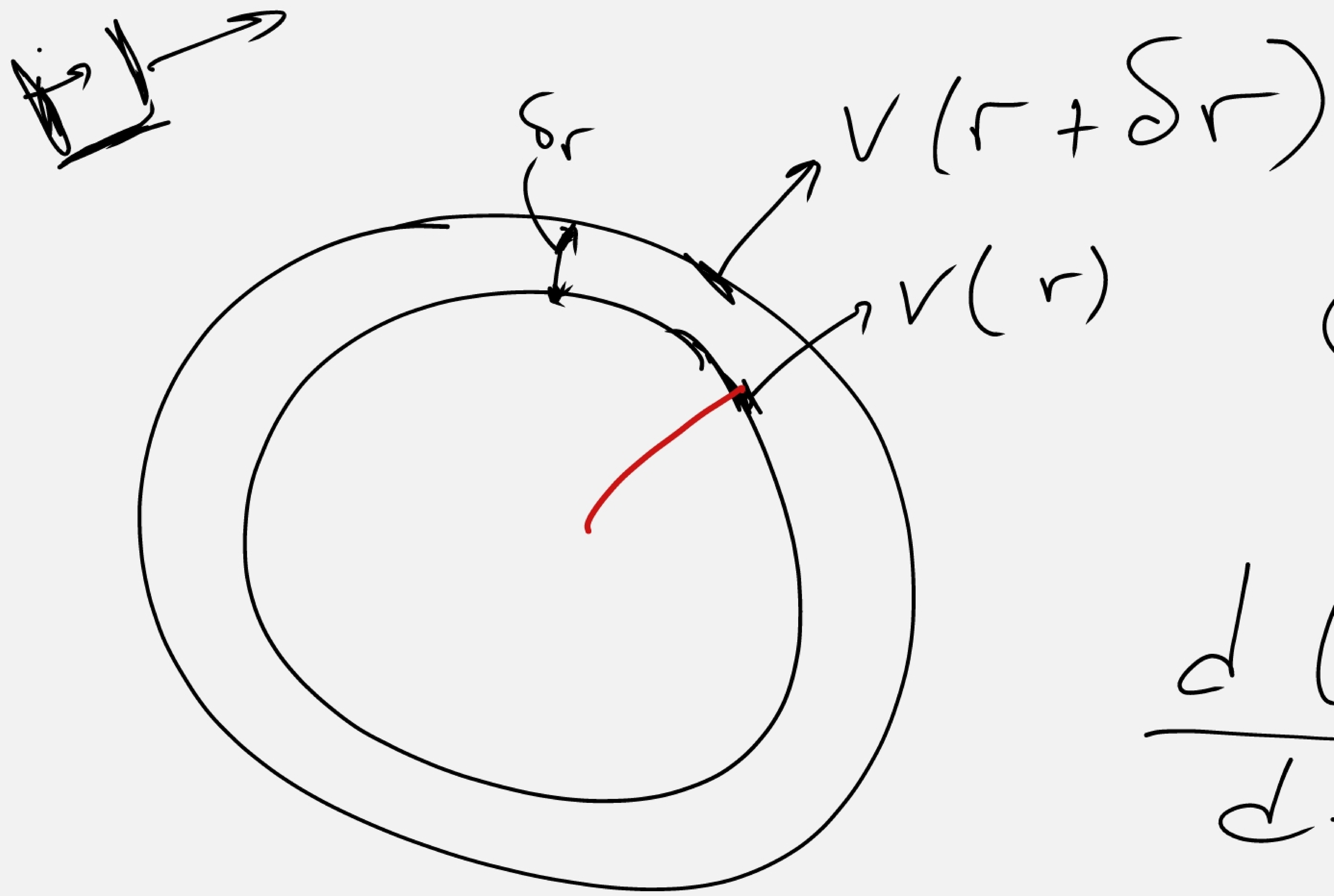


$$e_{rr} = \frac{v(r+\delta r) - v(r)}{\delta r} = \frac{\partial v}{\partial r}$$

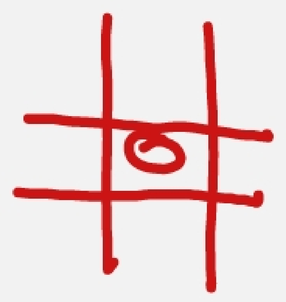
$$\varepsilon = \frac{\Delta L}{L_0}$$

$$\dot{\varepsilon} = \frac{\Delta V}{L_0}$$

$$\frac{d(\delta r)}{dt} =$$

$$e_{rr} = \frac{d(\delta r)/dt}{\delta r} = \frac{\partial v}{\partial r}$$

$$\frac{1}{\delta r}\frac{1}{dt}\delta r = \frac{\partial v}{\partial r}$$



$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = \alpha\frac{\partial^2 u}{\partial x^2} + \beta\frac{\partial^3 u}{\partial x^3}$$

1. $\alpha, \beta = 0$, Advection / convection:

Lax-Wendroff: $\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$

$$u_i^{n+1} = u_i^n - \frac{\Delta t}{2\Delta x} c[u_{i+1}^n - u_{i-1}^n] + \frac{\Delta t^2}{2\Delta x^2} c^2 [u_{i+1}^n - 2u_i^n + u_{i-1}^n] \quad - \ i = 1, \dots, N_x - 1$$

$c\frac{\partial u}{\partial x}\cdot\Delta t$

$\frac{\Delta t^2 c^2}{2}\frac{\partial^2 u}{\partial x^2}$?? $\quad \Delta t \to 0 \ \to 0$

$$u(x,0) = \begin{cases} f(x), & 0 \le x \le 1 \\ 0, & x > 1 \end{cases}$$

$u(0,t) = 0$

x = np.linspace(0, 10, шаг Δx)

$\frac{\Delta t}{2\Delta x} < 1$ – неточно, но какое-то ограничение есть

u = [. . . , . . .] ; u = np.zeros((Nx, Nt))

$u_i^{n+1} = u(x_i, t = n+1)$

$t \in [0, 20]$; 0(2)20 ⟷ 0, 2, 4, 6, 8, . . . , 18, 20.

$u_i^{n+1} =$ U[i, j] $= u(x = x_i, t = t_j)$

2) $\frac{\partial u}{\partial t} = \alpha\frac{\partial^2 u}{\partial x^2}$

FTCS: $\frac{u_i^{n+1} - u_i^n}{\Delta t} = \alpha\frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2}$

$$u_i^{n+1} = u_i^n + \frac{\alpha\Delta t}{\Delta x^2}(u_{i+1}^n - 2u_i^n + u_{i-1}^n)$$

≤ 1 – неточно, УТОЧНИТЬ!

$u(0,t) = u(1,t) = 0$

$x \in [0, 1]$

3) $\frac{\partial u}{\partial t} = \beta\frac{\partial^3 u}{\partial x^3}$

см Github

проверить $\boxed{n \text{ или } n-1}$

CN: $\frac{u_i^{n+1} - u_i^n}{\Delta t} = \frac{\beta}{2}\left(\frac{\partial^3 u_i^{n+1}}{\partial x^3} + \frac{\partial^3 u_i^{n-1}}{\partial x^3}\right)$ ⟼ составить матрицу:

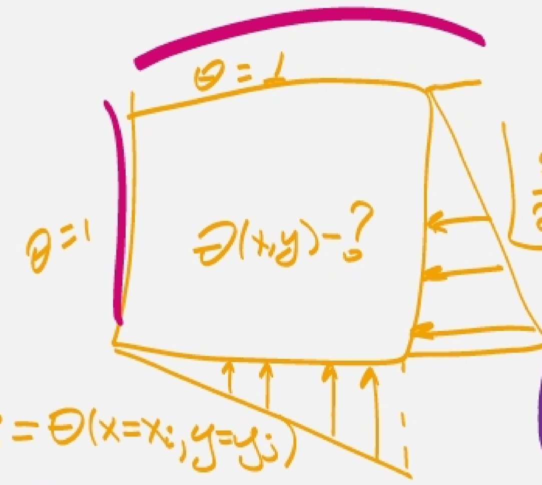
$\boxed{x_\infty = 10}$

$\boxed{u(x,t) \to 0}$ $x \to \infty$

$\boxed{u_{i+1}, u_{i+2}, \dots}$

$\boxed{u_{-1}, u_{-2} = 0}$ $\forall t$

$$\begin{pmatrix} & \\ & A \end{pmatrix}[u_i^{n+1}] = [u_i^n]$$



$\theta_{i,j} = \theta(x=x_i, y=y_j)$

$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0$

$\frac{\partial\theta}{\partial y}(x,0) = x$ (1)

$\frac{\partial\theta}{\partial x}(1,y) = 1-y$ (2)

$\theta(x,1) = 1$

$\theta(0,y) = 1$

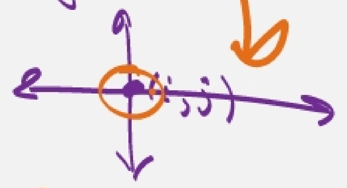
$\frac{\partial\theta_{i,0}}{\partial y} = x_i \Leftrightarrow \left.\frac{\partial\theta}{\partial y}\right|_{\substack{x=x_i\\ y=0}} = \left. x \right|_{x=x_i}$

$\frac{-3\theta_{i,0} + 4\theta_{i,1} - \theta_{i,2}}{2\Delta y} = x_i \Rightarrow \theta_{i,0} = -\frac{1}{3}(\dots)$

$\frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{\Delta x^2} + \frac{\text{аналогично}}{\Delta y^2} = 0$ — порядок $O(\Delta x^2, \Delta y^2)$

(1) - Forward difference 1-ой произв. 2-го порядка

(2) - Backward diff. 1-ой —II—



Jacobi: $\theta_{i,j}^{n+1}$ - с индексом n+1, остальные: $\theta_{i+1,j}^{n}; \theta_{i-1,j}^{n}; \theta_{i,j+1}^{n}; \theta_{i,j-1}^{n}$ - с индексом n.

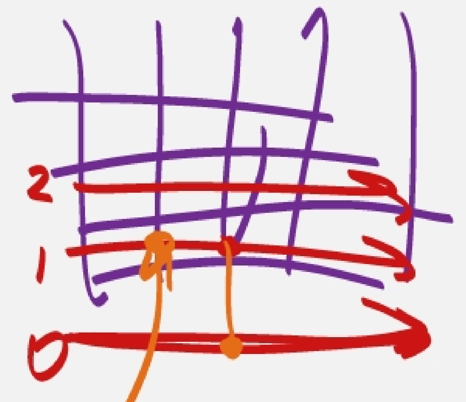
$\theta_{i,j}^{n+1} = f(\theta_{i+1,j}^{n}, \dots, \theta_{i,j-1}^{n})$

копия u-массива, либо всегда пред. шаг

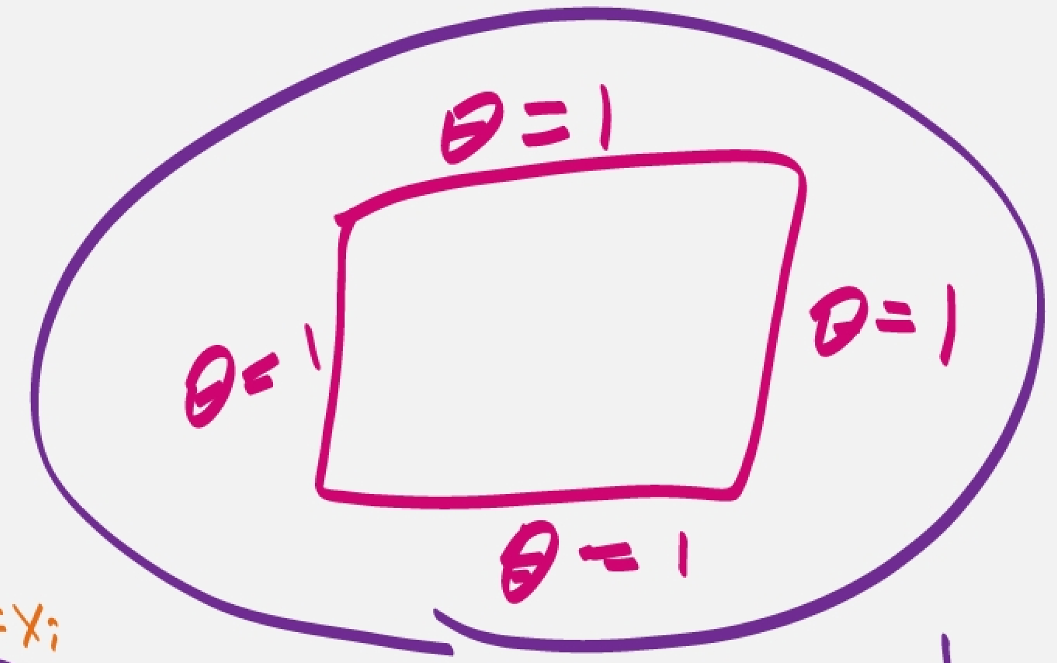
Условия $\theta(x,1)=1$, $\theta(0,y)=1$ - Сильнее/главнее (1), (2)

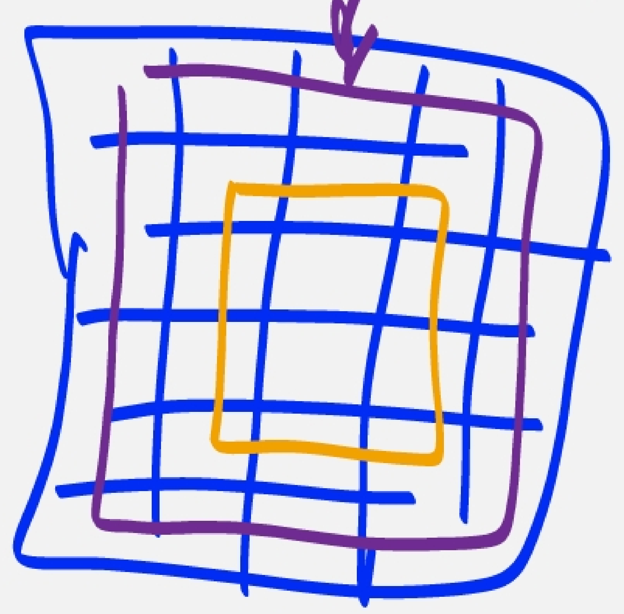
Gauss-Seidel:

$\theta_{i,j} = f(\theta_{i+1,j}, \dots, \theta_{i,j-1})$



нужно использовать.





Условие остановки:

1. $\max\limits_{i,j} |R_{i,j}| < \delta_R$

$\underbrace{\frac{\partial^2\theta_{i,j}}{\partial x^2} + \frac{\partial^2\theta_{i,j}}{\partial y^2}}_{\text{Finite diff}} = R_{i,j}$

2. $\max\limits_{i,j} |\theta_{i,j}^{n+1} - \theta_{i,j}^{n}| < \delta_\theta$

ОБА условия верны $\Rightarrow$ СТОП

S_1 - старая мат.

S_2 - здесь новые знач.

Алгор

Условие $\xrightarrow{\text{да}}$ СТОП

нет

$S_1 = S_2$

$$\frac{\partial u}{\partial t} = Fr + \frac{1}{Re}\underbrace{\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)}_{\text{опер. лапласа}}$$

$$u_{i,j}^n = u(x = x_i, y = y_j, t = t_n)$$

$$\boxed{\frac{u_{i,j}^{n+1} - u_{i,j}^n}{\Delta t} = Fr + \frac{1}{Re}\left(\frac{u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n}{\Delta x^2} + \text{анал}\right)}$$

$|\sigma| \le 1$

$$\boxed{u_{i,j}^n = \sigma^n e^{ik_1 x_i} e^{ik_2 y_j}} +$$

$$\boxed{u = T(t) \cdot X(x) \cdot Y(y)}$$

$$\boxed{\frac{\partial X(x)}{\partial x} = \frac{\partial Y(y)}{\partial y} = \frac{\partial T(t)}{\partial t}}$$

1) $$u_{i,j}^{n+1} = u_{i,j}^n + \frac{\partial u_{i,j}^n}{\partial t}\Delta t + \frac{1}{2}\frac{\partial^2 u_{i,j}^n}{\partial t^2}\Delta t^2 + O(\Delta t^3)$$ $u_{ex} - u_{числ} = O(\Delta t^2, \Delta x^2, \Delta y^2)$???

$$u_{i\pm1,j}^n = u_{i,j}^n \pm \frac{\partial u_{i,j}^n}{\partial x}\Delta x + \frac{1}{2}\cdot\frac{\partial^2 u_{i,j}^n}{\partial x^2}\Delta x^2 \pm (\,)\Delta x^3 + (\,)\Delta x^4 \pm (\,)\Delta x^5 + O(\Delta x^6)$$

$$\frac{\partial u_{i,j}^n}{\partial t} + \frac{1}{2}\frac{\partial^2 u_{i,j}}{\partial t^2}\Delta t + O(\Delta t^2) = Fr + \frac{1}{Re}\left(\frac{\partial^2 u_{i,j}}{\partial x^2} + (\,)\Delta x^2 + O(\Delta x^4) + \frac{\partial^2 u_{i,j}}{\partial y^2} + (\,)\Delta y^2 + O(\Delta y^4)\right)$$

Если $\Delta x, \Delta y, \Delta t \to 0$: $\frac{\partial u_{i,j}^n}{\partial t} = Fr + \frac{1}{Re}\left(\frac{\partial^2 u_{i,j}^n}{\partial x^2} + \frac{\partial^2 u_{i,j}^n}{\partial y^2}\right) \Rightarrow$ согласовано!

Accuracy: $O(\Delta t, \Delta x^2, \Delta y^2)$

2. Стабильность:

$$\boxed{u_{i,j}^n = \sigma^n e^{ik_1 x_i} \cdot e^{ik_2 y_j}}$$

Подставим в схему и сокращаем $\sigma^n e^{ik_1 x_i} e^{ik_2 y_j}$

$x_{i\pm1} = x_i \pm \Delta x$

$y_{j\pm1} = y_j \pm \Delta y$

$$\frac{\sigma - 1}{\Delta t} = \frac{1}{Re}\left(\frac{2\cos(k_1\Delta x) - 2}{\Delta x^2} + \frac{2\cos(k_2\Delta y) - 2}{\Delta y^2}\right) = \frac{1}{Re}\left(\frac{-4\sin^2\left(\frac{k_1\Delta x}{2}\right)}{\Delta x^2} + \frac{-4\sin^2\left(\frac{k_2\Delta y}{2}\right)}{\Delta y^2}\right)$$

$$\sigma = \frac{-4\Delta t}{Re}\left(\frac{\sin^2\left(\frac{k_1\Delta x}{2}\right)}{\Delta x^2} + \frac{\sin^2\left(\frac{k_2\Delta y}{2}\right)}{\Delta y^2}\right) + 1$$

$|\sigma| \le 1$

$$-1 \le \ldots \le 1 \Rightarrow \frac{4\Delta t}{Re}\underbrace{(\ldots)}_{max \Rightarrow \Delta t -} \le 2 \Rightarrow \boxed{\Delta t_{max} \le \frac{Re}{2}\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right)^{-1}}$$

$$\boxed{\frac{4\Delta t}{Re}\left(\frac{\sin^2\left(\frac{k_1\Delta x}{2}\right)}{\Delta x^2} + \ldots\right)} \le 2$$

Найти Δt_{max} при котором неравенство всегда верно

Δt – через $\Delta x, \Delta y$

$$a \cdot b \le 2 \Longleftrightarrow a \le \frac{2}{b}$$

Максим a для любого доступного b.

$$b = \frac{\sin^2}{\Delta x^2} + \frac{\sin^2}{\Delta y^2}$$

$$b_{max} = \frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}$$

$$Q_1(t) = \Delta x \Delta y \cdot \sum_{i,j} u_{i,j}^n$$

$$u_{i,j}^{n+1} = \Delta t\, Fr + u_{i,j}^n + \frac{\Delta t}{Re}\cdot f\left(u_{i+1,j}^n, \ldots u_{i,j-1}^n\right)$$

$$Q(t) = \int_0^{\ell}\int_0^{h} u(x, y, t)\, dy\, dx$$

СТАБИЛЬНОСТЬ!