Assignment on Propositional Logic

Question 1 (Ex.7.5, Russell & Norvig) (3 points)

Give the number of models for each of the following sentences.

(a) $(A \land B) \lor (B \land C)$

(b) $A \lor B$

(c) $(A \leftrightarrow B) \leftrightarrow C$

Question 2 (Ex.7.8, Russell & Norvig) (8 points)

Decide whether each of the following sentences is valid, unsatisfiable, or neither.

(a) $Smoke \rightarrow Smoke$

(b) $Smoke \rightarrow Fire$

(c) $(Smoke \rightarrow Fire) \rightarrow (\neg Smoke \rightarrow \neg Fire)$

(d) $Smoke \lor Fire \lor \neg Fire$

(e) $((Smoke \land Heat) \rightarrow Fire) \leftrightarrow ((Smoke \rightarrow Fire) \lor (Heat \rightarrow Fire))$

(f) $(Smoke \rightarrow Fire) \rightarrow ((Smoke \land Heat) \rightarrow Fire)$

(g) $Smoke \lor Fire \lor (Smoke \rightarrow Fire)$

(h) $(Smoke \land Fire) \lor \neg Fire$

Question 3 (Ex.7.4, Russell & Norvig) (2 points)

For arbitrary propositional sentences α and β, recall the definition of entailment: $\alpha \models \beta$ if every model of α is a model of β. Prove the following theorem: $\alpha \models \beta$ if and only if $\alpha \land \neg\beta$ is unsatisfiable.

Question 4 (Ex.12.11.4, Aho & Ullman) (6 points)

Using the theorem from Question 3, prove each of the following assertions by first converting $\alpha \land \neg\beta$ to CNF and then deriving the empty clause by resolution. For better readability you may write a CNF formula as a list of clauses (omitting the $\land$).

(a) $(P \rightarrow Q) \land (P \rightarrow R) \models P \rightarrow (Q \land R)$

(b) $(P \rightarrow (Q \lor R)) \land (P \rightarrow (Q \lor \neg R)) \models P \rightarrow Q$

(c) $(P \rightarrow Q) \land ((Q \land R) \rightarrow S) \models (P \land R) \rightarrow S$

Question 5 (6 points)

Recall the notion of an implication graph described in class, and that each cut through the graph that puts the conflict on one side and all the decisions on the other corresponds to a candidate clause to be learned. Show that each of these candidate clauses, if not already existing, can be derived by resolution from the existing clauses.

Question 6 (3 points)

Recall the 1-UIP learning method described in class, where each learned clause contains exactly one literal that was falsified in the latest decision level. Show that this method will never learn a clause that already exists.

Question 7 (12 points)

Convert the CNF formula $(A \vee B) \wedge (B \vee C)$ to a logically equivalent Boolean circuit in

(a) d-DNNF (deterministic decomposable NNF);

(b) DNNF (decomposable NNF) that does *not* satisfy determinism;

(c) d-NNF (deterministic NNF) that does *not* satisfy decomposability;

(d) sd-DNNF (smooth d-DNNF).