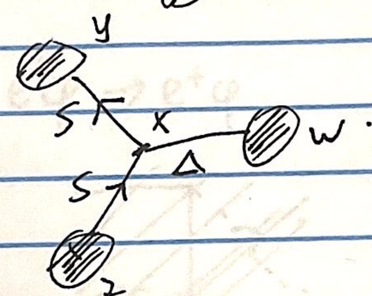
lec 20

But there is additional rules:
note that "charge is conserved" such that
each propagator is $\bar\eta(x)S(x-y)\eta(y)$

&

$\frac{\delta}{\delta\bar\eta(x)}\ \frac{\delta}{\delta\eta(y)}$ & $\bar\Psi(x)\,\bar\Psi(y)$

each vertex is $ig\int d^4x\ \underset{\eta(x)}{\bar\Psi(x)}\ \underset{\bar\eta(x)}{\Psi(x)}\ \varphi(x)$



$$ig\int d^4x\, d^4y\, d^4z\, d^4w.\ \bar\eta_{(y)}\, S(y-x)\underbrace{\ }_{\bar\eta(x)\,\eta(x)} S(x-z)\,\eta_{(z)}\,\Delta(z-w)J(w)$$

Removed by functional derivative

so there is a "direction" which is from $\bar\eta\to\eta$,
(left to right) in the Equation $\bar\Psi\to\eta$

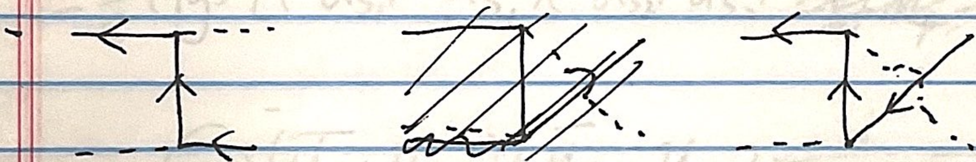
and we can ~~id~~ conveniently choose to be with the flow of charge.

The b-type $|e^-\rangle$ ~~flow~~ momentum "flow" is the same as charge and d-type $|e^+\rangle$ opposite. (just a notational choice).

Now we have

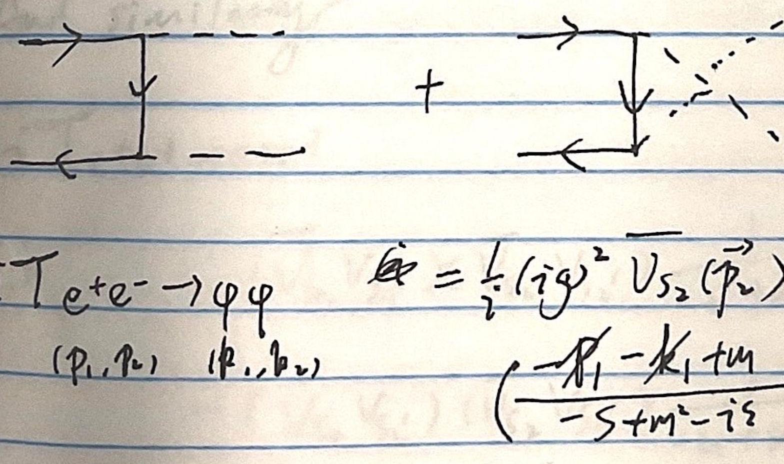
$$i\mathcal{T}_{e^-\varphi\to e^-\varphi} = \frac{1}{i}(ig)^2\, \bar{u}_{s'}(\vec{p}\,')\left[\frac{-\not{p}-\not{k}+m}{(p+k)^2+m^2-i\epsilon} + \frac{-\not{p}+\not{k}'+m}{(p-k')^2+m^2-i\epsilon}\right] u_s(\vec{p}).$$

Let's check $e^+\varphi \to e^+\varphi$.



$$i\mathcal{T}_{e^+\varphi\to e^+\varphi} = \frac{1}{i}(ig)^2\, \bar{v}_s(\vec{p})\left[\frac{-\not{p}-\not{k}+m}{(p+k)^2+m^2-i\epsilon} + \frac{-\not{p}+\not{k}'+m}{(p-k')^2+m^2-i\epsilon}\right] v_{s'}(\vec{p}\,')$$

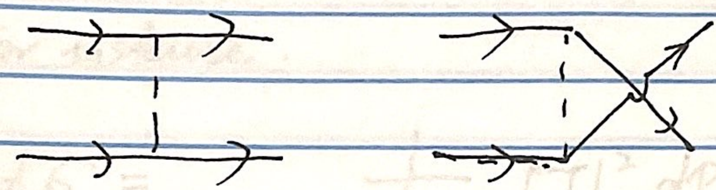
Now $e^+e^-\to\varphi\varphi$



$$i\mathcal{T}_{e^+e^-\to\varphi\varphi} \quad (p_1,p_2)\ (k_1,k_2) = \frac{1}{i}(ig)^2\, \bar{v}_{s_2}(\vec{p}_2)\left(\frac{-\not{p}_1-\not{k}_1+m}{-s+m^2-i\epsilon} + \frac{-\not{p}_1-\not{k}_2+m}{-u+m^2-i\epsilon}\right) u_{s_1}(\vec{p}_1)$$

And finnally 4 fermions

~~e~~ $e^-e^-\to e^-e^-$



$$iT_{e^-e^-\to e^-e^-} = \cancel{\frac{1}{2}(ig)^2\,\bar{u}_{s_1'}(\not p+\not k_1)\,(\ldots)}$$

$$= \frac{1}{2}(ig)^2\Big[(\bar{u}_{s_1'}\,u_{s_1})(\bar{u}_{s_2'}\,u_{s_2})\,\cancel{\frac{1}{-s+m_\varphi^2-i\varepsilon}}\,\frac{1}{-t+m_\varphi^2-i\varepsilon}$$

$$- (\bar{u}_{s_2'}\,u_{s_1})(\bar{u}_{s_1'}\,u_{s_2})\frac{1}{-u+m_\varphi^2-i\varepsilon}\Big]$$

$\downarrow$

exchange of fermions in the (final) state.

And similarly

$iT_{e^+e^+\to e^+e^+}$

$$= \ \cdots\ (\bar{V}_{s_1}V_{s_1'})(\bar{V}_{s_2}V_{s_2'})\frac{1}{-t+m_\varphi^2-i\varepsilon}$$

$$- (\bar{V}_{s_1}V_{s_2'})(\bar{V}_{s_2}V_{s_1'})\frac{1}{-u+m_\varphi^2-i\varepsilon}$$

~~The~~ In principle, we can calculate for observables now.

For instance.

$$d\sigma_{e^-_{s}\varphi \to e^-_{s'}\varphi} = \frac{1}{\text{flux}} |T|^2 \, dPS2$$

$$T_{ss'} = g^2 \bar{u}_{s'}(\vec{p}') \left[\frac{-\not{p} - \not{k} + m}{-s + m^2} + \frac{-\not{p} + \not{k}' + m}{-u + m^2} \right] u_s(\vec{p}).$$

plug in the expressions

plug in the expressions

You will get the matrix element for ~~one to one~~ a particular spin choices.

It is more often more convenient to ~~write~~ choose the spin quantization axis along the momentum direction, which is called helicity.

You will see in your HW it coincides with the chiral states in massless limit. (if you work slightly more)

But if we, experimentally, prepare only the unpolarised initial state and measure the rate in total (e.g., detector does not measure spin, which is typically in high energy), many things are further simplified.

Let's see.

$$\bar{u}_s \beta A^* \beta u_{s'} = \bar{u}_s \bar{A} u_{s'}$$

$$|M|^2 = g^4 \; \bar{u}_{s'} A u_s \cancel{\bar{u}_s \bar{A} u_{s'}} \; \boxed{u_s^* A^* \cancel{p^*} \beta^* u_{s'}}$$

$\bar{A} = A$ for this particular one.

$$A = \frac{-\cancel{p} + m}{\#}$$

$$A^* = \frac{-\cancel{p}^* + m}{\#}$$

$$\bar{A} = \beta A^* \beta = \frac{-\beta \cancel{p}^* \beta + m\beta^2}{\#}$$

$\beta^2 = \mathbb{1}$.

$$\gamma^{\mu *} = \gamma^0 \gamma^\mu \gamma^0$$

$$= g^4 \, \bar{u}_{s'} A u_s \bar{u}_s A u_{s'}$$

$$= g^4 \operatorname{Tr}[u_{s'} \bar{u}_{s'} A u_s \bar{u}_s A]$$

Note that $\sum_{s=\pm} \cancel{u_s \bar{u}_s}\ u_s(\vec{p})\bar{u}_s(\vec{p}) = -\not{p}+m$

$$\sum_{s=\pm} v_s(\vec{p})\bar{v}_s(\vec{p}) = -\not{p} \mp m.$$

$$\sum_{s,s'} |T|^2 = \cancel{\sum_{s,s'}} \mathrm{Tr}[(-\not{p}'-m)A(-\not{p}+m)A].$$

Now. γ - gymnastics.

(go back to page 10-12)

$$= \mathrm{Tr}\Big[(\not{p}'+m)\Big(\frac{(-\not{p}-\not{k}+m)}{-s+m^2} + \frac{(-\not{p}-\not{k}'+m)}{-u+m^2}\Big)$$

$$(\not{p}-m)\Big(\qquad\qquad \Big)\Big].$$

Only two types

$$\mathrm{Tr}[\not{p}\,\not{k}]\,m^2 + \mathrm{Tr}[\not{p}'\not{p}\not{p}\not{k}]$$

$$\not{p}'\cdots\not{k}' \qquad\qquad \underbrace{\not{p}\not{p}}_{p^2}\cdots$$