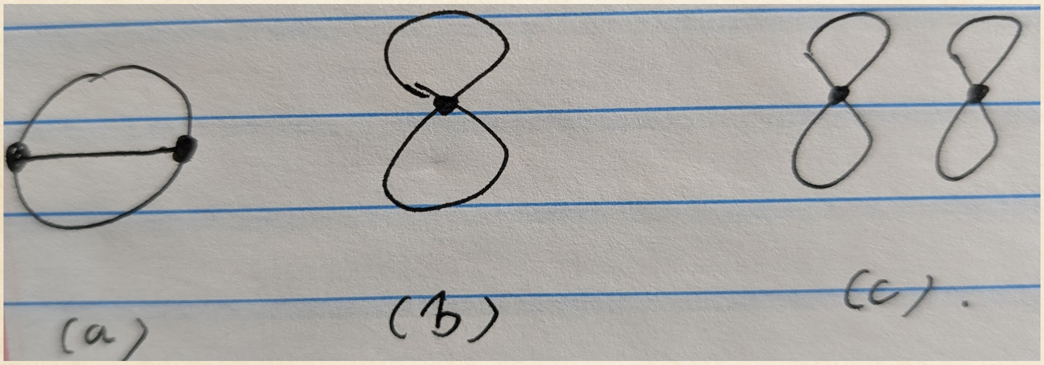
# 9.15 Symmetry Factors

In quantum field theory (QFT), symmetry factors are critical for correcting overcounting in perturbative expansions. This section supplements our discussion during lectures.

## Origin and Definition

Symmetry factors arise because multiple *Wick contractions* or permutations of vertices and lines in a Feynman diagram can produce identical configurations. These redundancies are quantified by the **automorphism group** of the diagram, which consists of all transformations (vertex/line permutations, rotations, reflections) that leave the diagram topologically unchanged. The **symmetry factor** $S$ is the order of this group: $S = |\text{Aut}(G)|$. Each diagram's contribution to amplitudes is weighted by $1/S$.

## Automorphism Groups and Examples




The order $|\text{Aut}(G)|$ counts distinct symmetries of a diagram. Key examples include:

- **"Sunset" diagram in $\phi^3$ theory** (a): Two vertices connected by three lines.
  - Symmetries: Vertex swap ($\mathbb{Z}_2$) and permutation of three lines ($S_3$).
  - $|\text{Aut}(G)| = 2 \times 6 = 12 \Rightarrow S = 12$.
- **Vacuum "figure-eight" diagram** (b): One vertex with two loops.
  - Symmetries: Loop swaps ($\mathbb{Z}_2$) and independent loop flips ($\mathbb{Z}_2 \times \mathbb{Z}_2$).
  - $|\text{Aut}(G)| = 2 \times 2 \times 2 = 8 \Rightarrow S = 8$.
- **Disconnected identical subdiagrams** (c): Two copies of a vacuum bubble.
  - Symmetries: Swap of subdiagrams ($\mathbb{Z}_2$) and individual bubble symmetries.

- $|\mathrm{Aut}(G)| = 2 \times (8 \times 8) = 128 \Rightarrow S = 128.$

### 9.15.1 Brief Summary

The contribution of a diagram to the amplitude includes:

$$\text{Contribution} = \frac{1}{S} \times (\text{Couplings}) \times (\text{Integrals}). \tag{9.15.1}$$

Symmetry factors ensure that each distinct physical process is counted exactly once. Errors in $S$ propagate to miscalculations of observables (e.g., cross-sections), making their correct determination essential. I directly write down the symmetry factors of many diagrams in my lecture notes and provide additional notes explaining different ways to get the symmetry factors. There is ultimately one method: carry out the functional derivative expansion in the path integral. There, you won't need to introduce symmetry factors but rather count how many ways to get the identical expression.