# Groups C&D: Monte Carlo Pricing Methods

## C. Monte Carlo 101

---

b). Run the MC program again with data from Batches 1 and 2.

Error Matrix (Batch 1, Call option, T=0.25)

| N_Simulations \| N_TimeSteps | 10 | 50 | 100 | 200 | 400 | 800 |
|---|---|---|---|---|---|---|
| 100 | 0.251878 | 0.108602 | -0.0384474 | -0.164607 | 0.0912756 | 0.0384025 |
| 1,000 | -0.0533561 | -0.0870517 | -0.0592721 | 0.0187694 | 0.0183932 | 0.00835902 |
| 10,000 | -0.0104041 | -0.0653066 | 0.00442991 | -0.0410344 | -0.0137536 | 0.00350434 |
| 100,000 | -0.0222026 | -0.0268527 | -0.0029445 | 0.0155129 | 0.0274011 | 0.00369452 |
| 100,0000 | -0.0166257 | -0.0001377 | -0.0006577 | 0.00018974 | -0.0027913 | 0.00461255 |

Error Matrix (Batch 1, Put option, T=0.25)

| N_Simulations \| N_TimeSteps | 10 | 50 | 100 | 200 | 400 | 800 |
|---|---|---|---|---|---|---|
| 100 | 0.178219 | 0.617421 | 0.741533 | 0.341041 | 0.237799 | 0.661955 |
| 1,000 | 0.181813 | -0.0541072 | 0.00223412 | 0.0468783 | 0.316352 | 0.186158 |
| 10,000 | -0.0045147 | 0.0319792 | 0.0617888 | 0.107535 | 0.0871251 | 0.100389 |
| 100,000 | -0.0006821 | 0.0100613 | 0.0269275 | 0.00615011 | -0.0106952 | 0.00188266 |
| 100,0000 | -0.0055359 | -0.0060801 | 0.00496558 | 0.0118842 | 0.00026649 | -0.0088976 |

Error Matrix (Batch 2, Call option, T = 1.0)

| N_Simulations \| N_TimeSteps | 10 | 50 | 100 | 200 | 400 | 800 |
|---|---|---|---|---|---|---|
| 100 | 0.477133 | 0.0995085 | -0.456498 | -0.494182 | 0.175358 | -0.146744 |
| 1,000 | -0.0958045 | -0.0822719 | -0.229996 | 0.0826504 | -0.243794 | -0.141424 |
| 10,000 | 0.0151157 | -0.180551 | -0.0993834 | -0.153064 | -0.0896099 | -0.0350671 |
| 100,000 | -0.0042699 | -0.0625969 | -0.0219536 | 0.0350313 | 0.0699948 | 0.00379866 |
| 100,0000 | 0.00365388 | 0.011206 | -0.0030687 | -0.0044384 | -0.0058721 | 0.017376 |

Error Matrix (Batch 2, Put option, T = 1.0)

| N_Simulations \| N_TimeSteps | 10 | 50 | 100 | 200 | 400 | 800 |
|---|---|---|---|---|---|---|
| 100 | 0.139968 | 1.15171 | -0.210553 | 0.637125 | 0.484443 | 1.17034 |
| 1,000 | 0.394808 | -0.0020975 | -0.0806299 | 0.138154 | 0.408891 | 0.240675 |
| 10,000 | 0.0182201 | 0.0393024 | 0.1908 | 0.170121 | 0.13184 | 0.170992 |
| 100,000 | 0.0411303 | 0.0197412 | 0.0423304 | 0.0108124 | -0.017423 | -0.0021193 |
| 100,0000 | 0.0254141 | -0.0024777 | 0.00882419 | 0.0202426 | 0.00097318 | -0.0133003 |

**CONCLUSIONS:**

1. For the fixed simulation times (NSim), increasing the time steps (NT) from 10 to 800 does not always improve the accuracy of Monte Carlo simulation, and sometime things may go worse.
2. Fox the fixed time steps (NT), increasing the simulation times (NSim) does improve the accuracy of MC simulation under most of the situation. The improvement is especially great when NSim goes from 100 to 1,000 and 1,000 to 10,000.
3. In order to get the stable result with accuracy **below 2 decimal**, we should always choose NT larger than 400 and NSim larger than **million**.

---

C). Run the MC program again with data from Batches 4 and get an accuracy to two places behind the decimal point.

Error Matrix (Batch 4, Call option, T = 30.0)

| N_Simulations \| N_TimeSteps | 10 | 50 | 100 | 200 | 400 | 800 |
|---|---|---|---|---|---|---|
| 100 | 6.8871 | 24.7918 | -15.6953 | -20.8759 | -10.0135 | 0.418346 |
| 1,000 | -18.3161 | -10.7502 | -6.39959 | -5.055 | -2.70881 | 1.63378 |
| 10,000 | -18.707 | -7.57187 | -4.13449 | -1.69468 | -0.843321 | -2.61702 |
| 100,000 | -20.5156 | -5.22228 | -2.7509 | 0.0367613 | 0.721127 | 1.13862 |
| 100,0000 | -20.5162 | -5.12025 | -2.65163 | -0.868016 | -0.674281 | 0.713892 |

Error Matrix (Batch 4, Put option, T = 30.0)

| N_Simulations \| N_TimeSteps | 10 | 50 | 100 | 200 | 400 | 800 |
|---|---|---|---|---|---|---|
| 100 | 0.233213 | 0.334768 | 0.500026 | 0.0326586 | 0.0509087 | 0.258818 |
| 1,000 | 1.15879 | 0.0961445 | -0.0189283 | 0.0845869 | 0.0828274 | 0.0367115 |
| 10,000 | 1.29945 | 0.101395 | 0.0744598 | 0.057336 | 0.036091 | 0.0381362 |
| 100,000 | 1.51365 | 0.100315 | 0.0485408 | 0.0249194 | 0.0131639 | 0.00148295 |
| 100,0000 | 1.54712 | 0.0997821 | 0.0452478 | 0.0229343 | 0.0102382 | 0.00162479 |

**ANALYSIS:**

We can see that the results of batch 4 are not so good as batch 1 and 2. The error of Put option is among the reasonable range, however, the performance of pricing call option is extremely poor.

That is becauseThe stock price may vary a lot in such long maturity time. However, the stock price cannot go below zero, but can go up to tremendous large value in 30 year.

We are able to reach every possible price between 0 and 100 given millions of simulation we implemented, but the condition that the price higher than100 might not be sufficiently considered.

Therefore, the accuracy of Put option looks somewhere sound but the accuracy of Call option is inevitably poor.

## D. Advanced Monte Carlo

---

b). Run the MC program again with data from Batches 1 and 2 to see SD and SE.

Standard Deviation Matrix (Batch 1, Call option, T=0.25)

| N_Simulations \| N_TimeSteps | 10 | 50 | 100 | 200 | 400 | 800 |
|---|---|---|---|---|---|---|
| 100 | 5.68432 | 5.01132 | 4.35468 | 4.10694 | 4.36427 | 4.73576 |
| 1,000 | 4.4872 | 4.22116 | 4.4872 | 4.50803 | 4.65114 | 4.66257 |
| 10,000 | 4.48636 | 4.43742 | 4.54229 | 4.54746 | 4.53902 | 4.5282 |
| 100,000 | 4.43418 | 4.49646 | 4.51298 | 4.55943 | 4.55427 | 4.54268 |

Standard Error Matrix (Batch 1, Call option, T=0.25)

| N_Simulations \| N_TimeSteps | 10 | 50 | 100 | 200 | 400 | 800 |
|---|---|---|---|---|---|---|
| 100 | 0.571296 | 0.503657 | 0.437662 | 0.412763 | 0.438626 | 0.475962 |
| 1,000 | 0.141969 | 0.133552 | 0.141969 | 0.142628 | 0.147156 | 0.147517 |
| 10,000 | 0.0448659 | 0.0443764 | 0.0454252 | 0.0454768 | 0.0453925 | 0.0452843 |
| 100,000 | 0.0140222 | 0.0142191 | 0.0142714 | 0.0144183 | 0.0144019 | 0.0143653 |

Standard Deviation Matrix (Batch 1, Put option, T=0.25)

| N_Simulations \| N_TimeSteps | 10 | 50 | 100 | 200 | 400 | 800 |
|---|---|---|---|---|---|---|
| 100 | 6.34558 | 6.24622 | 6.47135 | 6.11058 | 6.09186 | 6.32028 |
| 1,000 | 5.89554 | 6.14564 | 5.89554 | 6.05829 | 5.89946 | 6.02363 |
| 10,000 | 6.07762 | 6.06697 | 6.05095 | 6.08452 | 6.02585 | 6.09752 |
| 100,000 | 6.08728 | 6.05186 | 6.05774 | 6.05044 | 6.0447 | 6.05226 |

Standard Error Matrix (Batch 1, Put option, T=0.25)

| N_Simulations \| N_TimeSteps | 10 | 50 | 100 | 200 | 400 | 800 |
|---|---|---|---|---|---|---|
| 100 | 0.637755 | 0.627769 | 0.650395 | 0.614136 | 0.612255 | 0.635212 |
| 1,000 | 0.186527 | 0.194439 | 0.186527 | 0.191676 | 0.186651 | 0.190579 |
| 10,000 | 0.0607793 | 0.0606728 | 0.0605125 | 0.0608482 | 0.0602616 | 0.0609783 |
| 100,000 | 0.0192498 | 0.0191378 | 0.0191563 | 0.0191333 | 0.0191151 | 0.019139 |

Standard Deviation Matrix (Batch 2, Call option, T=0.25)

| N_Simulations \| N_TimeSteps | 10 | 50 | 100 | 200 | 400 | 800 |
|---|---|---|---|---|---|---|
| 100 | 15.8111 | 14.3125 | 12.8181 | 12.2344 | 12.8753 | 13.6987 |
| 1,000 | 13.1041 | 12.464 | 13.0688 | 13.1188 | 13.5045 | 13.4975 |
| 10,000 | 13.0898 | 12.9576 | 13.2117 | 13.1958 | 13.2049 | 13.1825 |
| 100,000 | 12.9674 | 13.0958 | 13.1477 | 13.25 | 13.2462 | 13.2104 |

Standard Error Matrix (Batch 2, Call option, T=0.25)

| N_Simulations \| N_TimeSteps | 10 | 50 | 100 | 200 | 400 | 800 |
|---|---|---|---|---|---|---|
| 100 | 1.58908 | 1.43846 | 1.28827 | 1.22961 | 1.29402 | 1.37677 |
| 1,000 | 0.414594 | 0.394344 | 0.413477 | 0.415061 | 0.427263 | 0.427043 |
| 10,000 | 0.130904 | 0.129583 | 0.132124 | 0.131965 | 0.132055 | 0.131831 |
| 100,000 | 0.0410068 | 0.0414126 | 0.0415769 | 0.0419003 | 0.0418883 | 0.0417752 |

Standard Deviation Matrix (Batch 2, Put option, T=0.25)

| N_Simulations \| N_TimeSteps | 10 | 50 | 100 | 200 | 400 | 800 |
|---|---|---|---|---|---|---|
| 100 | 11.2165 | 10.8075 | 11.2277 | 10.4938 | 10.4424 | 11.0178 |
| 1,000 | 10.8778 | 10.5699 | 10.1231 | 10.3915 | 10.1791 | 10.4152 |
| 10,000 | 10.5307 | 10.4749 | 10.427 | 10.5083 | 10.3665 | 10.5203 |
| 100,000 | 10.5416 | 10.4263 | 10.4359 | 10.4152 | 10.3994 | 10.4194 |

Standard Error Matrix (Batch 2, Put option, T=0.25)

| N_Simulations \| N_TimeSteps | 10 | 50 | 100 | 200 | 400 | 800 |
|---|---|---|---|---|---|---|
| 100 | 1.1273 | 1.08619 | 1.12842 | 1.05467 | 1.0495 | 1.10733 |
| 1,000 | 0.344159 | 0.334418 | 0.320282 | 0.328772 | 0.322052 | 0.329521 |
| 10,000 | 0.105312 | 0.104754 | 0.104275 | 0.105088 | 0.10367 | 0.105208 |
| 100,000 | 0.0333356 | 0.0329711 | 0.0330013 | 0.0329358 | 0.0328859 | 0.0329492 |

**CONCLUSIONS:**

1. Looks like the standard deviation of simulation results did not change along with the simulation times (NSim) and time steps (NT).
2. Fox the fixed time steps (NT), the Standard Error decreased significantly as the simulation times (NSim) increase.