



# Limits to arbitrage and CDS–bond dynamics around the financial crisis

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## ABSTRACT

An ostensibly broken cointegrating relationship between CDS and corporate bond spreads during the financial crisis is restored once Libor/OIS spread is included as a third component. The three-variable cointegrating relationship derives naturally from the arbitrage strategy that practitioners implement to exploit differences between the CDS and the underlying bond spread, known as the *basis*. In the presence of limits to arbitrage, the cointegration error is associated with the profit and loss (P&L) of the basis trade, which is why we use it as threshold variable in a regime-switching VECM to describe the joint CDS–bond dynamics. The model shows better in-sample fitting properties than competing specifications, whilst it improves the out-of-sample performance of hedging dynamically the mark-to-market risk of corporate bond portfolios with CDS. We also document destabilizing dynamics in the CDS market during the crisis that originate in supply shocks in the corporate bond market.

## 1. Introduction

Pre-crisis literature on credit default swaps (CDS) and corporate bond spreads postulates the existence of a common stochastic trend – i.e. the price of credit risk – that implies a pricing parity condition between the two credit markets (e.g. Duffie, 1999; Blanco et al., 2005). Practitioners exploited this prediction by trading the so-called *basis*, which is the difference between the CDS and the underlying bond spread. The basis trade consists of simultaneously buying the bond and CDS protection when CDS falls below the bond spread – i.e. when the basis becomes negative – or the opposite set of positions if the basis turns positive.

Conventional wisdom about spread convergence towards parity was challenged during the financial crisis when the basis reached unprecedented levels that persisted for more than a year. During the period 2007–2009 the basis moved deeply into negative territory for all but a few top-rated names, being particularly wide for those of lower credit quality. The result was a dramatic unraveling of basis trades and major losses to financial institutions in the likes of Merrill Lynch, Deutsche Bank, and Citadel.<sup>1</sup>

In this paper we develop an empirical framework motivated by the limits-to-arbitrage theory to investigate the mechanism that led to the breakdown of the basis trade. Theoretical literature on limits-to-arbitrage predicts that leveraged arbitrageurs may be forced to unwind their trades prematurely due to mark-to-market losses and margin requirements (Shleifer and Vishny, 1997; Gromb and Vayanos, 2002; Liu and Longstaff, 2004; Fostel and Geanakoplos, 2008; Brunnermeier and Pedersen, 2009; Acharya et al., 2013b).<sup>2</sup> Such liquidation of arbitrage trades prior to spread convergence can lead to widened mispricing, which may persist for some time if the flow of investment capital to exploit the arbitrage opportunity is slow (Duffie, 2010). In this sense, it could

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<sup>1</sup> See, for example, article by Susanne Craig and Carrick Mollenkamp, “At Merrill, Focus Is Now on Montag, Sales Chief”, Wall Street Journal, January 24, 2009.

<sup>2</sup> Gromb and Vayanos (2010) provide an excellent survey of theoretical work on limits-to-arbitrage.

be argued that basis arbitrage incurred losses simply because it was never a riskless trade. Or equivalently, it is riskless only from the point of view of an unleveraged arbitrageur who faces no funding constraints and no stop-loss limits, therefore can hold to the arbitrage trade until the bond's maturity (or default).

Recent empirical work supports the hypothesis that slow-moving capital leads to persistent market dislocations. Regarding the collapse of the basis trade, two papers stand out on the debate about its actual causes: [Mitchell and Pulvino \(2012\)](#) classify it into a general category of arbitrage crashes that occur because new capital is not swiftly deployed to fill the market gap caused by the unwinding of arbitrage trades.<sup>3</sup> [Bai and Collin-Dufresne \(2019\)](#) suggest several factors that pushed the basis into negative territory, including funding risk, counterparty risk and collateral quality. Both [Mitchell and Pulvino \(2012\)](#) and [Bai and Collin-Dufresne \(2019\)](#) regard the basis-trade unraveling as a departure from a predominant market equilibrium, rather than an equilibrium outcome of the economics of basis trade.

A key contribution of our paper is that our empirical design is motivated by the economics of the actual arbitrage strategy that practitioners implement to exploit the basis (e.g. [O'Kane and Turnbull, 2003](#); [Jersey et al., 2007](#); [Elizalde and Doctor, 2009](#); [Elizalde et al., 2009](#)). Specifically, traders exploit the negative CDS–bond basis by transforming the bond into a synthetic FRN using the asset-swap package. This consists of a long position in the bond and an asset-swap to hedge its interest rate risk. The purchase of the package is financed in the repo and Fed Funds market. This synthetic position replicates the risk profile of a CDS as it bears the credit risk of the bond issuer and is free of interest-rate risk. Consequently, arbitrageurs buy the synthetic bond and CDS protection with matching maturity to exploit a possible mispricing. [Appendix B](#) discusses the details of the basis arbitrage strategy showing that – absent limits to arbitrage – it implies a cointegrating relationship for the CDS, bond, and Libor/OIS spreads.

Therefore, we use a time-series framework to examine the impact of financing costs on basis dynamics and identify whether the observed basis dislocation during the crisis was caused by money-market illiquidity or bond supply shocks. Methodologically, this approach is fundamentally different from other studies that focus on descriptive evidence of arbitrage mispricings using regression methods. For instance, [Bai and Collin-Dufresne \(2019\)](#) follow a cross-sectional, regression-based approach to examine the impact of certain liquidity proxies on the level of the CDS–bond basis. By contrast, our time series approach allows to disentangle dynamic effects that are difficult to discern otherwise. For example, we can isolate the effect of bond supply shocks from the impact of money-market liquidity shocks, or examine the role of stop-loss limits on the joint dynamics of the two series.<sup>4</sup> At the same time, our framework allows to develop dynamic hedging strategies for corporate bonds using CDS.

The results contribute to the relevant literature in two ways: First, we confirm the existence of a stationary relationship between CDS and corporate bond spreads, even during the financial crisis. However, in contrast to the pre-crisis empirical literature, this relationship includes Libor/OIS spread (a proxy of money-market liquidity) as a third component.<sup>5</sup> We show that for the triplet of spreads (CDS, bond, Libor/OIS) a long-term equilibrium relationship remains valid during the crisis, even though for the pair (CDS, bond) the usual parity condition is no longer met. Second, we provide affirmative evidence that CDS contracts remain valid hedging instruments against the mark-to-market risk of corporate bonds also in periods of market stress.

In preliminary tests we confirm that the joint dynamics of CDS, bond and Libor/OIS spreads are characterized by at most two distinct common stochastic trends. A first cointegrating relationship between the three spreads is fully specified by the arbitrage strategy that practitioners implement to exploit the basis.<sup>6</sup> This is shown in the [Appendix](#) using a stripped-down model of basis arbitrage, from which Libor/OIS emerges naturally as part of the long-term equilibrium relationship between the CDS and bond market. Therefore, we show that an ostensibly broken equilibrium relationship between CDS and bond spreads is the result of an omitted-variable misspecification, which is restored once Libor/OIS is included as a third component in the relationship. Moreover, the price of credit risk across rating categories turns out to share a common component that is directly linked to Libor/OIS, and by extension to money-market risk.

Motivated by the economics of the basis trade under limits-to-arbitrage, we explore the dynamics of the basis using a framework of threshold cointegration. Specifically, we consider as threshold variable the deviation of CDS, bond and Libor/OIS spreads from their long-term equilibrium relationship. This deviation (mispricing) reflects the basis-trade carry received by a buy-and-hold arbitrageur who keeps the arbitrage positions till maturity. In other words, our threshold variable reflects the incentive to implement the basis trade in the absence of limits to arbitrage. Yet, if arbitrageurs face stop-loss limits on the mark-to-market value of their positions then a widening of the mispricing – for example, due to shocks in Libor/OIS or in bond supply – could trigger the unwinding of basis-tightening bets prior to spread convergence.

During normal market conditions, incidences of mispricing are expected to be relatively small and short-lived as the flow of investment capital to exploit arbitrage opportunities is fast-moving. But in periods of market stress, mispricing may be self-reinforcing as arbitrageurs hit stop-loss limits and capital flows are slow to fill the gap of those exiting the market. Overall, our framework posits that different degrees of mispricing define distinct regimes of market conditions. As these regimes are characterized by different

<sup>3</sup> Slow-moving capital is also identified as a driver of persistent mispricings in other markets, which may explain the strong commonality of arbitrage spreads. [Fleckenstein et al. \(2014\)](#) document that the TIPS-Treasury mispricing narrows when the capital available to hedge funds increases. It also correlates strongly with the CDS–bond basis and other arbitrage measures, which adds further support to the slow-moving-capital hypothesis. Relatedly, [Pasquariello \(2014\)](#) shows that an index of arbitrage deviations from parity conditions in stock, foreign exchange, and money markets has strong predictive power for stock and currency returns.

<sup>4</sup> For this reason, [Bai and Collin-Dufresne](#) employ causality analysis to discuss price discovery during the crisis.

<sup>5</sup> Libor/OIS spread is defined as the difference between the 3-month Libor and the 3-month overnight indexed swap rate (OIS). Historically it averaged at around 10 basis points before the collapse of Bear-Sterns, reaching an all-time high of 364 basis points in October 2008, in the aftermath of Lehman crisis. Since then, it dropped considerably below 100 basis points in mid-January 2009, returning to normal levels of 10–15 basis points by September 2009.

<sup>6</sup> A second cointegrating relationship is also identified for most rating categories but is unlikely to be interpretable since econometrically it is specified as orthogonal to the first.

speed of capital, they are also characterized by distinct patterns in the profit and loss (P&L) performance of basis trades. Qualitatively speaking, some regimes correspond to normal market dynamics while another to significant market dislocations.

Normal arbitrage activity in our framework takes place when the mispricing is inside a band. Outside this band, mark-to-market losses feed into an illiquidity spiral of destabilizing dynamics à la Brunnermeier and Pedersen (2009).<sup>7</sup> In this case, the basis dynamics switch into an error-correction mechanism where deviations from the usual parity condition between the CDS and bond market may not even “correcting” at all.<sup>8</sup>

Our framework spans models with multiple regimes and cointegrating vectors and encompasses the linear Vector Error-Correction Model (VECM) of Blanco et al. (2005), Zhu (2006) and Forte and Peña (2009).<sup>9</sup> Among the specifications considered, those with two cointegrating vectors and three regimes are found to have better in-sample fitting properties, in line with the predictions of the basis arbitrage model discussed in Appendix B. They are also found to outperform competing specifications when used for (out-of-sample) hedging the mark-to-market risk of corporate bonds with CDS, even in periods of market stress.

The first regime that we estimate tends to coincide with normal periods before the onset of the crisis (pre-crisis normal regime). The second regime corresponds to normal periods in the run-up and after the eruption of the crisis (post-crisis normal regime). The third regime points to the peak of the crisis, around the Lehman failure (distressed regime). In the pre- and post-crisis normal regimes CDS and bond spreads revert to their long-term equilibrium relationship, in line with the pre-crisis literature. In the pre-crisis normal regime, we also find evidence of price discovery taking place in the CDS market for AAA and cross-over, namely two rating categories of particular interest to speculators. For other rating categories we find no evidence of either the CDS or bond market leading the other. In the post-crisis normal regime, the CDS market no longer leads the price discovery process for any rating.

In the distressed regime we document a complete break from the usual pattern of joint dynamics. CDS spreads are now entirely driven by random shocks and by the disequilibrium error with the bond market. But instead of pulling CDS spreads towards parity with bond spreads, deviations from equilibrium now widen the basis even further. At the same time, bond spreads appear to be totally unaffected by anything that is happening in the CDS market. This pattern suggests that the breakdown of negative-basis trades was caused primarily by supply shocks in the bond market and by the concurrent increase in funding costs that negatively affected the P&L of basis trades. It is also consistent with the slow-moving-capital explanation of basis persistence: As arbitrageurs hit stop-loss limits and unwind their positions prior to spread convergence, new arbitrage capital is too slow to halt the widening of the mispricing.

The rest of the paper is organized as follows. Section 2 describes the data and Section 3 presents some preliminary analysis on the stability of CDS–bond cointegrating relationship during the financial crisis. Section 4 discusses alternative specifications for structural instability under threshold cointegration to allow for limits to arbitrage. Section 5 outlines the estimation procedure and Section 6 presents the results based on the best-fitting model. Section 7 discusses hedging implications and Section 8 concludes. Appendix A presents our metric of bond spread that directly compares with the CDS spread and is consistent with market practice of exploiting CDS–bond mispricings. Appendix B provides a stripped-down model of basis arbitrage and discusses the economic underpinnings of the cointegrating relationship between CDS, bond and Libor/OIS spreads. It also shows how limits-to-arbitrage in the form of stop-loss limits and slow-moving capital may produce spread dynamics that resemble our threshold models.

## 2. Data

The analysis draws on an extensive dataset of CDS and bond prices that covers three distinct sub-periods of comparable duration: The first sub-period (January 2005 to December 2006) corresponds to market conditions examined in the pre-crisis literature (e.g. Blanco et al., 2005). The second sub-period (January 2007–April 2010) includes all milestones of the financial crisis, i.e. the onset of the subprime crisis and ABCP market freeze, the failure of Bear Sterns and Washington Mutual, the Lehman crisis and AIG bailout. The third sub-period (April 2010–December 2012) is imbued by the crisis experience and bears the impact of regulatory response.<sup>10</sup> We use this sub-period exclusively for out-of-sample analysis, as a robustness check.

The bond sample consists of 1818 US corporate securities that appear in JP Morgan’s daily CDS-Basis Report.<sup>11</sup> In order to ensure sufficient bond and CDS contract liquidity and focus on basis trades that are actively pursued in the market, we use the following inclusion rules:

- Bonds are rated by Moody’s and Standard & Poor’s with issue sizes of at least \$300 million.
- The corporate entity issuing the bond has at least \$1 billion of fixed-rate bonds outstanding.
- The CDS contract of the bond issuer is included to at least one of the CDX indices.

<sup>7</sup> The negative basis could be particularly persistent in the low-rating spectrum where repo funding is relatively scarce. For top-rated names, the positive basis could persist as (lower rated) arbitrageurs may not be able to sell CDS protection, or short-sell top-quality bonds due to flight to safety and scarcity of top-quality names.

<sup>8</sup> Notice that the threshold mechanism in our framework operates quite the opposite to the standard threshold cointegration model of Balke and Fomby (1997) where error correction is triggered outside a “band of inaction” around a parity condition.

<sup>9</sup> A similar approach is used by Acharya et al. (2013a) to study the exposure of US corporate bond returns to liquidity shocks of stocks and treasury bonds over the period 1973–2007.

<sup>10</sup> For example, new capital regulations introduced at the aftermath of the 2008 crisis (e.g. Volcker Rule and Basel III requirements) raised the costs of negative basis trades, as the upkeep of bond inventories became more capital-demanding. CDS contracts also became more capital-intensive instruments compared to the pre-crisis period as they adopted upfront payment structure and more conservative rehypothecation practices, especially in the UK, in line with US Federal Reserve Board’s Regulation T and SEC Rules 15c3-3.

<sup>11</sup> This is distributed to customers by the Credit Research Department through the JP Morgan Markets platform.

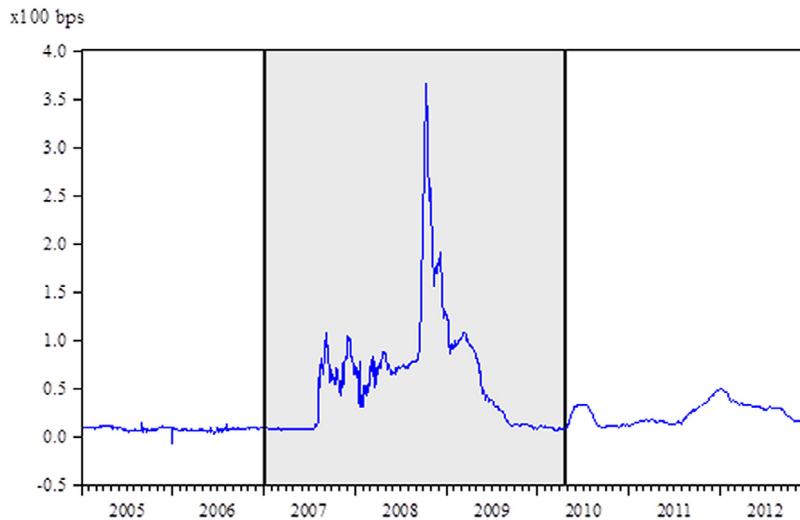


Fig. 1. Three-month Libor/OIS spread, for the period 3/1/2005 to 17/12/2012. This figure shows the Libor/OIS spread for three distinct sub-periods of the data sample: (i) the first sub-period (3/1/2005–31/12/2006) corresponds to market conditions discussed in the pre-crisis literature; (ii) the second sub-period (3/1/2007–17/4/2010), shown in shade, includes the global financial crisis, the breakdown of normal market conditions, the peak of financial distress and the return to a new “steady state”; (iii) the third sub-period (17/4/2010–17/12/2012) corresponds to the post-crisis “normalcy”, which differs from the pre-crisis one in various institutional aspects, such as limits to rehypothecation and new capital adequacy rules for financial institutions (Basel III).

- Bonds pay a non-zero coupon semi-annually and have a bullet (no optionality) maturity longer than 13 months from the index-commencement date but no longer than 31 years.
- All index-constituent bonds are “hand” priced daily, at least by JP Morgan’s trading desk, one of the largest US corporate bond dealers.

The sample selection process guarantees that prices are close to the actual market quotes. CDS spreads refer to mid-spreads on senior unsecured debt with modified restructuring clause, obtained from Markit at 1, 2, 3, 5, 7, and 10-year maturities and quoted for \$10 million notional as of close of business.<sup>12</sup> Bond pricing data refer to senior unsecured bonds with no embedded options, obtained from Bloomberg and completed where necessary with prices from Thomson Reuters Datastream.

Using the full CDS curve that corresponds to the issuer of each bond, we calculate the bond-implied CDS spread, as discussed in Appendix A. Then we pair bond-implied and CDS spreads by interpolating on the CDS-fitted forward hazard rate. The number of pairs per date is time-varying, ranging from a minimum of 411 to a maximum of 1448 (median 822). For each trading day we construct 8 rating pairs of bond portfolios. Each bond portfolio consists of equally weighted positions in bonds, coupled with a corresponding CDS portfolio in the same positions.

### 3. Preliminary results for linear specifications during the financial crisis

In this section we explore the existence and stability of a cointegrating relationship between CDS and bond spreads during the financial crisis. We examine their time-series properties specifically for the period 3/1/2007 – 17/4/2010 and find evidence of instability in their cointegrating relationship attributable to money-market liquidity. Notice that it would be inappropriate to apply standard unit root/cointegration tests for the whole sample period 3/1/2005 – 17/12/2012 as it would disguise market dislocations during the crisis and are central to our analysis. For example, although Libor/OIS spread contains a unit root in all 3 subsamples (i.e. before, during, and after the crisis) it appears to be stationary when testing for unit root on the whole sample.<sup>13</sup>

Fig. 1 shows this is a problem of scale that biases the test towards rejecting the null of a unit root, as Libor/OIS starts from a low-volatility/low-level environment and returns to a similar state in 2009. Once the null includes the assumption of a structural break during the sample period – using the Break Point Unit Root test of Perron (1989) – we confirm that Libor/OIS *does* contain a unit root throughout the entire sample. Therefore, we focus first on the crisis period and reconcile our evidence with pre- and post-crisis data using models adapted to structural breaks.

#### 3.1. Unit root/cointegration analysis

We start by applying a battery of unit root/stationarity tests (ADF, ERS and KPSS) on the CDS, bond and Libor/OIS series to verify the presence of unit roots for the sub-period 3/1/2007 – 17/4/2010.<sup>14</sup> As expected, these tests point to the existence of unit

<sup>12</sup> This is approximately 5:15 pm New York time.

<sup>13</sup> Indeed, it would be a rather poor attempt at consolation for, say a trader who lost millions on basis-tightening bets, to remind that market dislocations are transient and money-market liquidity eventually returns to normal levels.

<sup>14</sup> To economize on space, we omit presenting unit root results, which are available from the authors upon request.

**Table 1**

Johansen and Gregory–Hansen cointegration tests, for the period 3/1/2007 – 17/4/2010.

Panel A: Johansen cointegration test						
	(CDS, bond)	(bond Libor/OIS)	(CDS, Libor/OIS)	(CDS, bond, Libor/OIS)		
Null hypothesis (trace statistic):	No cointegration	No cointegration	No cointegration	No cointegration	At most one cointegrating vector	# Lags
AAA	25.94*	17.15	16.20	43.03**	17.41	3
AA	15.95	10.94	10.56	47.95**	13.99	2
A	39.73**	24.00*	9.81	132.50**	22.85*	1
BBB	18.88	66.38**	18.16	137.77**	32.12**	1
XOVER	13.31	7.04	7.42	38.23**	13.75	2
BB	17.52	32.47**	18.22	72.95**	24.54*	3
B	22.80*	19.76	14.99	67.36**	26.72**	3
CCC-C	51.09**	19.90	21.03*	73.38**	23.22*	10
Panel B: Gregory–Hansen cointegration test for (CDS, bond)						
	Break in Constant		Break in Trend		Full Break	
	ADF* statistic	Breakpoint	ADF* statistic	Breakpoint	ADF* statistic	Breakpoint
AA	−4.40	16/1/2009	−4.96	14/1/2009	−5.83**	30/1/2009
BBB	−4.77*	13/8/2008	−5.14**	4/8/2008	−4.95*	19/11/2008
XOVER	−2.36	10/2/2009	−4.11	5/2/2009	−4.49	5/2/2009
BB	−5.09*	30/6/2008	−5.14*	30/6/2008	−5.53**	1/7/2008

Panel A shows Johansen cointegration test results for the null hypothesis of “no cointegration” for the pairs (CDS, bond), (bond, Libor/OIS), (CDS, Libor/OIS), and the triplet (CDS, bond, Libor/OIS), where the number of lags used is determined on the basis of AIC. For the triplet (CDS, bond, Libor/OIS) we also test the null hypothesis of “at most one cointegrating vector”. Panel B shows Gregory–Hansen ADF\* test results for the null of “no cointegration” against the alternative of “a structural break in an existing cointegrating relationship” affecting the constant, the trend, or both (full break). It refers to rating categories that failed the Johansen cointegration test in the (CDS, bond) pair, aiming to investigate if failure to reject the null of “no cointegration” was due to a structural break in cointegrating relationship. Breakpoints indicate the dates of estimated structural break.

\*Denoted statistical significance at 5% confidence level.

\*\*Denoted statistical significance at 1% confidence level.

root in all series. We then apply Johansen’s cointegration test to examine the existence of stationary linear relationships for the pair of spreads (CDS, bond), (bond, Libor/OIS), (CDS, Libor/OIS), and also for the triplet (CDS, bond, Libor/OIS). Cointegration test results are shown in Table 1 (Panel A) for all rating portfolios during the crisis period.<sup>15</sup>

For (CDS, bond) the null of “no cointegration” is rejected in only half of the rating categories. This finding shows that the long-term stationary relationship between CDS and bond spreads, which was recognized in the pre-crisis literature, is questioned for the crisis period. For (bond, Libor/OIS) the null is rejected in only 3 out of 8 rating categories, and specifically in those placed in the middle of the rating spectrum (A, BBB, BB). Finally, for (CDS, Libor/OIS) evidence for a long-term equilibrium relationship between CDS and money-market liquidity exists only for the CCC-C rating portfolios. In all these cases, the rejection of the null is indicative of a common stochastic trend in each pair of spreads.

For rating categories that fail to reject the null of Johansen’s cointegration test in the pair (CDS, bond) – i.e. for AA, BBB, XOVER, and BB – we investigate if failure to reject can be attributed to a structural break in their long-term equilibrium relationship. To this aim we apply the ADF\* test of “no cointegration” of Gregory and Hansen (1996), the alternative of which stipulates “a structural break in an existing cointegrating relationship” affecting the constant, the trend, or both (full break). Table 1 (Panel B) shows that the null of “no cointegration” is strongly rejected in 3 out of 4 tested categories – i.e. for AA, BBB, and BB – once we assume a full break in the (CDS, bond) cointegrating vector. In effect, this test leaves only the cross-over rating case failing both the Johansen’s and Gregory–Hansen’s cointegration tests.

Intuitively, this result implies that a final decoupling in the relationship between the two markets during the sub-period 3/1/2007 – 17/4/2010 is not a satisfactory interpretation for the lack of cointegration in the data. Far from it, it suggests that the relationship continues to exist, albeit distinctly different from what it was before the break. It must be noted though that the Gregory–Hansen test takes a rather general view, assuming a structural break that is exogenous in nature. For all we know, such a structural break could be indicative of an “omitted variable” problem.

Repeating the cointegration test for the triplet of spreads (CDS, bond, Libor/OIS) we reject the null of “no cointegration” in all rating portfolios. This is a key result as it confirms that money-market liquidity is the omitted variable that restores the equilibrium relationship between CDS and bond spreads. Moreover, we reject the null of “at most one cointegrating vector” (for the alternative of two cointegrating vectors) in 5 out of 8 rating categories. The existence of two cointegrating vectors is also very important, considering that the common stochastic trend between CDS and bond spreads captures the “price of credit risk” (Blanco et al., 2005). We add to this intuition by further identifying a common component among all stochastic trends across rating categories that is directly linked to the state of money market liquidity (Libor/OIS spread).

Summarizing, our results confirm the existence of a long-term equilibrium relationship between CDS and bond spreads, even during the financial crisis of 2007–2009. However, this equilibrium relationship is described more efficiently once it includes Libor/OIS spread as a third component. In theory, the presence of cointegration between CDS and bond spreads is indicative of

<sup>15</sup> We choose the number of lags using the Akaike Information Criterion (AIC).



arbitrage activity spanning the two markets.<sup>16</sup> In practice, it provides us with a useful guide to impose restrictions on VECM models that may lead to more efficient estimation, forecasting, or hedging.

### 3.2. Robustness checks for cointegrating models with alternative money-market spreads

The basis arbitrage strategy implies a cointegrating relationship for CDS, bond and Libor/OIS spreads, as discussed in [Appendix B](#). In this sense, Libor/OIS impacts directly on the equilibrium relationship between CDS and bond spreads through the arbitrage strategy that traders implement in practice. Yet for robustness reasons, we compare in terms of model fitness (in-sample) the Libor/OIS with alternative money-market spreads discussed in the literature. These include the 3-month Libor minus the 3-month GC repo rate (Libor/GC repo), the 3-month Libor minus the 3-month Treasury Bill rate (TED), and the 3-month GC repo minus the 3-month Treasury Bill rate (GC repo/Tbill). [Table A.1](#) in the Appendix shows comparisons based on the linear VECM for CDS, bond, and different money-market spreads, with two cointegrating vectors and two lags.<sup>17</sup>

Overall, differences in model fitness across money-market spreads turn out to be rather small, suggesting that (at least in-sample) results are robust to the choice of money-market liquidity measure. But Libor/OIS turns out to be more frequently the best performer, supporting our modeling choice based on the economics of basis trade.

## 4. An encompassing view: structural instability under threshold cointegration

In theory, limits to arbitrage may lead to persistent mispricings, even in the breakdown of no-arbitrage relationships due to self-reinforcing deviations from equilibrium. As discussed in [Appendix B](#), stop-loss limits and slow-moving capital may imply different regimes for the joint dynamics of CDS, bond, and Libor/OIS spreads, depending on the magnitude of deviations from the cointegrating relationship. Therefore, in this section we extend the linear VECM to a regime-switching framework that allows for different market conditions within a single setting.

The threshold cointegration model of [Balke and Fomby \(1997\)](#) permits non-linearities in standard cointegrating relationships. Its applications usually assume a band of inaction in the error-correction mechanism where small deviations from equilibrium are not corrected unless the benefit of correction outweighs the cost.<sup>18</sup> However, limits to arbitrage may lead to a breakdown in the error-correction mechanism when the mispricing moves *outside* a band where arbitrageurs are normally active. The basic idea is that an initial widening of the mispricing may force arbitrageurs to liquidate positions prior to spread convergence as mark-to-market losses hit stop-loss limits. If capital is slow-moving to exploit the mispricing, then forced unwinding of arbitrage positions widens the mispricing further, driving more arbitrageurs out of the market.

More specifically, let  $z_1$ ,  $z_2$ , and  $z_3$  denote the deviations of CDS, bond, and Libor/OIS spreads from the equilibrium relationships tested in the previous section:

$$\begin{aligned} z_{1t} &= CDS_t - b_0 - b_1 \cdot Bond_t \\ z_{2t} &= Bond_t - c_0 - c_1 \cdot Libor/OIS_t \\ z_{3t} &= CDS_t - d_0 - d_1 \cdot Bond_t - d_2 \cdot Libor/OIS_t \end{aligned} \quad (1)$$

where  $b_i$ ,  $c_i$ ,  $d_i$  ( $i = 1, 2$ ) are cointegrating vector coefficients.

Based on disequilibrium errors (1) we discuss alternative models of the joint dynamics of CDS, bond, and Libor/OIS spreads. These vary from the simplest (linear) VECM to more structured (regime-switching) specifications, as summarized in [Table 2](#). To explain the joint dynamics around the financial crisis we select the best fitting model using AIC and BIC.

**Model 1: Bivariate linear VECM with 1 cointegrating vector.** This is the benchmark model used, among others, by [Blanco et al. \(2005\)](#) and [Zhu \(2006\)](#). It assumes the existence of a cointegrating relationship between the CDS and bond spreads that is stable over time and is not affected by any third variable, such as money-market liquidity.

$$\begin{bmatrix} \Delta CDS \\ \Delta Bond \end{bmatrix}_t = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \cdot z_{1,t-1} + \sum_{k=1}^2 \Phi_{t-k} \begin{bmatrix} \Delta CDS \\ \Delta Bond \end{bmatrix}_{t-k} \quad (\text{Model 1a})$$

where,  $a_i$  ( $i = 1, 2$ ) are mean-reversion coefficients towards the long-term equilibrium relationship,  $\Phi$  is coefficient matrix capturing short-term dynamics, and  $z_1$  is defined in (1).

To capture the possibility of money-market liquidity causing transient supply/demand shocks in the CDS and bond market, we augment (Model 1a) to include lagged changes in Libor/OIS spread.

$$\begin{bmatrix} \Delta CDS \\ \Delta Bond \end{bmatrix}_t = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \cdot z_{1,t-1} + \sum_{k=1}^2 \Phi_{t-k} \begin{bmatrix} \Delta CDS \\ \Delta Bond \\ \Delta(Libor/OIS) \end{bmatrix}_{t-k} \quad (\text{Model 1b})$$

<sup>16</sup> The inverse does not necessarily hold as various factors influencing the arbitrage trade may contain unit roots.

<sup>17</sup> AIC and BIC are computed separately for the CDS and bond equations of the jointly estimated VECM.

<sup>18</sup> Applications of threshold cointegration in the finance literature include the law of one price, the transmission of prices between substitute markets and the detection of arbitrage opportunities ([Enders and Falk, 1998](#); [Baum et al., 2001](#); [Lo and Zivot, 2001](#); [Taylor, 2001](#), among others).

Table 2

Basic features of linear and non-linear VECM specifications for the joint dynamics of CDS and bond spreads.

Model index number:	1a,b	2	3	4	5	6	7	8	9	10
(CDS, bond) single equilibrium	✓			✓	✓					
(CDS, bond, Libor/OIS) single equilibrium		✓				✓				
(CDS, bond) and (bond, Libor/OIS) independent equilibria			✓				✓	✓	✓	✓
Money-market dislocations have transitory effect	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Money-market dislocations have permanent effect		✓	✓			✓	✓	✓	✓	✓
Arbitrage may break down when:				✓	✓	✓	✓	✓	✓	✓
... stop-loss limits lead investors to retract capital				✓	✓	✓	✓		✓	
... funding-liquidity constraints become binding						✓		✓	✓	
... no explicit reason										✓
Pre- and post-crisis dynamics differ					✓	✓	✓	✓	✓	✓

This table shows basic features of linear and non-linear VECM specifications indexed 1–10, as discussed in Section 4. Such features include long-term (equilibrium) relationships for the pairs (CDS, bond), (bond, Libor/OIS) and the triplet (CDS, bond, Libor/OIS), transitory and permanent effects of money-market dislocations, possible break-down in CDS–bond arbitrage relationship due to funding constraints, stop-loss, or other reasons, and differences in pre- and post-crisis joint dynamics of CDS and bond spreads.

(Model 1b) assumes no limits to arbitrage. The model is consistent with the idea of high speed of capital where arbitrageurs unable to implement the basis trade are swiftly replaced by others. Thus money-market shocks affect only the short-term dynamics of CDS and bond spreads.

**Model 2: Bivariate linear VECM with 1 cointegrating vector including Libor/OIS spread.** It builds on (Model 1) recognizing that the Libor/OIS spread is a component of the long-term equilibrium relationship between CDS and bond spreads.

$$\begin{bmatrix} \Delta CDS \\ \Delta Bond \end{bmatrix}_t = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \cdot z_{3,t-1} + \sum_{k=1}^2 \Phi_{t-k} \begin{bmatrix} \Delta CDS \\ \Delta Bond \\ \Delta(Libor/OIS) \end{bmatrix}_{t-k} \quad (\text{Model 2})$$

where,  $\alpha_i$  ( $i = 1, 2$ ) and  $\Phi$  are defined as in (Model 1), and  $z_3$  is defined in (1).

(Model 2) assumes no limits to arbitrage between the CDS and bond market. But recognizes Libor/OIS as a component of the equilibrium relationship between CDS and bond spreads.

**Model 3: Bivariate linear VECM with 2 cointegrating vectors.** It allows for a cointegrating vector exclusively between CDS and bond spreads while it also includes Libor/OIS in a second cointegrating vector for a more complete description of the long-term relationship.

$$\begin{bmatrix} \Delta CDS \\ \Delta Bond \end{bmatrix}_t = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}_{t-1} + \sum_{k=1}^2 \Phi_{t-k} \begin{bmatrix} \Delta CDS \\ \Delta Bond \\ \Delta(Libor/OIS) \end{bmatrix}_{t-k} \quad (\text{Model 3})$$

where,  $\alpha_{ij}$  ( $i, j = 1, 2$ ) are mean-reversion coefficients towards long-term equilibrium relationships,  $\Phi$  is coefficients matrix capturing short-term dynamics, and  $z_i$  ( $i = 1, 2$ ) are defined in (1).

(Model 3) is motivated by preliminary results in Section 3.1 that indicate the presence of two cointegrating vectors for the triplet of spreads (CDS, bond, Libor/OIS). It allows CDS and bond spreads to respond differently to deviations from the first equilibrium relationship compared to similar deviations from the second. In this sense, (Model 3) nests as special cases (Models 1) and (2) that may still be valid, although partial descriptions of true dynamics.

Previous literature paid no attention to the impact of funding costs on CDS–bond dynamics. Fig. 2 shows that this apparent omission is possibly explained by the fact that prior to 2007 the disequilibrium error  $z_1$  used to exhibit much higher volatility and persistence than  $z_2$ , thus used to dominate CDS–bond dynamics. But since the onset of financial crisis, both the volatility and persistence of  $z_2$  increased markedly.

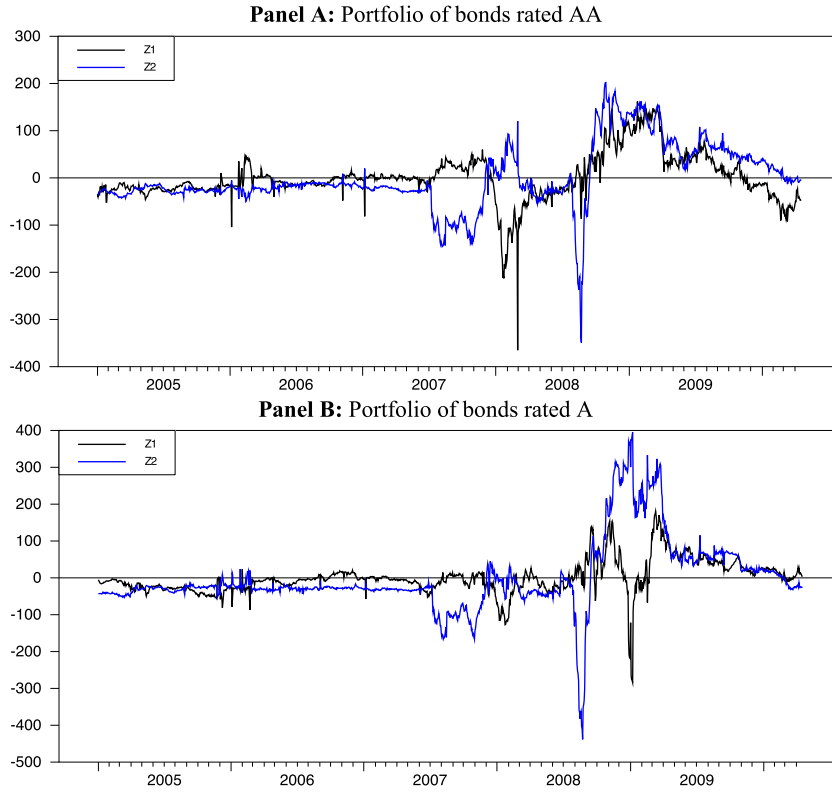
**Model 4: Bivariate threshold VECM with 1 cointegrating vector and 1 endogenous break.** This is the non-linear extension of (Model 1) with two regimes, as in Hansen and Seo (2002).

$$\begin{bmatrix} \Delta CDS \\ \Delta Bond \end{bmatrix}_t = \begin{bmatrix} a_1^{(i)} \\ a_2^{(i)} \end{bmatrix} \cdot z_{1,t-1} + \sum_{k=1}^2 \Phi^{(i)}_{t-k} \begin{bmatrix} \Delta CDS \\ \Delta Bond \end{bmatrix}_{t-k}, \quad (\text{Model 4})$$

$$i = \begin{cases} 1, & \text{if } z_{1,t-1} \leq \gamma \\ 2, & \text{if } z_{1,t-1} > \gamma \end{cases}$$

where,  $\gamma$  is threshold value that determines regime  $i$  at time  $t$ ,  $a_j^{(i)}$  ( $j = 1, 2$ ) are mean-reversion coefficients towards the equilibrium relationship,  $\Phi^{(i)}$  is coefficient matrix capturing short-term dynamics, and  $z_1$  is defined in (1).

(Model 4) introduces limits-to-arbitrage in our analysis as it assumes two regimes, one of which can possibly refer to “abnormal” market conditions. The underlying hypothesis is that the joint dynamics of CDS and bond markets shift radically from a “normal” to



**Fig. 2.** Disequilibrium errors  $z_1$  and  $z_2$  (in basis points) for portfolios of bonds rated AA and A. This figure shows disequilibrium errors  $z_1$  (between CDS and bond spreads) and  $z_2$  (between bond and Libor/OIS spreads) for portfolios of bonds rated AA (Panel A) and A (Panel B). Prior to 2007,  $z_1$  shows higher volatility and persistence than  $z_2$ , which appeared to be stationary. But since 2007, both volatility and persistence of  $z_2$  increased markedly, with a possible effect on basis.

a “distressed” regime after the basis crosses a given threshold  $\gamma$ . However, (Model 4) does not explicitly account for the impact of money-market liquidity on the equilibrium relationship between CDS and bond spreads, leaving the underlying arbitrage constraints unspecified.

**Models 5–6: Bivariate threshold VECM with 1 cointegrating vector and 2 endogenous breaks.** This is an extension of (Model 4) to the direction of including Libor/OIS both in short-term dynamics and in the cointegrating relationship between CDS and bond spreads.

$$\begin{bmatrix} \Delta CDS \\ \Delta Bond \end{bmatrix}_t = \begin{bmatrix} a_1^{(i)} \\ a_2^{(i)} \end{bmatrix} \cdot z_{1/3,t-d} + \sum_{k=1}^2 \Phi_{t-k}^{(i)} \begin{bmatrix} \Delta CDS \\ \Delta Bond \\ \Delta(Libor/OIS) \end{bmatrix}_{t-k} \quad (\text{Models 5-6})$$

$$i = \begin{cases} 1, & \text{if } z_{1/3,t-d} \leq \gamma_1 \\ 2, & \text{if } \gamma_1 < z_{1/3,t-d} \leq \gamma_2 \\ 3, & \text{if } z_{1/3,t-d} > \gamma_2 \end{cases}$$

where,  $\gamma_i$  ( $i = 1, 2$ ) are threshold values,  $d$  the lag of threshold variable that determines regime  $i$  at time  $t$ ,  $a_j^{(i)}$  ( $j = 1, 2$ ) and  $\Phi^{(i)}$  are defined as in (Model 4), and  $z_1, z_3$  defined in (1).

When  $z_1$  is used as threshold variable, the model (Model 5) assumes that the joint dynamics of CDS and bond markets enter the distressed regime once the basis widens beyond a certain level. Intuitively, this could be a result of stop-orders placed on the level of the basis as in (Model 4).

When  $z_3$  is used as threshold variable, the model (Model 6) assumes that normal market conditions are associated with modest levels of CDS–bond mispricing relative to the no-arbitrage condition that includes Libor/OIS spread as a third component. But when the mispricing ( $z_3$ ) widens too much, the P&L of basis trades moves into negative territory, triggering stop-loss orders and driving



arbitrageurs out of the market.<sup>19</sup> If capital is slow-moving to exploit the mispricing, large levels of  $z_3$  characterize a distressed regime where forced unwinding of trades widens the mispricing further, feeding into self-reinforcing market dislocations.

**Models 7–9: Bivariate threshold VECM with 2 cointegrating vectors and 2 endogenous breaks.** This is a non-linear extension of (Model 3) allowing for different regimes in the joint dynamics of CDS and bond spreads. It also extends (Models 5–6) to the direction of including one cointegrating vector exclusively between CDS and bond spreads, and a second one between bond and Libor/OIS spreads for a more complete description of the long-term relationship.

$$\begin{bmatrix} \Delta CDS \\ \Delta Bond \end{bmatrix}_t = \begin{bmatrix} a_{11}^{(i)} & a_{12}^{(i)} \\ a_{21}^{(i)} & a_{22}^{(i)} \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}_{t-1} + \sum_{k=1}^2 \Phi^{(i)}_{t-k} \begin{bmatrix} \Delta CDS \\ \Delta Bond \\ \Delta(Libor/OIS) \end{bmatrix}_{t-k} \quad (\text{Models 7-9})$$

$$i = \begin{cases} 1, & \text{if } z_{m,t-d} \leq \gamma_1 \\ 2, & \text{if } \gamma_1 < z_{m,t-d} \leq \gamma_2 \\ 3, & \text{if } z_{m,t-d} > \gamma_2 \end{cases}$$

where,  $a_{ij}^{(i)}$  ( $i, j = 1, 2$ ) and  $\Phi^{(i)}$  are defined as in (Model 3),  $\gamma_i$  ( $i = 1, 2$ ) and  $d$  determine regime  $i$  at time  $t$  as in (Models 5–6), and  $z_m$  ( $m = 1, 2, 3$ ) defined in (1).

Threshold variable  $z_m$  ( $m = 1, 2, 3$ ) allows for structural breaks in the CDS–bond arbitrage relationship to depend on stop-loss orders based on the level of basis ( $z_1$ ), on bond-funding costs ( $z_2$ ), or on the cumulative P&L of the basis trade ( $z_3$ ).

**Model 10: Bivariate threshold VECM with 2 cointegrating vectors and 2 exogenous breaks.** This model is very similar to (Models 7–9), except that regimes are now determined by *exogenous* structural breaks corresponding to specific points in time, defining time  $t$  as threshold variable.

$$\begin{bmatrix} \Delta CDS \\ \Delta Bond \end{bmatrix}_t = \begin{bmatrix} a_{11}^{(i)} & a_{12}^{(i)} \\ a_{21}^{(i)} & a_{22}^{(i)} \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}_{t-1} + \sum_{k=1}^2 \Phi^{(i)}_{t-k} \begin{bmatrix} \Delta CDS \\ \Delta Bond \\ \Delta(Libor/OIS) \end{bmatrix}_{t-k} \quad (3d - \text{Model 10})$$

$$i = \begin{cases} 1, & \text{if } t \leq t_1 \\ 2, & \text{if } t_1 < t \leq t_2 \\ 3, & \text{if } t > t_2 \end{cases}$$

where,  $a_{ij}^{(i)}$  ( $i, j = 1, 2$ ) and  $\Phi^{(i)}$  defined as in (Model 3).

(Model 10) serves as a robustness check to examine if threshold variables in (Model 4)–(Model 9) are indeed informative. If it proves to be more successful than previous models, it would imply that we have not sufficiently explained the reason of arbitrage brake-down in our sample.

## 5. Estimation procedure

Before proceeding with model estimation, we estimate cointegrating vectors and respective disequilibrium errors  $z_1$ ,  $z_2$  and  $z_3$  using Hansen's Fully Modified OLS.<sup>20</sup> Only exception is (Model 4) where  $z_1$  and threshold  $\gamma$  are estimated jointly using a double grid-search procedure that maximizes the log-likelihood function.

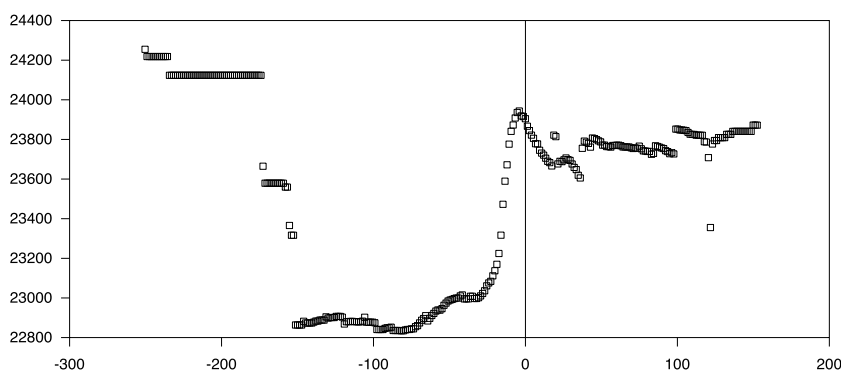
Threshold effects are tested in two ways: For (Model 1) we test the null hypothesis of “no-threshold effects” (single regime) against the alternative of a single threshold (2 regimes) using the LM test of Hansen and Seo (2002). Hansen–Seo LM test is a formal model-based statistical test that is heteroskedasticity-robust with higher power than comparable non-parametric tests.<sup>21</sup> However, it cannot be applied on larger specifications, i.e. with more than 2 endogenous variables and/or more regimes. Therefore, for multiple regime extensions of (Model 1)–(Model 3) we apply the threshold test of Tsay (1998) in the following successive steps: First we reject the null hypothesis of “no-threshold effects” for the alternative of a single threshold. Then we repeat the test on each of the two subintervals to examine if there is a second threshold in either of them. The procedure ends when the null is not rejected. To economize on space, we only present model comparisons that pass this multistage procedure.

Test results for all three specifications (Model 1)–(Model 3) suggest rejection of the null of “no-threshold effects” at 0.01 confidence level for all rating portfolios and for all values of  $d$  (delay in the threshold variable) ranging from 1 to 7. It is worth noting that the alternative hypothesis of threshold effects implies both the existence of regime effects *and* the ability of the assumed threshold variable to distinguish between the two regimes. While we cannot formally test for the existence of three regimes against

<sup>19</sup> Appendix B shows that mark-to-market losses on basis trades increase as the deviation ( $z_3$ ) of CDS, bond and Libor/OIS spreads from their cointegrating relationship widens.

<sup>20</sup> Notice that when both  $z_1$ ,  $z_2$  are included in the same model they are estimated orthogonal to each other.

<sup>21</sup> For a discussion, see Balke and Fomby (1997) and Lo and Zivot (2001).



**Fig. 3.** AIC values of two-regime specification against alternative threshold levels. This figure plots the AIC value for testing a two-regime specification of (Model 3) against alternative threshold levels (in basis points) for the portfolio of AA rated bonds, using  $z_1$  as threshold variable. A wide range of threshold values (between  $-150$  and  $-40$  basis points) correspond to almost equally informative models, i.e. with very low AIC values. The absence of an undisputable global minimum is indicative of the existence of more than two regimes.

**Table 3**

Comparison of linear and non-linear VECM specifications for the joint dynamics of CDS and bond spreads.

	# of Regimes = 2		# of Regimes = 3								
Cointegrating vector(s):	(CDS, bond)		(CDS, bond)		(bond, Libor/OIS)		(CDS, bond) & (bond, Libor/OIS)				
Threshold variable:	None	$z_1$	None	$z_1$	None	$z_3$	None	$z_1$	$z_2$	$z_3$	t
Model index number:	1a	4	1b	5	2	6	3	7	8	9	10
AAA	(11)	(7)	(9)	(4)	(8)	(5)	(10)	(2)	(3)	(1)	(6)
AA	(11)	(6)	(9)	(2)	(8)	(4)	(10)	(1)	(5)	(3)	(7)
A	(11)	(6)	(8)	(4)	(10)	(3)	(9)	(1)	(5)	(2)	(7)
BBB	(9)	(7)	(11)	(4)	(10)	(3)	(8)	(1)	(5)	(2)	(6)
XOVER	(11)	(6)	(10)	(2)	(8)	(4)	(8)	(1)	(3)	(5)	(7)
BB	(9)	(5)	(11)	(3)	(10)	(4)	(8)	(1)	(6)	(2)	(7)
B	(11)	(6)	(10)	(4)	(9)	(3)	(8)	(2)	(5)	(1)	(7)
CCC-C	(9)	(6)	(11)	(5)	(10)	(4)	(8)	(3)	(1)	(2)	(7)
Estimation method:	Johansen	Hansen-Seo	Johansen	Tsay	Johansen	Tsay	Johansen	Tsay	Tsay	Tsay	Tsay

This table shows comparisons of linear and non-linear VECM specifications with model index number 1a, b–10, as discussed in Section 4. The ordering is based on AIC and BIC, with ranking (1) corresponding to best fit and (11) to worst.

the null of two regimes, our procedure is able to distinguish if the basis dynamics are indeed better explained by a three-rather than two-regime process.

An indication that a three-regime process is possibly more accurate representation of CDS–bond dynamics than a two-regime process is shown in Fig. 3. It plots AIC values of a two-regime specification of (Model 3) against alternative threshold levels (in basis points).<sup>22</sup> We observe a wide range of threshold values (between  $-150$  and  $-40$  basis points) corresponding to almost equally informative models, i.e. with very low AIC values. This could be interpreted as absence of an undisputable global minimum, indicating the existence of more than two regimes.

Following Tong (1990) and Tsay (1998) we estimate (Model 5)–(Model 10) using the AIC approach for threshold cointegration. According to this approach, we first fix the number of lags and regimes and then select the model based on AIC. In practice, this approach is asymptotically equivalent to selecting the model with the smallest generalized residual variance using the conditional least squares method. Tsay (1998) shows that the estimates obtained by this method are strongly consistent and independent of threshold value priors.

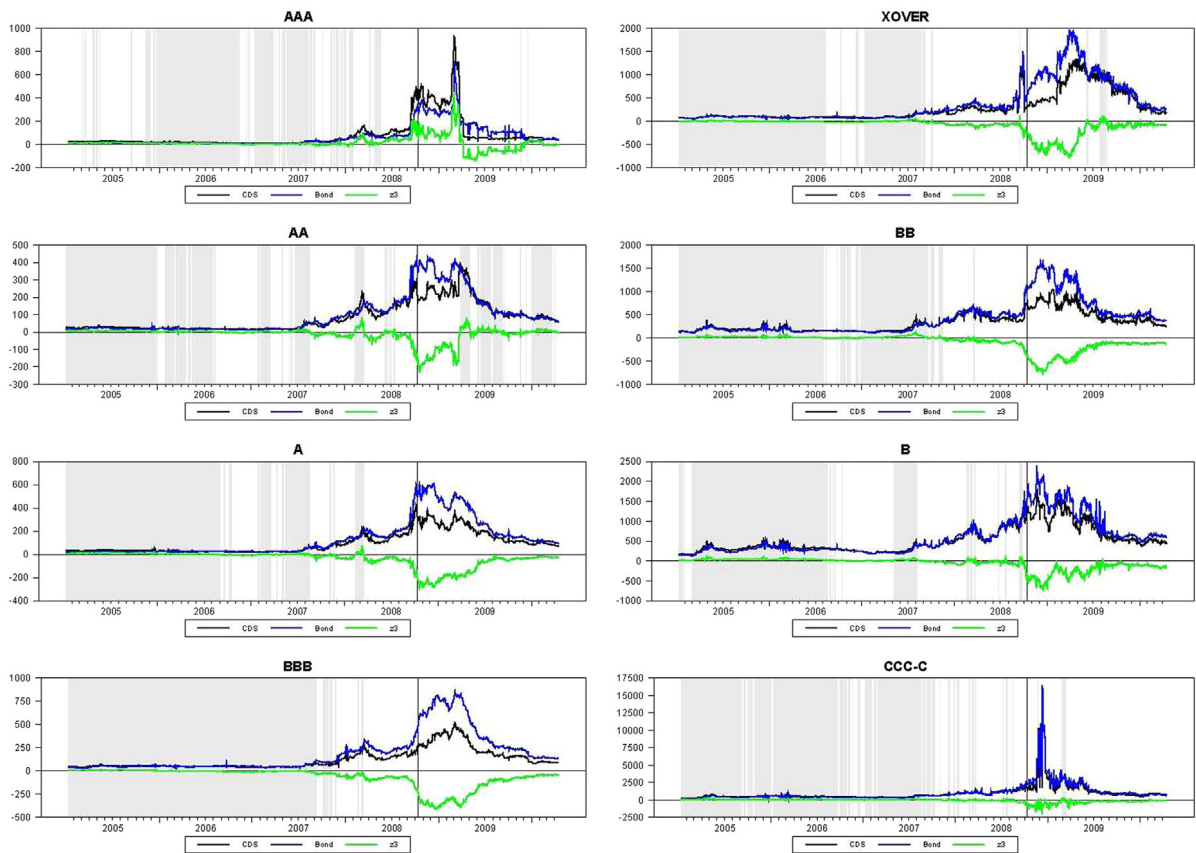
Table 3 presents model comparisons for each rating portfolio based on AIC and BIC, with ranking 1 corresponding to best fit and 11 to worst.<sup>23</sup> If the two criteria give conflicting results, then we consider it as tie. Overall, we find that in all cases both information criteria point to the same model ordering, except for the cross-over rating category for (Model 2) and (Model 3).

## 6. Results

Our main results (Table 3) can be summarized as follows:

<sup>22</sup> Fig. 3 refers to the portfolio of AA rated bonds, using  $z_1$  as threshold variable.

<sup>23</sup> Even though the usual information criteria (AIC, BIC, HQ) require all candidate models to belong to a single parametric family, previous empirical research suggests that they can be very effective for the purpose we use them. For example, Kapetanios (2001) shows that AIC, BIC are better able to choose the appropriate threshold model compared to other criteria (e.g. ICOMP or GIC), which – at least in theory – are not subject to similar constraints.



**Fig. 4.** In-sample estimates of regime 1 (pre-crisis normal regime) per rating category, for the period 3/1/2005–17/4/2010. This figure shows (in shade) in-sample estimates of regime 1 across rating categories, based on (Model 9) where  $z_3$  is used as threshold variable. They mostly coincide with normalcy periods before the eruption of financial crisis in 2007, thus regime 1 is referred as “pre-crisis normal regime”.

Specifications with two cointegrating vectors are generally superior to single cointegrating vector models, other things being equal (e.g. number of regimes). In line with preliminary tests in Section 3, including a second cointegrating vector between the bond and Libor/OIS spread offers a more accurate description of the joint CDS–bond dynamics for all rating categories.

Moreover, multiple-regime (Model 4)–(Model 10) exhibit consistently better fit than linear (single-regime) (Model 1)–(Model 3), supporting the limits-to-arbitrage explanation of the breakdown of basis arbitrage during the crisis, as already discussed. Econometrically this result is rather expected given our preliminary test results that indicate threshold effects for all rating portfolios.

Among multiple-regime models, regime-switching mechanisms that are linked to dynamic threshold variables  $z_i$ ,  $i = 1, 2, 3$  exhibit better fit across rating categories compared to (Model 10). While the former mechanism implies a link to the basis ( $z_1$ ), to bond-funding costs ( $z_2$ ), or to cumulative P&L of the basis trade ( $z_3$ ), the latter is “agnostic” to the transition mechanism between regimes assuming static break points in time. Intuitively, this result confirms our conjecture that the unraveling of the basis trade during the 2007–09 crisis was a direct product of its very economics. Furthermore, the fact that  $z_1$  and  $z_3$  are more successful choices for the threshold variable than  $z_2$  (except for the CCC rating category) points to a regime-switching mechanism that is better described by stop-loss limits rather than by funding constraints alone.

All 3-regime specifications (Model 5)–(Model 9) turn out to dominate the 2-regime Hansen–Seo model (Model 4), indicating the need to account for more structure than just a normal and a distressed regime. Figs. 4–6 show (in shade) the in-sample estimates of regimes 1 to 3 across rating categories based on (Model 9), where  $z_3$  is used as threshold variable. Regime 1 tends to coincide with normal periods before the eruption of financial crisis in 2007 (pre-crisis normal regime). Regime 2 roughly corresponds to an intermediate stage of normal periods *after* the onset of financial crisis in 2007 (post-crisis normal regime). Regime 3 points to the peak of the financial turmoil, right before and after the Lehman failure (distressed regime).<sup>24</sup>

Notice that  $z_3 > \gamma_2$  implies “pre-crisis normal regime” for all rating categories except AAA, for which it implies “distressed regime”. For  $z_3 \leq \gamma_1$  the basis is most negative and implies “distress regime” for all rating categories except AAA, for which it corresponds to “pre-crisis normal regime”. The “post-crisis normal regime” corresponds to  $\gamma_1 < z_3 \leq \gamma_2$  for all rating categories.

<sup>24</sup> Bai and Collin-Dufresne (2019) refer roughly to the same periods as “Before Crisis” for regime 1, “Crisis I” and “Post-Crisis” for regime 2, and “Crisis II” for regime 3.

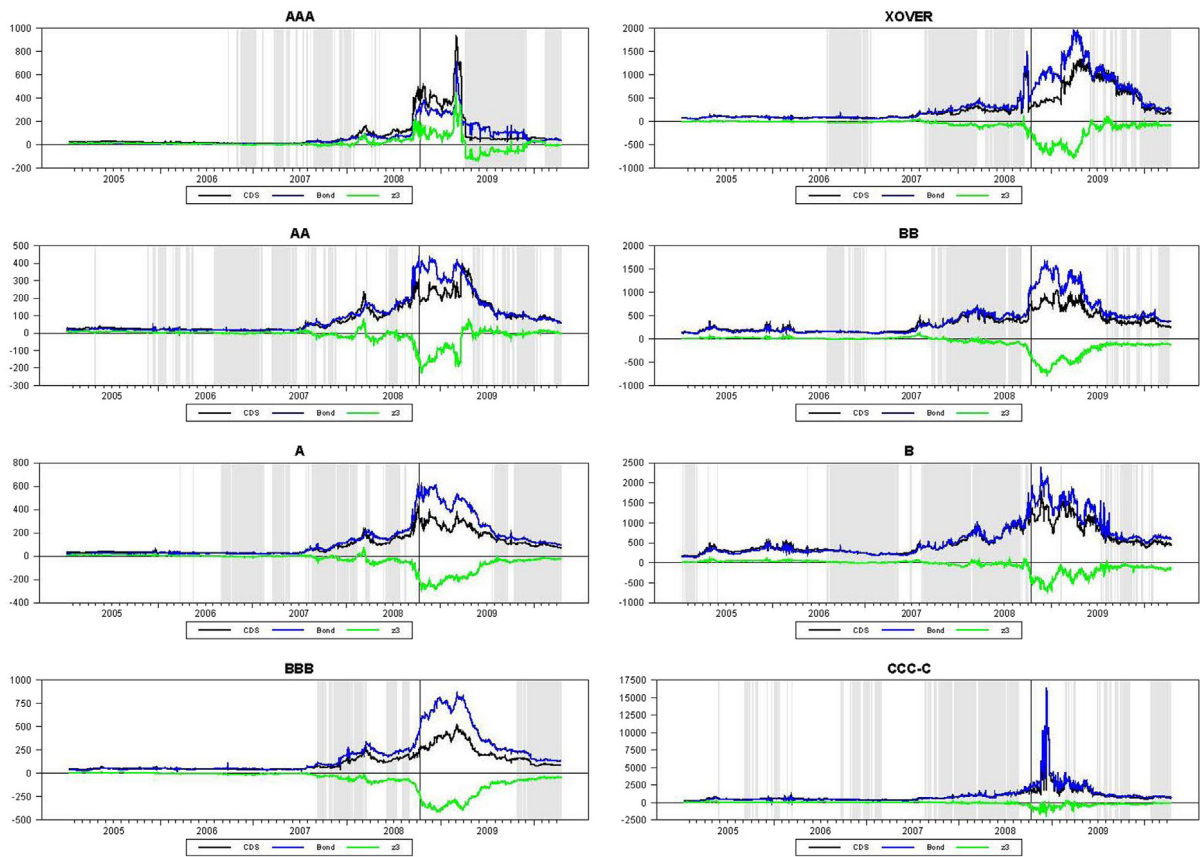


Fig. 5. In-sample estimates of regime 2 (post-crisis normal regime) per rating category, for the period 3/1/2005–17/4/2010. This figure shows (in shade) in-sample estimates of regime 2 across rating categories, based on (Model 9) where  $z_3$  is used as threshold variable. They mostly coincide with normalcy periods after the eruption of financial crisis in 2007, thus regime 2 is referred as “post-crisis normal regime”.

Tables 4 and 5 present estimated coefficients for CDS and bond spread dynamics using Eicker–White heteroskedasticity robust estimates of the 3-regime model, using as threshold variable the cointegration error  $z_3$  (Model 9).<sup>25</sup>

Tables 4 and 5 show that in normal regimes (1 and 2) CDS and bond spreads tend to revert towards their long-run equilibrium relationship, in line with the pre-crisis literature. This is indicated by the negative and statistically significant effect of disequilibrium error  $z_1$  on CDS dynamics. Furthermore, the effect of liquidity-related disequilibrium error  $z_2$  on bond spreads is mostly negative and statistically significant implying that an increase in Libor/OIS spread (i.e. a fall in money-market liquidity) relative to its long-term equilibrium relationship with the bond market tends to push bond spreads higher. Yet, short-term (possibly offsetting) effects of money-market liquidity obscure the economic interpretation of spread dynamics in normal regimes, except for causality effects that we discuss below.

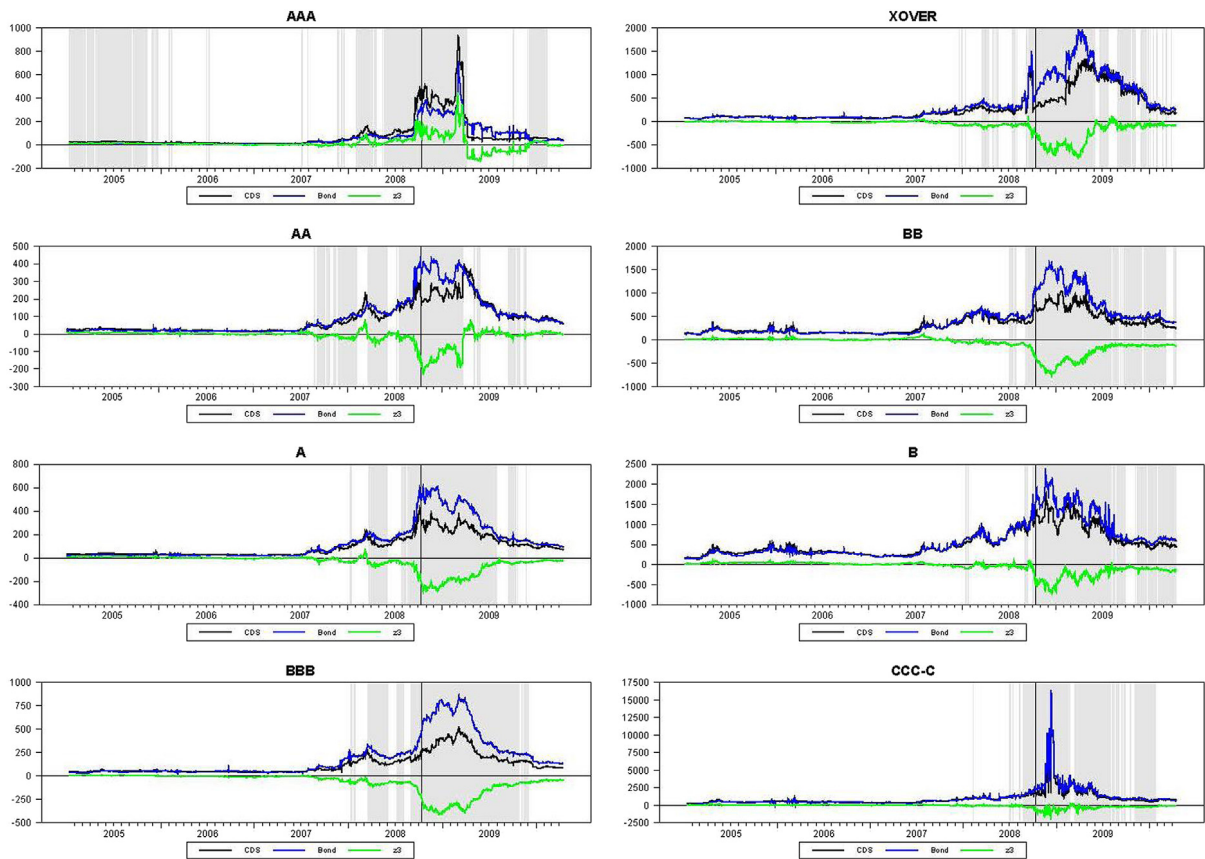
The pre-crisis normal regime also coincides with periods of “low risk premia”, as indicated by the mostly negative and statistically significant trend (constant) across the rating spectrum, especially in the CDS market (Table 4). In the post-crisis normal regime this negative trend flattens out, or moves into positive territory, especially for the BB rating category where it increases markedly (0.47 and 0.28 for CDS and bond spreads, respectively). The distressed regime is associated with a clear trend of widening spreads, especially in the CDS market.

The distressed regime is the most interesting of the three, presenting a complete break from the usual pattern of joint dynamics. Table 4 shows that changes in CDS spreads are now entirely driven by random shocks and the disequilibrium error with the bond market, as the effect of  $z_1$  is the only statistically significant.<sup>26</sup> The positive coefficient of  $z_1$  signifies that a widening of the basis drives CDS spreads away from the long-run equilibrium relationship with bond spreads, widening the basis even further. A plausible explanation for such destabilizing dynamics is the unwinding of basis trades due to activation of stop-loss orders or funding-liquidity constraints.

Yet forced unwinding of basis trades could accelerate the momentum of CDS spreads away from equilibrium with bond spreads only if new investment capital to replace arbitrageurs who leave the market is slow-moving. Under normal market conditions, a

<sup>25</sup> Similar results obtain when  $z_1$  is used as threshold variable (Model 7).

<sup>26</sup> The AAA rating category is the only exception and we discuss below.



**Fig. 6.** In-sample estimates of regime 3 (distressed regime) per rating category, for the period 3/1/2005–17/4/2010. This figure shows (in shade) in-sample estimates of regime 3 across rating categories, based on (Model 9) where  $z_3$  is used as threshold variable. They mostly coincide with the heights of financial crisis right before and after the Lehman failure, thus regime 3 is referred as “distressed regime”.

widening basis would attract enough capital to reduce any mispricing and push the basis towards regular levels. But under conditions of exceedingly slow-moving capital, the activation of stop-loss orders could accelerate the market towards new clusters of stop-loss limits, creating a virtual snowball effect (Shleifer and Vishny, 1997; Liu and Longstaff, 2004; Mitchell and Pulvino, 2012; Bai and Collin-Dufresne, 2019). When this happens, the basis goes on increasing almost uncontrollably.

Interestingly, for AAA names in the distressed regime, CDS spreads are not affected by  $z_1$ . Instead, the component with the highest impact is short-term changes in money-market liquidity: For every basis point widening in Libor/OIS there is a fourfold reduction in CDS spread. This is possibly due to reduced demand for CDS protection against AAA names at the height of the financial crisis when protection sellers considered not safe enough and sell protection at a lower spread.<sup>27</sup> This effect is moderated by the negative coefficient ( $-0.32$ ) of disequilibrium error  $z_2$  (between bond and Libor/OIS spreads), which falls as Libor/OIS spread increases.

Also in the distressed regime, the effect of  $z_1$  on bond spreads across the rating spectrum is mostly insignificant, as well as the effect of short-term changes in money-market liquidity. A special case is CCC-C rated bonds where short-term increases in Libor/OIS seem to reduce bond spreads substantially. Although this effect is rather counterintuitive, it is worth keeping in mind that quoted prices for CCC-C bonds in the distressed regime may be quite far from accurate.

Overall, the observed pattern in distress regime supports the hypothesis that the basis breakdown was caused by bond supply shocks contaminating the CDS market through the unraveling of basis trades: CDS spreads are almost exclusively driven by the P&L of basis trades, whereas bond spreads seem completely impervious to anything that is happening in the CDS market.

We also consider the extent to which price discovery takes place in the CDS market. During the pre-crisis normal regime, spread innovations in the CDS market transmit into the bond market for AAA and cross-over portfolios, but not the other way round. This is shown by the positive and statistically significant effect of short-term changes in CDS spread on AAA and cross-over rated bonds (0.15 and 0.28, respectively) in Table 5. Such causality is suggestive of price discovery taking place in the CDS market for rating categories that are of special interest to speculators. In particular, AAA bonds attract a liquidity premium due to their special role of acting as collateral in a wide array of financial transactions. On the other hand, the cross-over rating category is affected by

<sup>27</sup> Tightening money-market conditions during the financial crisis also coincides with broader unwinding of synthetic CDO structures that led to increased supply of CDS protection against AAA names and lower spreads.



Table 4

Eicker–White estimated coefficients of (Model 9) for changes in CDS spread under regimes  $i = 1, 2, 3$   $\Delta CDS_t = a_{11}^{(i)} z_{1,t-1} + a_{12}^{(i)} z_{2,t-1} + \sum_{k=1}^2 [\phi_{11,k}^{(i)} \Delta CDS_{t-k} + \phi_{12,k}^{(i)} \Delta Bond_{t-k} + \phi_{13,k}^{(i)} \Delta (Libor/OIS)_{t-k}]$ .

$Lag\ (k)$	$Const.$	$\Delta CDS_{t-k}$		$\Delta Bond_{t-k}$		$z_{1,t-1}$	$z_{2,t-1}$	$\Delta(Libor/OIS)_{t-k}$		$Obs.$
	–	1	2	1	2	1	1	1	2	
Pre-crisis normal regime ( $i = 1$ )										
AAA	–0.20**	0.03	–0.03	–0.20	–0.09	–0.45**	–0.49**	1.94	–4.58**	662
AA	–0.07**	0.31**	–0.12	–0.13	0.07	–0.13**	0.00	0.12	0.00	610
A	–0.21**	–0.12	–0.10	–0.01	0.07	–0.48**	0.00	0.21	0.69**	556
BBB	–0.02**	0.03	0.07	–0.23**	–0.06	–0.16**	–0.01**	–0.15*	0.04	714
XOVER	–0.10**	0.03	–0.06	–0.22	–0.02	–0.10**	–0.01	–0.09	0.34	602
BB	–0.10**	–0.12*	–0.17**	–0.16**	0.10	–0.40**	–0.05**	0.20	0.68**	639
B	–0.35**	–0.16	0.19*	0.13	–0.28**	–0.57**	–0.11**	–0.61	0.09	466
CCC-C	–8.29**	0.47	0.01	–1.84**	–0.49	–0.98**	–0.30**	–6.88	–1.55	635
Post-crisis normal regime ( $i = 2$ )										
AAA	0.00	0.07**	–0.17**	0.09*	0.25**	–0.03**	–0.02**	0.06	–0.15**	386
AA	0.00	0.06	0.09**	–0.08	–0.16**	–0.16**	0.03**	–0.30**	–0.55**	402
A	0.00	0.11**	0.05	–0.21**	0.08*	–0.11**	0.00	–0.31**	–0.19**	484
BBB	0.05**	–0.03	0.00	–0.24**	–0.07	–0.75**	0.01**	0.17**	0.00	261
XOVER	0.06**	–0.18**	–0.05	0.01	0.09	–0.19**	0.03**	–0.46	–1.17**	453
BB	0.47**	–0.26*	–0.33**	0.26	0.51**	–1.09**	–0.01	–0.02	0.31	356
B	0.09**	0.02	–0.33**	–0.59**	0.23*	–0.37**	–0.01	2.36**	–0.70	556
CCC-C	–0.08*	0.04	0.05	–0.66**	–0.35**	–0.22**	–0.14**	–1.13	0.66	418
Distressed regime ( $i = 3$ )										
AAA	0.01**	0.24	–0.39	1.04	0.05	–0.24	–0.32**	–4.00**	2.38	329
AA	0.31**	–0.03	–0.16	0.10	0.40	0.50**	0.16**	–0.06	0.31	365
A	0.40**	–0.35	0.53	0.34	–0.41	0.55**	0.02	–0.30	–0.16	337
BBB	0.12**	–0.19	0.17	0.17	–0.17	0.20**	0.00	–0.11	0.14	402
XOVER	2.45**	0.17	–0.66	0.56	0.54	1.14	0.17	–2.70	–1.32	322
BB	1.10**	0.05	–0.22	–0.28	–0.13	0.91**	0.01	–0.06	–0.06	382
B	1.81**	–0.08	–0.25	–0.37	0.39	0.69**	0.09	–2.79	–1.03	354
CCC-C	30.73**	–0.39	–0.21	1.28	–0.96	0.39	1.23	–9.53	–27.55	321

This table shows Eicker–White heteroskedasticity robust estimates for CDS spread dynamics across rating categories using the 3-regime model with threshold variable  $z_3$  (Model 9). Disequilibrium errors  $z_1$ ,  $z_2$ , and  $z_3$  are defined as  $z_{1t} = CDS_t - b_0 - b_1 \cdot Bond_t$ ,  $z_{2t} = Bond_t - c_0 - c_1 \cdot Libor/OIS_t$ , and  $z_{3t} = CDS_t - d_0 - d_1 \cdot Bond_t - d_2 \cdot Libor/OIS_t$ , where  $b_1$ ,  $c_1$ ,  $d_1$  ( $i = 1, 2$ ) are cointegrating vector coefficients. “Pre-crisis normal regime” ( $i = 1$ ) tends to coincide with normalcy periods before the eruption of financial crisis. “Post-crisis normal regime” ( $i = 2$ ) tends to coincide with normalcy periods around the full-blown financial crisis. “Distress regime” ( $i = 3$ ) tends to coincide with the heights of the financial crisis, before and after the Lehman failure.

\*Denoted statistical significance at 5% confidence level.

\*\*Denoted statistical significance at 1% confidence level.

funding discontinuity in repo market. It is also particularly sensitive to rating downgrades as it defines the investment–subinvestment dichotomy.

For AA and A rating categories we find no evidence of causality during the pre-crisis normal regime, where bond spreads appear unaffected both by short-term changes in CDS spreads and disequilibrium error  $z_1$ . Moreover, changes in CDS spreads are driven by disequilibrium error  $z_1$  (–0.13 and –0.48, respectively) with the A rating category depending also on short-term changes in money-market liquidity proxy (0.69). Therefore, while the two markets are cointegrated and basis trades seem to play a key role in CDS convergence towards bond spreads, we find no evidence of price discovery taking place primarily in any of the two markets, in line with [Zhu \(2006\)](#). Also, we find no one-way causality effects for BBB, BB and CCC-C portfolios during the pre-crisis normal regime. Instead, we observe (two-way) feedback effects between the CDS and bond spreads, where innovations in one market are transmitted into the other and vice versa.

In the post-crisis normal regime, causality effects disappear also for the AAA and cross-over rating category, indicating that the CDS market no longer leads the price discovery process for any rating category. This coincides with a period that is characterized by a substantial drop in the notional amount of outstanding CDS (from around \$58tn in 2008 down to \$30tn in 2010), constraints to rehypotheation and widespread distrust of CDS contracts to deliver on their intended purpose of insuring against default.<sup>28</sup>

## 7. Hedging implications: an out-of-sample exercise

The analysis so far has focused on models estimated in-sample. In this section we examine model performance in an out-of-sample framework. More specifically, we investigate if a threshold model governed by disequilibrium error  $z_3$  (Model 9) provides practical gains for the purpose of dynamically hedging a diversified portfolio of bonds of given credit rating.

<sup>28</sup> For example, according to FT article “Credit default swaps: a \$10tn market that leaves few happy” (July 25, 2017) distrust on the outcome of auctioning defaulted bonds to determine CDS payouts weighs negatively on CDS market.



Table 5

Eicker–White estimated coefficients of (Model 9) for changes in bond spread under regime  $i = 1, 2, 3$   $\Delta Bond_t = a_{21}^{(i)} z_{1,t-1} + a_{22}^{(i)} z_{2,t-1} + \sum_{k=1}^2 [\phi_{21,k}^{(i)} \Delta CDS_{t-k} + \phi_{22,k}^{(i)} \Delta Bond_{t-k} + \phi_{23,k}^{(i)} \Delta (Libor/OIS)_{t-k}]$ .

Lag (k)	Const.	$\Delta CDS_{t-k}$		$\Delta Bond_{t-k}$		$z_{1,t-1}$	$z_{2,t-1}$	$\Delta(Libor/OIS)_{t-k}$		Obs.
	–	1	2	1	2	1	1	1	2	
Pre-crisis normal regime (i = 1)										
AAA	0.13**	0.15**	0.03	–0.48**	–0.17*	–0.07*	–0.29**	0.91	–2.58**	662
AA	0.04*	0.14	–0.01	0.03	0.09	0.05	–0.02	–0.04	0.12	610
A	–0.10	–0.14	–0.02	0.07	–0.01	–0.22	0.00	0.38	0.87**	556
BBB	–0.01**	–0.06	0.17**	–0.22**	–0.06	–0.05**	–0.01**	–0.17**	–0.05	714
XOVER	–0.03	0.28**	–0.02	–0.50**	–0.05	–0.04*	–0.01	–0.10	0.05	602
BB	–0.06**	0.15**	0.05	–0.37**	–0.09	–0.25**	–0.04**	0.02	0.61**	639
B	–0.11**	0.10	0.29**	–0.19*	–0.45**	–0.22**	–0.06**	0.01	0.44	466
CCC-C	–0.44	0.26*	0.05	–1.07**	–0.41	0.01	–0.13*	–3.13	–0.71	635
Post-crisis normal regime (i = 2)										
AAA	–0.02**	0.15**	0.01	–0.26**	0.03	0.16**	–0.01*	0.01	–0.05	386
AA	0.00	0.24**	–0.01	–0.20**	–0.06	0.03**	–0.02**	–0.11*	–0.36**	402
A	0.00	0.12**	0.04	–0.23**	0.05	0.03*	–0.02**	–0.36**	–0.10*	484
BBB	0.03**	–0.11	–0.01	–0.20**	–0.11	–0.41**	–0.01*	0.22**	0.11	261
XOVER	0.00	0.08	0.05	–0.23**	–0.02	0.00	–0.01	–0.48*	–0.79**	453
BB	0.28**	–0.11	–0.27**	0.07	0.43**	–0.66**	–0.01	0.15	0.03	356
B	0.02	0.21**	–0.12*	–0.80**	–0.04	–0.10**	–0.01*	2.05**	–0.46	556
CCC-C	–0.01	0.01	0.01	–0.38**	–0.20**	–0.05**	–0.03**	–0.04	0.20	418
Distressed regime (i = 3)										
AAA	–0.02	0.06	–0.12	0.85**	–0.14	0.12*	–0.07**	–0.48	0.84*	329
AA	0.01	0.03	–0.08	–0.09	0.40*	0.00	–0.01	0.04	0.59**	365
A	0.06	–0.27	0.94**	0.28	–0.67**	–0.07	0.05	0.42	0.27	337
BBB	0.05**	–0.22	0.17	0.29*	–0.10	–0.02	–0.01	0.09	0.36*	402
XOVER	0.61	0.29	–0.38	0.34	0.24	–0.14	–0.06	–1.72	0.78	322
BB	0.52**	0.27**	0.03	–0.59**	–0.51**	–0.37**	–0.02	0.66	–0.65	382
B	0.65**	–0.01	–0.08	–0.36	0.07	–0.09	–0.10	1.77	–0.81	354
CCC-C	1.92	–0.07	–0.08*	0.46*	0.66*	0.11	–0.85**	–34.01**	–24.21	321

This table shows Eicker–White heteroskedasticity robust estimates for bond spread dynamics across rating categories using the 3-regime model with threshold variable  $z_3$  (Model 9). Disequilibrium errors  $z_1$ ,  $z_2$ , and  $z_3$  are defined as  $z_{1t} = CDS_t - b_0 - b_1 \cdot Bond_t$ ,  $z_{2t} = Bond_t - c_0 - c_1 \cdot Libor/OIS_t$ , and  $z_{3t} = CDS_t - d_0 - d_1 \cdot Bond_t - d_2 \cdot Libor/OIS_t$ , where  $b_i$ ,  $c_i$ ,  $d_i$  ( $i = 1, 2$ ) are cointegrating vector coefficients. “Pre-crisis normal regime” ( $i = 1$ ) tends to coincide with normalcy periods before the eruption of financial crisis. “Post-crisis normal regime” ( $i = 2$ ) tends to coincide with normalcy periods around the full-blown financial crisis. “Distress regime” ( $i = 3$ ) tends to coincide with the heights of the financial crisis, before and after the Lehman failure.

\*Denoted statistical significance at 5% confidence level.

\*\*Denoted statistical significance at 1% confidence level.

Although there is a long empirical literature on estimating optimal hedge ratios between cash and forward markets, to the best of our knowledge this is the first study to consider dynamic hedging between bonds and CDS. At the same time, and in addition to usual measures of hedging efficiency such as RMSE and maximum draw-down, we compare dynamic out-of-sample hedge ratios using popular tools of the forecasting literature, such as the Diebold–Mariano test.

We consider 5 competing hedging strategies:

Strategy 1 (Naïve Static Hedging) stipulates a hedging ratio of  $[1, -1]$  for the pair of par CDS and *bond-implied* par-equivalent CDS contracts. It assumes that the hedger is concerned only about the default scenario where bond losses are fully covered by the protection seller. However, this strategy implies either an infinite trading horizon (e.g. a buy-and-hold investor not subject to mark-to-market accounting), or infinite stop-loss limits for the hedger.

Strategy 2 (Rolling Regression Hedging) is estimated by the slope coefficient  $\beta$  in

$$\Delta Bond_t = a + \beta \cdot \Delta CDS_t \quad (2)$$

Despite the absence of time index in the slope coefficient in (2), in practice this hedge ratio is time-varying given that is estimated on a daily basis using the sample of changes in CDS and bond spreads over the preceding year (260 daily observations). However, as the variance–covariance matrix practically changes very little from day to day, this model produces hedge ratios that are significantly autocorrelated by construction.

The next three hedging strategies stem from the Kroner and Sultan (1993) bivariate GARCH approach. It assumes a cointegrated pair of prices for which the joint dynamics of the mean vector are described by a VECM.

Strategy 3 (VECM-CV1) relies on a mean VECM, similar to (Model 1b) in Section 4, but with  $L$  lags to describe the short-term dynamics.

$$\begin{bmatrix} \Delta CDS \\ \Delta Bond \end{bmatrix}_t = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \cdot z_{1,t-1} + \sum_{k=1}^L \Phi_{t-k} \begin{bmatrix} \Delta CDS \\ \Delta Bond \\ \Delta (Libor/OIS) \end{bmatrix}_{t-k} + \begin{bmatrix} \varepsilon_{CDS} \\ \varepsilon_{Bond} \end{bmatrix}_t \quad (3)$$

Strategy 4 (VECM-CV2) is based on (Model 3) with  $L$  lags for the short-term dynamics.

$$\begin{bmatrix} \Delta CDS \\ \Delta Bond \end{bmatrix}_t = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}_{t-1} + \sum_{k=1}^L \Phi_{t-k} \begin{bmatrix} \Delta CDS \\ \Delta Bond \\ \Delta(Libor/OIS) \end{bmatrix}_{t-k} + \begin{bmatrix} \varepsilon_{CDS} \\ \varepsilon_{Bond} \end{bmatrix}_t \quad (4)$$

Strategy 5 (T-VECM) is based on (Model 9) with three regimes and  $L$  lags for the short-term dynamics.

$$\begin{bmatrix} \Delta CDS \\ \Delta Bond \end{bmatrix}_t = \begin{bmatrix} a_{11}^{(i)} & a_{12}^{(i)} \\ a_{21}^{(i)} & a_{22}^{(i)} \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}_{t-1} + \sum_{k=1}^L \Phi_{t-k} \begin{bmatrix} \Delta CDS \\ \Delta Bond \\ \Delta(Libor/OIS) \end{bmatrix}_{t-k} + \begin{bmatrix} \varepsilon_{CDS} \\ \varepsilon_{Bond} \end{bmatrix}_t \quad (5)$$

$$i = \begin{cases} 1, & \text{if } z_{3,t-1} \leq \gamma_1 \\ 2, & \text{if } \gamma_1 < z_{3,t-1} \leq \gamma_2 \\ 3, & \text{if } z_{3,t-1} > \gamma_2 \end{cases}$$

Kroner and Sultan (1993) propose as “optimal” hedge ratio  $\beta^*$  the ratio of conditional covariance  $h_{CDS/Bond}$  between changes of the two series divided by the conditional variance  $h_{CDS}$  of the hedging instrument’s changes (here, the CDS contract).

$$\beta_{t-1}^* = -\frac{h_{CDS/Bond,t}}{h_{CDS,t}} \quad (6)$$

Both numerator and denominator of the hedge ratio in (6) are elements of the conditional covariance matrix  $H_t$  of the residual series  $\varepsilon_{CDS}$  and  $\varepsilon_{Bond}$  in (5). In every application of this method  $H_t$  is assumed to follow a bivariate GARCH specification. Given the apparent skewness of spread changes, we estimate  $H_t$  using the asymmetrical BEKK GARCH(1,1) model of Brooks et al. (2002). This model captures any asymmetric response of volatility to positive or negative innovations of equal magnitude and describes the dynamics of the conditional covariance matrix as follows.

$$H_t = C_0' C_0 + A_{11}' \varepsilon_{t-1} \varepsilon_{t-1}' A_{11} + B_{11}' H_{t-1} B_{11} + D_{11}' \xi_{t-1} \xi_{t-1}' D_{11} \quad (7)$$

where,  $\xi_t = \begin{bmatrix} \min(\varepsilon_{CDS,t}, 0) \\ \min(\varepsilon_{Bond,t}, 0) \end{bmatrix}$ ,  $C_0 = \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix}$ ,  $A_{11} = \begin{bmatrix} a_{11} & a_{12} \\ a_{11} & a_{22} \end{bmatrix}$ ,  $B_{11} = \begin{bmatrix} b_{11} & b_{12} \\ b_{11} & b_{22} \end{bmatrix}$ ,  $D_{11} = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{11} & \delta_{22} \end{bmatrix}$  We start the iterative procedure

by estimating the size (number of lags  $L$ , number of regimes) and coefficients for each model over the 2-year period 3/1/2005–3/1/2007. The number of lags ranges from 0 to 10 (usually 2 or 3) and is determined on the basis of minimizing AIC. The number of regimes ranges from 1 to 3 and is determined by the Tsay (1998) testing procedure discussed in Section 5. At this stage we are not particularly concerned about model parsimony or the robustness of estimated coefficients, but rather about the model’s ability to “whiten” the residual series by removing all possible auto/cross-correlations. Thus, we may end up with models of more than 2 lags that we assumed in Section 5 for in-sample analysis. First we estimate the mean model to generate the residuals and, at a second stage, we estimate the conditional covariance matrix of the residual vector series. As long as the residual series are clean of auto/cross-correlation, the produced hedge ratios are unbiased.

Having estimated the last incidence of conditional covariance matrix  $H_t$  for each strategy during the 2-year period, we calculate and record the hedge ratio  $\beta_t^*$  on the last trading day  $t$  of the period and the realized hedged portfolio return  $R_{t+1}$  on the next trading day  $t+1$ . We repeat this process 1553 times recording each time the estimated hedge ratio and hedged-portfolio return for each strategy. In each iteration the estimation sample increases by one trading day, until 16/12/2012. Clearly the quality of estimated hedge ratios depends on the quality of conditional covariance matrix estimates. In that sense, our model comparisons are essentially comparisons of forecasts of the true daily covariance matrix and, by extension, of the “optimal” hedge ratio.

Since any out-of-sample forecasting exercise requires an in-sample (ex-post) benchmark to evaluate different forecasts against, we derive the ex-post “optimal” hedge ratio using Kalman filter fixed-interval smoothing. Kalman smoothing runs only in off-line mode because to produce smoothed estimates of state variables at any point in time  $t$  requires not only processing of prior observations but also those that follow. Its main benefit is that yields the “optimal” estimate of a state variable, in a least squares sense, as long as the system is well-defined and identifiable. But when this process is used to forecast the state variable in “real time”, predictions are nowhere near as successful. As in all out-of-sample forecast exercises, Kalman-Filter covariance forecasts are no more but imperfect predictions of the true (unknown) covariance matrix.

Therefore, to filter the “optimal” hedge ratio in off-line mode we assume that the true hedge ratio  $\beta_t$  is an unobservable state variable in the following state-space setup that we estimate using maximum likelihood over the entire sample period 3/1/2005–17/12/2012.

$$\begin{aligned} \Delta Bond_t &= \Delta CDS_t \cdot \beta_t + v_t \\ \beta_t &= \beta_{t-1} + w_{1,t} \\ v_t &= \phi_v v_{t-1} + w_{2,t} \end{aligned} \quad (8)$$

The first equation denotes the so-called “measurement equation” in the state-space formulation and links the first state variable (i.e. the hedge ratio  $\beta_t$ ) with the observed variable ( $\Delta Bond_t$ ) through a linear time-varying relationship. The second equation describes the dynamics of state variable  $\beta_t$  which is the “optimal” hedge ratio, assuming that it follows a random walk. The random walk assumption is imposed not only for identification purposes, but also for the hedge-ratio process to have the highest possible

**Table 6**

Descriptive statistics of daily returns and turnover of hedging strategies for bond portfolios with CDS contracts per rating category, for the period 3/1/2007–16/12/2012..

		AAA	AA	A	BBB	XOVER	BB	B	CCC-C
Unhedged bond portfolio	Std.Deviation	0.15	0.09	0.05	0.04	0.08	0.05	0.06	0.13
	Skewness	−1.98	−1.89	−0.32	0.19	0.44	0.26	−0.12	−1.27
	Kurtosis	63.40	166.88	23.11	36.12	21.44	20.55	16.65	55.99
Kalman filter (in-sample)	Std.Deviation	0.06	0.02	0.02	0.02	0.02	0.01	0.01	0.02
	Skewness	−0.70	−0.14	−0.32	1.48	0.39	−0.42	0.32	0.04
	Kurtosis	13.76	7.71	8.52	17.99	8.40	4.55	5.42	21.09
	Turnover (% of Kalman filter)	100%	100%	100%	100%	100%	100%	100%	100%
Strategy 1: Naïve static	Std.Deviation	0.14	0.07	0.04	0.03	0.04	0.03	0.03	0.05
	Skewness	0.71	0.79	1.60	1.00	0.27	0.27	−0.35	−0.06
	Kurtosis	66.08	151.65	28.35	21.00	8.81	5.92	18.60	43.52
	Turnover (% of Kalman filter)	0%	0%	0%	0%	0%	0%	0%	0%
Strategy 2: Rolling regression	Std.Deviation	0.14	0.06	0.04	0.02	0.04	0.02	0.03	0.04
	Skewness	2.85	−0.36	1.21	1.81	0.71	0.12	−0.21	6.40
	Kurtosis	62.65	179.77	18.15	20.41	9.08	5.05	20.38	147.28
	Turnover (% of Kalman filter)	16%	11%	18%	42%	16%	50%	18%	23%
Strategy 3: VECM-CV1	Std.Deviation	0.09	0.04	0.03	0.02	0.03	0.02	0.02	0.04
	Skewness	2.10	−6.97	0.58	1.80	0.77	−0.01	0.02	0.82
	Kurtosis	48.03	210.05	18.30	18.38	13.98	6.73	9.92	33.71
	Turnover (% of Kalman filter)	202%	126%	153%	515%	226%	567%	197%	282%
Strategy 4: VECM-CV2	Std.Deviation	0.09	0.04	0.03	0.02	0.03	0.02	0.02	0.04
	Skewness	2.11	−3.18	0.79	1.48	0.78	−0.22	0.45	1.03
	Kurtosis	48.34	99.01	19.15	21.09	14.04	5.04	6.89	32.85
	Turnover (% of Kalman filter)	203%	109%	154%	561%	230%	162%	199%	277%
Strategy 5: T-VECM	Std.Deviation	0.09	0.04	0.03	0.02	0.03	0.02	0.02	0.04
	Skewness	1.26	−3.78	0.26	1.47	0.76	0.88	0.41	0.54
	Kurtosis	42.41	121.73	10.48	20.89	12.68	8.60	7.02	23.99
	Turnover (% of Kalman filter)	177%	128%	144%	434%	211%	155%	168%	247%

This table shows descriptive statistics of daily returns and turnover of hedging strategies for bond portfolios with CDS contracts, for the out-of-sample period 3/1/2007–16/12/2012. Out-of-sample hedging strategies considered are the *naïve* static hedging assuming hedging ratio [1 −1], rolling regression hedging based on a 1-year window, and three Kroner–Sultan dynamic hedging strategies assuming one cointegrating vector (VECM-CV1), two cointegrating vectors (VECM-CV2), as well as two cointegrating vectors and three regimes (T-VECM) for the joint CDS–bond dynamics. Descriptive statistics for the unhedged position and the in-sample Kalman-filter strategy benchmark are shown for comparison.

persistence. Given that persistent hedge ratios yield lower daily turnover compared to less persistent ones, we restrict our solutions to most economical in terms of transaction costs. The third equation describes the dynamics of state variable  $v_t$  that captures any first-order autocorrelation in measurement error, compensating for any short-term dynamics left unspecified in (8). Finally, we assume that  $w_{1,t}$  and  $w_{2,t}$  follow a bivariate normal distribution with diagonal covariance matrix.

We also define *turnover* ( $TO_i$ ) of strategy  $i$  as the sum of absolute daily changes in hedge ratios.

$$TO_i = \sum_{t=1}^T |\beta_{i,t+1} - \beta_{i,t}| \quad (9)$$

Turnover provides a measure to compare transaction costs of different hedging strategies. For example, if the average transaction cost for a particular CDS contract is 3 basis points (bps) then the total transaction cost of the hedging strategy over the whole forecasting period is roughly equal to  $3 \text{ bps} \times TO \times (\text{Bond Notional}) \times (\text{Spread Duration})$ .

Table 6 shows descriptive statistics of daily returns and turnover of different strategies for hedging portfolios of bonds of a given credit rating with CDS contracts, for the out-of-sample period 3/1/2007–16/12/2012. The out-of-sample hedging strategies considered are the naïve static hedging assuming hedging ratio [1 −1], the rolling regression hedging based on a 1-year window, and the three Kroner–Sultan dynamic hedging strategies assuming one cointegrating vector (VECM-CV1), two cointegrating vectors (VECM-CV2), and two cointegrating vectors and three regimes (T-VECM) for the joint CDS–bond dynamics. Descriptive statistics for the unhedged position and the in-sample Kalman-filter strategy benchmark are presented for comparison. For that purpose, turnover of each out-of-sample hedging strategy is expressed as a fraction of the turnover produced on the basis of the Kalman filter strategy.

To start with, we observe that all strategies reduce the variance of hedged portfolio returns compared to the unhedged position. This is despite the fact that the out-of-sample period (3/1/2007–16/12/2012) contains the Lehman failure and the ensuing turmoil in financial markets. This clearly shows that hedging bonds with CDS reduces mark-to-market risk compared to the unhedged portfolio, regardless of hedging strategy. Moreover, the in-sample Kalman filter smoothed estimates exhibit, as expected, the best performance in terms of daily return statistics (lowest standard deviation, kurtosis) compared to out-of-sample hedging strategies. This is also true in terms of total turnover, with the obvious exception of rolling regression that, by definition, is a low turnover strategy albeit with significant bias.

Table 7

Comparisons of out-of-sample hedging strategies for bond portfolios with CDS contracts per rating category, based on Diebold–Mariano modified statistic for the period 3/1/2007–16/12/2012..

Null hypothesis:	Neither rolling regression nor VECM-CV2 is better	Neither rolling regression nor T-VECM is better	Neither VECM-CV1 nor VECM-CV2 is better	Neither VECM-CV2 nor T-VECM is better
AAA	2.394**	3.282**	0.500	4.047**
AA	4.484**	4.029**	0.228	−0.228
A	2.388**	5.243**	1.867*	0.891
BBB	1.760*	2.136*	4.968**	4.734**
XOVER	3.491**	2.209*	1.681*	−0.030
BB	3.152**	−4.495**	2.199*	−4.496**
B	2.226*	3.885**	1.607	3.467**
CCC-C	−0.917	1.069	1.690*	1.805*

This table shows comparisons of out-of-sample hedging strategies for bond portfolios with CDS contracts, based on Diebold–Mariano modified statistic for the period 3/1/2007–16/12/2012, using as a proxy of the “true” hedge ratio the in-sample Kalman-filter benchmark. It tests the null of “neither of the two hedging strategies produces better hedge ratio forecast than the other”. A statistically significant *positive* statistic implies rejection of the null in favor of the *second* strategy. A statistically significant *negative* statistic implies rejection of the null in favor of the *first* strategy. The strategies considered are rolling regression hedging based on 1-year window, and three Kroner–Sultan dynamic hedging strategies assuming one cointegrating vector (VECM-CV1), two cointegrating vectors (VECM-CV2), and two cointegrating vectors and three regimes (T-VECM) for CDS–bond dynamics.

\*Denoted statistical significance at 5% confidence level.

\*\*Denoted statistical significance at 1% confidence level.

Regarding out-of-sample comparisons, the Kroner–Sultan strategies (VECM-CV1, VECM-CV2, T-VECM) clearly outperform static and rolling regression strategies, both in terms of daily-return volatility and kurtosis. It is also obvious that T-VECM performs better than VECM strategies in terms of total turnover, with only (slight) exception the AA rating portfolio.

However, by the basic statistics alone we cannot conclude if T-VECM is better hedging strategy than VECM. Therefore, we consider the hedge ratios produced by the out-of-sample hedging strategies as predictors of the “true” hedge ratio. As a proxy for the latter we use the in-sample Kalman-filter benchmark. To compare between forecasts of different hedging strategies we employ the Diebold–Mariano modified statistic based on the RMSE of forecast errors. The null of this test can be expressed as “neither of the two hedging strategies produces better hedge ratio forecast than the other”. A statistically significant *positive* statistic implies rejection of the null in favor of the second series of predictions (second strategy). In contrast, a statistically significant *negative* statistic implies rejection of the null in favor of the first series (first strategy). A non-statistically significant statistic implies that we cannot reject the null either way.

Table 7 shows comparisons of out-of-sample hedging strategies based on their Diebold–Mariano modified statistic and assuming error autocorrelation of 5 lags. The first column compares the rolling regression (strategy 2) against VECM-CV2 (strategy 4), in which case the null is soundly rejected in favor of VECM CV2. The second column shows that T-VECM (strategy 5) produces better hedge ratio forecasts than the rolling regression (strategy 2). The BB category is the only exception. The third column shows that VECM-CV2 (strategy 4) turns out to have better out-of-sample hedging performance than VECM-CV1 (strategy 3) for 5 out of 8 rating categories. Notice also that none of the remaining 3 categories shows an inverse relationship (i.e. negative Diebold–Mariano statistic). Finally, T-VECM (strategy 5) turns out to have better out-of-sample hedging performance than VECM-CV2 (strategy 4) in 4 out of 8 rating portfolios. In other words, assuming a threshold mechanism improves the out-of-sample hedging performance of VECM with two cointegrating vectors. Only for the BB rating category VECM-CV2 is clearly superior predictor than T-VECM.

Overall, our out-of-sample results corroborate the main points of the in-sample analysis regarding the need of a richer and regime-oriented description of the CDS–bond dynamics compared to the pre-crisis paradigm. Moreover, the use of two cointegrating vectors (i.e. both disequilibrium errors  $z_1$  and  $z_2$ ) improves the out-of-sample prediction of the “optimal” hedge ratio relative to a Kroner–Sultan framework with only one cointegrating vector.

## 8. Conclusions

In this paper, we examine the joint dynamics of CDS and corporate bond spreads based on an extensive dataset that covers the financial crisis and normal periods around it. The analysis is informed by the arbitrage strategy that practitioners implement to exploit the basis. This strategy specifies an equilibrium relationship between the CDS and the underlying bond spread where Libor/OIS spread emerges naturally as a third component. At a preliminary stage, we confirm the existence of such three-variable cointegrating relationship. We also show that the price of credit risk across rating categories shares a common component that directly links to Libor/OIS. These results are robust to the choice of alternative money-market spreads discussed in the literature.

In the presence of limits to arbitrage, the error in the cointegrating relationship between CDS, bond and Libor/OIS spreads is associated with the cumulative P&L of the basis trade. Therefore, we use it as threshold variable in a regime-switching VECM to describe the joint CDS–bond dynamics and investigate the mechanism that led to the unraveling of basis trades. The model shows better in-sample fitting properties than competing specifications, whilst it improves the out-of-sample performance of dynamic strategies to hedge the mark-to-market risk of corporate bonds with CDS. Moreover, it identifies destabilizing dynamics in the CDS market during the crisis that originate in supply shocks in the corporate bond market. At the peak of the crisis, bond spreads appear weakly exogenous to the system, while CDS spreads are entirely driven by random shocks and the disequilibrium error with the bond market. More importantly, deviations from the equilibrium relationship between the two credit markets push CDS spreads

further away from bond spreads, feeding into a self-reinforcing market dislocation. This is in line with practitioners' perception that the breakdown of basis trades during the financial crisis was caused by supply shocks in the corporate bond market and by elevated funding costs for arbitrageurs.

Finally, we find evidence of price discovery taking place in the CDS market before the crisis for key rating categories, such as the AAA and cross-over. But in the post-crisis period, the CDS market no longer leads the price discovery process for any rating category. This coincides with a fall of about 50 percent in the notional amount of outstanding CDS contracts between 2008 and 2010, as well as new regulations and market practices in the aftermath of the crisis introducing wider adoption of up-front CDS payments and limits to collateral rehypothecation.

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## Appendix A. Calculating bond-implied par-equivalent CDS spreads

CDS spreads are paid on quarterly or semi-annual frequency until contract maturity or default of the reference entity, whichever occurs first. Duffie (1999) shows that bond spreads are not directly comparable with CDS spreads unless the bond is a floating rate note (FRN) issued by the reference entity, trades at par and can be shorted without restrictions in a liquid repo market.<sup>29</sup> The more a bond deviates from these assumptions the more incompatible the metric of bond spreads (yield spread, zero spread, or asset-swap spread) with the CDS spread.<sup>30</sup>

To address this issue we use a survival-based valuation approach to derive the “bond-implied CDS spread”, similar to “par-equivalent CDS spread” (PECS) of JP Morgan and the “arbitrage price difference” (APD) of Credit Suisse.<sup>31</sup> By deriving the bond-implied CDS spreads within the same framework we employ a consistent relative value measure across the bond and CDS market, which does not depend on bond characteristics. Furthermore, it is a consistent metric for arbitrage-free interpolation between different CDS maturities.<sup>32</sup>

At inception of the CDS contract, arbitrage-free pricing implies the expected stream of CDS spread and protection payments are equally priced:

$$S_N \sum_1^N PS(t_i) \cdot \Delta_i \cdot d(t_i) + AI = (1 - R) \cdot \sum_1^N [PS(t_{i+1}) - PS(t_i)] \cdot d(t_{i+1}) \quad (A.1)$$

where,  $S_N$  is the CDS spread,  $\Delta_i$  is the CDS payment period, and  $R$  the recovery rate of par value in case of default. The survival probability from time 0 to time  $t_i$  is denoted as  $PS(t_i)$ , while the risk-free discount factor as  $d(t_i)$ .<sup>33</sup> Finally, AI stands for the accrued interest on default.

Similarly, the dirty price of a bullet bond issued by the same corporate entity, with  $M$  remaining coupons until maturity, is given by:

$$\text{bond price} = C \cdot \sum_1^M \Delta_i \cdot PS(t_i) \cdot d(t_i) + PS(t_M) \cdot d(t_M) + R \cdot \sum_1^M [PS(t_{i+1}) - PS(t_i)] \cdot d(t_{i+1}) \quad (A.2)$$

where,  $C$  denotes the bond coupon rate.

For obvious reasons, a function of maturity  $f(t)$  can be a survival probability curve  $PS(t)$  only if it is strictly positive, decreasing and satisfying  $PS(0) = 1$ . To avoid imposing such constraints when fitting a parametric curve to bond prices or CDS spreads, we fit instead of  $PS(t)$  the market-implied forward hazard rate curve  $H(t, t_{i-1}, t_i)$  in the spirit of Schonbächer (2003) and Houweling and Vorst (2005), which only needs to be positive. Berd et al. (2004) note that violation of this constraint usually signals violations of arbitrage conditions.

In a variant of their reduced-form model approach, we model the observed at time  $t$  forward hazard rate  $H(t, t_1, t_2)$  that spans the interval  $t_2 - t_1$  as a deterministic piecewise-constant function,<sup>34</sup> which remains constant between monthly intervals, and whose monthly value follows the Nelson and Siegel (1987) parametric form:

$$H\left(t, t_1, t_1 + \frac{1}{12}\right) = b_0 + b_1 \cdot \exp\left(-\frac{t_1}{b_3}\right) + b_1 \cdot \left(-\frac{t_1}{b_3}\right) \cdot \exp\left(-\frac{t_1}{b_3}\right) \quad (A.3)$$

<sup>29</sup> Even in that case some divergence in payoffs is still expected. For example, in case of default, the protection buyer will receive payment of the bond, but the protection seller will not receive the accrued premium.

<sup>30</sup> For a more detailed description of the various spread metrics see for example Jersey et al. (2007).

<sup>31</sup> Par-equivalent and APD are discussed in Beinstein and Scott (2006) and Jersey et al. (2007).

<sup>32</sup> An arbitrage-free interpolation scheme is conducted at the level of the hazard rate. It is arbitrage-free because it coincides with the market-practice that is used for calculating forward CDS spreads.

<sup>33</sup> These discount factors are bootstrapped from the Libor/Swap curve following Blanco et al. (2005), and Hull et al. (2004) among others.

<sup>34</sup> Schonbächer (2003) notes that using a more complicated stochastic hazard rate process – that is independent of the interest rate process – for pricing bullet bonds or credit default swaps with no embedded optionality offers no clear advantage to a deterministic specification of the hazard rate curve.

The forward hazard rate  $H\left(t, t_1, t_1 + \frac{1}{12}\right)$  corresponds to the conditional probability that the issuer will default in the interval  $\left[t_1, t_1 + \frac{1}{12}\right]$  given that she has survived until  $t_1$ .

Our method produces relatively smooth and intuitive shapes of the survival probability function.<sup>35</sup> This is in sharp contrast with the CDSW method in Bloomberg, which often yields coarse grids of implausible forward hazard rates and highly discontinuous survival probabilities.

Having defined the forward hazard rate curve from (A.2), it is easy to calculate the implied survival probability from

$$PS(t = t_n) = \prod_{i=1}^n \frac{1}{1 + \delta_i H(t, t_{i-1}, t_{i-1})} \quad (\text{A.4})$$

We define the bond-implied CDS spread similarly to par-equivalent spread of JP Morgan and arbitrage price difference (APD) of CSFB's, following three steps:

First, we use all available CDS maturities to fit through Eqs. (A.1) and (A.4) the forward hazard curve described in (A.3). Second, we use this curve to price any bond of the same issuer and seniority. To do this, we shift the forward hazard rate curve in parallel until we obtain the correct market price. With the adjusted hazard rate curve in hand, we calculate the bond-implied CDS spread from (A.1) and (A.4) having the same maturity and default risk as the reference bond.

The above-described algorithm provides a credit spread metric for a specific bond that takes into account both the term structure of default probabilities implied by the CDS market and the recovery rate of the bond's par value at default. An important advantage of this approach is that the CDS–bond basis is not affected either by bond characteristics or discontinuities of the bootstrapped hazard rate. The latter could potentially cause sudden shifts in the interpolated CDS when seeking to match a particular bond.

## Appendix B. Stripped-down model of the CDS–bond basis arbitrage

Here we describe the arbitrage trade that practitioners implement to exploit the CDS–bond basis. The outcome of this trade is the cointegrating relationship between the bond spread (PECS), the CDS and the Libor/OIS spread. We also illustrate that limits-to-arbitrage in the form of stop-loss limits and slow-moving capital produce spread dynamics that resemble our threshold models.

Since our sample exhibits significant negative basis, we focus on the economics of the negative basis trade. As already discussed in Appendix A, there is no perfect arbitrage between the CDS and cash-bond market unless the bond is a floating rate note (FRN) with a maturity matching exactly that of the CDS contract. However, traders transform corporate bonds into synthetic FRNs by trading the *asset-swap package*. This consists of a long position in the risky bond and an asset-swap to hedge its interest rate risk. The asset swap is an interest rate swap that matches the maturity of the bond, on its fixed leg pays the bond coupons while on its floating leg usually receives 3-month Libor ( $L_{3m}$ ) plus a spread, known as the Asset Swap Spread. On the trade date the asset-swap package is valued at par, meaning that fluctuations in bond price due to credit risk are fully reflected in the Asset Swap Spread (ASW – in Bloomberg notation).

To understand the economic underpinnings of the cointegrating relationship for the triplet of spreads (CDS, bond, Libor/OIS) we consider the case of arbitrageurs who implement the negative-basis trade by keeping their positions till maturity (buy-and-hold).<sup>36</sup> This involves:

1. Transforming the bond into a synthetic FRN by purchasing the asset-swap package.
2. Funding the asset swap in the repo market at the short-term repo rate. The term of the repo typically ranges from one day (overnight - O/N) to one month. Because of haircuts ( $h$ ) applying in the repo market, only part  $(1 - h)$  of the asset swap package can be financed this way. The rest is financed at the unsecured rate of the arbitrageur, typically the O/N Fed Funds rate ( $ff$ ) plus a funding spread ( $fs$ ).
3. Buying protection on the bond's notional. To do this, the arbitrageur pays a periodic CDS premium (every 3 months) and deposits cash in a margin account equal to proportion  $M$  of the notional. The margin account earns the O/N Fed Funds rate, while it is financed at the arbitrageur's unsecured money-market rate ( $ff + fs$ ).

We denote by  $\overline{ff}$ ,  $\overline{repo}_{O/N}$  and  $\overline{fs}$  the 3-month averages of the overnight rates for Fed Funds, repo and funding spread of arbitrageurs, respectively. For simplicity, we assume that the haircut ( $h$ ) and margin ( $M$ ) remain constant throughout the 3-month period. Then the accumulated 3-month carry of the negative-basis trade is given by:

$$\text{carry} = - \underbrace{(1 - h) \cdot \overline{repo}_{O/N}}_{\text{Funding cost of asset swap (secured)}} - \underbrace{h \cdot (\overline{ff} + \overline{fs})}_{\text{Funding cost of asset swap (unsecured)}} - \underbrace{\left[ CDS + M \cdot (\overline{ff} + \overline{fs}) - M \cdot \overline{ff} \right]}_{\text{Funding cost of CDS spread and margin (net unsecured)}} \quad (\text{B.1})$$

In the US, the (geometric) average of the effective O/N Fed Funds rate determines the overnight indexed swap rate (OIS). Therefore, we substitute  $\overline{ff}$  for OIS in (B.1). Moreover, following Garleanu and Pedersen (2011), the Libor/OIS spread is used as a proxy for the average funding spread of arbitrageurs. Thus substituting  $\overline{fs}$  for Libor/OIS in (B.1) and recasting it becomes:

$$\text{carry} = ASW - CDS + (1 - h - M) \cdot \text{Libor/OIS} + (1 - h) \cdot (OIS - \overline{repo}_{O/N}) \quad (\text{B.2})$$

<sup>35</sup> Berd et al. (2004) propose a smooth instantaneous hazard rate which requires numerical integration when fitting it to market prices. This imposes quite a computational burden without offering any clear advantage in the results. Indeed, our results show that both methods produce fitting errors in CDS spreads that are fractions of a basis point.

<sup>36</sup> This is equivalent to assume that arbitrageurs face stop-loss limits on the mark-to-market value of their positions, but the speed of capital is high enough to ensure that when they hit those limits and forced to unwind their trades they are quickly replaced by other arbitrageurs.



It can be shown that ASW equals a parameter ( $b$ ) times the par-equivalent CDS spread (PECS) implied by the bond price, as discussed in Appendix A. Thus (B.2) becomes:

$$\text{carry} = b \cdot \text{PECS} - \text{CDS} + (1 - h - M) \cdot \text{Libor/OIS} + (1 - h) \cdot (\text{OIS} - \overline{\text{repo}}_{\text{O/N}}) \quad (\text{B.3})$$

We expect the carry not to be driven by a stochastic trend, otherwise significant and persistent arbitrage opportunities would occur with high probability. It is also reasonable to assume that the difference between the OIS and the average O/N repo rate – which appears in the last term of (B.3) – is a stationary process. Therefore, (B.3) implies the first cointegrating relationship between CDS, PECS and Libor/OIS spread:

$$b \cdot \text{PECS}_t - \text{CDS}_t + (1 - h - M) \cdot \text{Libor/OIS}_t \sim I(0) \quad (\text{B.4})$$

Pre-crisis literature focused on the CDS–bond dynamics alone as the Libor/OIS spread at the time was low both in value (hovering around 10 bps) and in volatility. But once Libor/OIS became a solid stochastic trend itself, it could no longer be ignored when analyzing the basis trade dynamics. Similarly, in Section 4 we show that the disequilibrium error ( $z_1$ ) between CDS and bond spreads dominated the CDS–bond dynamics in the pre-crisis period, as it exhibited much higher volatility and persistence than disequilibrium error ( $z_2$ ) between bond and Libor/OIS spreads. But since the onset of crisis, both volatility and persistence of  $z_2$  increased markedly, as shown in Fig. 2.

Since CDS, bond and Libor/OIS spreads contain a unit root, their joint dynamics could be characterized by *at most* two distinct common stochastic trends. Thus, in addition to (B.4) – which is fully specified by the economics of the basis trade – there may be also a second cointegrating vector relating funding costs to CDS and bond spreads. But a second cointegrating relationship is unlikely to be interpretable, since it is specified as orthogonal to the first, capturing components not subsumed by (B.4). Clearly, an attempt to explain theoretically such a residual relationship is unlikely to yield intuitive results.

Section 3 shows empirically that, besides the no-arbitrage relationship, Libor/OIS is related to bond and CDS spreads through an alternative channel. Because bond prices depend on investors' financing costs, CDS are subject to margin requirements and Libor/OIS is a proxy for the average funding spread, we may conjecture that a second cointegrating relationship is consistent with the illiquidity premia literature.

#### Limits-to-Arbitrage: The effects of stop-loss limits and slow-moving capital

We now consider the case where leveraged arbitrageurs face mark-to-market losses and are subject to stop-loss limits. Moreover, the “speed” of capital is not high enough, thus arbitrageurs who leave the market are not swiftly replaced by others. Let  $D_{\text{CDS}}$  and  $D_{\text{ASW}}$  denote the durations of CDS and asset-swap package, respectively. Then, the daily P&L of the basis trade becomes<sup>37</sup>:

$$\begin{aligned} P\&L_{t,t+1} = & \underbrace{D_{\text{CDS},t} \cdot (\text{CDS}_{t+1} - \text{CDS}_t)}_{\text{Mark-to-market value of CDS}} - \underbrace{D_{\text{ASW},t} \cdot (\text{ASW}_{t+1} - \text{ASW}_t)}_{\text{Mark-to-market value of asset swap}} \\ & - \underbrace{(1 - h_t) \cdot \overline{\text{repo}}_{\text{O/N},t}}_{\text{Funding cost of asset swap (secured)}} - \underbrace{(h_t + M_t) \cdot (f f_t + f s_t)}_{\text{Funding cost of asset swap and CDS margin (unsecured)}} \end{aligned} \quad (\text{B.5})$$

Summing over a  $t$ -day period we obtain the cumulative P&L:

$$\begin{aligned} P\&L_{0,t} = & D_{\text{CDS},t-1} \cdot \text{CDS}_t - D_{\text{ASW},t-1} \cdot \text{ASW}_t \\ & + (D_{\text{CDS},0} - D_{\text{CDS},t-1}) \cdot \overline{\text{CDS}}_{(0 \rightarrow t)} - (D_{\text{ASW},0} - D_{\text{ASW},t-1}) \cdot \overline{\text{ASW}}_{(0 \rightarrow t)} \\ & - \overline{\text{repo}}_{\text{O/N},(0 \rightarrow t)} \sum_i (1 - h_{t-i}) - \overline{f f}_{(0 \rightarrow t)} \sum_i (h_{t-i} + M_{t-i}) - \overline{f s}_{(0 \rightarrow t)} \sum_i (h_{t-i} + M_{t-i}) \end{aligned} \quad (\text{B.6})$$

$$\text{where, } \overline{\text{CDS}}_{(0 \rightarrow t)} = \frac{\sum_i (D_{\text{CDS},t-i-1} - D_{\text{CDS},t-i}) \cdot \text{CDS}_{t-i}}{(D_{\text{CDS},0} - D_{\text{CDS},t})}, \quad \overline{\text{ASW}}_{(0 \rightarrow t)} = \frac{\sum_i (D_{\text{ASW},t-i-1} - D_{\text{ASW},t-i}) \cdot \text{ASW}_{t-i}}{(D_{\text{ASW},0} - D_{\text{ASW},t})},$$

$$\overline{\text{repo}}_{\text{O/N},(0 \rightarrow t)} = \frac{\sum_i (1 - h_{t-i}) \cdot \overline{\text{repo}}_{\text{O/N},t-i}}{\sum_i (1 - h_{t-i})}, \quad \overline{f f}_{(0 \rightarrow t)} = \frac{\sum_i (h_{t-i} + M_{t-i}) \cdot f f_{t-i}}{\sum_i (h_{t-i} + M_{t-i})}, \quad \overline{f s}_{(0 \rightarrow t)} = \frac{\sum_i (h_{t-i} + M_{t-i}) \cdot f s_{t-i}}{\sum_i (h_{t-i} + M_{t-i})}$$

Similar to (B.2), approximating the average Fed Funds rate  $\overline{f f}_{(0 \rightarrow t)}$  with OIS and the average overnight funding spread  $\overline{f s}_{(0 \rightarrow t)}$  with Libor/OIS <sub>$t$</sub> , (B.6) becomes:

$$\begin{aligned} P\&L_{0,t} = & D_{\text{CDS},t-1} \cdot \text{CDS}_t - D_{\text{ASW},t-1} \cdot \text{ASW}_t \\ & + (D_{\text{CDS},0} - D_{\text{CDS},t-1}) \cdot \overline{\text{CDS}}_{(0 \rightarrow t)} - (D_{\text{ASW},0} - D_{\text{ASW},t-1}) \cdot \overline{\text{ASW}}_{(0 \rightarrow t)} \\ & - \overline{\text{repo}}_{\text{O/N},(0 \rightarrow t)} \sum_i (1 - h_{t-i}) - \text{OIS}_t \sum_i (h_{t-i} + M_{t-i}) - \text{Libor/OIS}_t \sum_i (h_{t-i} + M_{t-i}) \end{aligned} \quad (\text{B.7})$$

If haircuts and margins remain relatively unchanged, (B.7) simplifies as follows:

$$\begin{aligned} P\&L_{0,t} = & D_{\text{CDS},t-1} \cdot \text{CDS}_t - D_{\text{ASW},t-1} \cdot \text{ASW}_t - t \cdot (h + M) \cdot \text{Libor/OIS}_t \\ & + (D_{\text{CDS},0} - D_{\text{CDS},t-1}) \cdot \overline{\text{CDS}}_{(0 \rightarrow t)} - (D_{\text{ASW},0} - D_{\text{ASW},t-1}) \cdot \overline{\text{ASW}}_{(0 \rightarrow t)} \\ & - t \cdot (h + M) \cdot \text{OIS}_t - t \cdot (1 - h) \cdot \overline{\text{repo}}_{\text{O/N},(0 \rightarrow t)} \end{aligned} \quad (\text{B.8})$$

<sup>37</sup> This is different from the P&L of the trading strategy described in Bai and Collin-Dufresne (2019), which does not entail asset swap hedging and is exposed to interest-rate risk. The trading strategy considered here isolates the credit-risk component of the bond from interest-rate risk, therefore is closer to basis arbitrage.

**Table A.1**

Robustness checks for alternative money-market spreads based on model fitness for the bond and CDS equation of the linear VECM, for the period 3/1/2007–17/4/2010..

		Libor/OIS	Libor/GC repo	TED	GC repo/Tbill
AAA	AIC (bond)	<b>8.330</b>	8.334	8.334	8.336
	BIC (bond)	<b>8.380</b>	8.384	8.383	8.386
	AIC (CDS)	<b>9.077</b>	9.080	9.081	9.089
	BIC (CDS)	<b>9.127</b>	9.129	9.130	9.139
AA	AIC (bond)	7.076	7.077	<b>7.075</b>	7.075
	BIC (bond)	7.121	7.122	7.125	<b>7.120</b>
	AIC (CDS)	7.632	<b>7.630</b>	7.636	7.651
	BIC (CDS)	7.676	<b>7.674</b>	7.686	7.695
A	AIC (bond)	<b>7.544</b>	7.560	7.554	7.577
	BIC (bond)	<b>7.594</b>	7.610	7.604	7.627
	AIC (CDS)	7.476	7.482	<b>7.469</b>	7.473
	BIC (CDS)	7.526	7.532	<b>7.519</b>	7.522
BBB	AIC (bond)	<b>7.472</b>	7.484	7.490	7.528
	BIC (bond)	<b>7.522</b>	7.533	7.540	7.578
	AIC (CDS)	7.325	7.328	<b>7.322</b>	7.330
	BIC (CDS)	7.375	7.378	<b>7.372</b>	7.380
XOVER	AIC (bond)	10.973	10.971	<b>10.968</b>	10.974
	BIC (bond)	11.017	11.015	<b>11.012</b>	11.018
	AIC (CDS)	10.743	10.740	<b>10.738</b>	10.751
	BIC (CDS)	10.788	10.784	<b>10.783</b>	10.796
BB	AIC (bond)	<b>10.224</b>	10.225	10.236	10.250
	BIC (bond)	<b>10.274</b>	10.275	10.286	10.300
	AIC (CDS)	10.015	<b>10.011</b>	10.019	10.021
	BIC (CDS)	10.065	<b>10.061</b>	10.069	10.071
B	AIC (bond)	<b>11.562</b>	11.568	11.570	11.582
	BIC (bond)	<b>11.612</b>	11.618	11.620	11.632
	AIC (CDS)	<b>11.298</b>	11.302	11.300	11.304
	BIC (CDS)	<b>11.349</b>	11.352	11.350	11.354
CCC-C	AIC (bond)	<b>15.589</b>	15.591	15.591	15.591
	BIC (bond)	<b>15.639</b>	15.641	15.641	15.641
	AIC (CDS)	<b>15.699</b>	15.702	15.702	15.702
	BIC (CDS)	<b>15.749</b>	15.752	15.752	15.752

This table compares alternative money-market spreads in terms of model fitness (in-sample) based on AIC and BIC. In addition to the difference of 3-month Libor minus 3-month overnight indexed swap rate (Libor/OIS), we also consider the 3-month Libor minus 3-month GC repo rate (Libor/GC repo), 3-month Libor minus 3-month Treasury Bill rate (TED), and 3-month GC repo rate minus 3-month Treasury Bill rate (GC repo/Tbill). Comparisons are based on the linear VECM for CDS, bond, and money-market spread, with two cointegrating vectors and two lags. AIC and BIC are computed separately for the CDS and bond equations of the jointly estimated VECM. The best performing money-market spread per rating category is reported in bold. Overall, differences in model fitness across money-market spreads are rather small, suggesting that (at least in-sample) results are robust to the choice of money-market spread. But Libor/OIS turns out to be more frequently the best performer, supporting our modeling choice based on the economics of basis trade (see [Appendix B](#)).

Eq. (B.8) states that the cumulative P&L of a negative basis trade depends on the current level of CDS and ASW spreads, as well as on their moving averages since the inception of the arbitrage strategy. As the linear function of current spreads becomes too negative relative to their moving average part, arbitrageurs may start to hit stop-loss limits. In fact, forced unwinding of basis-tightening bets is triggered by a linear function of CDS, ASW and Libor/OIS spreads that resembles cointegrating vector (B.4) albeit with the opposite sign. When (B.4) becomes positive – i.e. (B.8) tends to become negative – it indicates an opportunity to enter a buy-and-hold negative basis trade. But when (B.8) becomes too negative it triggers unwinding of negative basis trades. This imposes a limit on the no-arbitrage condition (B.4), which prevails only when new arbitrageurs replace those leaving the market (fast-moving capital). If new capital is slow-moving then, unwinding of arbitrage trades pushes (B.8) deeper into negative territory, causing a break-down of arbitrage activity.

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