Forecast at the Appropriate Level of Aggregation

Given that aggregate forecasts are more accurate than disaggregate forecasts, it is important to forecast at a level of aggregation that is appropriate, given the supply chain decision that is driven by the forecast. Consider a buyer at a retail chain who is forecasting to select an order size for shirts. One approach is to ask each store manager the precise number of shirts needed and add up all the requests to get an order size with the supplier. The advantage of this approach is that it uses local market intelligence that each store manager has. The problem with this approach is that it makes store managers forecast well before demand arises at a time when their forecasts are unlikely to be accurate. A better approach may be to forecast demand at the aggregate level when ordering with the supplier and ask each store manager to forecast only when the shirts are to be allocated across the stores. In this case, the long lead time forecast (supplier order) is aggregate, thus lowering error. The disaggregate store-level forecast is made close to the sales season, when local market intelligence is likely to be most effective.

Establish Performance and Error Measures for the Forecast

Companies should establish clear performance measures to evaluate the accuracy and timeliness of the forecast. These measures should be linked to the objectives of the business decisions based on these forecasts. Consider a mail-order company that uses a forecast to place orders with its suppliers, which take two months to send in the orders. The mail-order company must ensure that the forecast is created at least two months before the start of the sales season because of the two-month lead time for replenishment. At the end of the sales season, the company must compare actual demand to forecasted demand to estimate the accuracy of the forecast. Then plans for decreasing future forecast errors or responding to the observed forecast errors can be put into place.

In the next section, we discuss techniques for static and adaptive time-series forecasting.

7.5 TIME-SERIES FORECASTING METHODS

The goal of any forecasting method is to predict the systematic component of demand and estimate the random component. In its most general form, the systematic component of demand data contains a level, a trend, and a seasonal factor. The equation for calculating the systematic component may take a variety of forms:

- ***Multiplicative:*** Systematic component = level × trend × seasonal factor
- ***Additive:*** Systematic component = level + trend + seasonal factor
- ***Mixed:*** Systematic component = (level + trend) × seasonal factor

The specific form of the systematic component applicable to a given forecast depends on the nature of demand. Companies may develop both static and adaptive forecasting methods for each form. We now describe these static and adaptive forecasting methods.

Static Methods

A static method assumes that the estimates of level, trend, and seasonality within the systematic component do not vary as new demand is observed. In this case, we estimate each of these parameters based on historical data and then use the same values for all future forecasts. In this section, we discuss a static forecasting method for use when demand has a trend as well as a seasonal component. We assume that the systematic component of demand is mixed; that is,

$$\text{Systematic component} = (\text{level} + \text{trend}) \times \text{seasonal factor}$$

A similar approach can be applied for other forms as well. We begin with a few basic definitions:

L = estimate of level at $t = 0$ (the deseasonalized demand estimate during Period $t = 0$)

T = estimate of trend (increase or decrease in demand per period)

TABLE 7-1 Quarterly Demand for Tahoe Salt

Year	Quarter	Period, t	Demand, D_t
1	2	1	8,000
1	3	2	13,000
1	4	3	23,000
2	1	4	34,000
2	2	5	10,000
2	3	6	18,000
2	4	7	23,000
3	1	8	38,000
3	2	9	12,000
3	3	10	13,000
3	4	11	32,000
4	1	12	41,000

S_t = estimate of seasonal factor for Period t

D_t = actual demand observed in Period t

F_t = forecast of demand for Period t

In a static forecasting method, the forecast in Period t for demand in Period $t + l$ is a product of the level in Period $t + l$ and the seasonal factor for Period $t + l$. The level in Period $t + l$ is the sum of the level in Period 0 (L) and ($t + l$) times the trend T. The forecast in Period t for demand in Period $t + l$ is thus given as

$$F_{t+l} = [L + (t + l)T]S_{t+l} \tag{7.1}$$

We now describe one method for estimating the three parameters L, T, and S. As an example, consider the demand for rock salt used primarily to melt snow. This salt is produced by a firm called Tahoe Salt, which sells its salt through a variety of independent retailers around the Lake Tahoe area of the Sierra Nevada Mountains. In the past, Tahoe Salt has relied on estimates of demand from a sample of its retailers, but the company has noticed that these retailers always overestimate their purchases, leaving Tahoe (and even some retailers) stuck with excess inventory. After meeting with its retailers, Tahoe has decided to produce a collaborative forecast. Tahoe Salt wants to work with the retailers to create a more accurate forecast based on the actual retail sales of their salt. Quarterly retail demand data for the past three years are shown in Table 7-1 and charted in Figure 7-1.

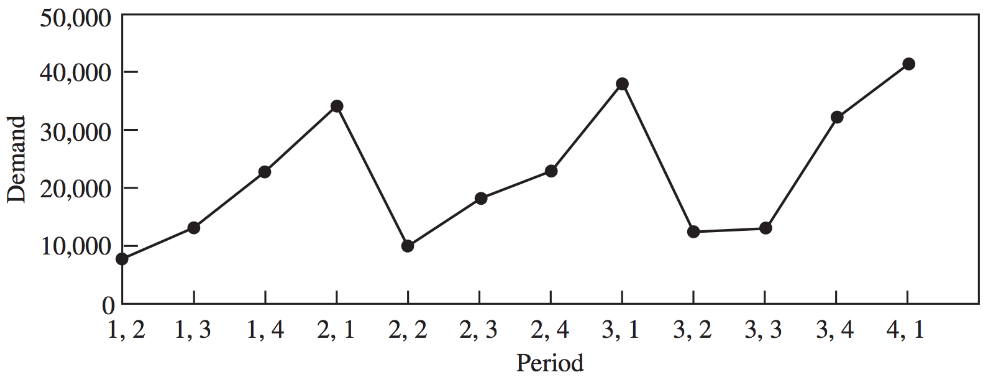

FIGURE 7-1 Quarterly Demand at Tahoe Salt

In Figure 7-1, observe that demand for salt is seasonal, increasing from the second quarter of a given year to the first quarter of the following year. The second quarter of each year has the lowest demand. Each cycle lasts four quarters, and the demand pattern repeats every year. There is also a growth trend in the demand, with sales growing over the past three years. The company estimates that growth will continue in the coming year at historical rates. We now describe the following two steps required to estimate each of the three parameters—level, trend, and seasonal factors.

1. Deseasonalize demand and run linear regression to estimate level and trend.
2. Estimate seasonal factors.

ESTIMATING LEVEL AND TREND The objective of this step is to estimate the level at Period 0 and the trend. We start by *deseasonalizing* the demand data. *Deseasonalized demand* represents the demand that would have been observed in the absence of seasonal fluctuations. The *periodicity* (p) is the number of periods after which the seasonal cycle repeats. For Tahoe Salt's demand, the pattern repeats every year. Given that we are measuring demand on a quarterly basis, the periodicity for the demand in Table 7-1 is $p = 4$.

To ensure that each season is given equal weight when deseasonalizing demand, we take the average of p consecutive periods of demand. The average of demand from Period $l + 1$ to Period $l + p$ provides deseasonalized demand for Period $l + (p + 1)/2$. If p is odd, this method provides deseasonalized demand for an existing period. If p is even, this method provides deseasonalized demand at a point between Period $l + (p/2)$ and Period $l + 1 + (p/2)$. By taking the average of deseasonalized demand provided by Periods $l + 1$ to $l + p$ and $l + 2$ to $l + p + 1$, we obtain the deseasonalized demand for Period $l + 1 + (p/2)$ if p is even. Thus, the deseasonalized demand, $\overline{D}_t$, for Period t, can be obtained as follows:

$$\overline{D}_t = \begin{cases} \left[D_{t-(p/2)} + D_{t+(p/2)} + \sum_{i=t+1-(p/2)}^{t-1+(p/2)} 2D_i \right] \Big/ (2p) \text{ for } p \text{ even} \\ \sum_{i=t-[(p-1)/2]}^{t+[(p-1)/2]} D_i/p \text{ for } p \text{ odd} \end{cases} \tag{7.2}$$

In our example, $p = 4$ is even. For $t = 3$, we obtain the deseasonalized demand using Equation 7.2 as follows:

$$\overline{D}_3 = \left[D_{t-(p/2)} + D_{t+(p/2)} + \sum_{i=t+1-(p/2)}^{t-1+(p/2)} 2D_i \right] \Big/ (2p) = D_1 + D_5 + \sum_{i=2}^{4} 2D_i \Big/ 8$$

With this procedure, we can obtain deseasonalized demand between Periods 3 and 10 as shown in Figures 7-2 and 7-3 (all details are available in the accompanying spreadsheet *Chapter 7-Tahoe-salt*).

The following linear relationship exists between the deseasonalized demand, $\overline{D}_t$, and time t, based on the change in demand over time:

$$\overline{D}_t = L + Tt \tag{7.3}$$

In Equation 7.3, $\overline{D}_t$ represents deseasonalized demand and not the actual demand in Period t, L represents the *level* or deseasonalized demand at Period 0, and T represents the rate of growth of deseasonalized demand or *trend.* We can estimate the values of L and T for the deseasonalized demand using linear regression with deseasonalized demand (see Figure 7-2) as the dependent variable and time as the independent variable. Such a regression can be run using Microsoft Excel (Data | Data Analysis | Regression). This sequence of commands opens the Regression

	A	B	C
1	*Period* t	*Demand* D_t	*Deseasonalized Demand*
2	1	8,000	
3	2	13,000	
4	3	23,000	19,750
5	4	34,000	20,625
6	5	10,000	21,250
7	6	18,000	21,750
8	7	23,000	22,500
9	8	38,000	22,125
10	9	12,000	22,625
11	10	13,000	24,125
12	11	32,000	
13	12	41,000	

Cell	Cell Formula	Equation	Copied to
C4	=(B2+B6+2*SUM(B3:B5))/8	7.2	C5:C11

FIGURE 7-2 Excel Workbook with Deseasonalized Demand for Tahoe Salt

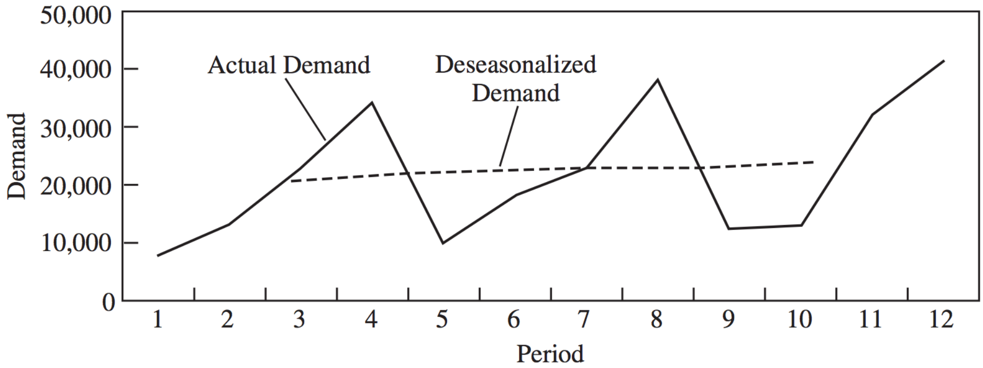

FIGURE 7-3 Deseasonalized Demand for Tahoe Salt

dialog box in Excel. For the Tahoe Salt workbook in Figure 7-2, in the resulting dialog box, we enter

Input *Y* Range:C4:C11

Input *X* Range:A4:A11

and click the OK button. A new sheet containing the results of the regression opens up (see worksheet *Regression-1*). This new sheet contains estimates for both the initial level L and the trend T. The initial level, L, is obtained as the *intercept coefficient,* and the trend, T, is obtained as the *X variable coefficient* (or the slope) from the sheet containing the regression results. For the Tahoe Salt example, we obtain $L = 18{,}439$ and $T = 524$ (all details are available in the worksheet *Regression-1* and numbers are rounded to integer values). For this example, deseasonalized demand $\overline{D}_t$ for any Period t is thus given by

$$\overline{D}_t = 18{,}439 + 524t \quad \textbf{(7.4)}$$

It is not appropriate to run a linear regression between the original demand data and time to estimate level and trend because the original demand data are not linear and the resulting linear regression will not be accurate. The demand must be deseasonalized before we run the linear regression.

	A	B	C	D
1	Period t	Demand D_t	Deseasonalized Demand (Eqn 7.4) $\overline{D}_t$	Seasonal Factor (Eqn 7.5) $\overline{S}_t$
2	1	8,000	18,963	0.42
3	2	13,000	19,487	0.67
4	3	23,000	20,011	1.15
5	4	34,000	20,535	1.66
6	5	10,000	21,059	0.47
7	6	18,000	21,583	0.83
8	7	23,000	22,107	1.04
9	8	38,000	22,631	1.68
10	9	12,000	23,155	0.52
11	10	13,000	23,679	0.55
12	11	32,000	24,203	1.32
13	12	41,000	24,727	1.66

Cell	Cell Formula	Equation	Copied to
C2	=18439+A2*524	7.4	C3:C13
D2	=B2/C2	7.5	D3:D13

FIGURE 7-4 Deseasonalized Demand and Seasonal Factors for Tahoe Salt

ESTIMATING SEASONAL FACTORS We can now obtain deseasonalized demand for each period using Equation 7.4 (see Figure 7-4). The seasonal factor $\overline{S}_t$ for Period t is the ratio of actual demand D_t to deseasonalized demand $\overline{D}_t$ and is given as

$$\overline{S}_t = \frac{D_t}{\overline{D}_t} \tag{7.5}$$

For the Tahoe Salt example, the deseasonalized demand estimated using Equation 7.4 and the seasonal factors estimated using Equation 7.5 are shown in Figure 7-4 (see worksheet *Figure 7-4*).

Given the periodicity p, we obtain the seasonal factor for a given period by averaging seasonal factors that correspond to similar periods. For example, if we have a periodicity of $p = 4$, Periods 1, 5, and 9 have similar seasonal factors. The seasonal factor for these periods is obtained as the average of the three seasonal factors. Given r seasonal cycles in the data, for all periods of the form $pt + i$, $1 \leq i \leq p$, we obtain the seasonal factor as

$$S_i = \frac{\sum_{j=0}^{r-1} \overline{S}_{jp+i}}{r} \tag{7.6}$$

For the Tahoe Salt example, a total of 12 periods and a periodicity of $p = 4$ imply that there are $r = 3$ seasonal cycles in the data. We obtain seasonal factors using Equation 7.6 as

$$S_1 = (\overline{S}_1 + \overline{S}_5 + \overline{S}_9)/3 = (0.42 + 0.47 + 0.52)/3 = 0.47$$
$$S_2 = (\overline{S}_2 + \overline{S}_6 + \overline{S}_{10})/3 = (0.67 + 0.83 + 0.55)/3 = 0.68$$
$$S_3 = (\overline{S}_3 + \overline{S}_7 + \overline{S}_{11})/3 = (1.15 + 1.04 + 1.32)/3 = 1.17$$
$$S_4 = (\overline{S}_4 + \overline{S}_8 + \overline{S}_{12})/3 = (1.66 + 1.68 + 1.66)/3 = 1.67$$

At this stage, we have estimated the level, trend, and all seasonal factors. We can now obtain the forecast for the next four quarters using Equation 7.1. In the example, the forecast for the next four periods using the static forecasting method is given by

$$F_{13} = (L + 13T)S_{13} = (18{,}439 + 13 \times 524)0.47 = 11{,}868$$
$$F_{14} = (L + 14T)S_{14} = (18{,}439 + 14 \times 524)0.68 = 17{,}527$$

$$F_{15} = (L + 15T)S_{15} = (18{,}439 + 15 \times 524)1.17 = 30{,}770$$
$$F_{16} = (L + 16T)S_{16} = (18{,}439 + 16 \times 524)1.67 = 44{,}794$$

Tahoe Salt and its retailers now have a more accurate forecast of demand. Without the sharing of sell-through information between the retailers and the manufacturer, this supply chain would have a less accurate forecast, and a variety of production and inventory inefficiencies would result.

Adaptive Forecasting

In adaptive forecasting, the estimates of level, trend, and seasonality are updated after each demand observation. The main advantage of adaptive forecasting is that estimates incorporate all new data that are observed. We now discuss a basic framework and several methods that can be used for this type of forecast. The framework is provided in the most general setting, when the systematic component of demand data has the mixed form and contains a level, a trend, and a seasonal factor. It can easily be modified for the other two cases, however. The framework can also be specialized for the case in which the systematic component contains no seasonality or trend. We assume that we have a set of historical data for n periods and that demand is seasonal, with periodicity p. Given quarterly data, wherein the pattern repeats itself every year, we have a periodicity of $p = 4$.

We begin by defining a few terms:

L_t = estimate of level at the end of Period t

T_t = estimate of trend at the end of Period t

S_t = estimate of seasonal factor for Period t

F_t = forecast of demand for Period t (made in Period $t - 1$ or earlier)

D_t = actual demand observed in Period t

$E_t = F_t - D_t$ = forecast error in Period t

In adaptive methods, the forecast for Period $t + l$ in Period t uses the estimate of level and trend in Period t (L_t and T_t respectively) and is given as

$$F_{t+l} = (L_t + lT_t)S_{t+l} \tag{7.7}$$

The four steps in the adaptive forecasting framework are as follows:

1. ***Initialize:*** Compute initial estimates of the level (L_0), trend (T_0), and seasonal factors ($S_1, \ldots, S_p$) from the given data. This is done exactly as in the static forecasting method discussed earlier in the chapter with $L_0 = L$ and $T_0 = T$.
2. ***Forecast:*** Given the estimates in Period t, forecast demand for Period $t + 1$ using Equation 7.7. Our first forecast is for Period 1 and is made with the estimates of level, trend, and seasonal factor at Period 0.
3. ***Estimate error:*** Record the actual demand D_{t+1} for Period $t + 1$ and compute the error E_{t+1} in the forecast for Period $t + 1$ as the difference between the forecast and the actual demand. The error for Period $t + 1$ is stated as

$$E_{t+1} = F_{t+1} - D_{t+1} \tag{7.8}$$

4. ***Modify estimates:*** Modify the estimates of level (L_{t+1}), trend (T_{t+1}), and seasonal factor (S_{t+p+1}), given the error E_{t+1} in the forecast. It is desirable that the modification be such that if the demand is lower than forecast, the estimates are revised downward, whereas if the demand is higher than forecast, the estimates are revised upward.

The revised estimates in Period $t + 1$ are then used to make a forecast for Period $t + 2$, and Steps 2, 3, and 4 are repeated until all historical data up to Period n have been covered. The estimates at Period n are then used to forecast future demand.