

This test is a *review* of material covered in prerequisite math and computer science classes. Students familiar with the material should finish in less than 10 mins.

Each problem is worth one point: out of 30.

Discrete Math

Q1: What is the intersection of the interval $\mathcal{A} = [-1, 3]$ and $\mathcal{B} = [0, 4]$? $\mathcal{A} \cap \mathcal{B} =$

Q2: In a balanced binary tree with n leaf nodes, how many internal nodes are there?

Q3: In a balanced binary tree with n leaf nodes, how deep is the tree?

(i.e., number of nodes on a shortest path from root to leaf)

Q4: What is $5!$ (five factorial)?

Q5: How many different ways are there to choose k items from a set of n items?

Linear Algebra

Q6: What size is the result of multiplying a 2×3 matrix and a 3×4 matrix?

Q7: What is the dot product between the vector $\mathbf{a} = [1, 2, 3]$ and $\mathbf{b} = [4, 5, 6]$? $\mathbf{a} \cdot \mathbf{b} =$

Q8: What is the cross product between the vector $\mathbf{a} = [4, 0, 0]$ and $\mathbf{b} = [0, 0, -2]$? $\mathbf{a} \times \mathbf{b} =$

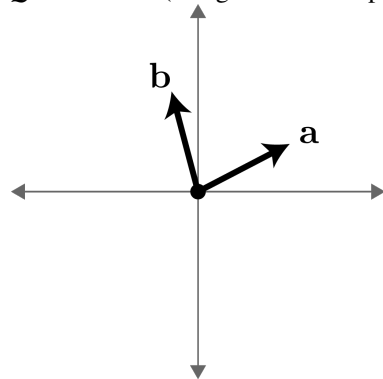
Q9: What is the matrix product of the matrix $\mathbf{A} = \begin{pmatrix} 1 & 0 & 4 \\ 0 & 2 & 1 \\ 0 & -3 & 0 \end{pmatrix}$ and the vector $\mathbf{b} = \begin{pmatrix} -4 \\ 5 \\ 6 \end{pmatrix}$?

$\mathbf{A} \mathbf{b} =$

Q10: Using matrices, express the solution to the following system of equations:

$3x_1 + 2x_2 - 1x_3 = 10$, $2x_1 - 10x_3 = 0$, $-9x_2 + 9x_3 = -1$. (Form the system, but do not solve).

Q11: Sketch (using PDF markup or PDF highlighter) the results of $\mathbf{c} = \mathbf{a} + \mathbf{b}$ and $\mathbf{d} = \mathbf{b} - \mathbf{c}$ on the plot:



Mappings

Q12: Suppose we have $f : \mathcal{A} \rightarrow \mathcal{B}$, what needs to be true to call f a *well-defined function*?

Q13: Is $f(x) = x^2$ invertible? Why (not)?

Q14: If x and y are real numbers, what sets are the domain and range of $f(x, y) = x^2 + y^2$? (use \mathbb{R})

Logarithms

Q15: Express $\log(a^b)$ in terms of $\log a$ and b . $\log(a^b) =$

Q16: Express $\log(ab)$ in terms of $\log a$ and $\log b$. $\log(ab) =$

Trigonometry

Q17: Express $\cos(a + b)$ in terms of $\cos a$ and $\cos b$. $\cos(a + b) =$

Q18: Given a triangle with side lengths a, b, c and opposite angles A, B, C , express $\sin A$ in terms of a, b , and $\sin B$.

$\sin A =$

Q19: Given a right triangle with acute angles A and B , express the following in terms of opposite side length a and b :

$\sin A =$

$\cos A =$

$\tan A =$

Quadratic equation

Q20: How many real-value solutions for x are possible to an equation of the form $ax^2 + bx + c = 0$?

A) zero; B) one; C) two; D) three; E) zero, one or two; or F) zero, one, two or three.

Q21: Identify the solution(s), if any, to $x^2 - 4x - 5 = 0$.

Linear interpolation

Q22: What is *the* linear function $f : [0, 1] \rightarrow \mathbb{R}$, so that $f(0) = a$ and $f(1) = b$?

Q23: What is *the* linear vector-valued function $f : [0, 1] \rightarrow \mathbb{R}^3$, so that $f(0) = \mathbf{a}$ and $f(1) = \mathbf{b}$?

Derivatives

Q24: What is the derivative of the function $f(t) = 3t^2$? $\frac{df}{dt} =$

Q25: What is the second derivative of the function $f(t) = 3t^2$? $\frac{d^2f}{dt^2} =$

Q26: What is the partial derivative of the function $f(x, y) = 3x^2 - 3y^2$ with respect to y ? $\frac{df}{dy} =$

Q27: Given a function $f : \mathbb{R}^3 \rightarrow \mathbb{R}$, how many dimensions does its gradient have?

Q28: Using partial derivative and vector notations express the gradient of a function $f(x, y, z) : \mathbb{R}^3 \rightarrow \mathbb{R}$.

$\nabla f =$

C++ Programming

Q29: We want to call `func` on `y`, what should be written in place of the question mark in the following C++ code?

A) `*`; B) `&`; or C) nothing.

```
bool func(double & x);
...
int main()
{
    double * y = new double();
    func( ? y);
}
```

Geometry

Q30: What is the volume of a sphere with radius r ?