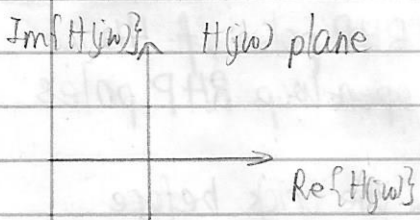
ECE264A Lect 10 Stability

1. ~~ECE486 UIUC Lect 18~~
Nyquist Criterion

1.1. mapping

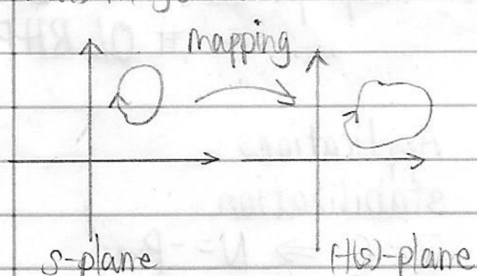
$$H(s)=\frac{(s-z_1)\cdots(s-z_m)}{(s-p_1)\cdots(s-p_n)} \quad m<n$$
(strictly proper)

$$H(j\omega)=\frac{(j\omega-z_1)\cdots(j\omega-z_m)}{(j\omega-p_1)\cdots(j\omega-p_n)}$$



which is like the Γ-plane in RF

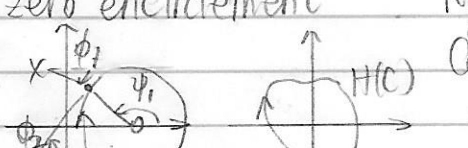
But, in general



1.2. phase change contour traversal

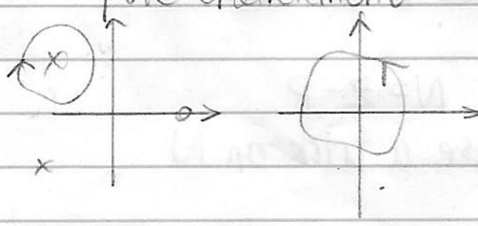
$$\angle H(s)=\sum_{i=1}^{m}\angle(s-z_i)-\sum_{j=1}^{n}\angle(s-p_j)$$
$$=\sum_{i=1}^{m}\psi_i-\sum_{j=1}^{n}\phi_j$$

1.2.1. zero encirclement and zero pole encirclement



ψ_1 $-360°$
other ψ and ϕ no change
$\Rightarrow \angle H(s)$ $-360°$
$\Rightarrow$ H(C) encircle origin once in H(s)-plane, ↻ (cw)

1.2.2. pole encirclement



ϕ_1 $-360°$
other no change
$\Rightarrow \angle H(s)$ $+360°$
$\Rightarrow$ ↺ (ccw) encirclement of origin

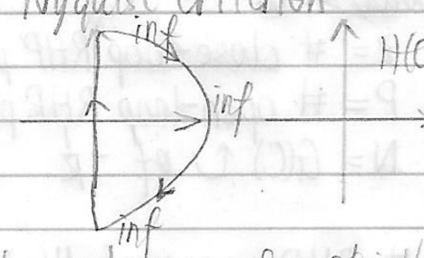
1.2.3. nothing encirclement
H(C) no encirclement of origin

1.2.4. Argument Principle

$$N=Z-P$$
H(C) ↻ (cw) = (# of z − # of p) in C

1.3. Nyquist Criterion

1.3.1. Nyquist Contour and Nyquist Plot

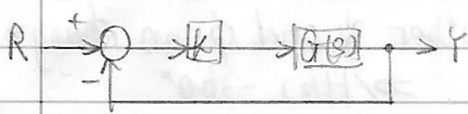


H(C) is the "Nyquist" plot

for strictly proper H(s), $H(\infty)=0$

1.3.2. ECE486 FB Sys

In ECE 486, the fb sys is



$$\frac{KG(s)}{1+KG(s)}$$

Let $H(s) = KG(s)+1$

$N = Z - P$

use a trick on N

$N = H(C) \circlearrowright$ of 0

$= 1+KG(C) \circlearrowright$ of 0

$= KG(C) \circlearrowright$ of -1

$= G(C) \circlearrowright$ of $-\frac{1}{K}$

1.3.3. Convert the problem to ...

$Z = N + P$

will show

Z = # close-loop RHP poles

P = # open-loop RHP poles

$N = G(C) \circlearrowright$ of $-\frac{1}{K}$

1.3.3.1 Z: # RHP zeros of $H(s)$

= # closed-loop RHP poles of $\frac{KG(s)}{1+KG(s)}$

1.3.3.2 P = # RHP poles of $H(s)$

= ?

This is not immediately obvious.

Need to link it to open-loop quantities.

$G(s) = \frac{q(s)}{p(s)}$

$H(s) = 1 + KG(s) = \frac{p(s)+Kq(s)}{p(s)}$

Thus,

P = # RHP poles of $H(s)$

= # open-loop RHP poles

1.3.3.3 N is the trick before

1.3.4. Nyquist Theorem

$Z = N + P$

CL RHP poles = $G(C) \circlearrowright -\frac{1}{K}$ + # OL RHP poles

1.4. Applications

1.4.1. stabilization

$Z = 0 \Rightarrow N = -P$

$G(C) \circlearrowleft -\frac{1}{K}$ = # OL RHP poles

Nyquist Criterion

$T(C)$ ↺ of $(-1,0)$ = # RHP poles of $T(s)$

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2. Rosenstark

2.1. $G_f(s) = \frac{1}{1+T(s)}(G_\infty T(s) + G_0)$

den $= F(s) = 1 + T(s)$

$T(s) = H\frac{N(s)}{D(s)}$

$G_f(s) = \dfrac{G_\infty H\frac{N(s)}{D(s)} + G_0}{1 + H\frac{N(s)}{D(s)}}$

$= \dfrac{G_\infty H N(s) + G_0 D(s)}{D(s) + H N(s)}$

$C(s) = D(s) + H N(s)$

2.2. encirclement theorem

more later

3. Galton

$A(s) = A_0 \dfrac{1 + a_1 s + \cdots + a_m s^m}{1 + b_1 s + \cdots + b_n s^n}$

$= A_0 \dfrac{a(s)}{1 + a(s) f(s)}$

$R(s) = 1 + T(s)$

$R(C)$ ↻ of $(0,0)$

$=$ # $T(C)$ ↻ of $(-1,0)$ (trick 3)

$= (\# z - \# p)$ of $1 + T(s)$ in C

(by the arg prin)

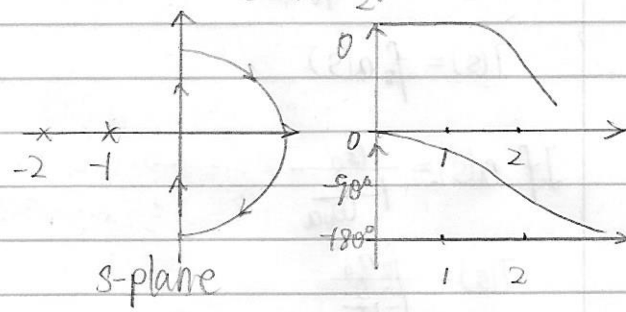
trick 1 zeros of $R(s)$ = poles of $A(s)$

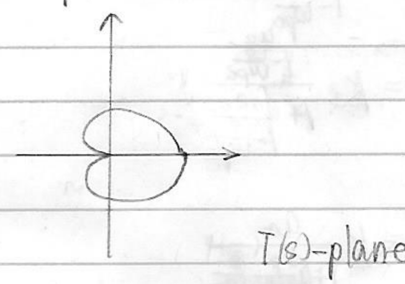
trick 2 poles of $R(s)$ = poles of $T(s)$

$1 + T(s)$

4. Examples

4.1. $T(s) = \dfrac{1}{(1+s)(1+\frac{s}{2})}$





3. Galton

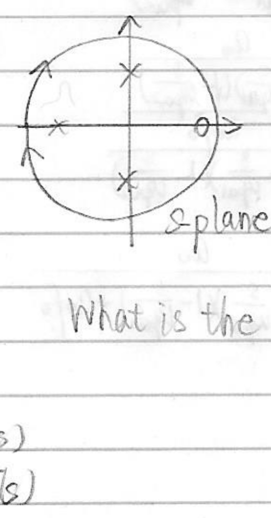
3.1. Encirclement Property

= Argument Principle (in ECE486)

4. Examples

4.2. encirclement property

$T(s) = \dfrac{s-1}{1+s+s^2+s^3} = \dfrac{s-1}{(s+1)(s^2+1)}$



What is the $T(s)$-plane contour (i.e. $T(C)$)

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4. Examples

4.3. $$A(s) = A_0 \frac{a(s)}{1+f_0 a(s)}$$

$$T(s) = f_0 a(s)$$

If $$a(s) = \frac{a_0}{1-\frac{s}{\omega_{pa}}}$$

$$T(s) = \frac{f_0 a_0}{1-\frac{s}{\omega_{pa}}}$$

$$A(s) = K \frac{\frac{a_0}{1-\frac{s}{\omega_{pa}}}}{1+\frac{f_0 a_0}{1-\frac{s}{\omega_{pa}}}}$$

$$= K \frac{a_0}{1+a_0 f_0 - \frac{s}{\omega_{pa}}}$$

$$= A_0 \frac{1}{1-\frac{s}{\omega_{pA}}}$$

$$\omega_{pA} = (1+a_0 f_0)\omega_{pa}$$

4.4. If $$a(s) = \frac{a_0}{(1-\frac{s}{\omega_{pa1}})(1-\frac{s}{\omega_{pa2}})}$$

$$T(s) = \frac{a_0 f_0}{(1-\frac{s}{\omega_{pa1}})(1-\frac{s}{\omega_{pa2}})}$$

$$A(s) = K \frac{\frac{a_0}{(1-\frac{s}{\omega_{pa1}})(1-\frac{s}{\omega_{pa2}})}}{1+\frac{a_0 f_0}{(1-\frac{s}{\omega_{pa1}})(1-\frac{s}{\omega_{pa2}})}}$$

$$= K \frac{a_0}{(1-\frac{s}{\omega_{pa1}})(1-\frac{s}{\omega_{pa2}}) + a_0 f_0}$$