$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = \alpha\frac{\partial^2 u}{\partial x^2} + \beta\frac{\partial^3 u}{\partial x^3}$$

1. $c=0{,}1$. LW scheme:

$$u_i^{n+1} = \underbrace{u_i^n - \frac{\Delta t}{2\Delta x}c\left[u_{i+1}^n - u_{i-1}^n\right] + \frac{\Delta t^2}{2\Delta x^2}c^2\left[u_{i+1}^n - 2u_i + u_{i-1}^n\right]}_{\text{Previous time step}}$$

iteration in time

$u(x,0)=f(x)$ Λ or $\cap$

$u(0,t)=0 \quad \forall t$

Нужно: $\boxed{\frac{\Delta t}{\Delta x}c < 1}$

1. Задаете Δx и сетку
2. Задаете Δt по Δx.

2. Diffusion: $\frac{\partial u}{\partial t} = \alpha\frac{\partial^2 u}{\partial x^2}$, $\alpha = \frac{0.1}{\pi^2}$

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \alpha\frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2}$$

$u_i^{n+1} = \dots$

$\frac{\Delta t \cdot 2\alpha}{\Delta x^2} \leq 1$ – отсюда находите нужный Δt.

$u(0,t) = u(1,t) = 0$, $x \in [0,1]$, $\forall t \geq 0$

3. $\frac{\partial u}{\partial t} = \beta\frac{\partial^3 u}{\partial x^3}$

CN: $$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \beta\left(\left.\frac{\partial^3 u}{\partial x^3}\right|_{i,n+1} + \left.\frac{\partial^3 u}{\partial x^3}\right|_{i,n}\right)\cdot\frac{1}{2}$$

$$\left.\frac{\partial^3 u}{\partial x^3}\right|_{i,n+1} = \frac{u_{i-1}^{n+1} - 3u_i^{n+1} - 3u_{i+1}^{n+1} - u_{i+2}^{n+1}}{2\Delta x^3}$$

точность Δx

$$\frac{u_{i+1}^n - u_i^n}{\Delta t} = \beta\left(\frac{u_{i-1}^{n+1} - 3u_i^{n+1} - 3u_{i+1}^{n+1} - u_{i+2}^{n+1}}{2\Delta x^3} + \frac{u_{i-1}^n - 3u_i^n - 3u_{i+1}^n - u_{i+2}^n}{2\Delta x^3}\right)$$

что-то от $n+1$ = что-то от n.

$$Au^{n+1} = 0$$

Любые $\Delta t, \Delta x$

$\boxed{u(x,t) \to 0}$ при $x \to \infty$

$u_{-1}^n = u_{-2}^n = 0 \quad \forall n$

бесконечность ≈ 5, тогда

$u(0,t) = 0 \quad \forall t$

$u(x,0) = f(x)$