Celestial Mechanics Applications

The Restricted 4-Body Problem and Optimization for Spaceship Mission Design

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Mission

Goal: Create a fuel efficient spaceship trajectory from Earth to one of the outer planets.

Solution: 4-body model + Optimization techniques = efficient mission design

Problem Background

Fuel a spaceship can carry is limited.

How do we assign the trajectory to minimize energy usage?

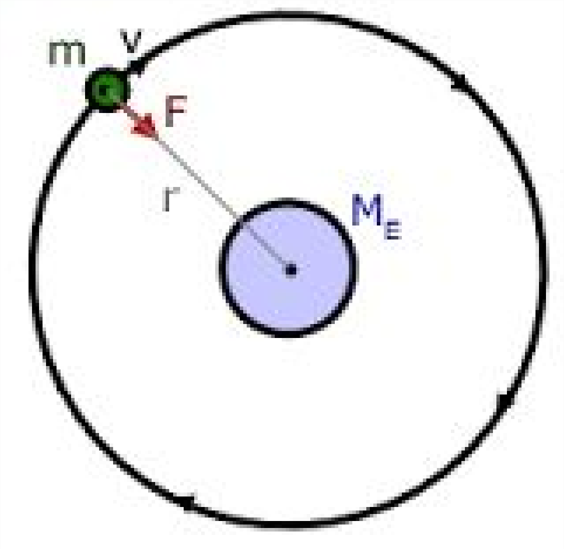
Use Gravitational pulls of local massive objects to control trajectory

2-body to 4-body

2 body: earth, rocket

3 body: earth, moon, rocket

4 body: earth, moon, sun, rocket



Fuel Optimization

$\sum \Delta V$= total changes of velocity as a result of fuel burned

Minimizing $\sum \Delta V$ gives you fuel efficient path

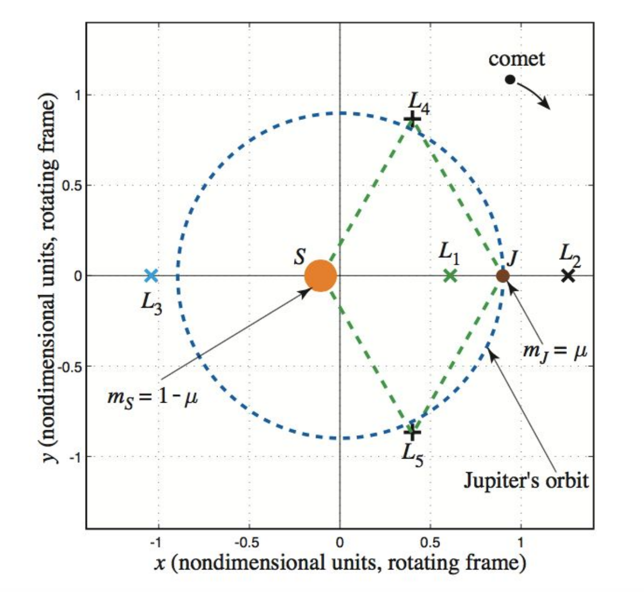
3-body model equations

$$V(x, y) = -\frac{1-\mu}{r_1} - \frac{\mu}{r_2}.$$

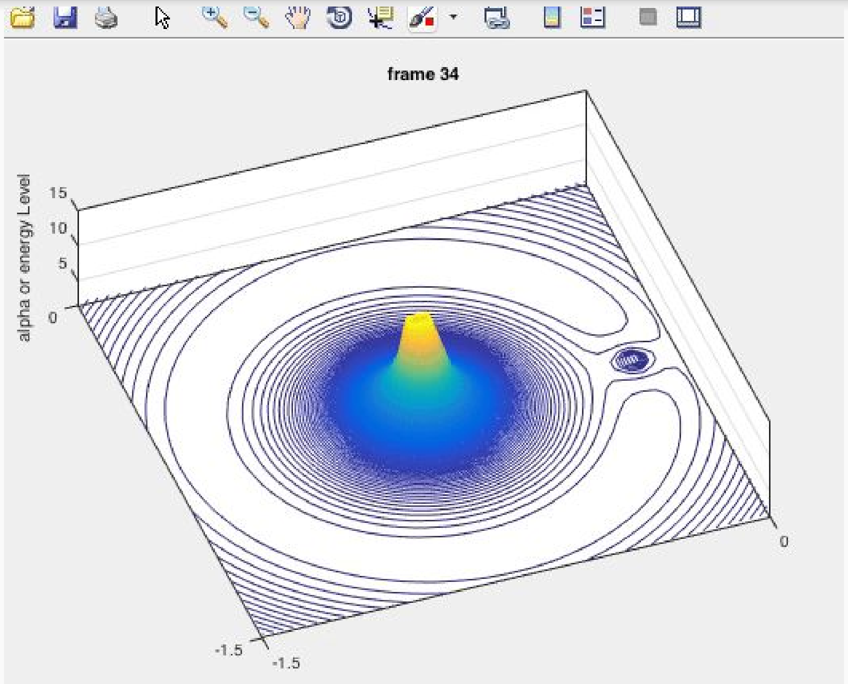
$$\overline{V} = V - \frac{x^2 + y^2}{2}.$$

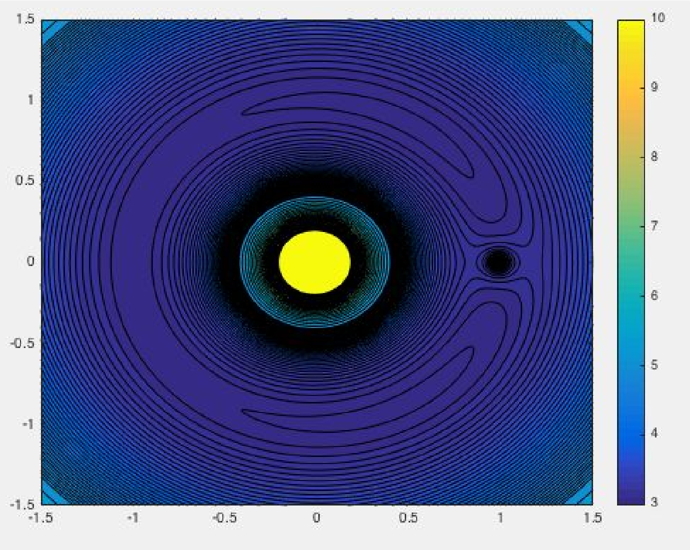
$$\ddot{x} - 2\dot{y} = -\frac{\partial \overline{V}}{\partial x}, \qquad \ddot{y} + 2\dot{x} = -\frac{\partial \overline{V}}{\partial y}$$

$$E = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) + \overline{V}(x, y).$$

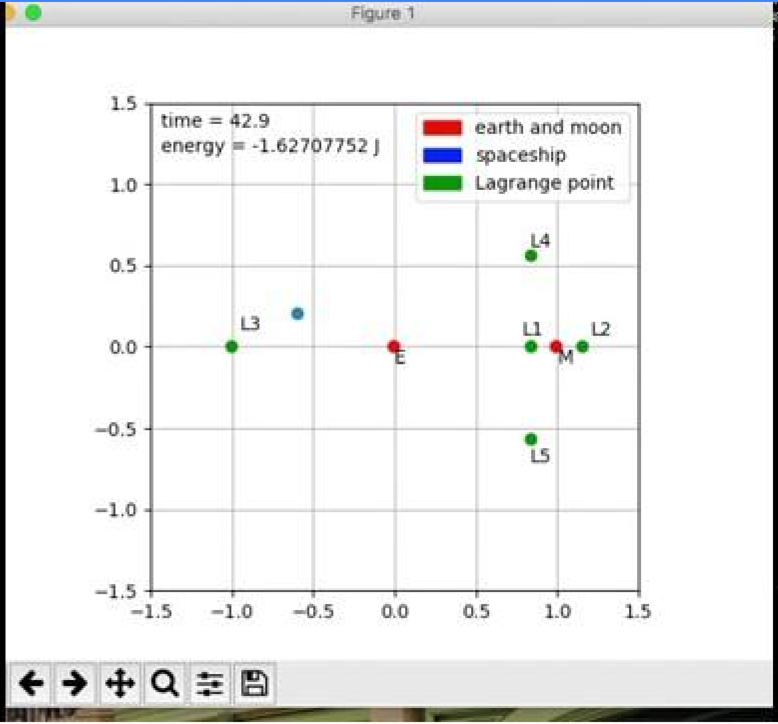


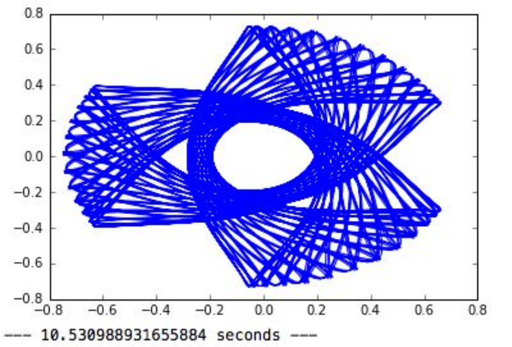
Step 1; Find the Lagrange point [github]





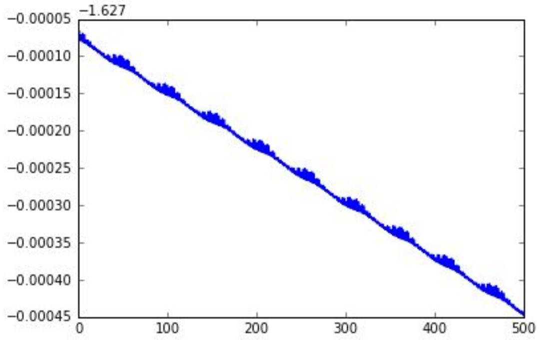
Step 2; A trajectory without using fuel



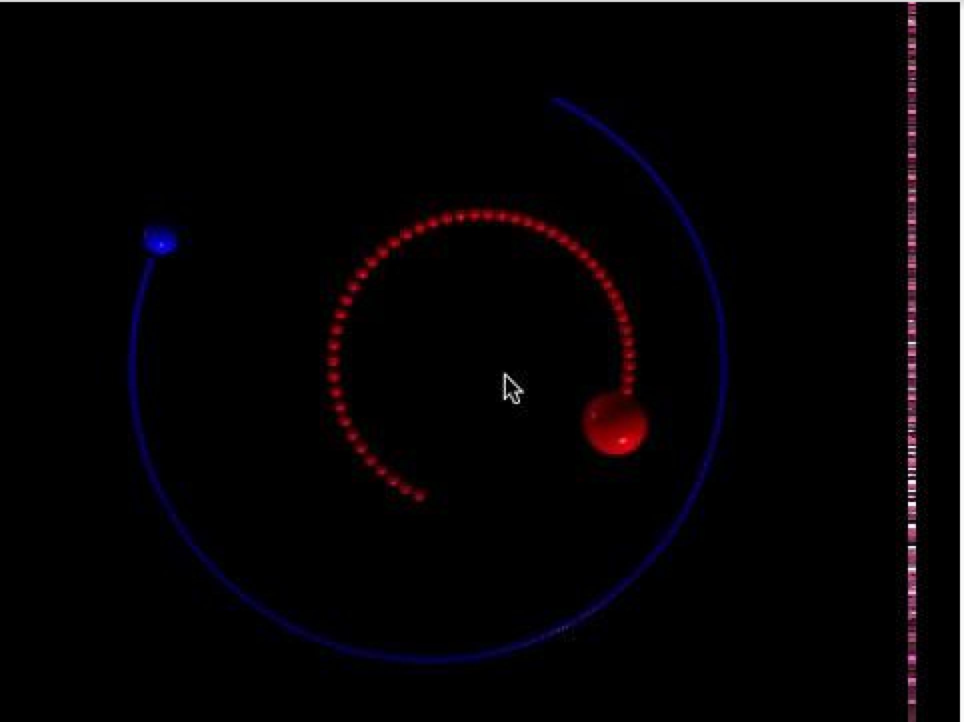


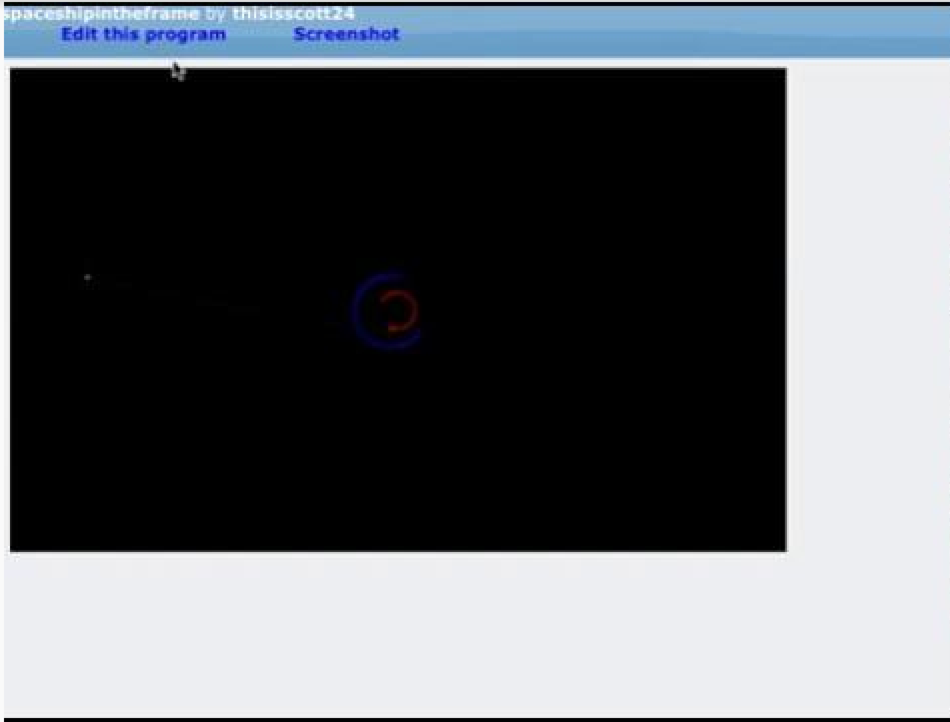
Final trajectory

Energy change

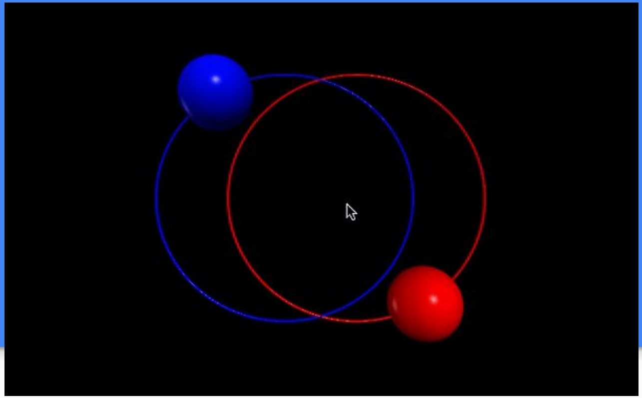


Step 3; escalate to a general three-body problem Test case 1

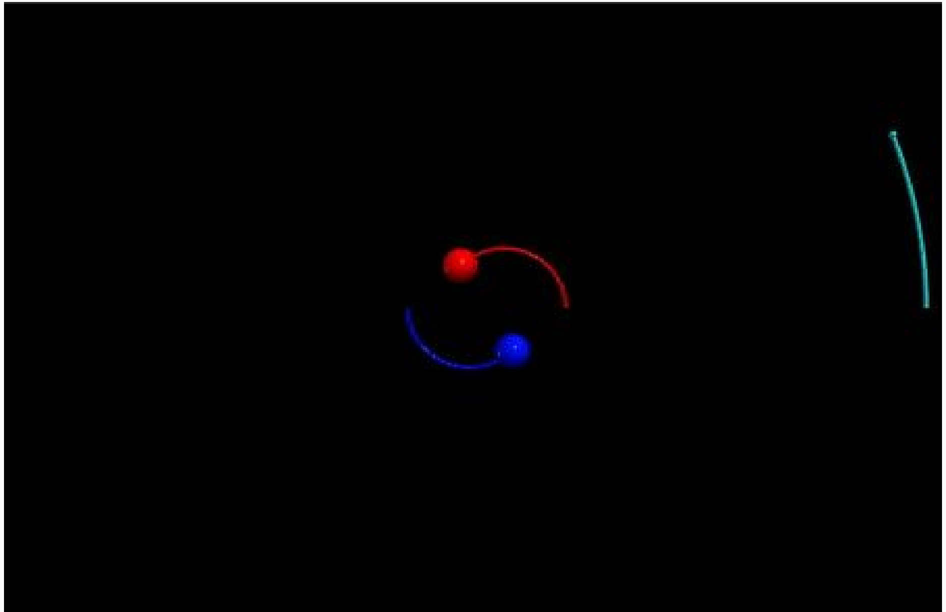




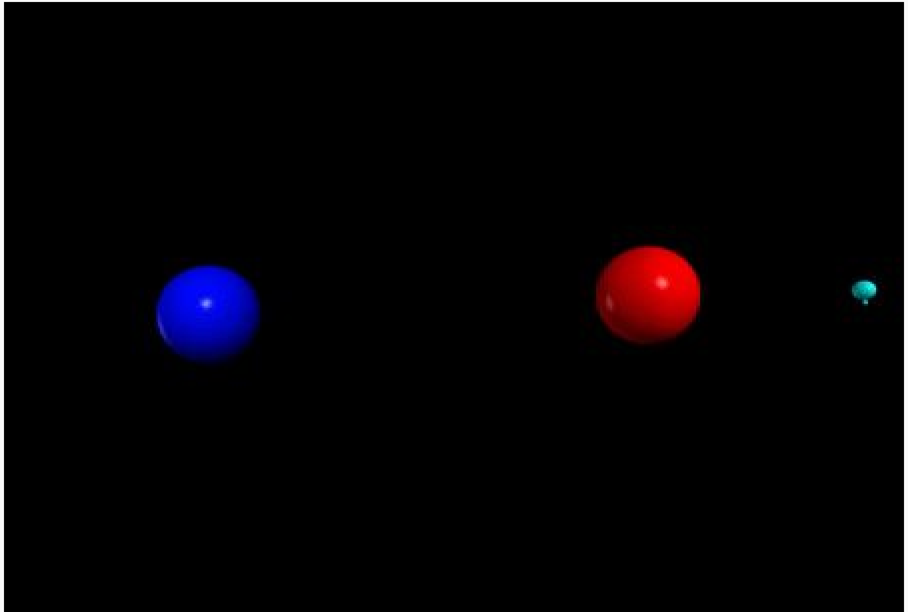
Test case 2



Stable trajectory



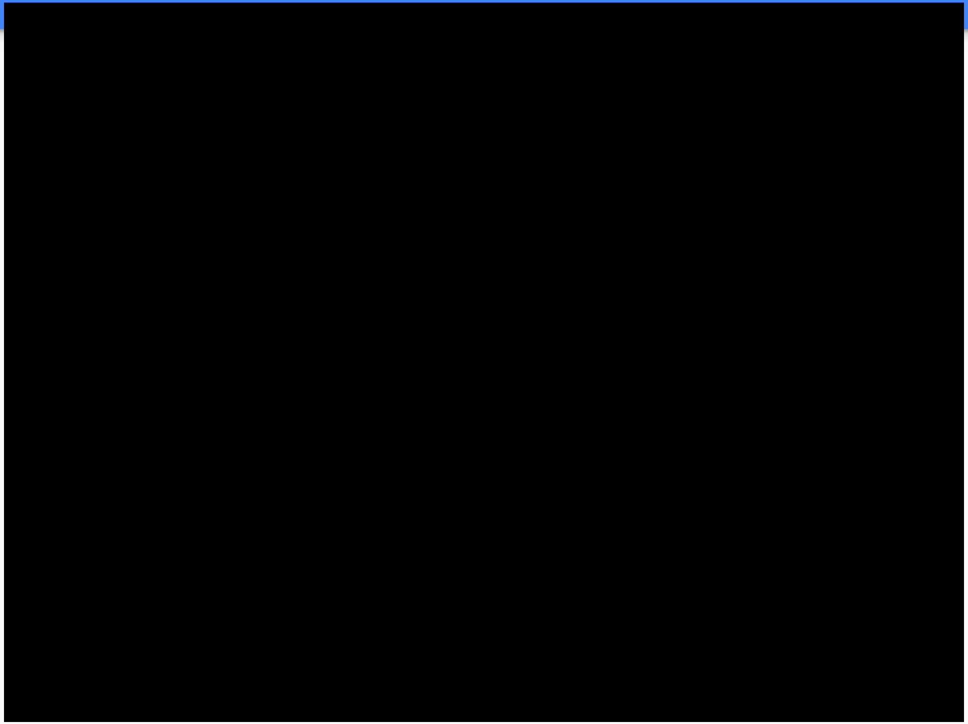
Unstable trajectory



Step 4 ; a program allow change in speed

[presentation sites]

The deterministic Chaos

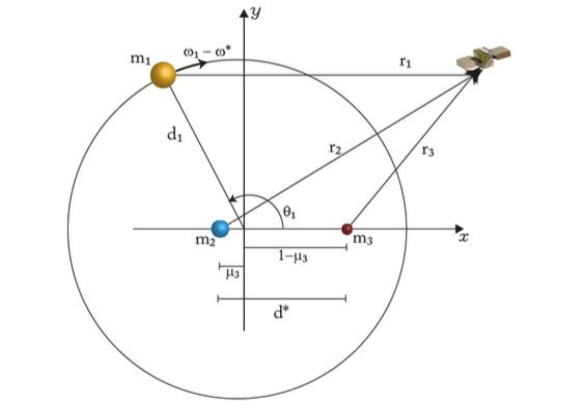


4-body model equations

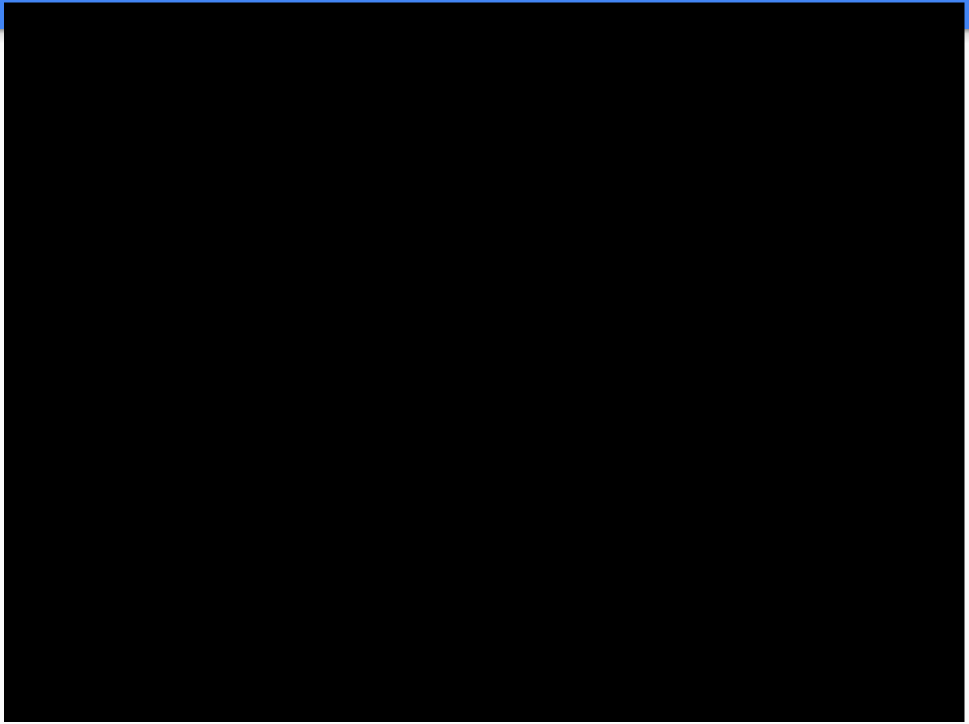
$$\ddot{\mathbf{r}} = \nabla U - 2\boldsymbol{\omega}_i \times \mathbf{v} + \boldsymbol{I},$$

$$U = \frac{(x^2 + y^2)}{2} + \frac{\mu_1}{r_1} + \frac{\mu_i}{r_i} + \frac{\mu_j}{r_j}$$

$$\boldsymbol{I} = -\left(\mu_i \frac{\mathbf{r}_{1i}}{|\mathbf{r}_{1i}|^3} + \mu_j \frac{\mathbf{r}_{1j}}{|\mathbf{r}_{1j}|^3}\right).$$



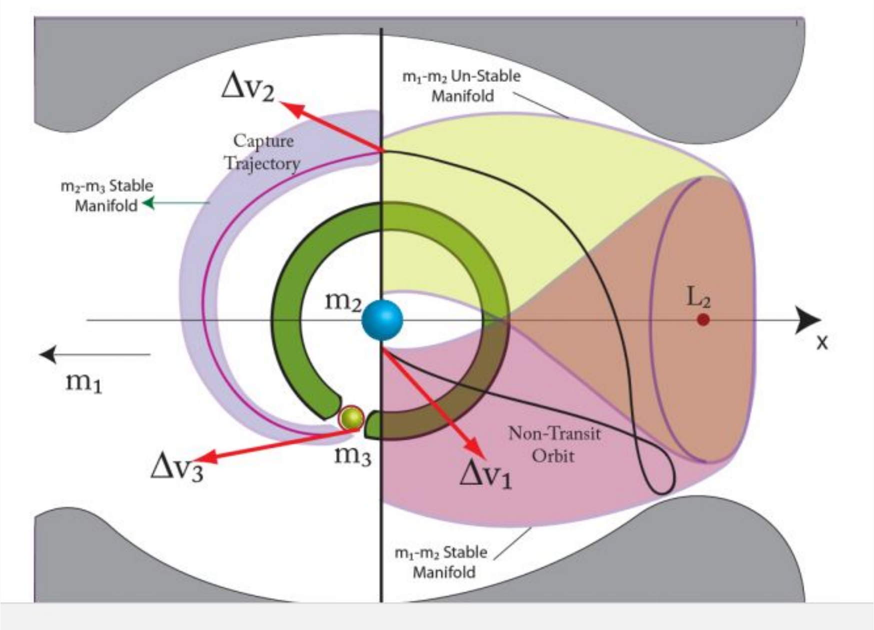
Again , travel without fuel in a 4-body system



Optimization theory for orbit to orbit transfer

Break total trajectory into 2 trajectories, can only change velocity at start and end of trajectories

Theory states that if you minimize delta-V capture and delta-V escape, you have the optimal trajectory



Delta-V capture algorithm

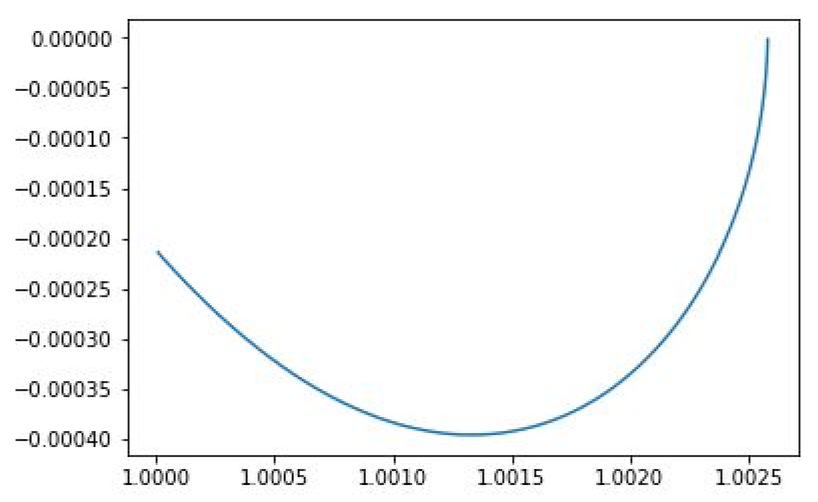
Calculate in the Earth-Moon Frame

Initial Parameters: z = tau, delta_V

End conditions: $x_f \leq x_{L1}$

$v'_f \leq 0$

Switch to M1 M2 frame and solve for when spaceship crosses x position of earth



Delta-V Escape Non-Transit Orbit Search

Calculate in the Earth-Moon Frame

Initial Parameters: z = tau, delta_V, initial time

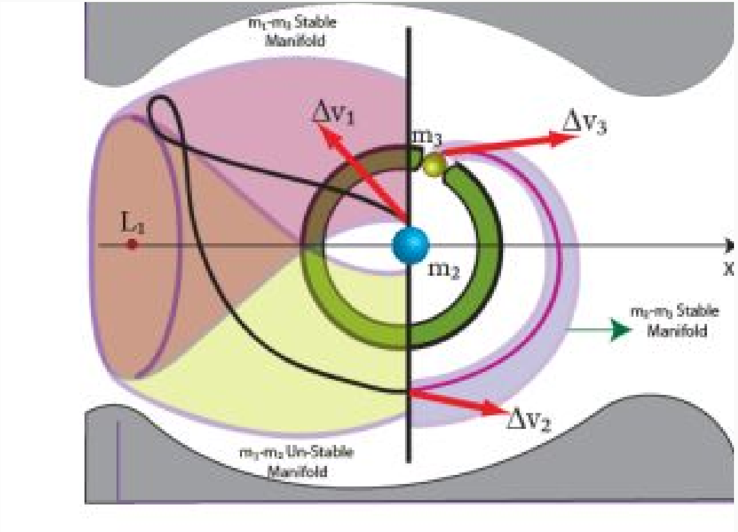
End conditions:

$x_f \le x_{earth}$

$y_f \le y_{intermediate}$

$v_f \ge 0$

Switch to M1 M2 frame and solve for when spaceship crosses x position of earth



References

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https://repositories.lib.utexas.edu/bitstream/handle/2152/ETD-UT-2010-12-2370/MORCOS-DISSERTATION.pdf?sequence=2&isAllowed=y

All the animation is available on www.github.com/zhekaijin