This file provides a list of project topic examples. Both reference lists and descriptions of projects are not complete. Interested students are expected to clarify details with teachers (and assistants). We want to draw your attention to the fact that a student can suggest a project theme if her/his area of scientific interest nontrivially intersects with linear algebra.

1. **Singular decomposition for face recognition**

Write a program that performs a person recognition from a photograph using a singular value decomposition (SVD) (see ref.[1]). Write a program that constructs SVD for rectangular matrices of an arbitrary size and test this program on small matrices. Apply the program to the face recognition problem (see ref. [2]). One can convert a photo to a matrix using any standard application or package. What will change if one modifies individual entries/factors of a singular decomposition?

References:

1. Shevtsov G. S. "Linear algebra: Theory and Applied aspects": textbook, 2010 — 525 sec.

2. J.Demmel "Applied Numerical Linear Algebra"

3. http://link.springer.com/chapter/10.1007%2F978-1-4020-6264-3_26

4. TianY, TanT, WangY, FangY: Do singular values contain adequate information for face recognition? Pattern Recogn. 2003, 36: 649–655.

2. Linear Algebra and the Search Problem

Write a program that implements an official open version of the algorithm of the most famous search website. You can choose the degree of implementation complexity yourself. We hope that the dataset will be really non-trivial.

References:

1. *K Bryan, T Leise The $25,000,000,000 eigenvector: The linear algebra behind Google - Siam Review, 2006 - SIAM*

3. Quadratic optimization

The problem of quadratic optimization (or quadratic programming) is a generalization of the least squares method. It requires finding a solution to a system of linear equations that minimizes some quadratic function (the generalized version of the “squared error”) under constraints defined in the form of linear inequalities. Several effective methods for solving this problem are known. We suggest to study these methods, implement them (or use already existing packages) and compare their effectiveness for the case of economic statistics - the reconstruction of tables of intersectoral transactions in the scale of national economy.

References:

1. For quadratic optimization algorithms, see Pshenichny BN, Danilin Yu.M. “Numerical methods in extreme problems”, chap. III, paragraph 1, or

2. Hadley J; “Nonlinear and dynamic programming”, chap. 7

3. For the more contemporary examples of general optimization methods, see: Y.E. Nesterov. “Convex optimization methods” (in Russian)

4. An example of economic challenges can be found here: http://www.wiod.org/publications/papers/wiod2.pdf, paragraphs 2.7 and 2.8

4. Multidimensional arrays, TT decomposition and big data analysis

Illustrate the use of the “tensor-train decompositions” or their analogues in problems of big data analysis.

References:

1. http://epubs.siam.org/doi/abs/10.1137/090752286

2. I. V. Oseledets and E. E. Tyrtyshnikov. “TT-Cross approximation for multidimensional arrays” INM RAS Preprint, 2009-05.

3. http://people.csail.mit.edu/moitra/docs/bookex.pdf

5. Solution of systems of algebraic equations and inequalities

The modern symbolic methods for solving systems of algebraic equations and inequalities are associated with the Gröbner bases and the Cylindrical Algebraic Decomposition concepts. We assume that the basic algorithms will be analyzed and illustrated. It is possible to analyze only one of the two theories in the project. See chapters 11 and 12 in [1], as well as [2] and [3] for information.

References:

1. Basu, Saugata; Pollack, Richard; Roy, Marie-Françoise “Algorithms in real algebraic geometry.” Second edition. Algorithms and Computation in Mathematics, 10. Springer-Verlag, Berlin, 2006. x+662 pp. ISBN 978-3-540-33098-1; 3-540-33098-4 ; author's edition — https://perso.univ-rennes1.fr/marie-francoise.roy/bpr-ed2-posted3.pdf

2. I.V. Arzhantsev. “Gröbner bases and systems of algebraic equations” — https://www.mccme.ru/free-books/dubna/arjantsev.pdf

3. Mats Jirstrand. “Cylindrical Algebraic Decomposition – an Introduction” — http://www.diva-portal.org/smash/get/diva2:315832/FULLTEXT02

6. Methods of the Krylov subspace for solving linear systems

It is proposed to numerically solve the equations of mathematical physics (for example, the Poisson equation) using one of the Krylov subspace methods.

References:

1. J.Demmel “Applied Numerical Linear Algebra”

7. Epsilon spectrum and the construction of spectral portraits

The calculation of the eigenvalues of a real matrix on a computer is always associated with errors. How can we visualize the actual results of computer calculations at a given accuracy?

References:

1. Godunov S.K. "Lectures on modern aspects of linear algebra" - Nauchnaya Kniga, Novosibirsk, 2002

8. Tropical linear algebra and scheduling

Illustrate the use of operations on a tropical semiring.

References:

1. P. Butkovic. Max-Linear Systems: Theory and Algorithms. Springer, 2010.

2. Francois Louis Baccelli, Guy Cohen, Geert Jan Olsder, Jean-Pierre Quadrat, "Synchronization and Linearity: An Algebra for Discrete Event Systems" John Wiley & Sons, 1993, pp. 514

9. Spectral clustering

Clustering - the task of dividing objects into groups of "similar" elements - is one of the most widely used approaches for data analysis, which has found its application in many tasks, ranging from statistics, computer science, biology to social sciences or psychology.

As a project, it is proposed to study the spectral clustering method and try to apply this algorithm to real data. Implementation of the algorithm can be found in the python package *sklearn*. As the data, one can use the most famous data set "Karate Club" from the python *networkx* package.

References:

1. *http://people.csail.mit.edu/dsontag/courses/ml14/notes/Luxburg07_tutorial_spectral_clustering.pdf*

2. *https://scikit-learn.org/stable/modules/generated/sklearn.cluster.SpectralClustering.html*