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NICTA and ANU

August 14, 2013

CPMP 4630/COMP 6463
Assignment "Automated Reasoning in First-Order Logic"

Due date: August 30, 2013, 5pm.

Marking scheme: Full marks for a formulation that correctly answers the question and clearly shows the steps to obtain the solution.

Solutions to be submitted electronically by Email to `Peter.Baumgartner@nicta.com.au`, or on paper to a lecturer of this course. Neatly hand-written solutions are of course acceptable.

Question 1 (Structural induction, 8 pts). Let T be a tautology, B an atom, $\mathcal{A}$ an assignment, and F a propositional formula that contains the connectives $\wedge$ and $\vee$ only (no $\neg$). Prove by structural induction the following property:

> If $\mathcal{A} \models F$ and $\mathcal{A}$ is suitable for T then $\mathcal{A} \models F[B/T]$, where $F[B/T]$ is the formula obtained from F by replacing every occurrence of B by T.

Question 2 (Semantics, 6 pts). Is the following propositional clause set M satisfiable? Justify your answer by a proof. (*Hint:* consider finite subsets and the use of Theorem 25 (Compactness).)

$$M = \{A_1 \vee B_1\} \cup \{\neg A_1 \vee A_2,\ \neg B_1 \vee B_2,\ \neg A_2 \vee A_3,\ \neg B_2 \vee B_3, \ldots\}\ .$$

Question 3 (Clause normal form, 6 pts). Transform the formula $\forall x\ \exists y\ ((R(y,x) \wedge \neg R(x,y)) \vee \exists y\ Q(x,y))$ into clause normal form. All intermediate formulas (Prenex normal form, Skolemized form, CNF) must be given explicitly.

Question 4 (Herbrand interpretation, 2 + 4 pts). Let $F = \forall x\ (P(a) \wedge \neg P(f(b)) \wedge (\neg P(x) \vee P(f(x))))$ be a sentence in Skolem normal form.

1. Describe the Herbrand universe for F and the Herbrand expansion of F.
2. Find a Herbrand structure $\mathcal{A}$ such that $\mathcal{A} \models F$.

Question 5 (Unification, 3 + 3 pts). Apply the unification algorithm presented in class to these sets of equations and read off the result, i.e., either FAIL or the unifier (a is a constant, x and y are variables):

1. $U = \{x = y,\ g(f(x)) = g(y)\}$
2. $U = \{a = x,\ g(f(x,z)) = y,\ y = g(f(z,x))\}$

Question 6 (First-order resolution, 8 pts). Find a Resolution refutation of the following clause set. As for the mgus used, it suffices to only state them, you do not need to show the details of the runs of the unification algorithm. As in question 5, a is a constant, x and y are variables.

(1)	$\neg P(x) \vee Q(f(x)) \vee Q(y)$	(3)	$P(a)$
(2)	$\neg Q(x) \vee R(x)$	(4)	$\neg R(f(a))$