Some more Drawings & Explanertions of Symmtry factors.

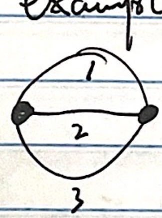
Recall that in "Getting Rid of" $\frac{1}{V!}\left(\frac{1}{3!}\right)^V \frac{1}{P!}\left(\frac{1}{2!}\right)^P$

We viewed that each propagator and each vertex identically exchangible to get $V!\ P!$ and then the contraction of $\left(i\frac{\delta}{\delta J}\right)^3$ ~~and~~ on $J\Delta J$ provides $(3!)^V (2!)^P$ additional possibilitygies. cancelling exactly the above factor.

But, this is obviously an over counting.

For example

Example A:



① the $(3!)^2$ permutations of $\left[\left(i\frac{\delta}{\delta J}\right)^3\right]^2$ gives one only $3!$ diffent product of Δ^3 if one "distinguish" the propagators. There is an over counting of $3!$

② the $(2!)^3$ flipping of propagators is already equivalent to the exchange of the two verticies, which I assigned $V! = 2!$ to already. There is hence and overcounting of 2.

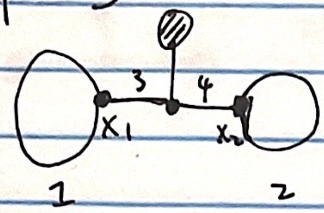
③ The total overcounting is then $2 \times 3! = 12$. And $S = 12$.

④ One can also get the overcounting by "Symmtry" analysis. There are two symmtries in this diagram:

the exchange symmtries of three $\Delta =$ —— : $3!$

the reflection/exchange symmtry of two vertices • : 2

Example B:



① for loop 1 and loop 2, the propagator is $\Delta(x_1-x_1)$ and $\Delta(x_2-x_2)$ there is no $\Delta(x-y)\to\Delta(y-x)$ reflection, so I over counted the weight by 2×2

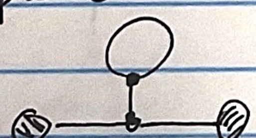
② If I exchange propagator 3 and 4 and as well vertex x_1, x_2, it is the same process and this only correspond to how I arrange the functional derivative, which do not result in more weigh (two seperate configurations) so I overcounted it by 2.

③ The symmetry factor is then $2\times2\times2=8$

④ Again one can make a symmetry analysis.
There is a reflection symmetry between left and right And there are two up-down reflection symmetry ~~betwe~~ in the two ○ ↕ so the overall symmetry factor is

$$S=2\times2\times2=8$$

Example C:

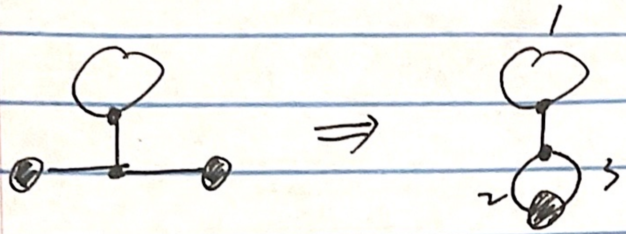


① Clearly, the bubble yields overcounting of 2

② Left-right reflection gives another 2

③ $S=2\times2=4$.

④ For multiple sources, there is a deformation method in discussing the symmetry factor. First, one deforms to make the sources joint (but they are not vertices)

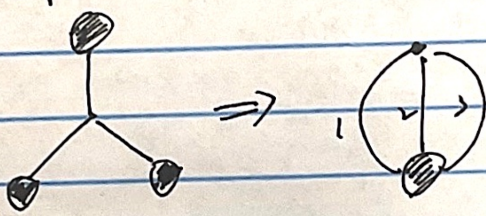


Then, it is clear that
For propagator 1, there is a factor of 2

And for propagator 2 and 3, there is an exchange symmetry of 2!

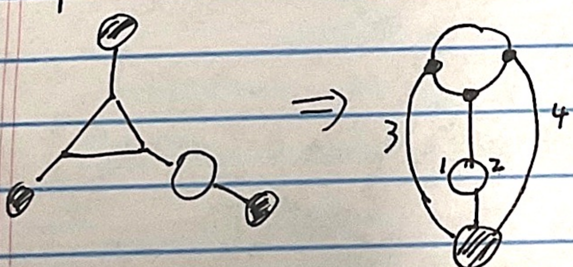
So the total symmetry factor is $S = 2 \times 2 = 4$.

Example D: Let's do a more complex one using the above method



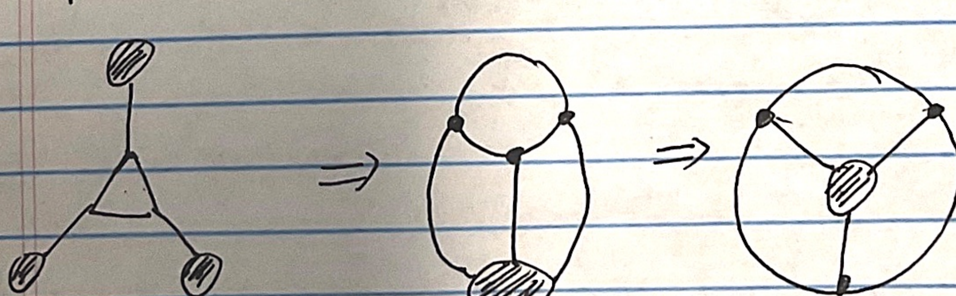
Clearly, one can exchange 1, 2, 3 in permutation
$S = 3! = 6$.

Example E:



Clearly, one can exchange $1 \leftrightarrow 2$, $3 \leftrightarrow 4$ (Reflections)
So the symmetry factor is
$S = 4$.

Example F:



This is a cyclic symmetry of 3 (rotate by $120°$, $240°$ degrees)

and ~~three~~ one reflection symmetry along each internal Δ's $\cancel{2^3 = 8}$

(the three reflections are related by cyclic × reflection)

$S = 3 \times 2^{\cancel{3}} = \cancel{8}\ \cancel{24}\ 6$