Australian National University
Department of Computer Science

Computer Science COMP4600 2014 – Assignment Two

Part II: Complexity Theory

Due: Sunday 21 September
Late Penalty: 10% per day

No programming is needed for this assignment. *Be precise. Avoid long-winded answers. Answer clearly and legibly. Indicate clearly the question and part to which you are providing an answer for.*

Submit a typed or scanned copy of your assignment by email to `paulette.lieby@anu.edu.au`, or a hard copy to Paulette Lieby, room N222 (under the door is ok), CSIT Building 108 ANU. You may also drop your hard copy into the COMP4600 assignment box on level 1 in the CSIT Building.

The total mark of this assignment is 100 points.
Don't forget to add your name/student ID.

Question 1 (20/100).

1. Let ϕ be a boolean formula constructed from the boolean input variables x_1, x_2, $\ldots$, x_k, negations ($\neg$), ANDs ($\wedge$), ORs ($\vee$), and parentheses. The formula ϕ is a tautology if it evaluates to 1 for every assignment of 1 and 0 to the input variables. Define TAUTOLOGY as the language of boolean formulas that are tautologies. Show that TAUTOLOGY $\in$ co-NP.

2. Show that NP $\cap$ co-NP is closed under complement, that is, $L \in$ NP $\cap$ co-NP implies that $\overline{L} \in$ NP $\cap$ co-NP.

Question 2 (30/100).

Assume that there is an algorithm A that, given a graph G and a positive integer k, decides in polynomial time whether G has a vertex cover of size k.

1. Use algorithm A to construct a polynomial-time algorithm that determines the size of a minimum vertex cover for a graph.

2. Use algorithm A to find, in polynomial time, a minimum vertex cover for a graph.

Question 3 (30/100).

1. Show that the following problem is in NP:
   Given a graph G with positive integer edge weights and a positive integer k, decide whether G has a spanning tree whose weight is exactly k.

2. Show that the following problem is in co-NP:
   Given a graph G and a positive integer k, decide whether the clique number $\omega(G)$ is at most k.

3. Show that the following problem is in NP $\cap$ co-NP:
   Given two positive integers n and k, decide whether n and k are relative prime (also called coprime).

Question 4 (20/100).

Prove that the following problem is NP-complete:
Given a simple graph G and positive integers n and m, decide whether G has a subgraph with n vertices and at least m edges. (Hint: Use a problem for which we proved NP-completeness in the lectures.)