Sec. 12. Dimensional Analysis.

Näive.

$\hbar = c = 1.$

$\hbar$: $E \cdot \cancel{L} T$, c: L/T.

$\hbar c = E \cdot L.$

$= 0.2\ \mu m \cdot eV.$

So we can convert anything to ~~"mass of~~ mass (energy) dimension.

$[m] = +1$

$[x^\mu] = -1$, $[dx] = -1.$

$[\partial^\mu] = +1$

$[d^d x] = \cancel{+d}\ -d.$

for a general theory

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\varphi\,\partial_\mu\varphi - \frac{1}{2}m^2\varphi^2 - \sum_{n=3}^{N}\frac{1}{n!}g_n\varphi^n.$$

$$S = \int d^d x\,\mathcal{L}.$$

$$Z(J) = \int \mathcal{D}\varphi\, e^{i\int d^d x(\mathcal{L} + J\varphi)}$$

because $[S] = 0$ as it is the ~~exponential of~~ exponent.

$[\mathcal{L}] = d.$

$[\varphi] = \frac{1}{2}(d-2).$

$[g_n] = d - \frac{1}{2}n(d-2).$

$[g_3] = \frac{1}{2}(6-d).$

One concept is the non-trivial dependence ~~out~~ on couplings for scattering amplitudes. need to be dimensionless argument of a function.

For high energy, it means

$$f(\bar{g}\, s^{-[g]/2}). \qquad \bar{g} = \left(\frac{g}{\Lambda}\right)^{[g]}$$

So for $[g] < 0$, $f(\bar{g}\, s^{-[g]/2})$ has a blowing up arguement.

so for $[g] > 0$, $f(\bar{g}\, s^{-[g]/2})$ becomes trivial at high energy.

for $[g] = 0$, $f(g)$ will be fun across scales.

Lec 13 ~~...~~. Propagator & Corrections

Before begin the class, ask for ~~feet~~ feedback for the first part of the course.

(Basic of QFT, scalar theories, Feynman Rules, etc.)

~~Emphasize that~~ QFT is to be digested all the time.

We've defined the "exact" propagator via

$$\Delta(x-y) \equiv i\langle 0|T(\varphi(x)\varphi(y))|0\rangle$$

with

$$\langle 0|\varphi(x)|0\rangle = 0, \quad \langle k|\varphi(x)|0\rangle = e^{-ikx}$$

In d-spacetime dimensions

$$\langle k|k'\rangle = (2\pi)^{d-1}\, 2\omega\, \delta^{d-1}(\vec{k}-\vec{k}').$$

[Here I assumed only 1 time direction $SO^+(1,d-1)$]

where $\omega = \sqrt{|\vec{k}|^2+m^2}$

The corresponding completeness relation yields

$$\int \widetilde{dk}\, |k\rangle\langle k| = I_1$$

identity op in one-particle subspace.

$$\widetilde{dk} \equiv \frac{d^{d-1}\vec{k}}{(2\pi)^{d-1}\, 2\omega}.$$

is the Lorentz invariant phase-space differential

We also define the exact momentum space propagator

$\hat{\Delta}(k^2)$ via.

$$\Delta(x-y) = \int \frac{d^d k}{(2\pi)^d} e^{ik(x-y)} \hat{\Delta}(k^2)$$

In free field theory, the momentum-space propagator is

$$\hat{\Delta}(k^2) = \frac{1}{k^2+m^2-i\epsilon}$$

It has an isolated pole at $k^2 = -m^2$ with residue one

m is the actual, physical mass of the free particle that enters into the energy momentum relation.

Let's insert a complete set of energy eigenstates.

(including $|0\rangle$, $|k\rangle$, and multiparticle state $|k, n\rangle$

$\uparrow$ total momentum with mass M.

Then (Note that I haven't used perturbative expansion in the following)

$$\langle 0|\varphi(x)\varphi(y)|0\rangle$$

$$= \langle 0|\varphi(x)|0\rangle\langle 0|\varphi(y)|0\rangle + \int \widehat{dk}\, \langle 0|\varphi(x)|k\rangle\langle k|\varphi(y)|0\rangle$$

$$+ \underbrace{\boxed{\sum_n \int \widehat{dk}}}\, \langle 0|\varphi(x)|k,n\rangle\langle k,n|\varphi(y)|0\rangle$$

Recall that we previously showed (not fully but pretty good) $\langle k,n|k\rangle = 0$.

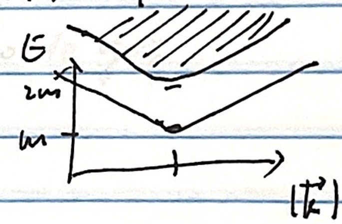
mean integrating over all phase space.

$\int dk$ would also work.

Noting that $\varphi(x) = e^{-i\hat{P}^\mu x_\mu}\varphi(0)\, e^{i\hat{P}^\mu x_\mu}$

$$\langle k,n|\varphi(x)|0\rangle = e^{-ikx}\langle k,n|\varphi(0)|0\rangle$$

where $k^0 = (\vec{k}^2 + M^2)^{1/2}$



$$\langle 0|\varphi(x)\varphi(y)|0\rangle = \int \widehat{dk}\, e^{-ik(x-y)} + \sum_n \int \widehat{dk}\, e^{-ik(x-y)} |\langle k,n|\varphi(0)|0\rangle|^2.$$

We can define a quantity called spectral density

$$\rho(s) \equiv \sum_n |\langle k,n|\varphi(0)|0\rangle|^2 \delta(s - M^2)$$

Recall earlier we shown that $\lim_{t\to\pm\infty} \langle k,n|\varphi(x)|0\rangle \to 0$. Here is not that limit. And $\vec{k}$ goes "off-shell" from 1 particle state.

$\boxed{\rho(s) \geq 0}$ for $s \geq 4m^2$ and $\rho(s) = 0$ for $s < 4m^2$

preserves unitarity (non-negative probability)

ensures microcausility

Now we get

$$\langle 0|\varphi(x)\varphi(y)|0\rangle = \int d\tilde{k}\, e^{ik(x-y)} + \int_{4m^2}^{\infty} ds\, \rho(s) \int d\hat{k}\, e^{-ik(x-y)}$$

here $k^0 = (\vec{k}^2 + m^2)^{1/2}$

$k^0 = (\vec{k}^2 + s)^{1/2}$

We can swap x and y to get

$$\langle 0|\varphi(y)\varphi(x)|0\rangle = \int d\hat{k}\, e^{-ik(x-y)} + \int_{4m^2}^{\infty} ds\, \rho(s) \int d\tilde{k}\, e^{-ik(x-y)}$$

We can combine the ~~about~~ above two to get time-ordered product.

$$\langle 0|T\varphi(x)\varphi(y)|0\rangle = \theta(x^0-y^0)\langle 0|\varphi(x)\varphi(y)|0\rangle + \theta(y^0-x^0)\langle 0|\varphi(y)\varphi(x)|0\rangle$$

We've already seen a useful $\boxed{\text{math}}$ in early lecture on path integral for free theory, that.

$$\int \frac{d^dk}{(2\pi)^d} \frac{e^{ik(x-y)}}{k^2+m^2-i\epsilon} = i\theta(x^0-y^0)\int d\tilde{k}\, e^{ik(x-y)} + i\theta(y^0-x^0)\int d\tilde{k}\, e^{-ik(x-y)}$$

So we get

$$i\langle 0|T\,\varphi(x)\varphi(y)|0\rangle$$

$$= \int \frac{d^dk}{(2\pi)^d}\, e^{-ik(x-y)}$$

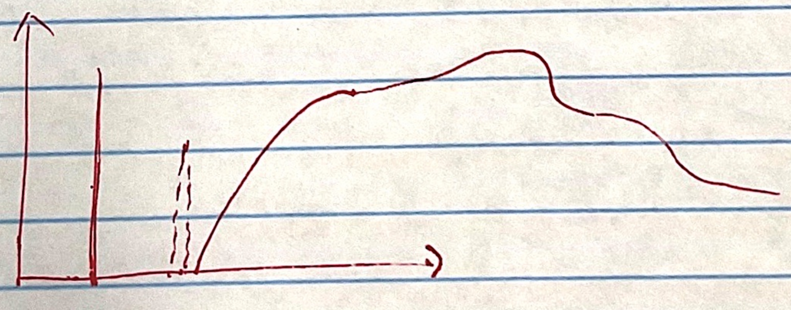
$$\left[\frac{1}{k^2+m^2-i\epsilon} + \int_{4m^2}^{\infty} ds\, \rho(s)\, \frac{1}{k^2+s-i\epsilon}\right].$$

Hence

$$\underbrace{\tilde{\Delta}(k^2) = \frac{1}{k^2+m^2-i\epsilon} + \int_{4m^2}^{\infty} ds\, \rho(s)\, \frac{1}{k^2+s^2-i\epsilon}}$$

Lehmann-Källén form of the exact propagator. [Note this is actually also true for ~~non-perturbative~~ strongly coupled systems]

free theory pole.



m

Next, we explore the meaning & definition of m.