2-person Stackelberg pricing games

GROUP 36

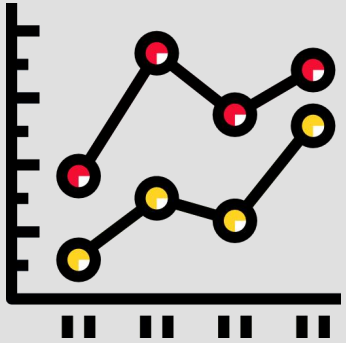
BUSHUIZHANG , WEILUE LUO,
YECHENG CHU, ZHAOYU ZHANG

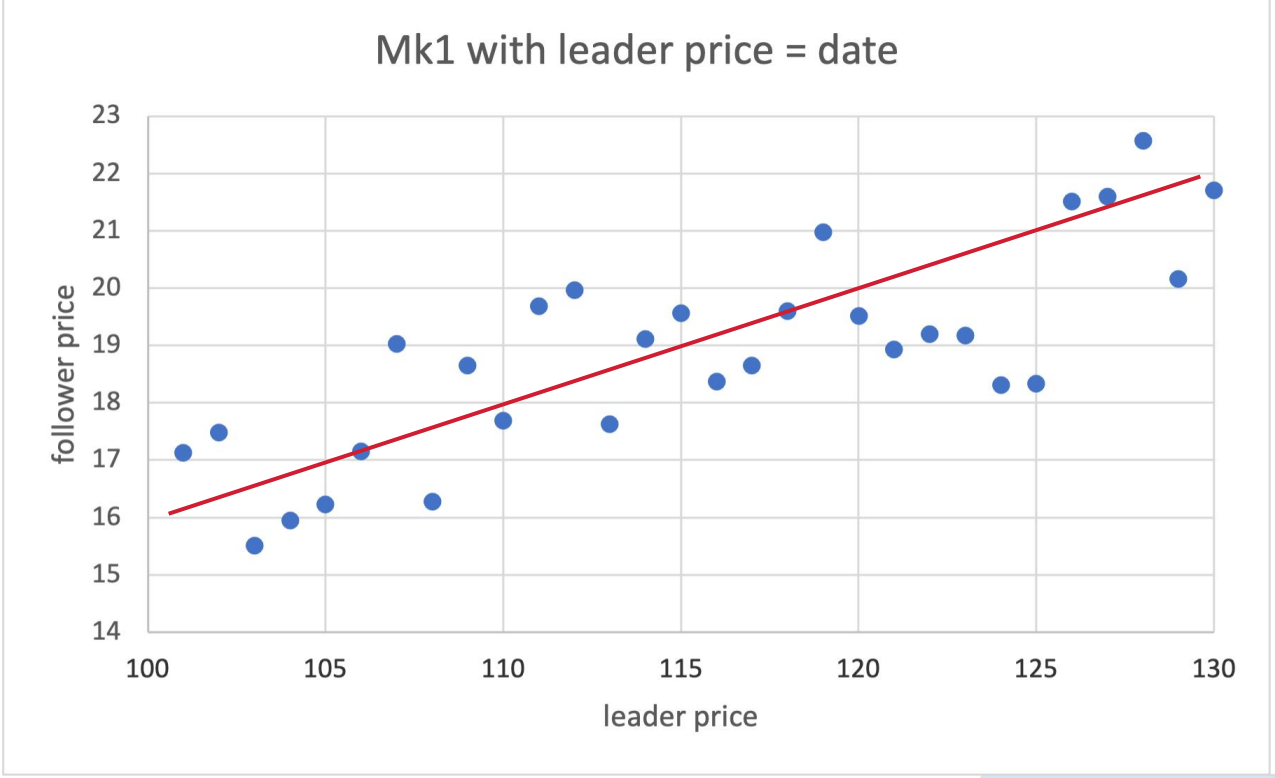
Analyze the behavior of the three followers

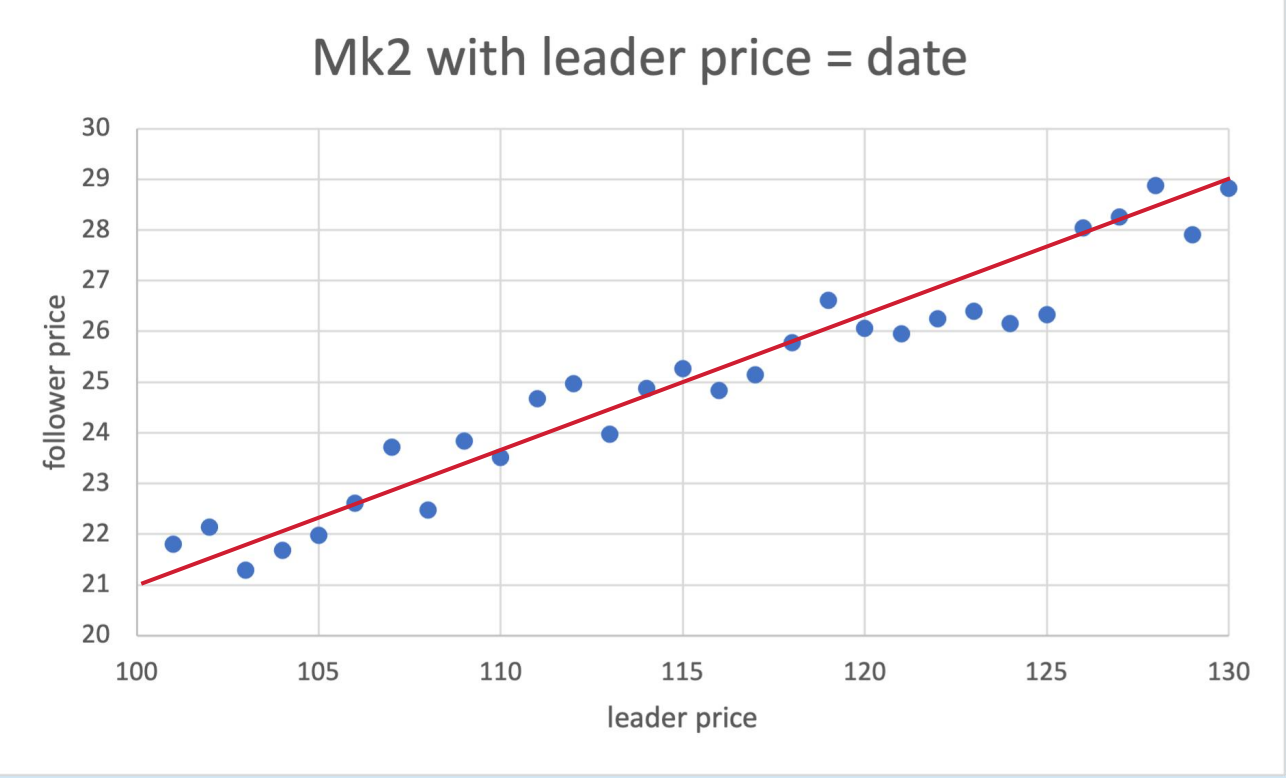
- Modify SimpleLeader.java

- Set the leader price to be **date**
- Plot for follower price against leader price

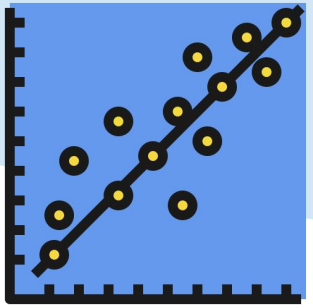
Linear or Non-Linear

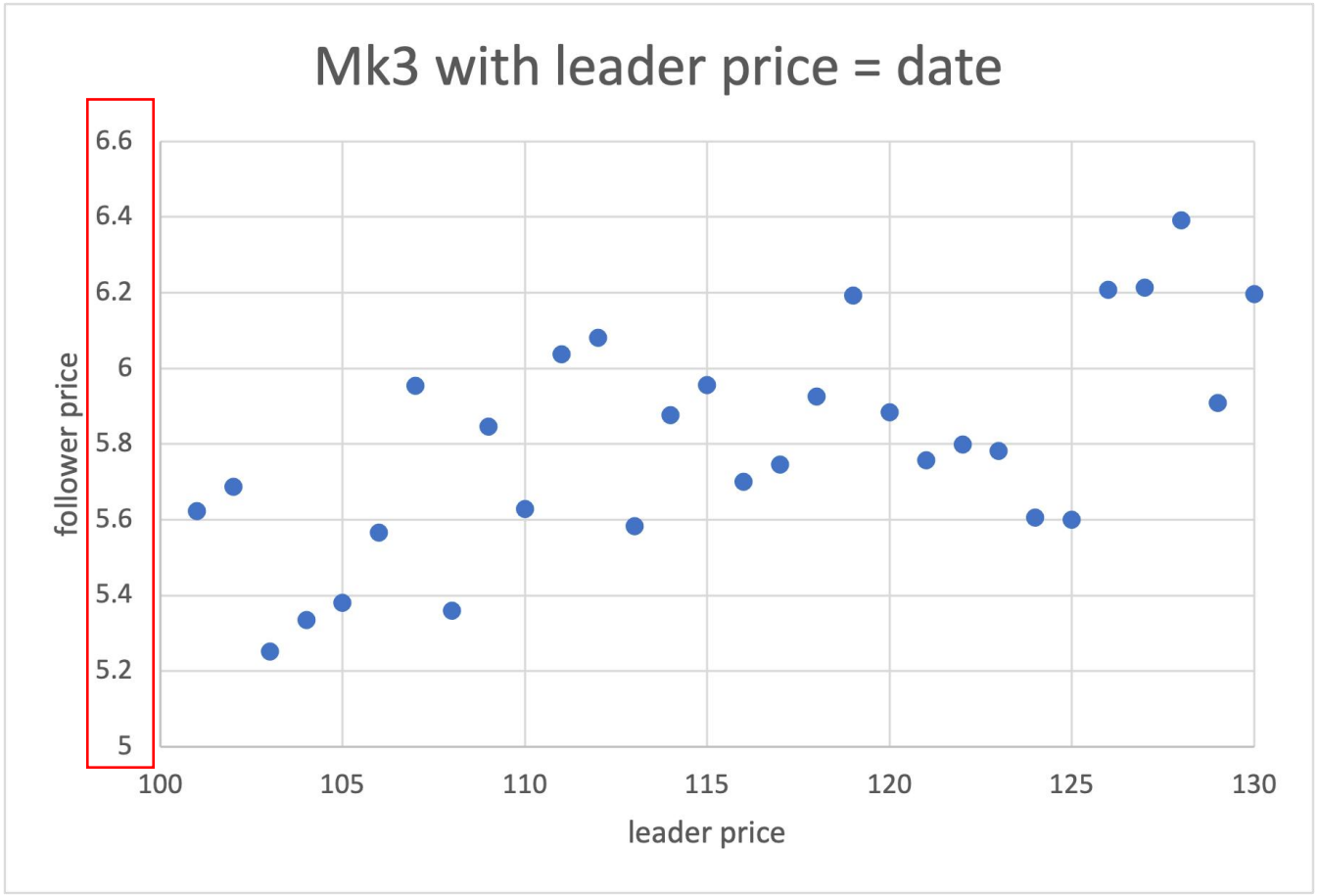




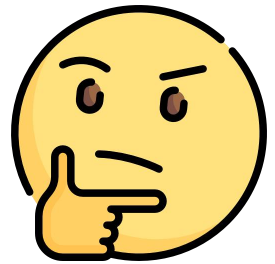


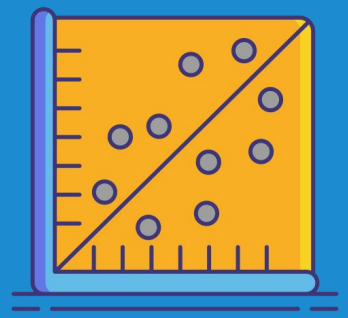
Mk1 and Mk2 are both linear





Mk3 could be *non-linear*





Linear Approaches

Goal: find the follower reaction function

- ALL HISTORICAL DATA
- MOVING WINDOW
- MODIFIED MOVING WINDOW
- WEIGHTED LEAST SQUARE WITH FORGETTING FACTOR

Best leader strategy given follower reaction

Given follower reaction

$$u_F = \hat{a} + \hat{b}u_L$$

Substitute into Leader profit equation

$$\begin{aligned}&(u_L - c_L)S_L(u_L, u_F)\\ &= (u_L - 1)(2 - u_L + 0.3u_F)\\ &= (u_L - 1)(2 - u_L + 0.3(\hat{a} + \hat{b}u_L))\end{aligned}$$

Group common terms

$$\begin{aligned}&2u_L - u_L^2 + 0.3\hat{a}\,u_L + 0.3\hat{b}\,u_L^2 - 2 + u_L - 0.3\hat{a} - 0.3\hat{b}\,u_L\\ &= (0.3\hat{b} - 1)\,u_L^2 + (3 + 0.3\hat{a} - 0.3\hat{b})\,u_L + (-2 - 0.3\hat{a})\end{aligned}$$

Make 1st derivative equal to zero and solve the leader price

$$\frac{\partial}{\partial u_L}(0.3\hat{b} - 1)\,u_L^2 + (3 + 0.3\hat{a} - 0.3\hat{b})\,u_L + (-2 - 0.3\hat{a}) = 0$$

$$2(0.3\hat{b} - 1)u_L + (3 + 0.3\hat{a} - 0.3\hat{b}) = 0$$

$$u_L = -\frac{(3 + 0.3\hat{a} - 0.3\hat{b})}{2(0.3\hat{b} - 1)}$$

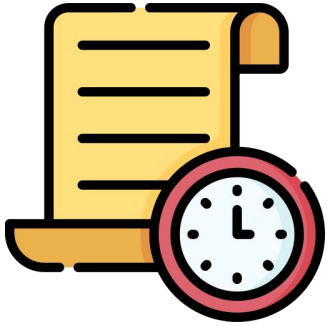
(Referenced from Lecture03)

All Historical Data Approach

- Use all the historical data to do the estimation

$$\sum_{t=1}^{T} \{y(t) - [a^* + b^* x(t)]\}^2 = min_{\{\hat{a},\hat{b}\}} \sum_{t=1}^{T} \{y(t) - [\hat{a} + \hat{b}\, x(t)]\}^2$$

In the equation x(t) is the leader price at day t
And y(t) is the follower price at day t



Convert the minimization problem to maximization problem.
Set the 1st order partial derivative equal to 0 to solve this problem.

$$\hat{a}^* = \frac{\sum_{t=1}^{T} x^2(t) \sum_{t=1}^{T} y(t) - \sum_{t=1}^{T} x(t) \sum_{t=1}^{T} x(t)y(t)}{T\sum_{t=1}^{T} x^2(t) - (\sum_{t=1}^{T} x(t))^2}$$

$$\hat{b}^* = \frac{T\sum_{t=1}^{T} x(t)y(t) - \sum_{t=1}^{T} x(t) \sum_{t=1}^{T} y(t)}{T\sum_{t=1}^{T} x^2(t) - (\sum_{t=1}^{T} x(t))^2}$$

(Referenced from Lecture04 Slide

Moving Window Approach

- Similar to All Historical Data Approach
- Instead of looping through all the historical data
- Use a window to loop only the recent data
- For example: a window size of 30 will just loop the data of past 30 days

(Referenced from Lecture06 Slide 6-7)

Evaluation and Analysis

	All historical data	Moving window Size = 100	Modified moving window Size = 30 Lambda = 0.95	Weighed least square with forgetting factor Lambda = 0.99	ML	Best approach
Mk1	**17.557161331 1768**	17.555740356 4453	17.55332946777 34	17.55502319335 94	17.555046081 5429	All historical data
Mk2	16.956459045 4102	16.955856323 2422	**16.95775985717 77**	16.95581626892 09	16.946899414 0625	Modifeid moving window
Mk3	19.488283157 3486	19.488330841 0645	19.48791313171 39	19.48825645446 78	**19.488372802 7343**	ML

	Analysis
Mk1	**Linear** and **not depend** on date
Mk2	**Linear** and **depend** on date
Mk3	**Non-linear**

Thank you !

GROUP 36

BUSHUI ZHANG

WEILUE LUO

YECHENG CHU

ZHAOYU ZHANG