

# COMSCI 33 (AUTOMATA THEORY)

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## Lesson 2: Regular Expression

# Regular Language

- In theoretical computer science and formal language theory, a **regular language** is a formal language that can be expressed using a regular expression

# Regular Expression

- A language-defining notation
- A declarative way to express the string we want to accept
- The Regular Expressions (RE) over an alphabet  $\Sigma$  are the strings over the alphabet  $\Sigma$  union with parentheses, star operator and plus sign.

$$r = \Sigma \in \{ (, ), *, + \}$$

# Definition of a Regular Expression

- Let  $\Sigma$  be an alphabet. The regular expressions over  $\Sigma$  are:
  - $\emptyset$  Represents the empty set  $\{ \}$
  - $\epsilon$  Represents the set  $\{\epsilon\}$
  - $a$  Represents the set  $\{a\}$ , for any symbol  $a$  in  $\Sigma$

Let  $r$  and  $s$  be regular expressions that represent the sets  $R$  and  $S$ , respectively.

- $r+s$  Represents the set  $R \cup S$
  - $rs$  Represents the set  $RS$
  - $r^*$  Represents the set  $R^*$
  - $(r)$  Represents the set  $R$
- If  $r$  is a regular expression, then  $L(r)$  is used to denote the corresponding language.

# Example: Let $\Sigma = \{0, 1\}$

$(0 + 1)^*$  All strings of 0's and 1's

$01^*$  0 followed by any number 1's

$0(0 + 1)^*$  All strings of 0's and 1's, beginning with a 0

$(0 + 1)^*1$  All strings of 0's and 1's, ending with a 1

# Practice Problems

- What is the RE of the following, Let  $\Sigma = \{a, b\}$ 
  - $L = \{b, ba, baa, baaa, \dots\}$
  - $L = \{aa, bb, ab, ba\}$
  - $L = \{a, aaa, aaaaa, aaaaaaa, aaaaaaaaa, \dots\}$
- $L = \{w \in \{a,b\}^* / w \text{ starts with } a\}$
- $L = \{w \in \{a,b\}^* / w \text{ contains the substring } aa\}$
- $L = \{w \in \Sigma^* / \text{the third to the last symbol in } w \text{ is an } a\}$
- $L = \{w \in \Sigma^* / |w| < 2\}$
- $L = \{w \in \Sigma^* / w \text{ contains exactly 3 a's}\}$

# Assignment

- $L = \{ w \in \{a,b\}^* / w \text{ contains } aa \text{ or } bb \}$
- $L = \{ w \in \{a,b\}^* / w \text{ contains the substring } aba \}$
- $L = \{ w \in \{a,b\}^* / w \text{ starts and ends in } b \}$
- $L = \{ w \in \{a,b\}^* / |w| = 2k \text{ where } k=0,1,2,3,\dots \}$
- $L = \{ w \in \{a,b\}^* / w \text{ ends in } ab \}$
- $L = \{ w \in \{a,b\}^* / w \text{ contains at most 1 } b \}$
- $L = \{ w \in \{a,b\}^* / w \text{ contains the substring } aa \}$
- $L = \{ w \in \{a,b\}^* / w \text{ contains 0 or more clumps of } aba's \}$
- $L = \{ w \in \{a,b\}^* / \text{second symbol in } w \text{ is an } a \}$
- $L = \{ w \in \{a,b\}^* / |w| = 3k + 1 \text{ where } k=0,1,2,3,\dots \}$
- $L = \{ w \in \{a,b\}^* / w \text{ contains all } a's \text{ or all } b's \}$
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