures at the county level. The 1993 data is sourced from the Financial Statistics of Cities and Counties Nationwide, compiled by the Ministry of Finance in China, while data from 2016 to 2019 are manually collected from the official websites of county-level governments. We merge the two datasets using region codes. Since the classification of items in the Chinese fiscal budget experienced a significant reform in 2007, we use total fiscal expenditures and revenues to ensure comparability of fiscal gaps across the two periods.

The result that transfers positively correlate with local public expenditures reported by local governments does not contradict the formula method employing objective factors to prevent the endogenous influence of local fiscal statistics. The reasons are twofold. First, there is a soft budget problem in the Chinese transfer system. As discussed by Qian and Roland (1998), Chinese governments always lack the capacity to commit to their budgets. When local governments overspend, the superior governments are more likely to provide temporary transfers to support those jurisdictions. Hence some supplementary clauses in the document of formula methods state that the computed results can be adjusted based on the actual fiscal expenditures and revenues of local governments, thereby enabling such ex-post transfer subsidies. Second, every few years, the transfer formula would be revised based on the redistribution outcomes of the previous policy. This revised rule tends to favor regions with larger fiscal gaps, admitting the fiscal expansion behavior of these regions. Moreover, local governors can negotiate with the central ministry through internal reports or personalized visits to Beijing, and the outcomes of these informal negotiations are reflected in adjustments to the weights of indices or coefficients in the formula[40]. Thus local govern-

[40]Such informal negotiation can be influenced by the political network in China. Literature has revealed the influence of political connections on the transfer distribution (Jiang and Zhang, 2020).

ments can take advantage of their private information to seek additional transfer funds to support their expenditures.

E.4 Basic Public Service

Transfer payment policies are concerned with the basic public services of China's grassroots governments. The definition of the scope of basic sectors of local government expenditures in the document "*Administration Measures for the Management of Reward Funds for the Central Government's Basic Financial Support Mechanism for County-level Governments*" (which is an item of general transfer payment) contains the following parts: (i) personnel expenses, including the basic salary, bonuses and allowances for officials, pension insurance expenditures, staff retirement expenses and other items. (ii) Public funds, including the county-level organs and institutions office expenses, office equipment purchase and other capital expenditures. (iii) The residents' livelihood expenditure, mainly including the policies designated by the central government, involving agriculture, education, culture, social security, health care, science and technology, family planning, environmental protection, housing security and village organizations operating expenses. (iv) Other necessary expenditures.

The document "*State Council Notice on the Issuance of the 13th Five-Year Plan to Promote Equalization of Basic Public Services*(13FYP)" includes seven major areas of basic public services, including education, labor, employment and entrepreneurship, social insurance, health care, social services, housing security, culture and sports, and basic public services for the disabled. The document lists some indicators to evaluate the expenditure results of the seven sectors, such as the number of new jobs in urban areas, infant and maternal mortality, annual circulation of public libraries and number of people who regularly participate in physical exercise.

The general public budget expenditure items are numerous, each of which also contains many sub-items. The public financial settlement data of county-level governments only discloses the first-level expenditure items, but will not disclose the sub-items of these first-level items. Therefore we should determine whether the the expenditure items belong to basic public services.

In this paper, each expenditure item is judged according to the following criteria: (i) whether the main expenditure purpose and direction of the expenditure item can directly affect the production of enterprises, industries, or society in the **short term**. If the answer is no, then it belongs to basic public services. (ii) whether this type of expenditure is primarily domestic. (iii)Whether this expenditure is difficult to directly affect both production and residents' utilities, or the probability to affect production and utilities is low. If it is true, then this expenditure is not within the scope of this paper. Table E.1 presents the selection results for basic public service expenditures. The basic public services in Table E.1 are the expenditure items we choose as the proxy. The nine items account for about 70.12% of total expenditures in the general budget in 2019 according to our data.

Table E.1: Selection of Basic Public Service Expenditure

Expenditure Item	**Cretiria I**	**Cretiria II**	**Cretiria III**	**13FYP**	**Basic Public Services**
Science	✓	✗	✗	✗	✗
Transportation	✓	✗	✗	✗	✗
Agriculture, Forestry and Water	Partially	✗	✗	✗	✗
Finance	Partially	✗	✗	✗	✗
Resource Exploration and Industrial Information	Partially	✗	✗	✗	✗
Natural Resources and Marine Meteorological	Partially	✗	✗	✗	✗
Business and Service Industry	Partially	✗	✗	✗	✗
Energy Saving and Environmental Protection	Partially	✗	✗	✗	✗
Diplomatic	✗	✓	✓	✗	✗
General Public Service	✗	✗	✗	Partially	✓
National Defense	✗	✗	✗	✗	✓
Public Safety	✗	✗	✗	✗	✓
Education	✗	✗	✗	✓	✓
Cultural, Sports and Media	✗	✗	✗	✓	✓
Social Security and Employment	Partially	✗	✗	✓	✓
Medical and Health Care	✗	✗	✗	✓	✓
Urban and Rural Community Affairs	✗	✗	✗	Partially	✓
Housing Security	✗	✗	✗	✓	✓
Food and Oil Reserve	✗	✗	✓	✗	✗
Disaster Prevention and Control	✗	✗	✓	✗	✗

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