- For large companies, the bond that they issue will behave very much like a stock
  - Very liquid
  - Many buyers
- For mid cap companies, the bond that they issue will not be very liquid
  - Do not trade that much in the market
  - A lot weaker
  - These companies behave a lot like a retail consumer even though they sell bonds
    - We can't use data science tools to measure the risk of these bonds effectively
    - But deep learning could do it more correctly

- The traditional way of managing bonds will rely a lot on the rating agencies

Bond Pricing

- Big companies can raise money by going to the market and issue a bond
  - Skip the bank (middleman)
  - But banks usually work as the underwriter of the bond
    - Bank is the one actually taking the risk
    - The company get the money, the bank takes a cut and you pay back the banks
      - The company wants money, they don't buy and sell risks
      - So the bank said, I am taking a piece, and I am going to take care of the risk
      - The bank pays the company with some discount and take the bond and sell it to the market

- The buyer of the bond is buying the debt of the company, how do we evaluate risk?
  - Credit risk of the company (spread)
  - Risk free rates (yield curve)
  - The condition of the bond (coupon rate, face value, maturity)
  - Comparison against the market
    - Bond is comparatively good, the price of the bond will go up

- The bond can have properties
  - E.g. bond only pays when there is a natural disaster
  - E.g. with option to call bond which increases the value
  - E.g. with option to prepay the bond which decrease the value

- There are some bonds (e.g. negative coupon bonds, or premium zero-coupon bond) that are not sensible to buy and hold
  - Do it because you believe it's a good investment and the price of the bond might go up
    - Bond price is easier to foresee than stock
  - Or do it because you know the risk is virtually none, and the market is volatile

Bond pricing

- Theoretical method
  - ➤ Fixed interest rate

    Denote by $PV(t, t_i, C_{t_i})$: time t value of an amount C_{t_i} received at future time t_i

    Then value of cash flows is:

    $$V(t) = \sum_{i=1} PV(t, t_i, C_{t_i})$$

    We can simplify the past equation a bit by breaking down the present values into the present values of a single dollar
    - ○ $D(t, t_i)$ is the value at time t of a dollar that is certain to be received at time t_i.

    $$V(t) = \sum_{i=1} C_{t_i} \cdot D(t, t_i)$$

    Using a constant discount rate δ and fixed time periods (i.e. Months or days)

    $$V(t) = \sum_{i=1} C_{t_i} \cdot \delta^i$$

COUPON BOND	ZERO COUPON BOND
Principal X, m coupons per year (i.e. monthly if $m = 12$), rate per coupon c, interest rate r, term N.	No coupons. One payment at end of period. Principal X, interest rate r, term N.
Last coupon is paid at same time as principal	Pays only final coupon
May be very long term	Tends to be shorter term (exceptions! See Germany 2019 bond)
Value equation:	Value equation
$$V(0) = \frac{cX}{r} + X\left(1 - \frac{c}{r}\right)\left(1 + \frac{r}{2}\right)^{-N}$$	$$V(0) = X\frac{(1+c)}{(1+r)^N}$$

- Time value of money
  - ➤ Since the amount of money today will be less than amount of money tomorrow from all the money injection

- Yield curve
  - ➤ The yield takes into account
    - ✧ The market
    - ✧ The risk
    - ✧ The risk free rate
  - ➤ Taking one two year risk is not the same of taking two one year risks

YTM

In order to think about these relationships without getting too confused we need some simple measures of yield

Let's call our interest rate y. Bond price equation becomes:

$$V = \sum_{j=1}^{n} C\left(1+\frac{y}{2}\right)^{-j} + X * \left(1+\frac{y}{2}\right)^{-N}$$

If interest rate is a constant y across terms we can calculate V for each value of y. Look at today's market price, what's the value of y?

Internal Rate of Return (IRR), or Yield to Maturity (YTM)

- This is the interest rate that intrinsically promising you to pay if you are to replace this for a very safe investment
  - ➢ It is usually used to compare different bonds of the same characteristic (same level of risk)
  - ➢ It's a way of being neutral to risk
  - ➢ YTM ignores that fact that the company may default

Calculating Yield Curve

1. Use the 'bootstrapping' approach to get equivalent zero coupon prices for as many maturities as possible
2. Use some kind of interpolation technique to extend these prices to maturities which aren't directly represented
3. Overcome the difficulties inherent in implementing this conceptual picture.

See next week's lab.

Now, we have bond prices. What about risk???

Pricing Bonds with Varying Rates

The previous equation assumes fixed rates over the whole period. This is **not realistic**, as yields vary depending on the horizon.

In reality the interest rate r varies for every period, so it becomes r_t, and it also varies by the intrinsic risk.

Investors also charge a "spread" or the difference between the risk-free rate $r_{f,t}$ and the rate r_t. r_t then becomes $r_t = r_{f,t} + s_t$ and the price equation for a bond with coupon rate c and maturity M becomes

$$P_0 = \sum_{t}^{M} \frac{c}{(1+r_{f,t}+s_t)^t} + \frac{1}{(1+r_{f,M}+s_M)^M}$$

so the investors are willing to pay $P \times FV$ with FV the face value of the loan.

- The spread is the price of that particular credit risk compared to the market
  - ➢ The market spread is implied by the observed deals in the market using the same bootstrap

approach as calculating the yield curve

- Risk free rate can vary by country

Corporate Credit Risk

- Banks need to evaluate the risk of companies because they either lend them money (giving out loans)