1) SOR.

$u_{xx}+u_{yy} = -2\sin(y)\cos(x)$

$u_{ex} = \cos(x)\sin(y)$ ← define Dirichlet B.C.

use to define B.V.

$\Delta x = \Delta y = h$

G-S +

$u_{G\text{-}S}[i,j]$ = G.S - ordiniary ←

For SOR: $\omega = \dfrac{2}{1+\pi h}$

$u_{SOR}[i,j] = u_{G\text{-}S}\cdot\underline{\underline{\omega}} + u_{old}[i,j]\cdot\boxed{(1-\omega)}$

2.) Conjugate Gradient Method.

$r^n = f - Av^n$

numerical sol-n.

1. v^0 = Anywhat - random. / in our problem it's asked for zero $\boxed{u}$

• Apply B.C.

• $r^0 = f - Av^0$

• $q^0 = r^0$

It only works for symm. positive definite A

2. Loop:

$v^{p+1} = v^p + \alpha(p)\cdot q^p$, $\boxed{\alpha(p) = \dfrac{(r^p)^T\cdot r^p}{(q^p)^T A q^p}}$ $= \|r^p\|^2 = \sum (r^p_k)^2 = \sum_{i,j} (r^p_{ij})^2$ ✱

$r^{p+1} = r^p - \alpha(p)\cdot Aq^p$

$q^{p+1} = r^{p+1} + \beta(p)\cdot q^p$, $\beta = \dfrac{(r^{p+1})^T r^{p+1}}{(r^p)^T r^p}$ $= \|r^{p+1}\|^2$ - similiar

$N = \text{len}(v)$ - convergence

until some condition.

$$t_{ij} = -Av_{ij} = \underbrace{\frac{v^q_{i+1,j} - 2v^q_{ij} + v^q_{i-1,j}}{h^2}}_{u_{xx}} + \underbrace{\frac{v^q_{i,j+1} - 2v^q_{i,j} + v^q_{i,j-1}}{h^2}}_{u_{yy}} = \frac{v^q_{i+1,j} + v^q_{i-1,j} - 4v^q_{i,j} + v^q_{i,j-1} + v^q_{i,j+1}}{h^2}$$

$v_{(i,j)} = v_{i\cdot I+j}$

I - # points for x direction

$\underbrace{(q^p)^T}_{\text{vector}} \cdot \underbrace{\boxed{A\cdot q^p}}_{\text{vector}} = \sum q^p_k \cdot t_k = \sum q^p_{ij}\cdot t_{ij}$

1. 2D (v_{ij})

2D $f_{ij} = -2\sin(y_j)\cdot\cos(x_i)$

2D (r_{ij})

2D (q_{ij})