FA and RE

Theorem: if L is a language accepted by a DFA, then L is denoted by a regular expressions

$L(M) = \cup \{ R(i, j, n+1) / q_j \in F \}$

where $i \rightarrow$ start state

$j \rightarrow$ final state

$n \rightarrow$ number of states

$$R(i,j,1) = \begin{cases} \{ \sigma \in \Sigma / \delta(q_i, \sigma) = q_j \} & \text{if } i \neq j \\ \{\{\epsilon\} + \{\sigma \in \Sigma / \delta(q_i, \sigma) = q_j\}\} & \text{if } i = j \end{cases}$$

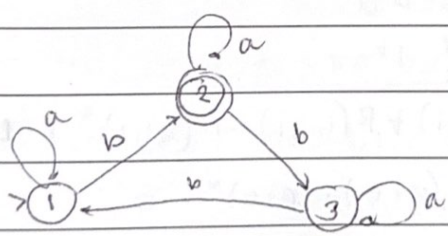
$$R(i,j,k+1) = R(i,j,k) + R(i,k,k) \cdot R(k,k,k)^* \cdot R(k,j,k)$$

where $i \rightarrow$ start state

$j \rightarrow$ final state

$k \rightarrow$ number of states

Example:



find $L(M) = R(1,2,4)$

note: if multiple final states use union; ie. if 3 is also final it would be

$L(M) = R(1,2,4) + R(1,3,4)$

step 1: with the move

$R(1,1,1) = a+e$ ~ +e because $i=j$ (from the rule)

$R(1,2,1) = b$

$R(1,3,1) = \emptyset$

$R(2,1,1) = \emptyset$

$R(2,2,1) = a+e$

$R(2,3,1) = b$

$R(3,1,1) = b$

$R(3,2,1) = \emptyset$

$R(3,3,1) = a+e$

step 2:

$R(1,2,4) = R(1,2,3) + R(1,3,3) \cdot R(3,3,3)^* \cdot R(3,2,3)$

$i=1;\ j=2;\ k=3+1$

solving for $R(1,2,3)$:

$R(1,2,3) = R(1,2,2) + R(1,2,2) \cdot R(2,2,2)^* \cdot R(2,2,2)$

$i=1;\ j=2;\ k=2$

$= a^*b + a^*b \cdot (a+e)^* \cdot (a+e)$

$= a^*b + a^*b\,a^*$

$= \underline{\underline{a^*b\,a^*}}$

$R(1,2,2) = R(1,2,1) + R(1,1,1) \cdot R(1,1,1)^* \cdot R(2,2,1)$

$i\ j\ k=1$

$= b + (a+e) \cdot (a+e)^* \cdot b$

$= b + a^*b$

$= a^*b$

$R(2,2,2) = R(2,2,1) + R(2,1,1) \cdot R(1,1,1)^* \cdot R(1,2,1)$

$i=2\ j=2\ k=1$

$= (a+e) + \cancel{\emptyset \cdot (a+e)^* \cdot b} \to \emptyset$

$= a+e$

~~$R(1,2,2) = R(1,2,1) + R(1,1,1) \cdot R(1,1,1)^* \cdot R(1,2,1)$~~

~~$i=1\ j=2$~~ $k=1$

Solving for R(1,3,3):

$$R(1,3,3) = R(1,3,2) + R(1,2,2)\cdot R(2,2,2)^{*}\cdot R(2,3,2)$$

$i=1\ j=3\ k=2$

$$= \emptyset + (a^{*}b)\cdot(a+e)^{*}(b)$$

$$= \emptyset + a^{*}ba^{*}b$$

$$= \underline{a^{*}ba^{*}b}$$

$$R(1,3,2) = R(1,3,1) + R(1,1,1)\cdot R(1,1,1)^{*} R(1,3,1)$$

$i=1\ j=3\ k=1$

$$= \emptyset + (a+e)\cdot(a+e)^{*}\cdot\emptyset$$

$$= \emptyset$$

$$R(2,3,2) = R(2,3,1) + R(2,1,1)\cdot R(1,1,1)^{*}\cdot R(1,3,1)$$

$i=2\ j=3\ k=1$

$$= b + \cancel{\emptyset\ (a+e)^{*}\cdot\emptyset} \rightarrow \emptyset$$

$$= b + \emptyset$$

$$= b$$

Solving for R(3,3,3):

$$R(3,3,3) = R(3,3,2) + R(3,2,2)\cdot R(2,2,2)^{*}\cdot R(2,3,2)$$

$i=3\ j=3\ k=2$

$$= (a+e) + (ba^{*}b)\,(a+e)^{*}\ b$$

$$= (a+e) + (ba^{*}b)\,a^{*}b$$

$$= \underline{a+e+ba^{*}ba^{*}b}$$

$$R(3,3,2) = R(3,3,1) + R(3,1,1)\cdot R(1,1,1)^{*}\cdot R(1,3,1)$$

$i=3\ j=3\ k=1$

$$= (a+e) + \cancel{b\cdot(a+e)^{*}\cdot\emptyset} \rightarrow \emptyset$$

$$= a+e+\emptyset$$

$$= a+e$$

$$R(3,2,2) = R(3,2,1) + R(3,1,1)\cdot R(1,1,1)^{*}\cdot R(1,2,1)$$

$i=3\ j=2\ k=1$

$$= \emptyset + b\,(a+e)^{*}\,b$$

$$= ba^{*}b$$

Solving for $R(3,2,3)$:

$R(3,2,3) = R(3,2,2) + R(3,2,2) \cdot R(2,2,2)^* \cdot R(2,2,2)$

$i=3\ j=2\ k=2$ $= ba^*b + ba^*b \cdot (a+e)^* \cdot (a+e)$

$= ba^*b + ba^*ba^*$

$= \underline{\underline{ba^*ba^*}}$

so: $R(1,2,4) = R(1,2,3) + R(1,3,3) \cdot R(3,3,3)^* R(3,2,3)$

$i=1\ j=2\ k=3$ $= a^*ba^* + (a^*ba^*b) \cdot (a+e+ba^*ba^*b)^* (ba^*ba^*)$

$= \underline{\underline{a^*ba^* + (a^*ba^*b)(a+ba^*ba^*b)^*(ba^*ba^*)}}$

which is:

$L = \{ w \in \{a,b\}^* \,/\, w$ contains $3k+1$ number of b's where $k = 0,1,2,\ldots \}$

$L(a^*ba^*(a^*ba^*ba^*ba^*)^*$

Assignment : ~~for Aug 4, 2007~~