# FOCS Homework 11, for Day 12

You may edit your answers into this file, or add a separate file in the same directory.

If you add a separate file, please include the following at the top:

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Student Name: Zhecan Wang [change to your name]

Check one:

[1] I completed this assignment without assistance or external resources.

[ ] I completed this assignment with assistance from \_\_\_

and/or using these external resources: \_\_\_

```

## I. Formal Proofs

Use the rules of inference from class (reprinted below) to construct formal, numbered proofs of the following:

### 1.

Assuming \_p -> q\_ and \_p -> r\_, prove \_p -> (q AND r)\_

1. p -> q assumption given

2. p -> r assumption given

3 p

4. q -> true, p->true

5. p ∧ q Conjunction rule

### 2.

Assuming \_p -> (q OR r)\_ and \_p -> (q OR NOT r)\_, prove \_p -> q\_

1. p -> (q OR r) assumption given

2. p -> (q OR NOT r) assumption given

3. p

4. (q OR r) AND (q OR NOT r) Conjuntion Rule

5. q OR (r AND NOT r) Distributive Rule

6. q OR False Negation Rule

7. q Identity Rule

## II. Truth tables

Use a truth table to show the equivalence of the assumptions and conclusion from problem 1, above: Assuming \_p -> q\_ and \_p -> r\_, prove \_p -> (q AND r)\_

[Style note: what is the canonical order of the rows in a truth table for three propositions?]

p q r p -> q p -> r q AND r

T T T T T T

T T F T F F

T F T F T F

T F F F F F

F T T T T T

F T F T T F

F F T T T F

F F F T T F

## III. Tautology, satisfiable, contradiction

Which of the following are tautologies? Satisfiable? Unsatisfiable (contradictions)?

### 1. \_(p AND q AND r) -> (p OR q)\_

p q r (p AND q AND r) p OR q (p AND q AND r) -> (p OR q)

T T T T T T

T T F F T T

T F T F T T

T F F F T T

F T T F T T

F T F F T T

F F T F F T

F F F F F T

it is a tautology

### 2. \_((p -> q) AND (q -> r)) -> (p -> r)\_

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | r | p -> q | q -> r | p -> r | ((p -> q) AND (q -> r)) | ((p -> q) AND (q -> r)) -> (p -> r) |
| T | T | T | T | T | T | T | T |
| T | F | T | F | T | T | F | T |
| T | T | F | T | F | F | F | T |
| T | F | F | F | T | F | F | T |
| F | T | T | T | T | T | T | T |
| F | F | T | T | T | T | F | F |
| F | T | F | T | F | T | F | T |
| F | F | F | T | T | T | T | T |

Satisfiable

### 3. \_(p -> q) -> p\_

|  |  |  |  |
| --- | --- | --- | --- |
| p | q | p -> q | (p -> q) -> p |
| T | T | T | T |
| T | F | F | T |
| F | T | T | F |
| F | F | T | F |

Satisfiable

### 4. \_(p OR q OR r) AND ((NOT p) OR (NOT q) OR (NOT r)) AND (p OR (NOT q)) AND (q OR (NOT r)) OR (r OR (not p))\_

|  |  |  |  |
| --- | --- | --- | --- |
| p | q | r | (p OR q OR r) AND ((NOT p) OR (NOT q) OR (NOT r)) AND (p OR (NOT q)) AND (q OR (NOT r)) OR (r OR (not p)) |
| T | F | F | T |
| F | T | F | F |

Satisfiable

### 5. [\*\* challenge/optional] \_(p <=> (q OR r)) -> ((NOT q) -> (p AND r))\_

## IV. CNF

Transform the following formula into conjunctive normal form. Show your steps.

\_p OR (q AND NOT (r AND (s -> t)))\_

p OR (q AND NOT (r AND (s -> t))) Given

p OR (q AND NOT (r AND ((NOT s) OR t))) Conditional Law

p OR (q AND ((NOT r) OR ((NOT s) OR t))) De Morgan's Law

(p OR q) AND ((NOT r) OR ((NOT s) OR t)) Distributive Law

## V. Short certificate

Demonstrate that the formula in problem IV is satisfiable. Explain whether you used a "short certificate" or exhaustive enumeration to make this determination.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| p | q | r | s | t | (p OR q) AND ((NOT r) OR ((NOT s) OR t)) |
| T | T | T | T | T | T |
| F | F | F | F | F | F |

Satisfiable

[several problems adapted from Ullman FOCS ch12]