

# Exploring Drug Candidates: All $\epsilon$ -Best Arms Identification in Linear Bandits

## Authors:

- Zhekai Li (SEIEE, Shanghai Jiao Tong University)
- Tianyi Ma (University of Michigan - Shanghai Jiao Tong University Joint Institute)
- Cheng Hua (Antai College of Economics and Management, Shanghai Jiao Tong University)
- Ruihao Zhu (Cornell SC Johnson College of Business)

***2024 INFORMS Annual Meeting***

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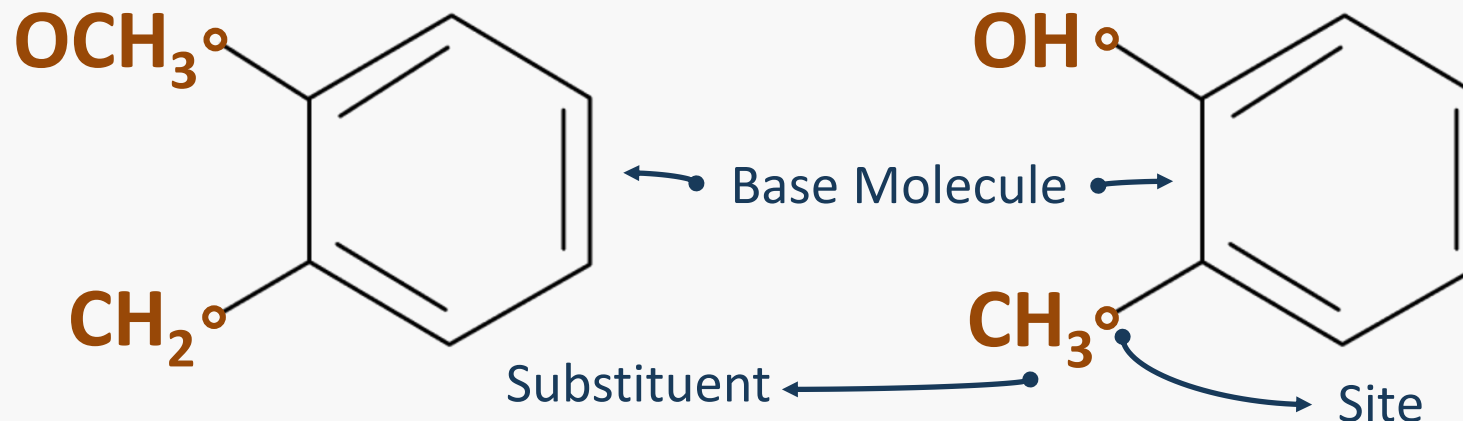
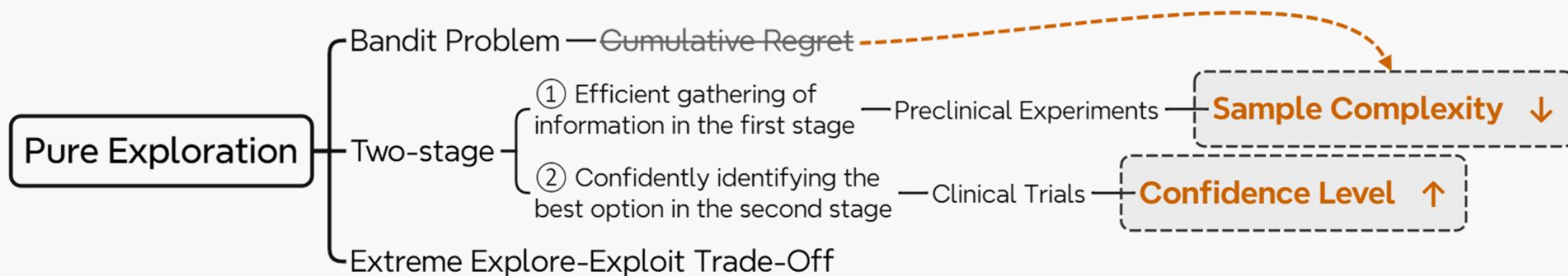
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## An Illustrative Example

- **In Drug discovery:** medical researchers start with a promising molecule for treating a given disease and then **test** potentially millions of variants of this molecule to identify the highly potent candidates for later **clinical trials**



## An Illustrative Example

H  
OH  
OCOCH<sub>3</sub>  
OCOCH<sub>2</sub>CH<sub>3</sub>  
OCH<sub>3</sub>  
NO<sub>2</sub>

Six Optional  
substituents  
for the first site

CH<sub>3</sub>  
CH<sub>2</sub>CH<sub>2</sub>(C<sub>6</sub>H<sub>5</sub>)  
CH<sub>2</sub>(C<sub>3</sub>H<sub>5</sub>)  
CH<sub>2</sub>CHC(CH<sub>3</sub>)<sub>2</sub>  
CH<sub>2</sub>  
NO<sub>2</sub>

Six Optional  
substituents  
for the second site

$$6 \times 6 = 36$$

## Combination

(H+CH<sub>3</sub>), (H+CH<sub>2</sub>CH<sub>2</sub>(C<sub>6</sub>H<sub>5</sub>)), (H+CH<sub>2</sub>(C<sub>3</sub>H<sub>5</sub>)),  
(H+CH<sub>2</sub>), (H+NO<sub>2</sub>)  
  
(OH+CH<sub>3</sub>), (OH+CH<sub>2</sub>CH<sub>2</sub>(C<sub>6</sub>H<sub>5</sub>)), (OH+CH<sub>2</sub>(C<sub>3</sub>H<sub>5</sub>)),  
(OH+CH<sub>2</sub>), (OH+NO<sub>2</sub>)  
  
(OCOCH<sub>3</sub>+CH<sub>3</sub>), (OCOCH<sub>3</sub>+CH<sub>2</sub>CH<sub>2</sub>(C<sub>6</sub>H<sub>5</sub>)),  
(OCOCH<sub>3</sub>+CH<sub>2</sub>(C<sub>3</sub>H<sub>5</sub>)), (OCOCH<sub>3</sub>+CH<sub>2</sub>),  
(OCOCH<sub>3</sub>+NO<sub>2</sub>)  
.....

How to handle this  
increasing scale of variants

Motivation

Model

Algorithm

Results

Experiment

# Motivation – Why Linear Bandits

## Free-Wilson – A Quantitative Structured Relationship (Free and Wilson 1964)

$$\text{Total Effectiveness} = [\text{Effectiveness of Base Molecule}] + \sum_i [\text{Effectiveness of Substituent } i]$$

### Combination - grow exponentially

(H+CH<sub>3</sub>), (H+CH<sub>2</sub>CH<sub>2</sub>(C<sub>6</sub>H<sub>5</sub>)), (H+CH<sub>2</sub>(C<sub>3</sub>H<sub>5</sub>)), (H+CH<sub>2</sub>),  
(H+NO<sub>2</sub>)

(OH+CH<sub>3</sub>), (OH+CH<sub>2</sub>CH<sub>2</sub>(C<sub>6</sub>H<sub>5</sub>)), (OH+CH<sub>2</sub>(C<sub>3</sub>H<sub>5</sub>)),  
(OH+CH<sub>2</sub>), (OH+NO<sub>2</sub>)

(OCOCH<sub>3</sub>+CH<sub>3</sub>), (OCOCH<sub>3</sub>+CH<sub>2</sub>CH<sub>2</sub>(C<sub>6</sub>H<sub>5</sub>)),  
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(OCOCH<sub>3</sub>+NO<sub>2</sub>)

.....

H  
OH  
OCOCH<sub>3</sub>  
OCOCH<sub>2</sub>CH<sub>3</sub>  
OCH<sub>3</sub>  
NO<sub>2</sub>  
  
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CH<sub>2</sub>CHC(CH<sub>3</sub>)<sub>2</sub>  
CH<sub>2</sub>  
NO<sub>2</sub>

Shared Information

Linear Bandits

Effectiveness of all the  
substituents

Much fewer

Motivation

Model

Algorithm

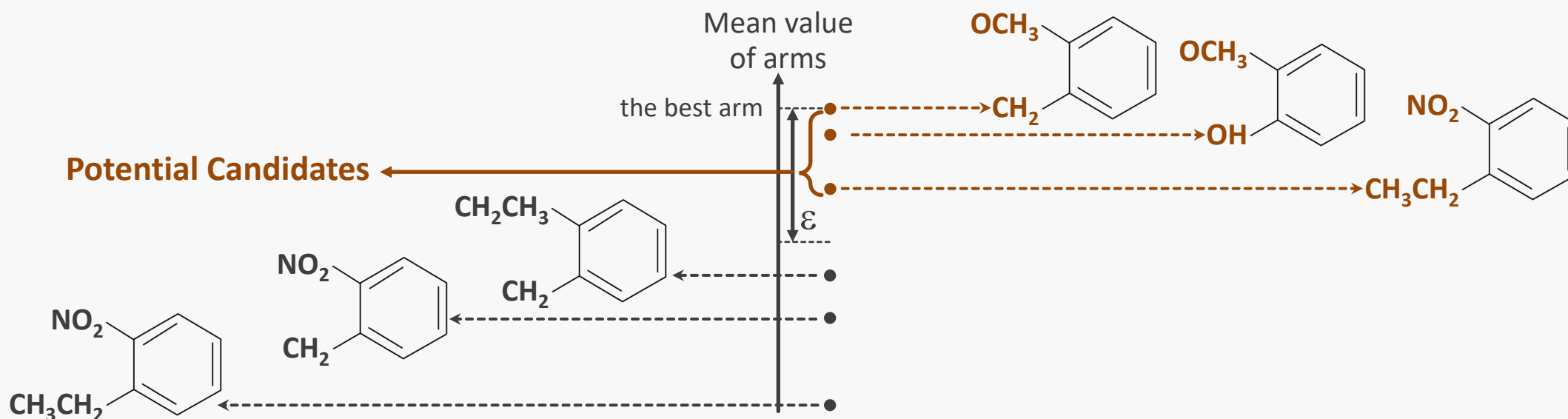
Results

Experiment

## Problem

- ## high cost and low efficacy

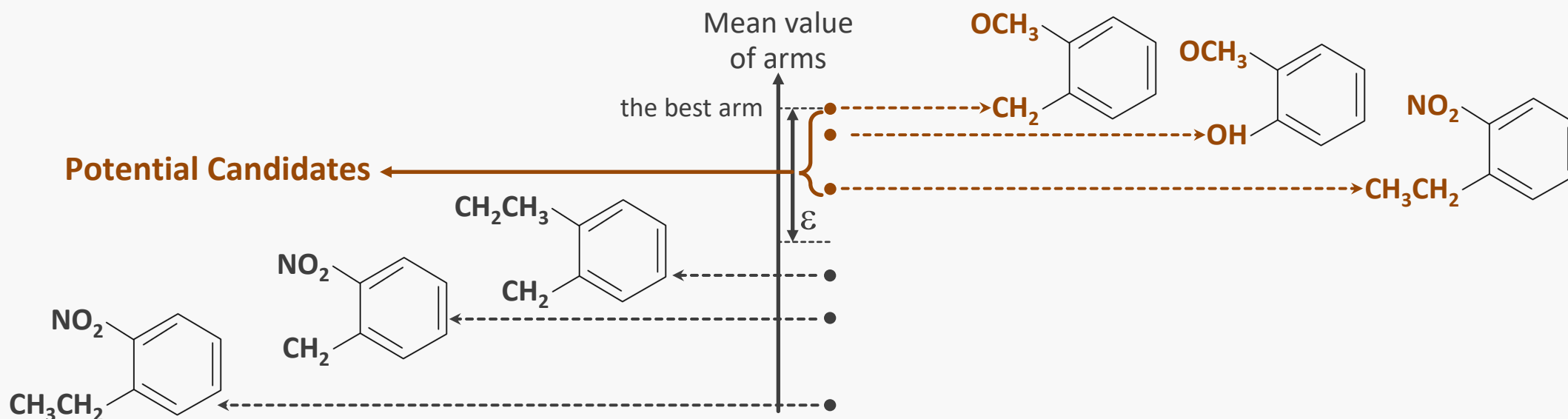
- **Identifying all candidates whose effectiveness is within a range of  $\varepsilon$  from the best one**



## Problem

- ## high cost and low efficacy

- **Identifying all candidates whose effectiveness is within a range of  $\varepsilon$  from the best one**



# Model - All $\epsilon$ -Best Arms Identification in Linear Bandits

Exploring Drug Candidates:  
All  $\epsilon$ -Best Arms Identification  
in Linear Bandits

Zhekai Li  
(SJTU)

Motivation

➤ A finite set of  $K$  arms, denoted as  $\mu = (\mu_1, \mu_2, \dots, \mu_K)$  and we have  $\mu_1 > \mu_2 \geq \dots \geq \mu_K$

➤ The Linear Structure

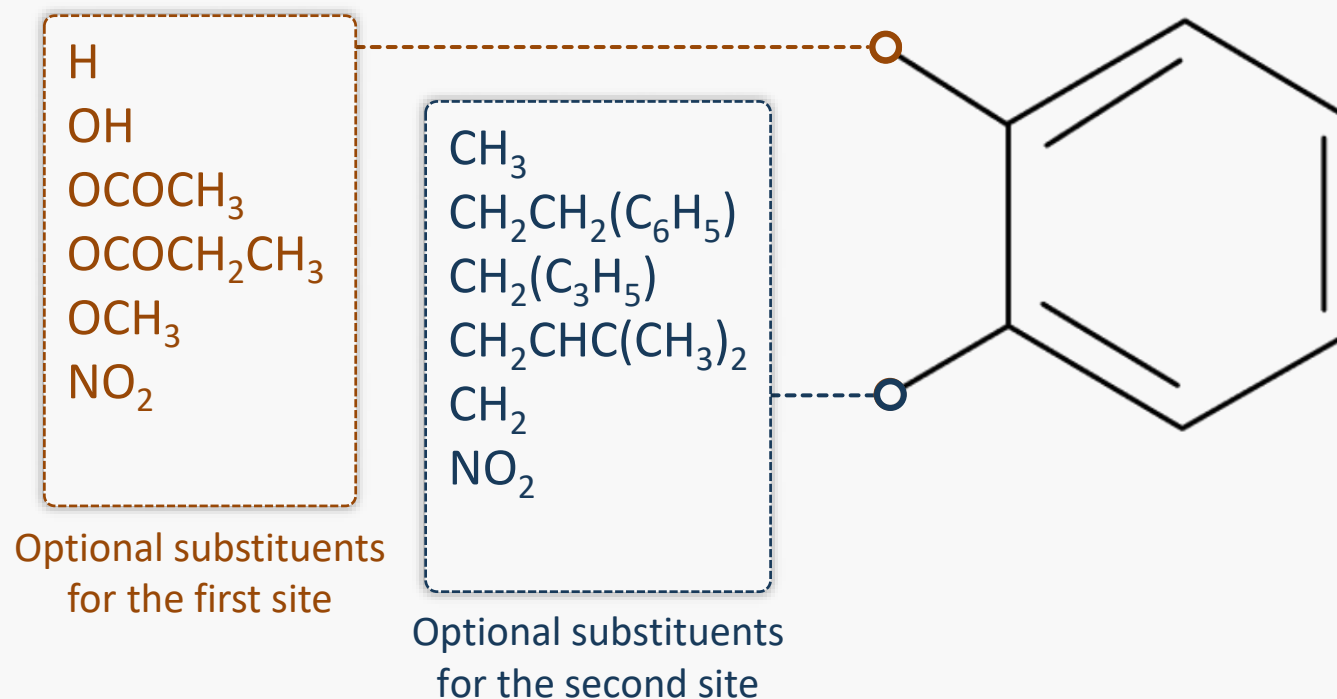
- Denote  $\theta$  as the unknown parameter vector
- Denote  $A = \{a_1, a_2, \dots, a_K\} \subset \mathbf{R}^d$  as the set containing the feature vectors of all arms
- Observe the reward  $X_t = a_{A_t}^\top \theta + \eta_t$ , where  $\eta_t$ , the noise, is conditionally 1-sub-Gaussian

Model

Algorithm

Results

Experiment



Previous Drug Example



Motivation

➤ A finite set of  $K$  arms, denoted as  $\mu = (\mu_1, \mu_2, \dots, \mu_K)$  and we have  $\mu_1 > \mu_2 \geq \dots \geq \mu_K$

➤ The Linear Structure

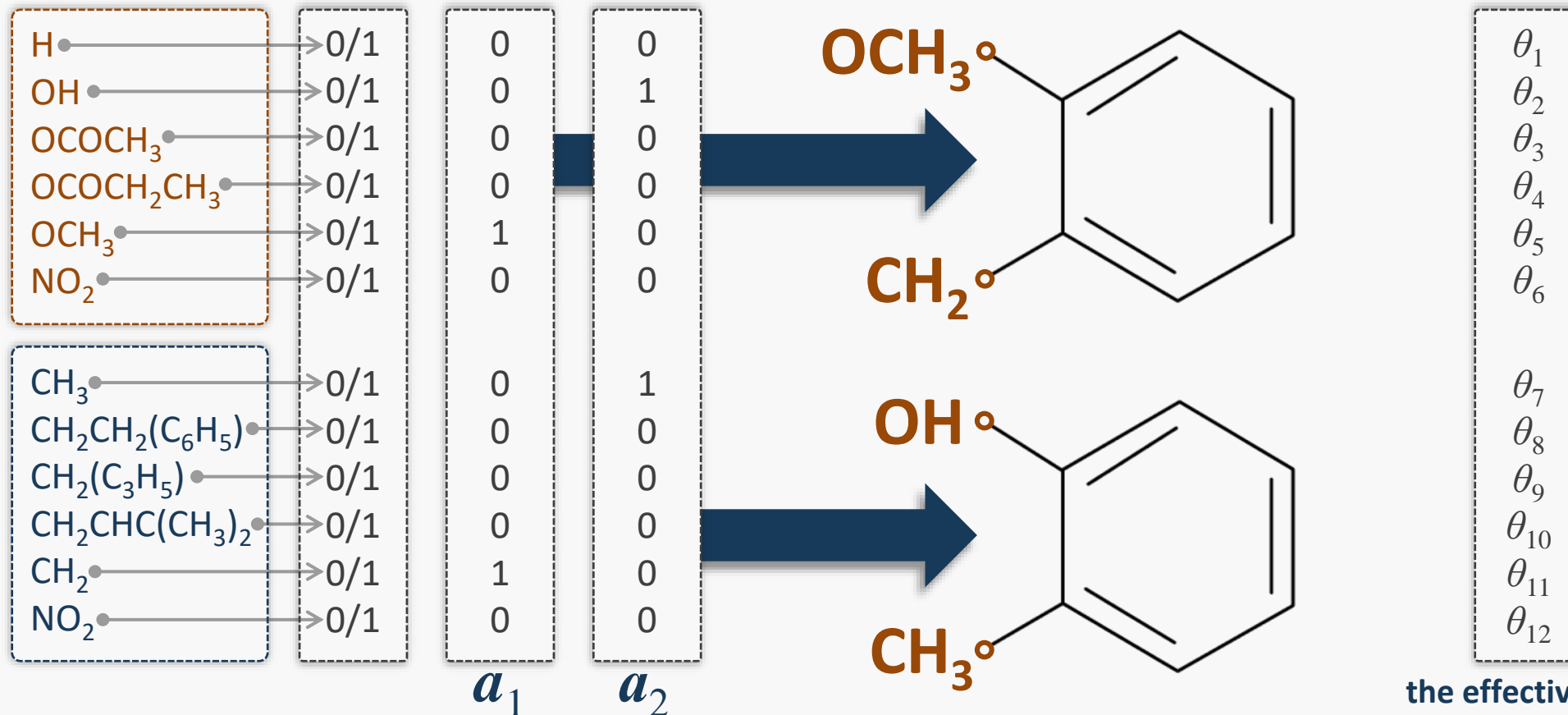
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Model

Algorithm

Results

Experiment



the effectiveness  
of all substituents

Motivation

➤ **A finite set of  $K$  arms**, denoted as  $\boldsymbol{\mu} = (\mu_1, \mu_2, \dots, \mu_K)$  and we have  $\mu_1 > \mu_2 \geq \dots \geq \mu_K$

➤ **The Linear Structure**

- Denote  $\boldsymbol{\theta}$  as the unknown parameter vector
- Denote  $\mathcal{A} = \{\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_K\} \subset \mathbf{R}^d$  as the set containing the feature vectors of all arms
- Observe the reward  $X_t = \mathbf{a}_{A_t}^\top \boldsymbol{\theta} + \eta_t$ , where  $\eta_t$ , the noise, is conditionally 1-sub-Gaussian

➤ **Task: Denote the set of all  $\varepsilon$ -best arms with mean vector  $\boldsymbol{\mu}$  as  $G_\varepsilon(\boldsymbol{\mu}) := \{i: \mu_i \geq \mu_1 - \varepsilon\}$**

- Additive  $\varepsilon$ -Best Arm: given  $\varepsilon > 0$ , an arm  $i$  is deemed  $\varepsilon$ -best if  $\mu_i \geq \mu_1 - \varepsilon$

➤ **Performance Metric: the sample complexity**

- Confidence level  $\delta$  is fixed  $\rightarrow$  Fixed-Confidence Setting


$$\begin{array}{ll} \min & \mathbf{E}_{\boldsymbol{\mu}} [\tau_\delta] \\ \text{s.t.} & \mathbf{P}_{\boldsymbol{\mu}}(\tau_\delta < \infty, \text{recommended set equals } G_\varepsilon(\boldsymbol{\mu})) \geq 1 - \delta \end{array}$$

Algorithm

Results

Experiment

Motivation

- Pure Exploration with different tasks. **Mannor and Tsitsiklis (2004), Even-Dar et al. (2006), Russo (2020), Komiyama et al. (2023), Kalyanakrishnan and Stone (2010), Kalyanakrishnan et al. (2012), Locatelli et al. (2016), Abernethy et al. (2016), Garivier and Kaufmann (2016),**
- Linear Bandits in Pure Exploration. **Abbasi-Yadkori et al. (2011), Gabillon et al. (2012), Hoffman et al. (2014), Soare et al. (2014), Fiez et al. (2019), Reda et al. (2021), Yang and Tan (2021), Azizi et al. (2023)**
- Model Misspecification. **Ghosh et al. (2017), Lattimore et al. (2020), Reda et al. (2021), Ahn et al. (2024)**
- All  $\epsilon$ -Best Arms Identification. **Mason et al. (2020), Al Marjani et al. (2022)**

Model

Algorithm

## Current Status of Research

Results

- All  $\epsilon$ -best arms identification in stochastic bandits **Mason et al. (2020) – lower bound and two algorithms**
- **Limited to the stochastic setting and is hard to be applied in problems with a large number of choices**

Experiment

## Research Question

- **How to solve the All  $\epsilon$ -Best Arms Identification in Linear Bandits?**
  - The description of the problem complexity
  - The algorithm and the upper bound
  - Extensions and other insights (Misspecification and GLM)

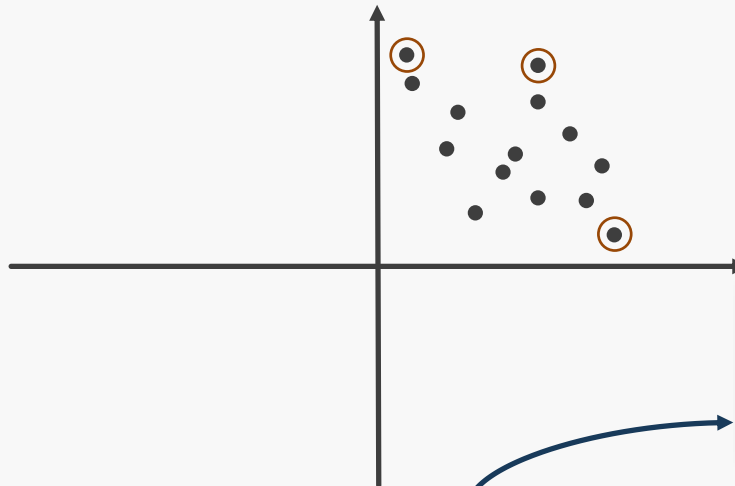
## General Structure

- LinFACTE is a **phase-based elimination and classification algorithm with five general components**
- Initialization → sampling → estimation → classification → stopping and decision

## Phase Iteration

## Sampling and Estimation

- A probabilistic guarantee that the true mean value is within a range of the estimated mean value for each arm
- Challenge: Extremely large arm space → solved by **the optimal design**  $\left\{ \begin{array}{l} \text{G-optimal (arm's confidence region)} \\ \text{xy-optimal (gap's confidence region)} \end{array} \right.$



2-Dimensional Arm Space

- Given confidence level → Minimized budget
- Only a small number of arms need to be sampled
- Dramatically reducing the problem's difficulty

Motivation

Model

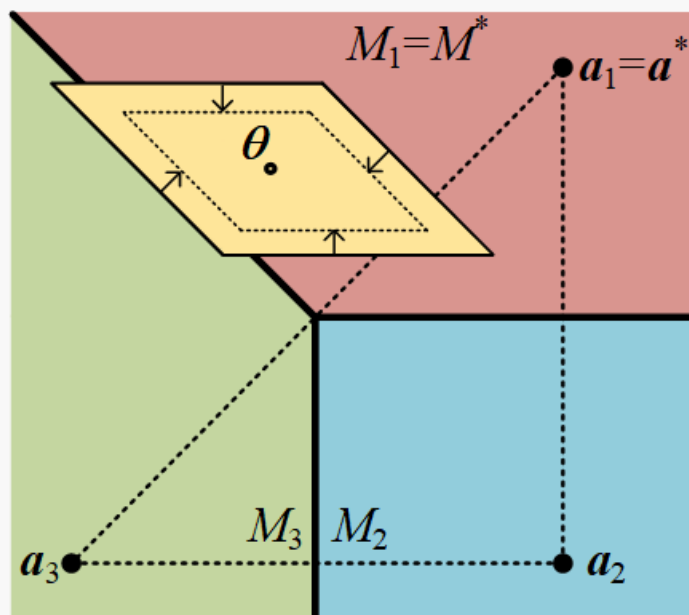
Algorithm

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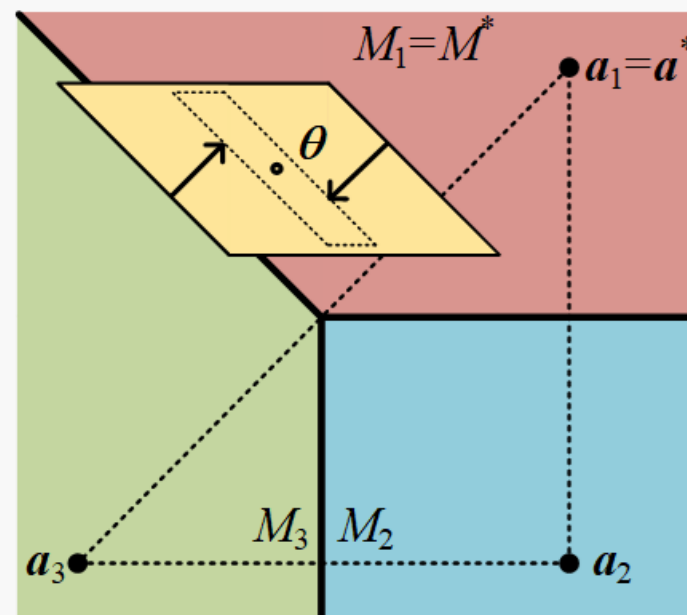
Experiment

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  - G-optimal (arm's confidence region)**
  - $\chi\chi$ -optimal (gap's confidence region)**



Uniform Contraction Based on  
G-Optimal Design



More Purposeful Contraction of  
 $\chi\chi$ -Optimal Design

## General Structure

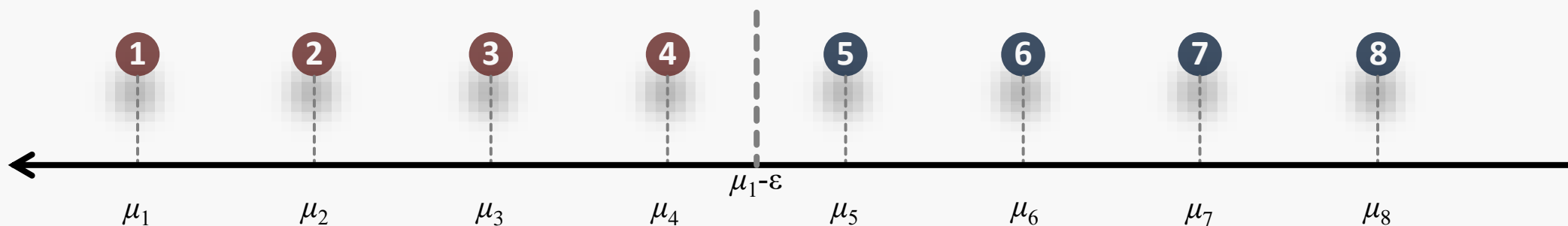
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## Toy Example of Eight Arms



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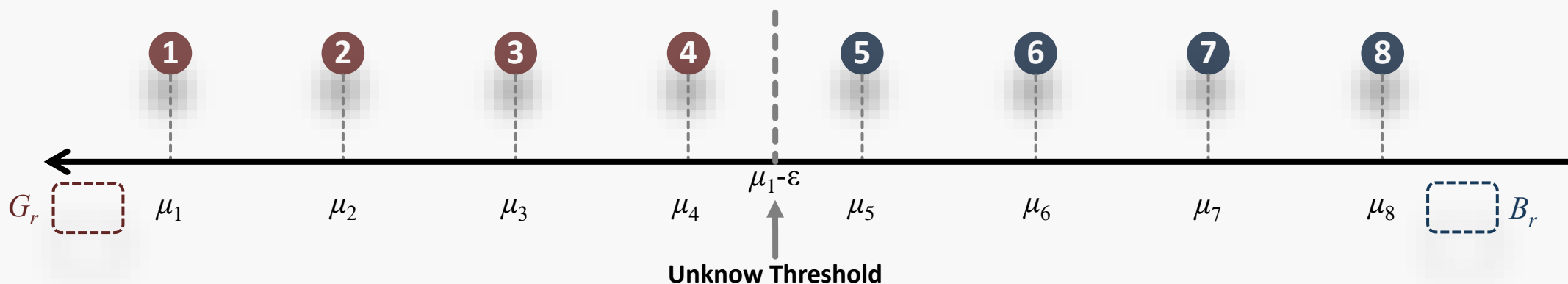
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- Update two sets of arms, that is,  $G_r$  and  $B_r$ , classifying arms that are empirically  $\varepsilon$ -best and not  $\varepsilon$ -best



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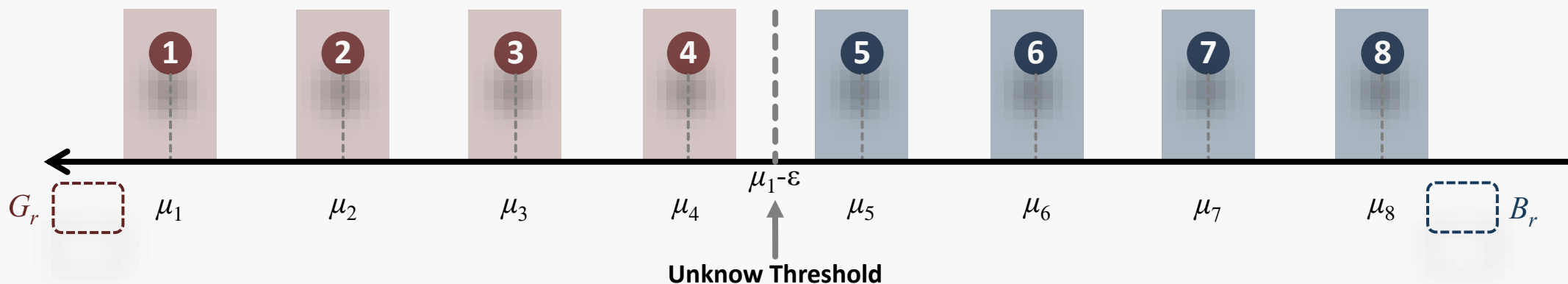
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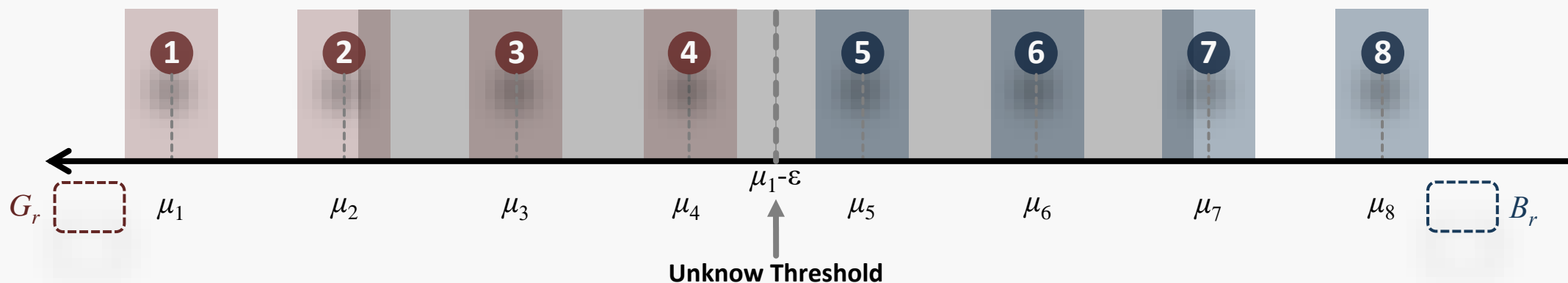
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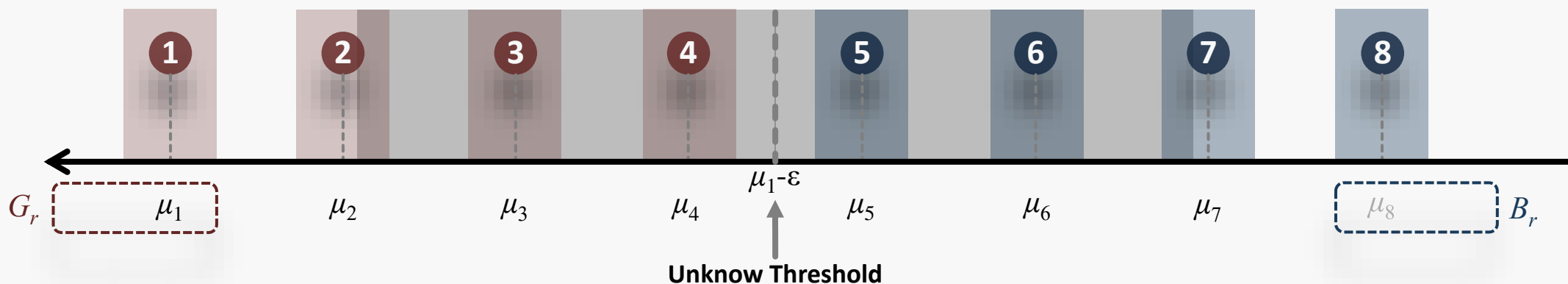
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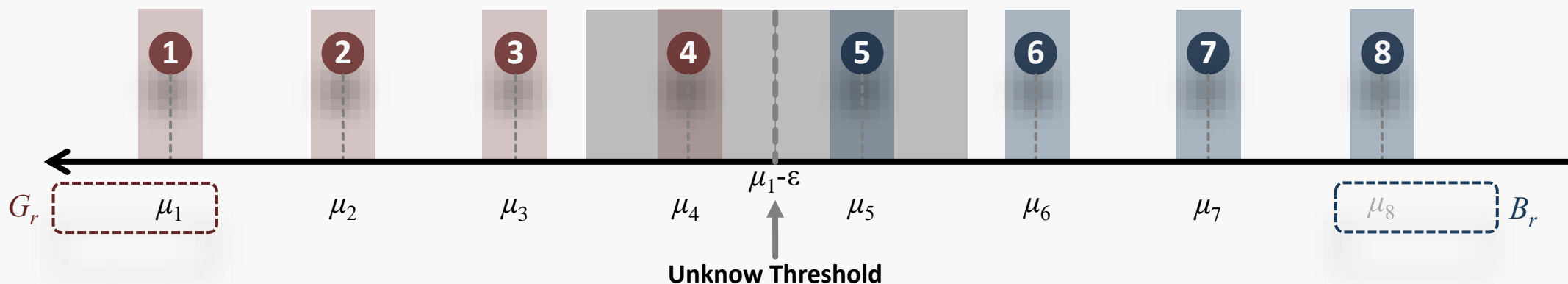
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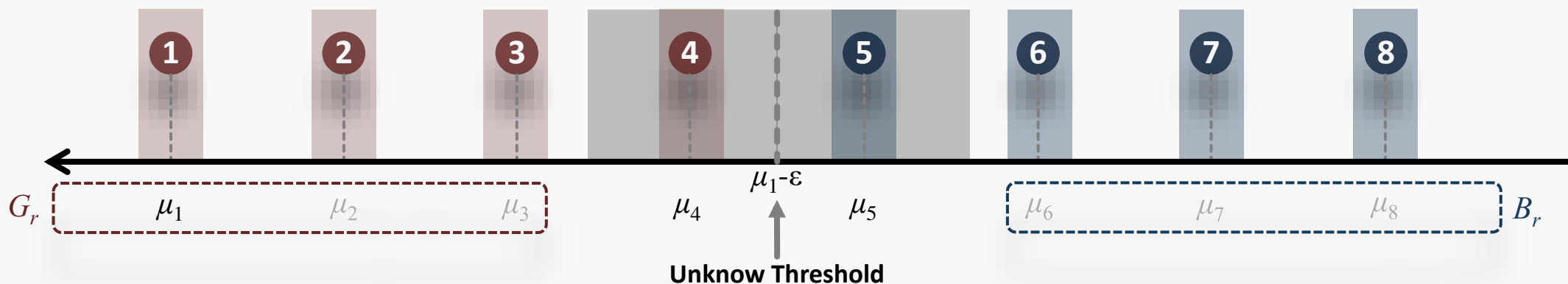
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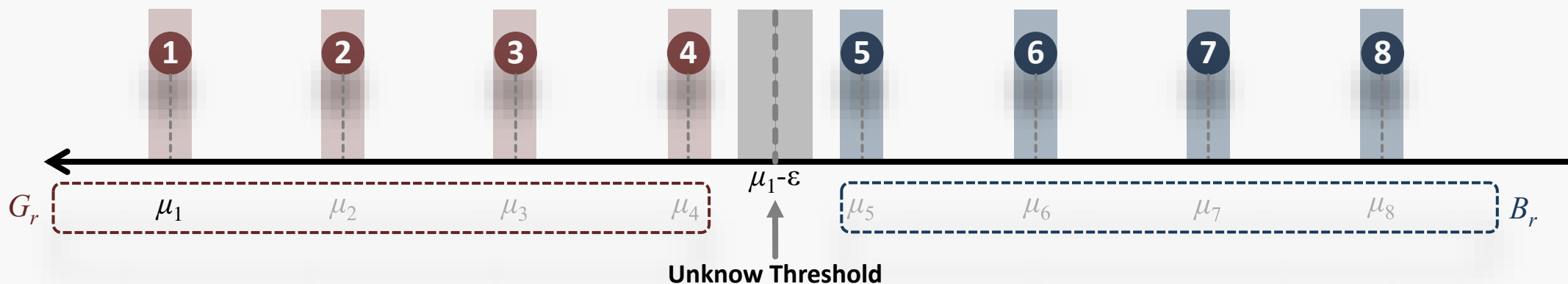
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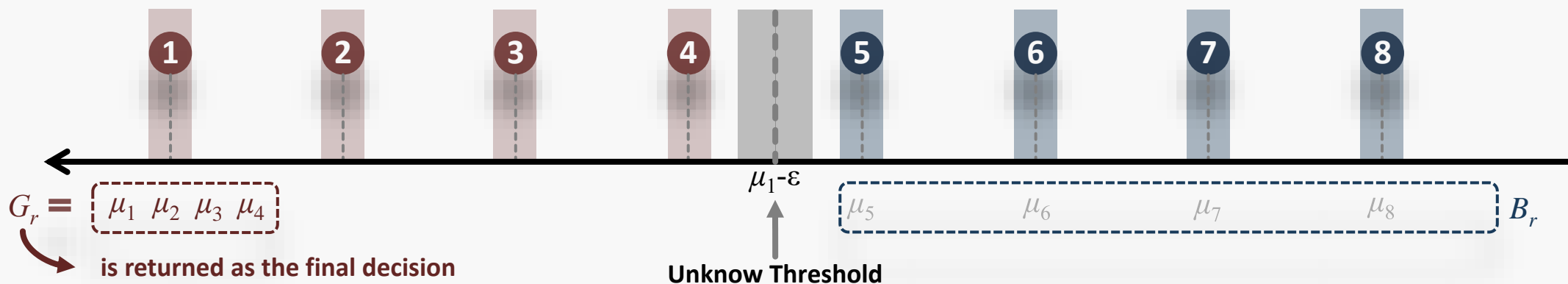
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## Lower Bound

➤ Consider a set of arms where arm  $i$  follows a normal distribution. Any  $\delta$ -PAC algorithm must satisfy

$$\frac{\mathbb{E}_{\mu} [\tau_{\delta}]}{\log(1/2.4\delta)} \geq (\Gamma^*)^{-1} = \min_{p \in S_K} \max_{(i,j,m) \in \mathcal{X}} \max \left\{ \frac{2\|\mathbf{a}_i - \mathbf{a}_j\|_{\mathbf{V}_p^{-1}}^2}{(\mathbf{a}_i^{\top} \boldsymbol{\theta} - \mathbf{a}_j^{\top} \boldsymbol{\theta} + \epsilon)^2}, \frac{2\|\mathbf{a}_1 - \mathbf{a}_m\|_{\mathbf{V}_p^{-1}}^2}{(\mathbf{a}_1^{\top} \boldsymbol{\theta} - \mathbf{a}_m^{\top} \boldsymbol{\theta} - \epsilon)^2} \right\}$$

## Upper Bound

Define  $\Delta = \min(\alpha_{\epsilon}, \beta_{\epsilon})/8$ . Based on the **G-optimal** design, with a probability of at least  $1 - \delta$ , the expected sampling budget of LinFACTE has the following upper bound

$$\mathbb{E}[T_G | \mathcal{E}] = O \left( d\Delta^{-2} \log \left( \frac{K}{\delta} \log_2(\Delta^{-2}) \right) + d^2 \log(\Delta^{-1}) \right)$$

Define  $\Delta = \min(\alpha_{\epsilon}, \beta_{\epsilon})/8$ . Based on the  **$\mathcal{XY}$ -optimal** design, with a probability of at least  $1 - \delta$ , the expected sampling budget of LinFACTE has the following upper bound

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Motivation

Model

Algorithm

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- Algorithm with  $\mathcal{XY}$ -optimal design → **near optimal up to some logarithmic factors**



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- Algorithm with  $\mathcal{XY}$ -optimal design → **near optimal up to some logarithmic factors**
- Model Extension → more general results → applicability of LinFACTE
- Extendable to **misspecified linear bandits**
  - Extendable to **generalized linear model (GLM)**

Motivation

## Baselines

- **Bayesian optimization** with a knowledge gradient acquisition function (**Negoescu et al. 2011**)
- **BayesGap**: a gap-based algorithm for the best arm identification (BAI) (**Hoffman et al. 2014**)
- **m-LinGapE** and **LinGIFA**: two gap-based algorithms for the top m identification (**R'eda et al. 2021**)
- **Lazy Track-Threshold-and-Stop**: track and stop algorithm for the threshold bandit (**Tewari et al. 2024**)

Model

## Dataset

- Synthetic Data → LinFACTE's superiority in various edge cases
- Real Data From Drug Discovery → LinFACTE's applicability in real-world applications

Algorithm

Results

Experiment

Motivation

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- **Bayesian optimization** with a knowledge gradient acquisition function (**Negoescu et al. 2011**)
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Model

## Dataset

- Synthetic Data → LinFACTE's superiority in various edge cases
  - Adaptive Setting
  - Static Setting
- Real Data From Drug Discovery → LinFACTE's applicability in real-world applications

Algorithm

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Experiment

## Synthetic Data – F1 Score

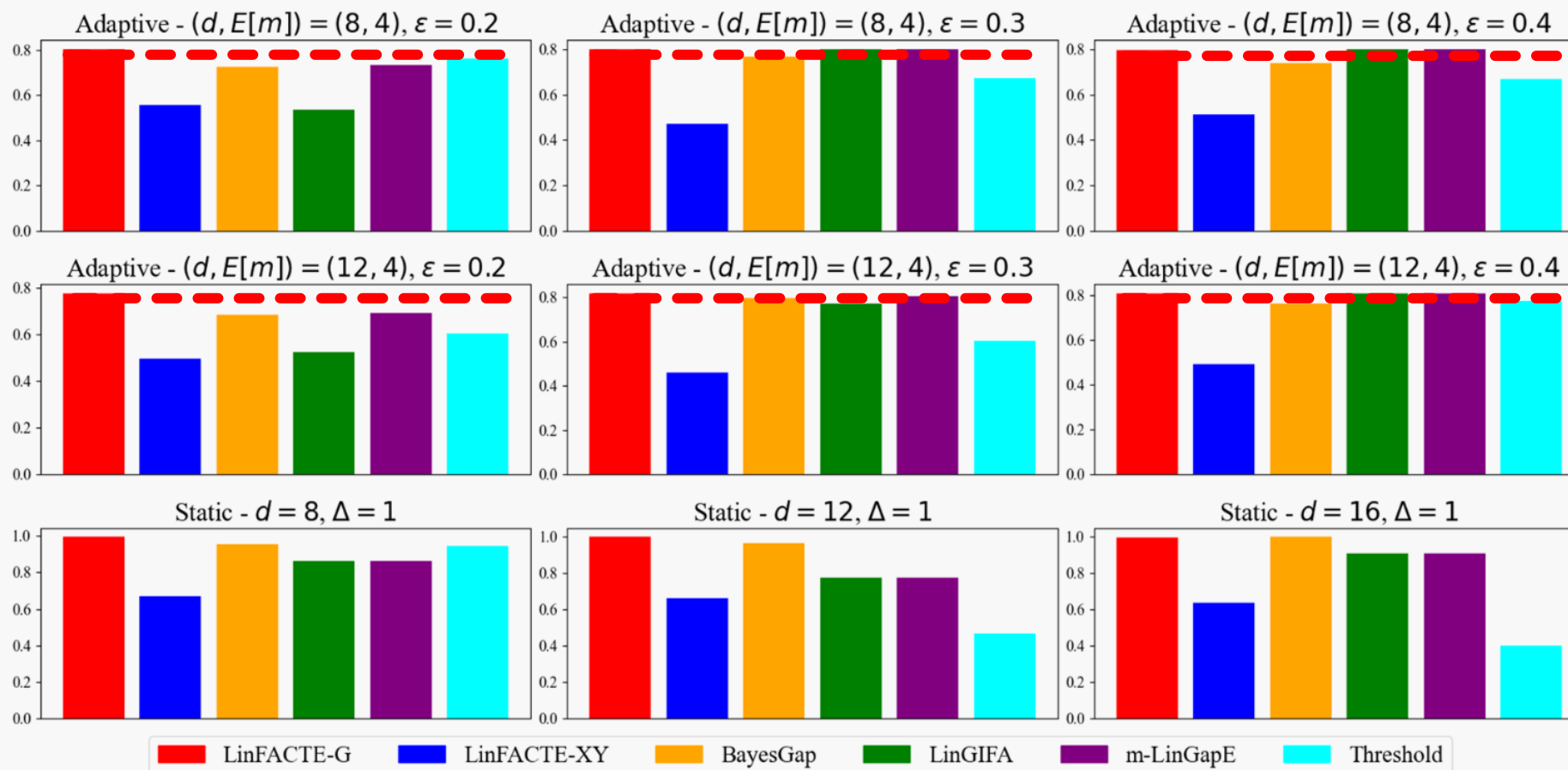
Motivation

Model

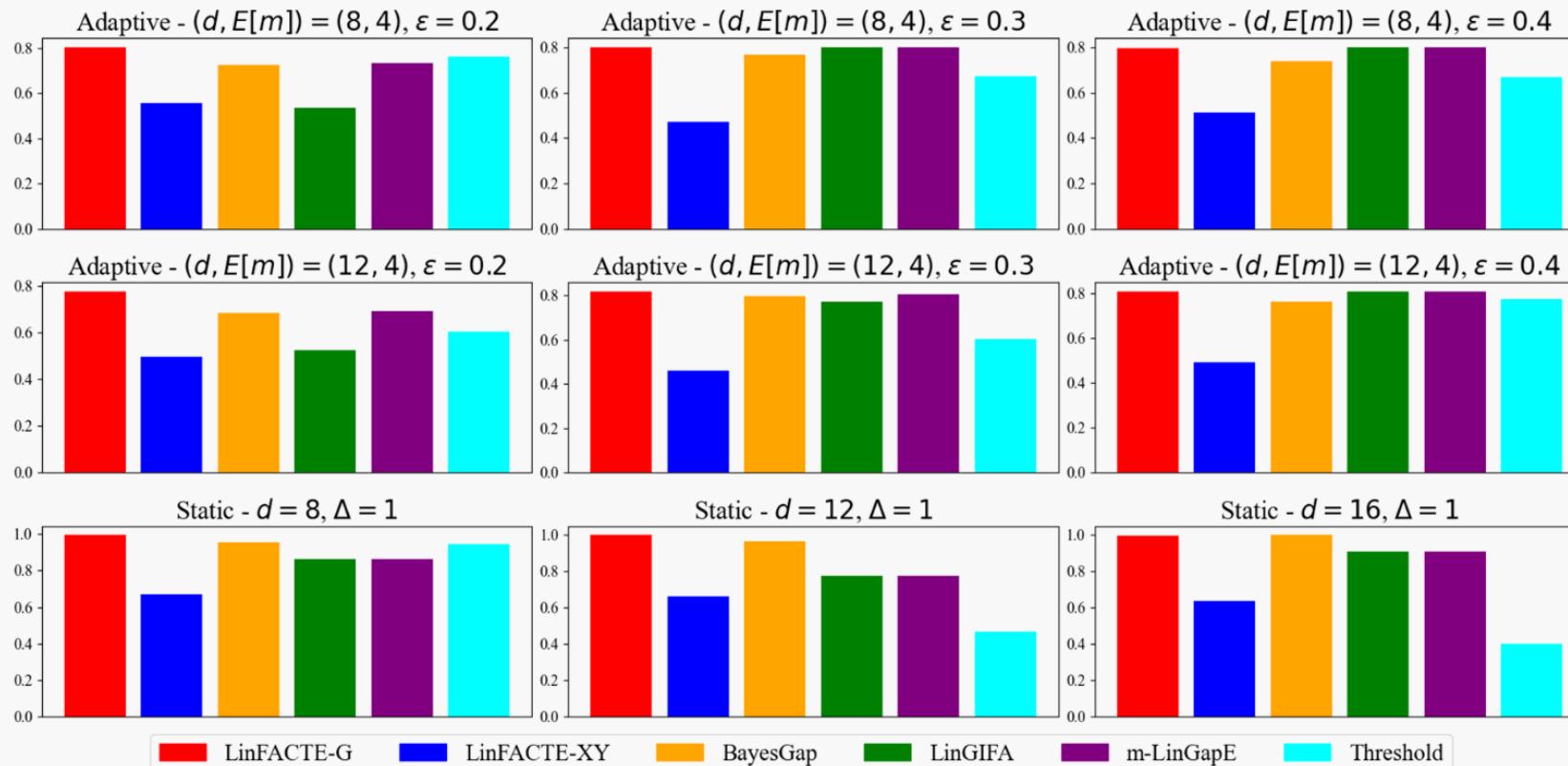
Algorithm

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Experiment



## Synthetic Data – F1 Score



Superiority and Adaptivity in Complex Edge Cases

## Synthetic Data – F1 Score

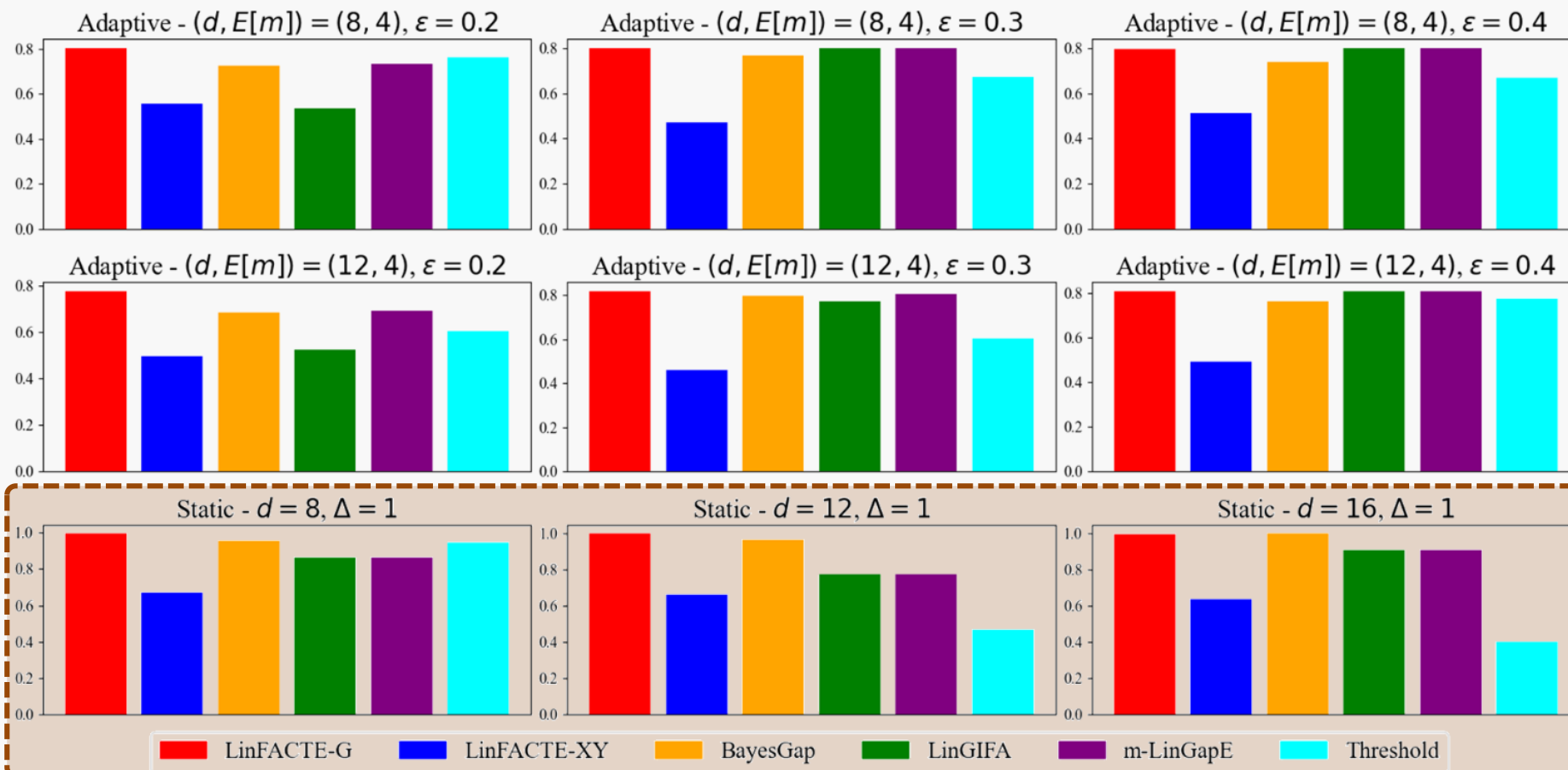
Motivation

Model

Algorithm

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## Synthetic Data – F1 Score

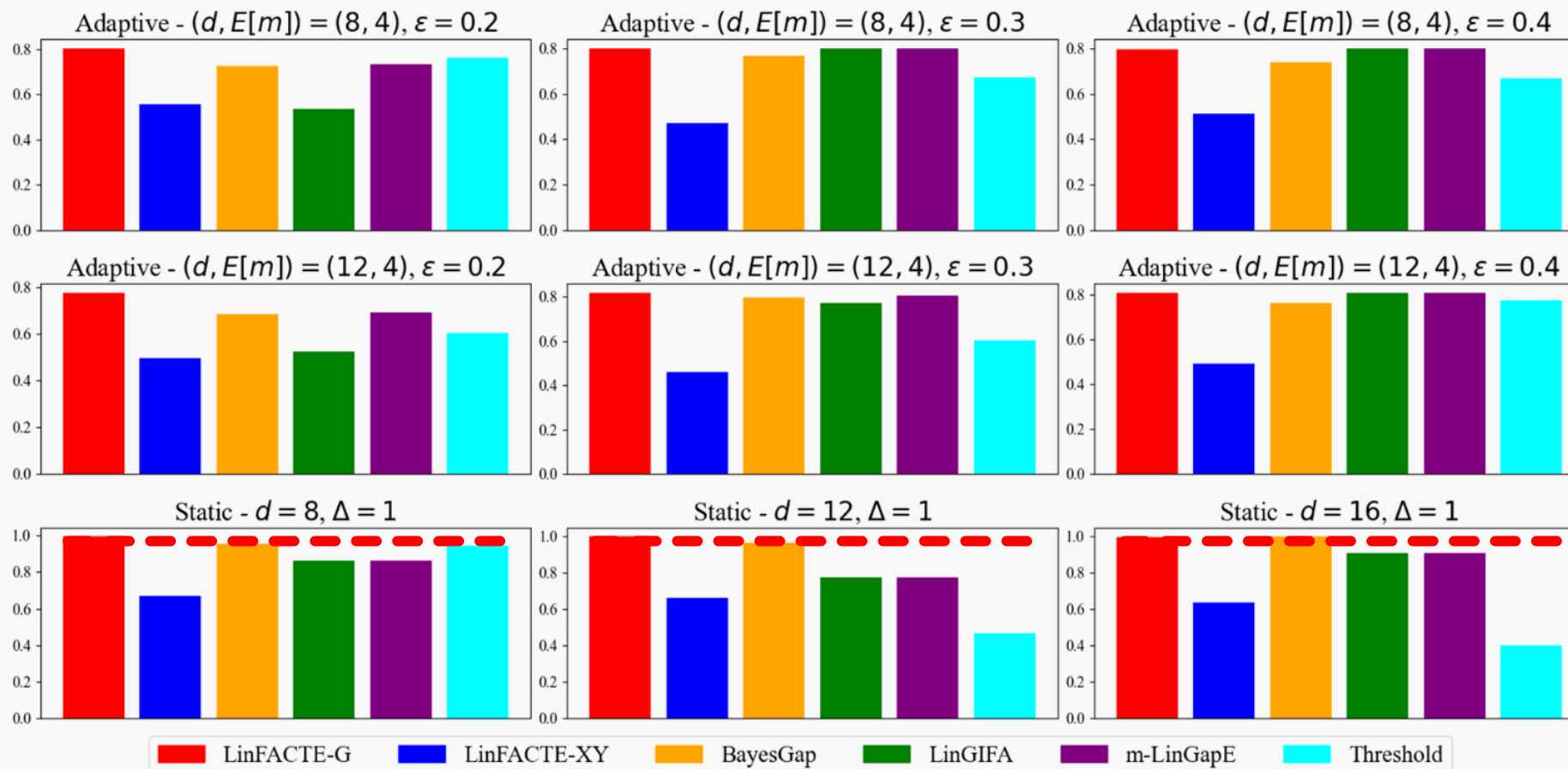
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Model

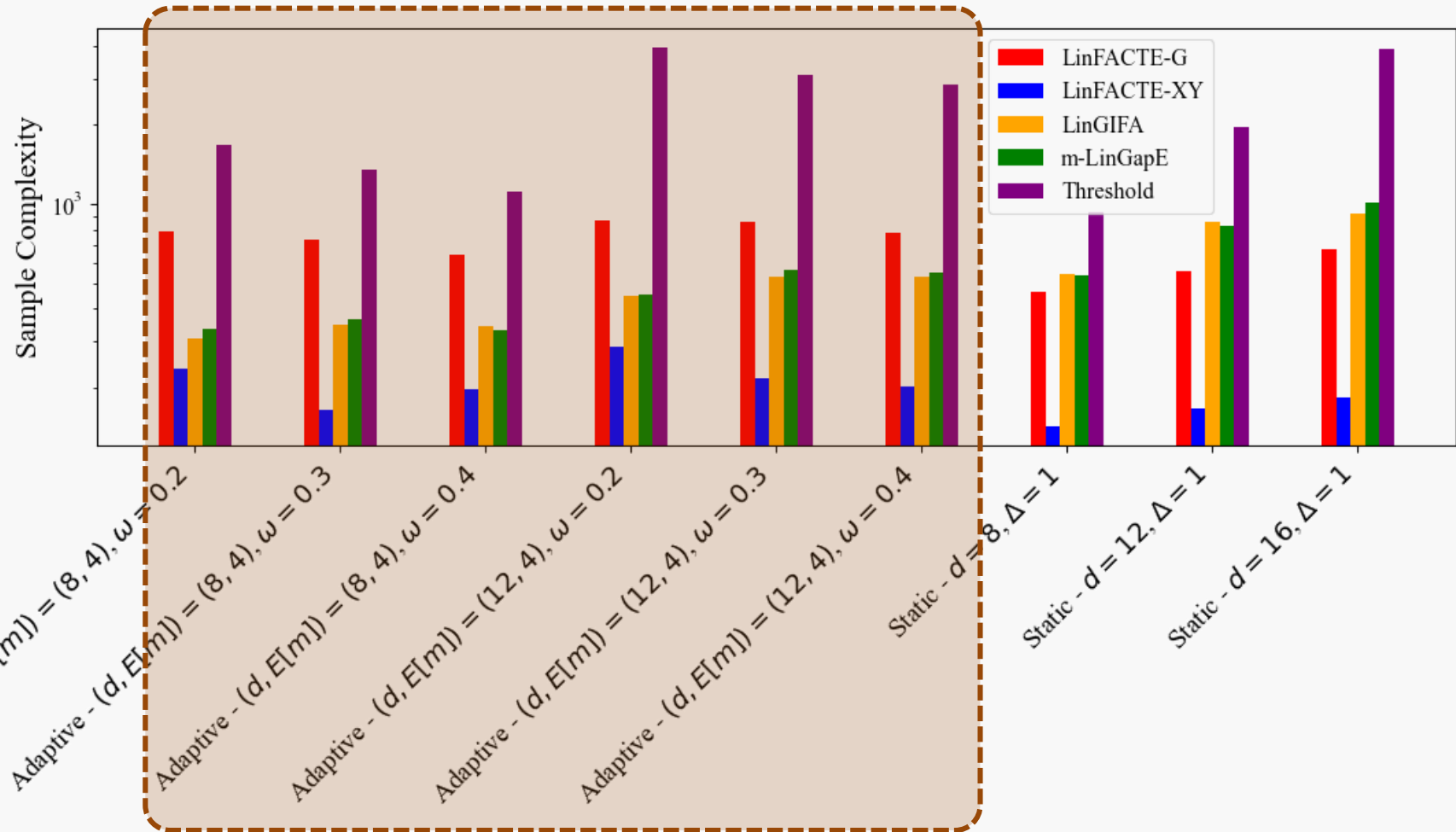
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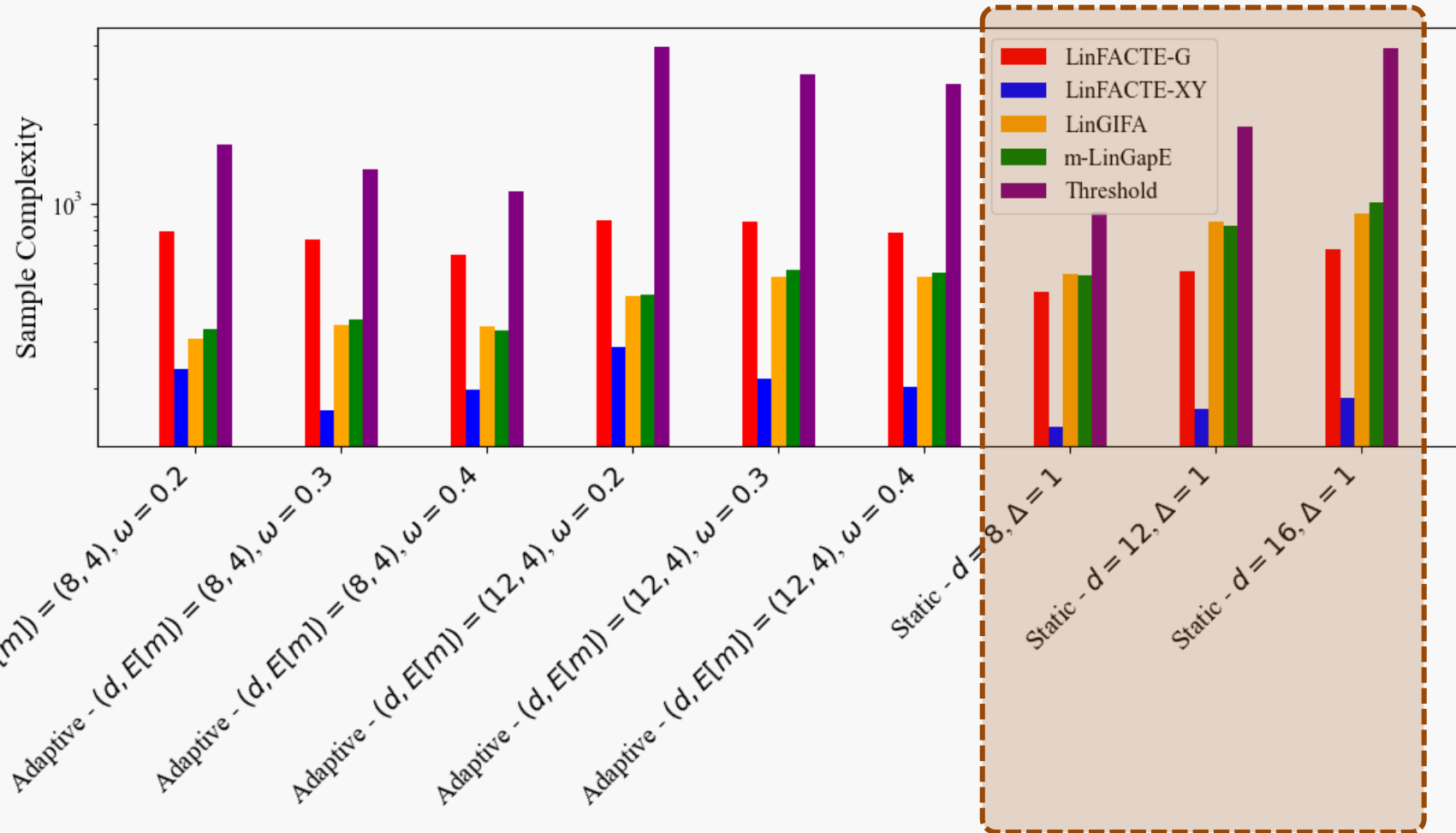


## Synthetic Data – Sample Complexity





## Synthetic Data – Sample Complexity



## Drug Discovery with Free-Wilson Model (Negoescu et al. 2011)

arm space is extremely large

Motivation

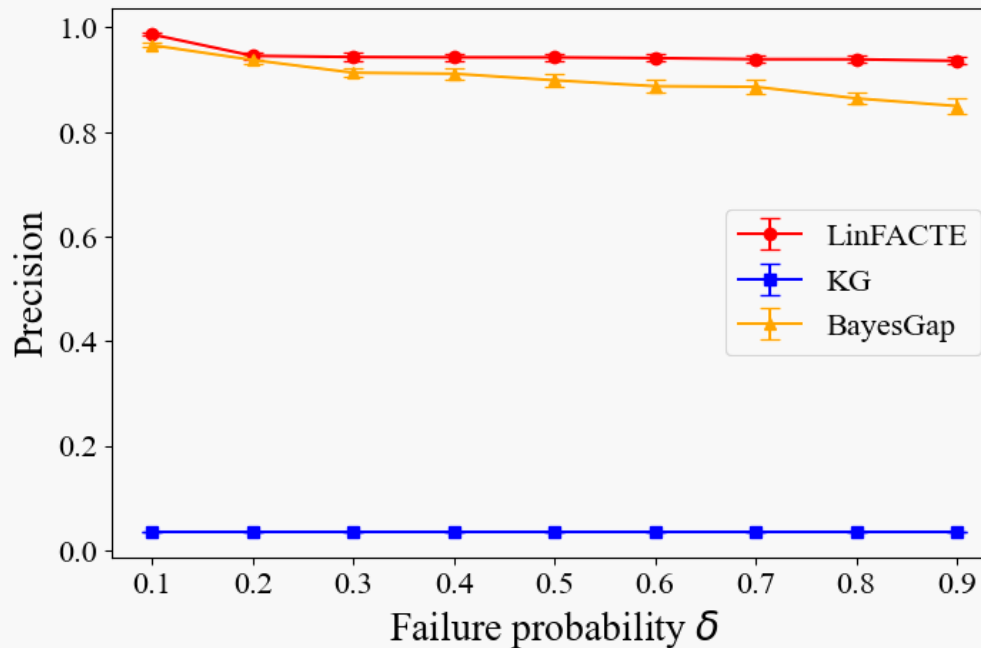
Model

Algorithm

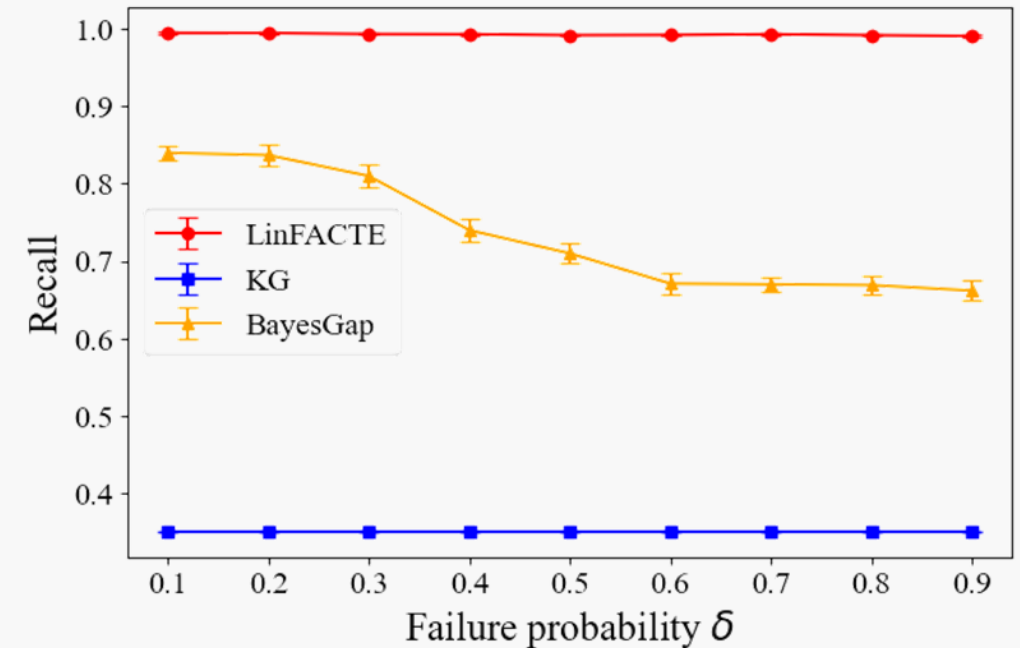
Results

Experiment

measures the accuracy of positive predictions



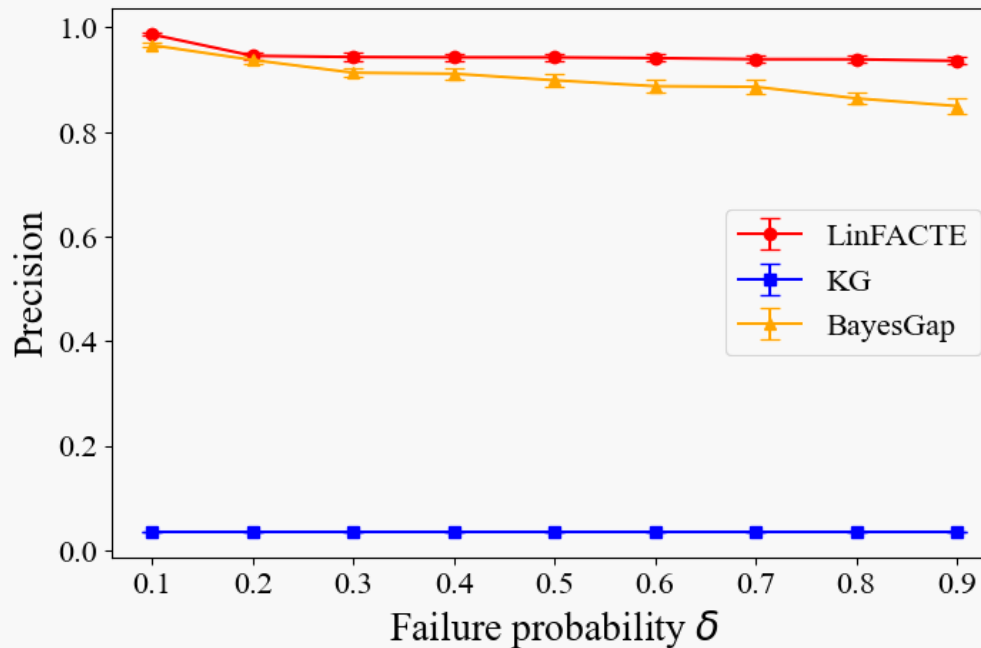
measures the ability to find all actual positive cases



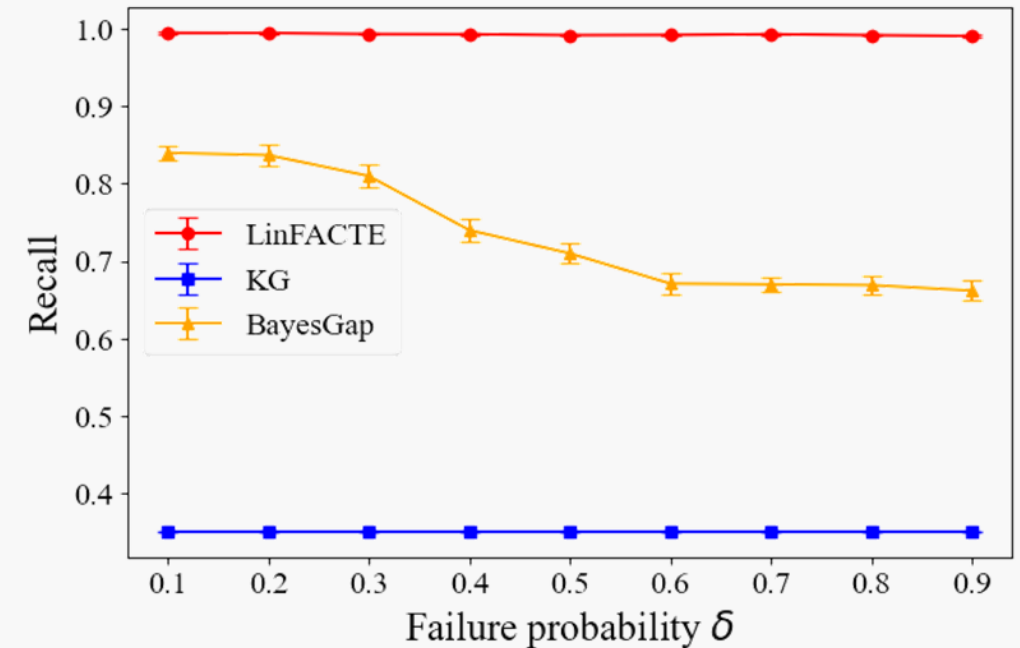
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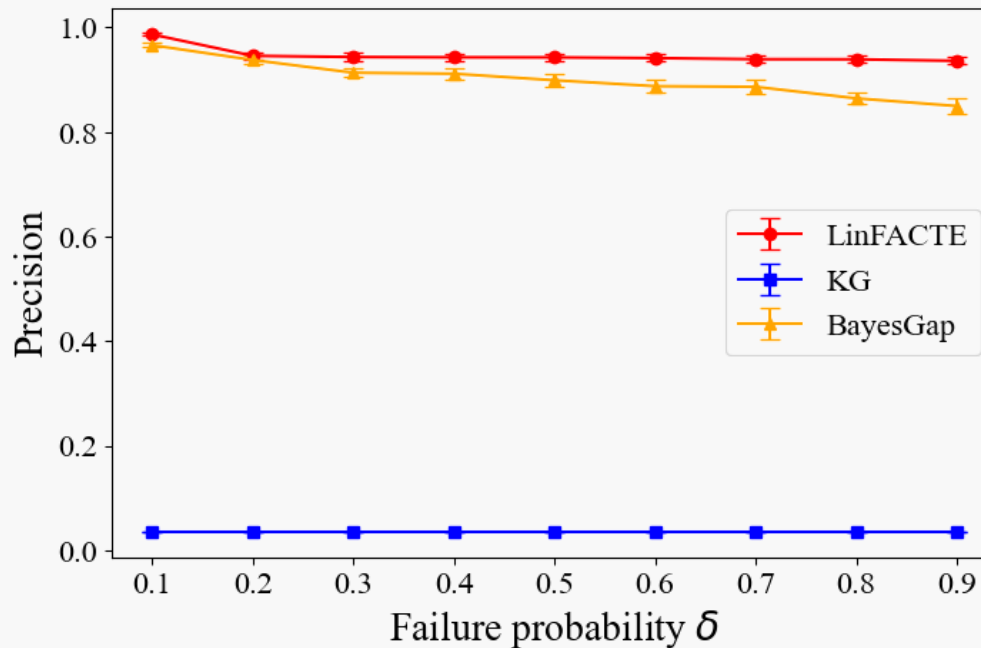


- LinFACTE shows outstanding advantages in computational complexity → 1min < 4mins << 2 hours

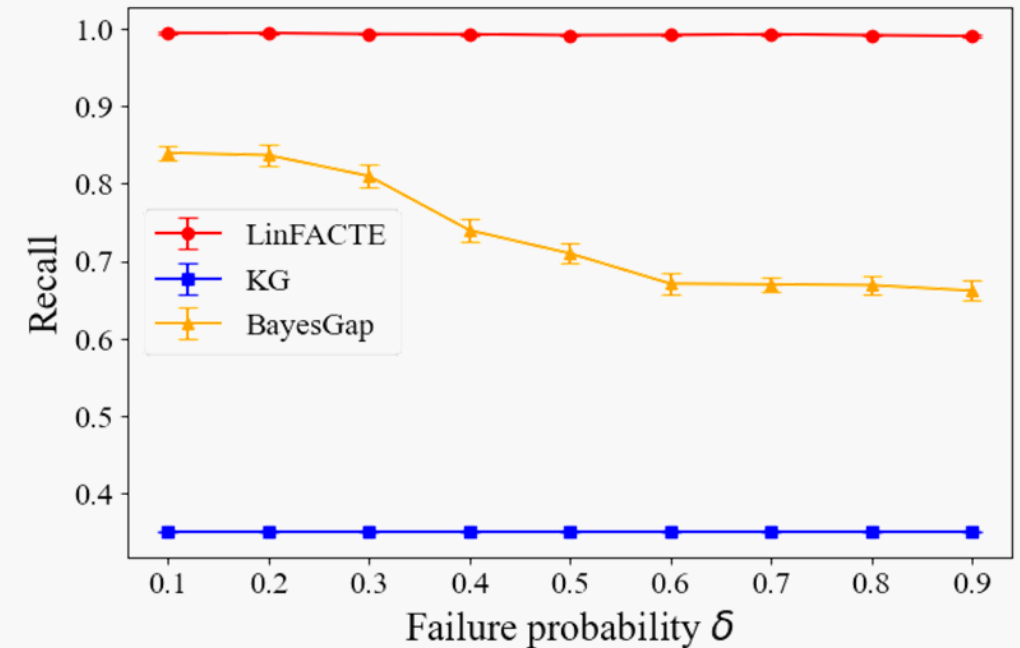
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measures the accuracy of positive predictions



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- LinFACTE shows outstanding advantages in computational complexity → **1min < 4mins << 2 hours**
- In fact, LinFACTE is more suitable for real experiment
  - All other algorithms propose one drug and do one experiment
  - LinFACTE can propose different drugs and do multiple experiments in a batch

# Conclusion

- New **Setting**: All  $\varepsilon$ -Best Arms Identification + Linear Bandits
- First Information-Theoretic **Lower Bound**
- Matching **Upper Bound**
- **Model Extensions** to Misspecified Linear Bandits and GLM
- **Numerical Simulations** with Synthetic Data and Real Data

# Thank you for your attention!

For any further questions, please contact: [\*\*zhikai\\_li.work@sjtu.edu.cn\*\*](mailto:zhikai_li.work@sjtu.edu.cn)