Event-Triggered State Estimation for Networked Switched Systems: An Output Predictor Approach (IEEE SJ 23)

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1. Motivation
2. Related work
3. Research contents
4. Results
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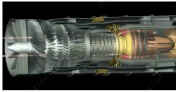
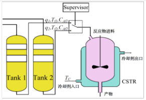
Switched systems

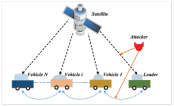


Air engine
[Chen 2023 NAHS]



Chemical process
[Ma 2010 Automatica]

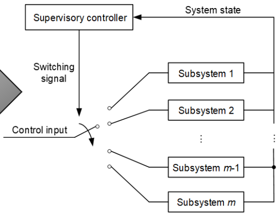
Supersonic aircraft
[Lian 2017 TIE]



Multi-agent formation
[Zhang 2021 TSMC-S]

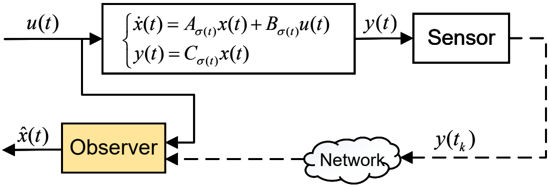
Modeled as

Switched systems



- Many practical systems exhibit multiple mode switching behaviors, e.g., a multi-agent formation with switching topology.
- Thus, single system model is not enough to handle the related control and observation problems.

Networked switched systems (NSS)



- Fast development of digitalization and networking technology (embedded and networked control).
- Network-induced phenomena arise unavoidably, such as aperiodic sampling, quantization, transmission delay, packet loss, and susceptibility to cyber attacks.
- Traditional state observers for switched systems are no longer applicable.

Sampled-data observer (three main structures)

1. Zero-order-hold (ZOH) observer, [Raff 2008 ACC], [Ferrante 2019 TAC].
2. Continuous-discrete observer (Impulsive observer) = state predictor + implulsive corrector, [Ahmed-Ali 2009 Automatica], [Ferrante 2022 CSL].
3. Output predictor, [Karafyllis 2009 TAC], [Karafyllis 2020 SCL].

Sampled-data observer design for NSS

- In [Etienne 2020 IFAC], asynchronous sampled-data observer is proposed.

However, event-triggered state estimation for switched system with (a)synchronous switching remains an open problem.

Problem formulation

Consider the switched linear system (SLS):

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t), \\ y(t) = C_{\sigma(t)}x(t) \end{cases} \tag{1}$$

- $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^p$ represent the state, input and output, respectively.
- $\sigma(t) : [0, +\infty) \to \mathcal{P} = \{1, 2, \cdots, M\}$ is the switching signal, a piecewise right-continuous constant function. Besides, $A_i, B_i, C_i, \forall i \in \mathcal{P}$ are constant matrices.
- $0 = t_0^\sigma < t_1^\sigma < \cdots < t_l^\sigma < t_{l+1}^\sigma < \cdots$ stands for the switching time sequences, with an average dwell time (ADT) constraint.
- $\{t_k\}_{k=0}^{\infty}$ denotes the aperiodic sampling times, generated by an event-triggered mechanism.

Problem formulation

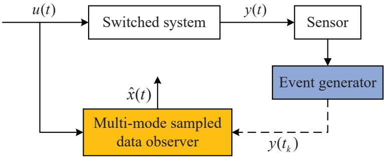


Figure 1: ET observer under synchronous switching.

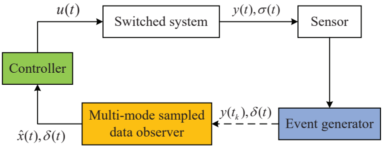


Figure 2: ET observer-based control under asynchronous switching.

Goal:

1) Construct a **multi-mode sampled-data observer** with input $u(t)$ and aperiodic sampled output $\{y(t_k)\}$ generated by an **event-triggered (ET) mechanism** under *synchronous* switching.
2) Besides, design an **observer-based controller** to stabilize the SLS under *asynchronous* switching.

1) Observer structure and ET mechanism under synchronous switching

Multi-mode sampled-data observer

For $\forall\ t \geq 0$,

$$\begin{cases} \dot{\hat{x}}(t) = A_{\sigma(t)}\hat{x}(t) + B_{\sigma(t)}u(t) + L_{\sigma(t)}(w(t) - C_{\sigma(t)}\hat{x}(t)) \\ \hat{y}(t) = C_{\sigma(t)}\hat{x}(t) \end{cases} \quad (2)$$

For $k \in \mathbb{N}$,

$$\begin{cases} \dot{w}(t) = C_{\sigma(t)}A_{\sigma(t)}\hat{x}(t) + C_{\sigma(t)}B_{\sigma(t)}u(t), \forall t \in [t_k, t_{k+1}) \\ w(t_{k+1}) = y(t_{k+1}) \leftarrow \text{reset with new sampled output} \end{cases} \quad (3)$$

where $\hat{x}(t) \in \mathbb{R}^n$ denotes the estimation of $x(t)$, $\hat{y}(t) \in \mathbb{R}^p$ is the estimation of $y(t)$, $w(t) \in \mathbb{R}^p$ denotes the prediction of $y(t)$, and $L_i, i \in \mathcal{P}$ are observer gains to be designed. $t_k, k \in \mathbb{N}$ is the aperiodic sampling and transmission instant.

1) Observer structure and ET mechanism under synchronous switching

In order to save communication and computing resources, the following ET mechanism is proposed.

ET mechanism

$$t_{k+1} = \inf\{\, t > t_k \,|\, \|e_w(t)\|^2 > \eta \|e_y(t)\|^2 \} \tag{4}$$

where $e_w(t) = y(t) - w(t)$ stands for the output prediction error, $e_y(t) = y(t) - \hat{y}(t)$ represents the output estimation error, and η is a positive event-triggered tuning parameter.

Define the state estimation error as $e(t) = x(t) - \hat{x}(t)$, one obtains

$$\dot{e}(t) = (A_{\sigma(t)} - L_{\sigma(t)} C_{\sigma(t)}) e(t) + L_{\sigma(t)} e_w(t) \tag{5}$$

1) Stability analysis and observer gain design under synchronous switching

Theorem 1

For given constants $\lambda > 0$, $\mu \geq 1$, $\eta > 0$, $\vartheta > 0$, if there exist matrices $G_i, P_i > 0, P_j > 0, \forall i \neq j \in \mathcal{P}$ satisfying

$$\begin{bmatrix} \Phi_i & G_i \\ * & -\vartheta I \end{bmatrix} < 0 \tag{6}$$

$$P_i \leq \mu P_j \tag{7}$$

with $\Phi_i = A_i^T P_i + P_i A_i - G_i C_i - C_i^T G_i^T + \vartheta \eta C_i^T C_i + \lambda P_i$, and $L_i = (P_i)^{-1} G_i$. Then, system (5) is GUES for any ADT switching signal satisfying

$$\tau_a \geq \tau_a^* = \frac{\ln \mu}{\lambda}. \tag{8}$$

1) Exclusion of Zeno Phenomenon

According to Theorem 1, the state estimation error $e(t)$ is known to be GUES, thus it is bounded, i.e., $\|e(t)\| \leq \Lambda$ for some constant $\Lambda > 0$.

Furthermore, owing to the precision of sensor, when implementing the proposed sampled-data observer, the event-triggered condition (4) can be detected only when $\|e_y(t)\|^2 \geq \rho$, which is a small constant related to sensor accuracy.

Theorem 2

With the event-triggered condition (4), for any small positive constant ρ, there exists a constant $\tau > 0$ satisfying the following relationship

$$t_{k+1} - t_k > \tau, \forall k \in \mathbb{N} \tag{9}$$

where $\tau = \frac{\sqrt{\eta\rho}}{\max\limits_{i\in\mathcal{P}}\|C_i\| \max\limits_{i\in\mathcal{P}}\|A_i\|\Lambda}$.

1) Illustrative example

Example 1: Consider the following electronic circuit system.

$$A_1 = \begin{bmatrix} -\frac{1}{R_1 C} & 0 \\ 0 & -\frac{R_2}{L} \end{bmatrix}, A_2 = \begin{bmatrix} -\frac{1}{R_1 C} & \frac{1}{C} \\ -\frac{1}{L} & 0 \end{bmatrix}$$
$$B_1 = B_2 = \begin{bmatrix} 0 & \frac{1}{L} \end{bmatrix}^T, C_1 = C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

where $R_1 = 3.33\Omega$, $R_2 = 0.05\Omega$, $C = 10\mathrm{mF}$, $L = 10\mathrm{mH}$.
Let $\lambda = 8$, $\mu = 1.1$, $\eta = 0.9$, $\vartheta = 1$, based on Theorem 1, the feasible observer gains and ADT are calculated as

$$L_1 = [-6.4138 \ -0.0916]^T, L_2 = [-2.9830 \ -1.6643]^T$$

and $\tau_a \geq \tau_a^* = 0.0119$. The input is chosen as $u(t) = 20\sin(50t)$. The initial conditions are $x(0) = [1, 3]^T$, $\hat{x}(0) = [0, 0]^T$, and the precision of sensor is set as $\rho = 10^{-5}$.

1) Illustrative example

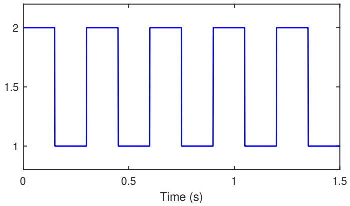


Figure 3: Switching signal with $\tau_a = 0.15$.

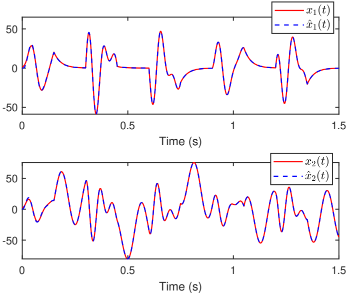


Figure 4: State trajectory and its estimation.

1) Illustrative example

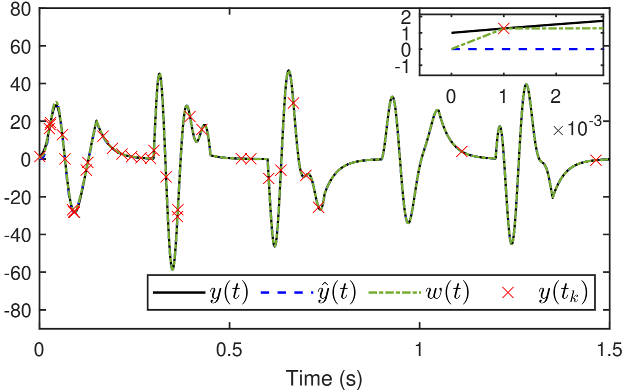


Figure 5: System output $y(t)$, output estimation $\hat{y}(t)$, output prediction $w(t)$ and sampled output $y(t_k)$.

2) Observer-based controller and ET mechanism under asynchronous switching

In this case, the output $y(t)$ and switching signal $\sigma(t)$ will be sampled and transmitted to the observer together, which may cause frequent switching during $[t_k, t_{k+1}), k \in \mathbb{N}$.
The multi-mode sampled-data observer (2-3) becomes
For $\forall\ t \geq 0$,

$$\begin{cases} \dot{\hat{x}}(t) = A_{\delta(t)}\hat{x}(t) + B_{\delta(t)}u(t) + L_{\delta(t)}(w(t) - C_{\delta(t)}\hat{x}(t)) \\ \hat{y}(t) = C_{\delta(t)}\hat{x}(t) \end{cases} \tag{10}$$

For $k \in \mathbb{N}$,

$$\begin{cases} \dot{w}(t) = C_{\delta(t)}A_{\delta(t)}\hat{x}(t) + C_{\delta(t)}B_{\delta(t)}u(t), \forall t \in [t_k, t_{k+1}) \\ w(t_{k+1}) = y(t_{k+1}) \end{cases} \tag{11}$$

where $\delta(t) = \sigma(t_k), t \in [t_k, t_{k+1})$.

2) Observer-based controller and ET mechanism under asynchronous switching

The ET mechanism (4) is modified as below:

$$t_{k+1} = \min\{\bar{t}_{k+1}, t_k + H\} \tag{12}$$

$$\bar{t}_{k+1} = \inf\{\, t > t_k \,|\, \|e_w(t)\|^2 > \eta \|e_y(t)\|^2\} \tag{13}$$

where $\eta > 0$ and $H > 0$ are given constants.
[*Remark*]. The parameter H in (12) represents the upper bound of asynchronous time in $[t_k, t_{k+1})$, which can be larger than the minimum dwell time τ_d, i.e., $t^{\sigma}_{l+1} - t^{\sigma}_{l} \geq \tau_d, \forall l \in \mathbb{N}$.
[*Assumption 1*]. For every $i \in \mathcal{P}$, there exist K_i such that $A_i + B_i K_i$ is Hurwitz.
The following feedback control law is adopted

$$u(t) = K_{\delta(t)} \hat{x}(t). \tag{14}$$

2) Stability analysis and observer gain design under asynchronous switching

Augment $x(t)$ and $e(t)$ as $\varsigma(t) = [x^T(t), e^T(t)]^T$, then one gets

$$\dot{\varsigma}(t) = \mathfrak{A}_{\sigma(t),\delta(t)}\varsigma(t) + \mathfrak{B}_{\delta(t)}e_w(t) \tag{15}$$

where

$$\begin{aligned}
\mathfrak{A}_{\sigma(t),\delta(t)} &= \begin{bmatrix} A_{\sigma(t)} + B_{\sigma(t)}K_{\delta(t)} & -B_{\sigma(t)}K_{\delta(t)} \\ \Xi_{\sigma(t),\delta(t)} & \Gamma_{\sigma(t),\delta(t)} \end{bmatrix}, \\
\mathfrak{B}_{\delta(t)} &= \begin{bmatrix} 0 \\ L_{\delta(t)} \end{bmatrix}, \\
\Xi_{\sigma(t)\delta(t)} &= A_{\sigma(t),\delta(t)} + B_{\sigma(t),\delta(t)}K_{\delta(t)} - L_{\delta(t)}C_{\sigma(t),\delta(t)}, \\
\Gamma_{\sigma(t),\delta(t)} &= A_{\delta(t)} - B_{\sigma(t),\delta(t)}K_{\delta(t)} - L_{\delta(t)}C_{\delta(t)}, \\
A_{\sigma(t),\delta(t)} &= A_{\sigma(t)} - A_{\delta(t)}, B_{\sigma(t),\delta(t)} = B_{\sigma(t)} - B_{\delta(t)}, \\
C_{\sigma(t),\delta(t)} &= C_{\sigma(t)} - C_{\delta(t)}.
\end{aligned}$$

2) Stability analysis and observer gain design under asynchronous switching

Theorem 3

Given constants $\lambda > 0$, $\kappa > 0$, $\mu \geq 1$, $\eta > 0$, $\vartheta > 0$ and matrices K_i satisfying Assumption 1, if there exist matrices $N_i, P_i > 0, P_j > 0, \forall i \neq j \in \mathcal{P}$ such that

$$\begin{bmatrix} \Psi_{ji}^{11} & \Psi_{ji}^{12} & 0 \\ * & \Psi_{ji}^{22} & N_i \\ * & * & -I \end{bmatrix} < 0 \tag{16}$$

$$\begin{bmatrix} \Psi_{jj}^{11} & \Psi_{jj}^{12} & 0 \\ * & \Psi_{jj}^{22} & N_j \\ * & * & -I \end{bmatrix} < 0 \tag{17}$$

$$P_j \leq \mu P_i \tag{18}$$

2) Stability analysis and observer gain design under asynchronous switching

Theorem 3 (continued)

Then, system (15) is GUES under Assumption 1 for any ADT switching signal satisfying

$$\tau_a \geq \tau_a^* = \frac{\ln \mu + (\lambda + \kappa)H}{\lambda} \tag{19}$$

where

$$\begin{aligned}
\Psi_{ji}^{11} = {} & P_i A_j + A_j^T P_i + P_i B_j K_i + K_i^T B_j^T P_i - \kappa P_i \\
& + \eta(1 + \vartheta) C_{ji}^T C_{ji}, \\
\Psi_{ji}^{12} = {} & - P_i B_j K_i + A_{ji}^T P_i + K_i^T B_{ji}^T P_i - C_{ji}^T N_i^T,
\end{aligned}$$

2) Stability analysis and observer gain design under asynchronous switching

Theorem 3 (continued)

$$
\begin{aligned}
\Psi_{ji}^{22} &= P_i A_i + A_i^T P_i - P_i B_{ji} K_i - K_i^T B_{ji}^T P_i - \kappa P_i \\
&\quad - N_i C_i - C_i^T N_i^T + \eta(1+\vartheta^{-1}) C_i^T C_i, \\
\Psi_{jj}^{11} &= P_j A_j + A_j^T P_j + P_j B_j K_j + K_j^T B_j^T P_j + \lambda P_j, \\
\Psi_{jj}^{12} &= -P_j B_j K_j, \\
\Psi_{jj}^{22} &= P_j A_j + A_j^T P_j - N_j C_j - C_j^T N_j^T + \eta C_j^T C_j + \lambda P_j, \\
L_i &= (P_i)^{-1} N_i.
\end{aligned}
$$

Similar to Theorem 2, a positive lower bound of inter-event intervals exists in this case.

2) Illustrative example

Example 2: Consider the switched linear system as follows.

$$A_1 = \begin{bmatrix} -0.8 & 0.2 & -0.2 \\ 0.2 & -0.2 & 0.3 \\ -0.3 & 0.1 & -0.1 \end{bmatrix}, A_2 = \begin{bmatrix} -0.8 & -0.1 & -0.2 \\ 0.2 & -0.7 & 0.3 \\ 0.2 & -0.1 & 0.1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 \\ 0.5 \\ 2.0 \end{bmatrix}, B_2 = \begin{bmatrix} 0.5 \\ 0.7 \\ 1.5 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0.8 & 1.0 & 1.5 \end{bmatrix}, C_2 = \begin{bmatrix} 0.6 & 1.2 & 0.3 \end{bmatrix}$$

In this case, both $y(t)$ and $\sigma(t)$ are sampled and transmitted through the network. We concern the ET observer-based control under asynchronous ADT switching.

2) Illustrative example

Let $\lambda = 0.25$, $\kappa = 1.3$, $\mu = 1.01$, $\eta = 0.6$, $\vartheta = 4$, $H = 0.25$, and the controller gains are set as

$$K_1 = \begin{bmatrix} -0.3964 & -1.0469 & -0.1151 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} -0.1849 & 0.0606 & -0.5333 \end{bmatrix}$$

From Theorem 3, it yields that the observer gains and ADT are

$$L_1 = \begin{bmatrix} 0.6520 & 2.5963 & 5.6169 \end{bmatrix}^T$$
$$L_2 = \begin{bmatrix} 0.2992 & 2.7699 & 5.7164 \end{bmatrix}^T$$

$\tau_a \geq \tau_a^* = 1.5898$. The initial states are chosen as $x(0) = \begin{bmatrix} 1 & -1.5 & -0.5 \end{bmatrix}^T$, $\hat{x}(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$, the sensor accuracy is $\rho = 10^{-5}$.

2) Illustrative example

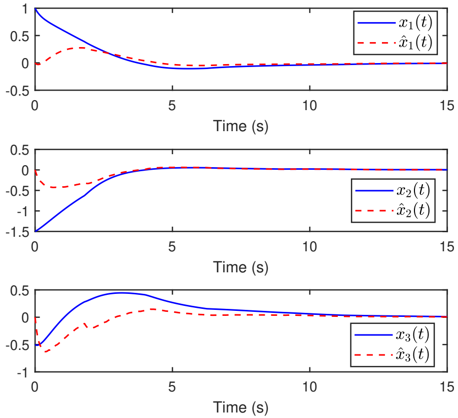


Figure 6: State trajectory and its estimation.

2) Illustrative example

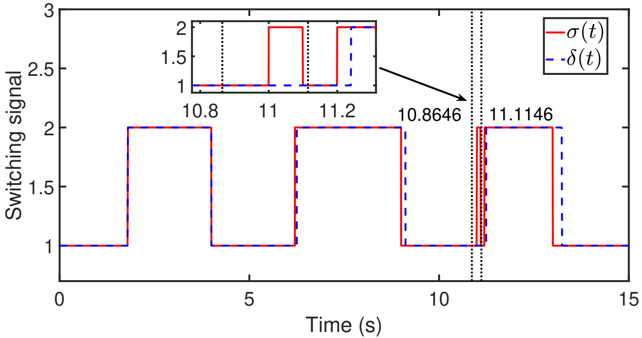


Figure 7: Switching signal of the system and observer.

2) Illustrative example

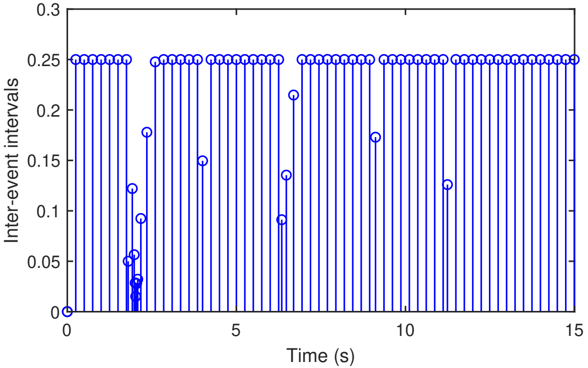


Figure 8: Event-triggered instants and inter-event intervals.

Conclusion

- A new multi-mode output predictor-based sampled-data observer is presented to reconstruct the continuous system state for networked switched systems.
- Co-design of the observer gains and a novel ET mechanism is established based on linear matrix inequalities (LMI) and ADT techniques.
- Both synchronous and frequent asynchronous switching scenarios are considered, and sufficient GUES conditions are obtained for the estimation error system.

Ongoing work

1. Observer design for switched linear systems with sampled and delayed output
2. Observability and observer design for aperiodic sampled switched linear systems with unknown switching signal (joint work with *Elena De Santis* at University of L'Aquila).

Thank you!