Documentation

Question 1

1.1

To turn the second- order equation to first order equation

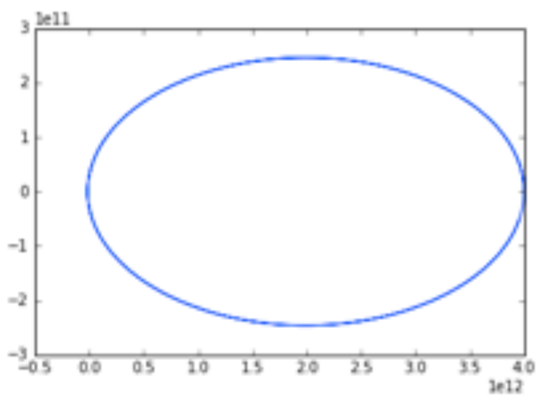
$$fx = q \qquad\qquad fq = -GMx/(\sqrt{x^2+y^2})^3$$

$$fy = w \qquad\qquad fw = -GMy/(\sqrt{x^2+y^2})^3$$

1.2

Referenced to the Runge–Kutta methods for the simultaneous equations introduced for simultaneous equations, just plug in the equations introduced in the question and generate the graph. The manipulation of time step h is tricky to get a whole eclipse shape.
Originally I use the earth year time 3.154e+7 to approximate but that clear is not a full cycle, and after a few tryouts the period 15e8 seconds seems to a pretty accurate approximation .

```
In [1]: runfile('/Users/scott/untitled37.py', wdir='/Users/scott')
```



Answers to the question:
step size chosen: 1e3

I saw an eclipse trajectory in the simulation and the full period is around15e8. Thus use b=3e9 will give an eclipse on top of another and b=75e7 will give a half eclipse.

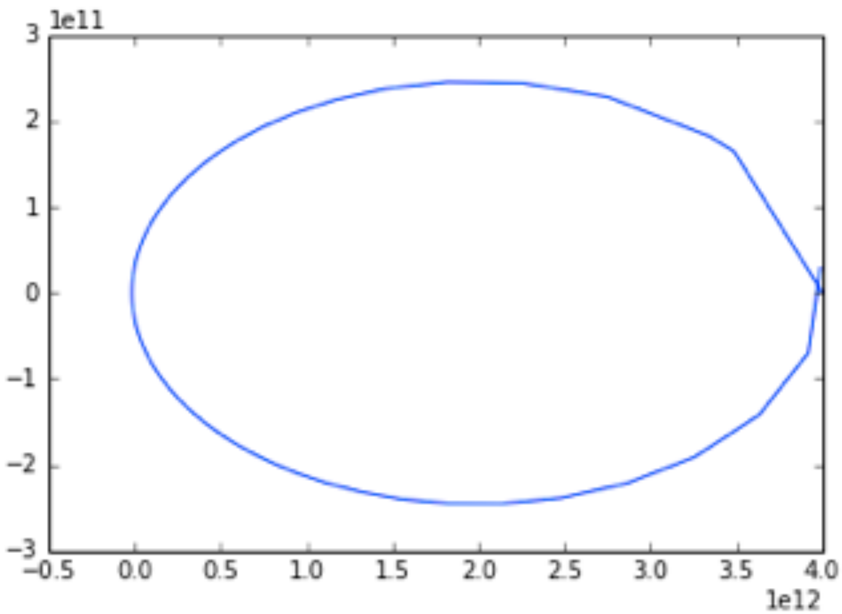
The time taken for the simulation is `--- 140.813227891922 seconds ---` (quite large)

Documentation

1.3

The question ask for an adaptive Runge-Kutta method. Here it is 4 simultaneous equations together so the next step size calculation should be using (8.54) on the book and only concern about the accuracy about x and y will be enough for this question.

The tricky part is the self determined pointer thing. Since we need use the same r two times for two the R1 and R2, in the function runge-Kutta I can't simply add to r but have to return another identity without changing r. And r still should be added at the end of operation. So there is the step r= R1 within the loop to allow the next step to go.
P.S the trivial version is my first workable version where everything is kind of stupidly listed and easy to follow. And I simply the program to the final version.
Graph generated like this



where the point far away (slow) use large step h and points near (fast)use smaller steps.

Answers

Obviously, the answers turned out to be calculated a lot faster than the 1.2 since the adaptive method has been used.

```
--- 0.19004011154174805 seconds ---
```

the step size at the left half of the graph is smaller while the right half is large and the accuracy for the left(fast period) is much higher than the slower period in the left.(clearly from the graph)

1.4

Just change the plot to scatter to give the answers and it testify the observation in 1.3.