Assignment for the Modal Logic component of Overview of Logic COMP 6463

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Note: this assignment is worth 16.5% (= 33% * 50) of the final mark!

Deadline: 1700 Thursday 31st of October 2013 at 1700. Hand it in to me in person in the RSISE building (ring me on 61258603 to get into the building) or email it to me as a (scanned) PDF or hand it in to the main CSIT office in Building 108.

Questions and marks: There are ten questions, each is worth 20 marks.

Late Penalty: There will be a penalty of 20 marks per day for late submissions and pro-rata for parts of days.

Good excuses: If you have a good excuse then email me as soon as you know that you will be late to submit so we can work out a later submission date. But I will be the judge of whether your excuse is good enough. "I have a lot of other assignments" will certainly fail to convince me.

Questions: if you want to ask questions about this assignment or point out typos or mistakes in the questions then email me.

1. Let $\langle W, R, \vartheta \rangle$ be a Kripke model and let $w \in W$. Use the semantic clauses for the definition of $\vartheta(w, \varphi)$ around Slide 9 to prove that if $\vartheta(w, []\neg p_0) = \mathbf{t}$ then $\vartheta(w, \neg\langle\rangle p_0) = \mathbf{t}$.

Hint: One way is to assume that the "if" part holds and that the "then" part does not hold, and derive a contradiction. A proof needs to be set out properly as in my lecture notes.

2. Show a model $\mathcal{M} = \langle W, R, \vartheta \rangle$ such that $\mathcal{M} \not\Vdash (\langle\rangle[]p) \to [][]p$. You must give concrete values of your W, R and ϑ as in my lecture notes. A diagram alone does not count but can be used to clarify your concrete values.

3. Prove that a frame validates the shape $\langle\rangle\varphi \to \langle\rangle\langle\rangle\varphi$ if and only if the frame is dense i.e. $\forall x, z.R(x, z) \Rightarrow (\exists y.R(x, y)\&R(y, z))$.

Hint: You need to show two proofs: the "if" part and the "only if" part. Look at the analogous proof for $[]\varphi \to \varphi$ and reflexivity of Lemma 13.

4. Use the tableau method for **K** with global assumptions to decide whether the formula $\langle\rangle p_1 \to [][]p_0$ is a logical consequence of $\{p_0, p_0 \to p_1\}$. Show the tableau(x).

Hint: remember to use nnf!

5. Give a tense logic model which satisfies $(\langle F\rangle\langle P\rangle p_0) \wedge (\langle F\rangle\langle P\rangle\neg p_0)$. You have to give a concrete W, R and ϑ. A diagram alone does not count but can be used to clarify your concrete values.

6. Prove that the following formula $(\langle F\rangle\langle F\rangle[P]p_1) \to \langle F\rangle p_1$ is Kt-valid.

Hint: Use a proof by contradiction.

7. Use the tableau method from the lecture notes for PLTL to decide whether the formula $(\top\,\mathcal{U}p) \to \langle F\rangle p$ is PLTL-valid. Show the tableau.

Hint: Build the appropriate PLTL tableau, pushing negations inside other connectives on the fly, and decide whether it is open or closed. Giving an answer without giving a tableau to support it will not count. In particular, do not try to convert the formula into nnf first as my lecture notes do not show you all the rules needed to do this.

8. Show that $([\alpha]q) \leftrightarrow (\neg\langle\alpha\rangle\neg q)$ is true in all PDL-models.

Hint: You have to show two parts, one for each direction of the bi-implication. The easiest way is to argue by contradiction for each part. Alternatively, use the definitions of $\tau_M(.)$ to show the equivalence directly.

9. Is the formula $\langle\gamma*\rangle q \to \langle\gamma; (\gamma*)\rangle q$ PDL-valid? Give your reasoning.

Hint: Use a proof by contradiction and use the definitions of $\tau_M(.)$ if it is PDL-valid or else give a PDL-model that falsifies this formula if it is not PDL-valid.

10. Let us call a tense-logical formula "purely modal" if it contains no occurrences of $\langle P\rangle$ nor $[P]$. Prove the following theorem: a purely modal formula φ is **Kt**-valid if and only if it is **K**-valid.

Hints: You have to show two things as set out below.

(a) if φ is **Kt**-valid then it is **K**-valid.

(b) if φ is **K**-valid then it is also **Kt**-valid.

This question is testing your ability to think rather than just follow a given procedure mechanically. I am also willing to give some (not all) marks to answers which are not formal proofs but which argue using intuitions only.