FX Modeling: HW 2

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Problem 1

Question:

Why are correlations of daily returns of spot vs daily returns of forward prices so high in the FX markets? What are the two requirements a market must support to enforce a high correlation across the forward curve?

Solution

(1)
People can arbitrage between Spot and Forward. So the spot and Forwards will move together with some relations.

(2)
First, you need to be able to store currencies and receive an interest rate for them.
Second, you need to be able to borrow/short currencies and pay an interest rate for them.

Problem 2

Question:

Why is risk management more complex for an FX forwards risk manager than for an FX spot risk manager?

Solution

Forwards have one more dimension, tenor, than the spot. It might be hard for them to hedge this perfectly. Reasons are as follows:
1. Not all trades are fungible with each other, like with spot
2. Benchmark tenors trade in the broker market for inter-dealer trades
3. Clients can trade any settlement date they like
4. Spreads are not tighter for benchmark settlement dates
5. Liquidity runs out to 2-3y for most currency pairs

Problem 3

Question:

Explain why risk to FX forward points can be expressed as risk to non-USD interest rates.

Solution

The forwards traders are really FX swap traders. They trade outright forward vs spot as their product. Not much spot risk; really they are trading interest rates.

Mathematically, due to the spot/forward arbitrage, the fair forward price is:

$$F(t,T) = S(t)e^{R(t,T)-Q(t,T)(T-t)} \tag{1}$$

Problem 3 [Solution] continued on next page. . .

which is related to the USD and non USD interest rate.

Problem 4

Question:

Assume a portfolio has just one FX forward position in it, settling on a date T which lies between two benchmark settlement dates T1 and T2. Derive the notionals N1 and N2 of the benchmark forwards which hedge the portfolio risk assuming triangle shocks to the benchmark non-USD interest rates, as shown on page 21 of the lecture notes.

Solution

From the note, we have the first order sensitivity.

$$\frac{\partial v}{\partial Q} = -STe^{-QT} \tag{2}$$

So, apply this formula into two benchmark forward contract, we have,

$$\frac{\partial v_1}{\partial Q} = -ST_1e^{-QT_1} \tag{3}$$

$$\frac{\partial v_2}{\partial Q} = -ST_2e^{-QT_2} \tag{4}$$

Therefore, in order to hedge the risk to zero coupon bond rate for asset currency, Q, we have to let the following formula holds:

$$\frac{\partial v}{\partial Q} = N_1\frac{\partial v_1}{\partial Q} + N_2\frac{\partial v_2}{\partial Q} \tag{5}$$

So, plug the respective formulas into the above one, we have:

$$Te^{-QT} = N_1T_1e^{-QT_1} + N_2T_2e^{-QT_2} \tag{6}$$

So, set the exponential term equal, we must have $e^{-Q(T-T_1)}$ in N_1 and $e^{Q(T_2-T)}$ in N_2.
Similarly, in order to match T term on both sides, we must have $\frac{T_2-T}{T_2-T_1}\frac{T}{T_1}$ in N_1 and $\frac{T-T_1}{T_2-T_1}\frac{T}{T_2}$ in N_2
Then, we have,

$$N_1 = \frac{T_2-T}{T_2-T_1}\frac{T}{T_1}e^{-Q(T-T_1)} \tag{7}$$

$$N_2 = \frac{T-T_1}{T_2-T_1}\frac{T}{T_2}e^{Q(T_2-T)} \tag{8}$$

where T lies between T_1 and T_2
Note: N_1 and N_2 are the absolute position we need to take in order to hedge one unit asset currency notional of a forward contract. BUT make sure to take the opposite position of the forward contract.

When T is prior to T_1, we can only use benchemark whose tenor is T_1 to hedge, so following formula need to hold:

$$\frac{\partial v}{\partial Q} = -STe^{-QT} = N_1\frac{\partial v_1}{\partial Q} = -N_1ST_1e^{-QT_1} \tag{9}$$

We have,

$$N_1 = \frac{T}{T_1}e^{-Q(T-T_1)} \tag{10}$$

Note: N_1 is the absolute position we need to take in order to hedge one unit asset currency notional of a forward contract. BUT make sure to take the opposite position of the forward contract.

When T is after T_2, we can only use benchemark whose tenor is T_2 to hedge, so following formula need to hold:

$$\frac{\partial v}{\partial Q} = -STe^{-QT} = N_2\frac{\partial v_2}{\partial Q} = -N_2ST_1e^{-QT_1} \tag{11}$$

We have,

$$N_2 = \frac{T}{T_2}e^{-Q(T-T_2)} \tag{12}$$

Note: N_2 is the absolute position we need to take in order to hedge one unit asset currency notional of a forward contract. BUT make sure to take the opposite position of the forward contract.

Problem 5

Question:

Explain principal component analysis and factor models, focusing on the differences between the two approaches to reduce dimensionality.

Solution

Principal component analysis is to look for most important (non-parametric) shocks that tend to drive moves in the whole curve, eg parallel shift, tilt move, bend move, etc involving extracting linear composites of observed variables.
Factor analysis is based on a formal model predicting observed variables from theoretical latent factors.
Difference:
1. Clearly, PCA is simply to pick up the most "important" variables you have "picked" automatically, while the factor models is to formulate a model with common factors you think that is important.
2. Factor models have fixed number of parameters while PCA might grow the number of parameters with the amount of training data.
3. Non-parametric, PCA, is harder to be understood properly. Can have unusual shapes due to specific data points in the history youre using.
4. Non-parametric shock shapes change over time.

Problem 6

Question:

Programming assignments. Monte Carlo simulation to see performance between different hedging strategies. See assignment document for details.

Solution

First, let us derive the two factor hedging notionals. When $T \neq T_1$ and $T \neq T_2$, we use two benchmark to hedge the factor risk, Suppose, N_1, N_2, we constructed the portfolio as follow:

$$\begin{aligned} P &= V_T - N_1V_{T_1} - N_2V_{T_2} \\ &= Se^{-QT} - Ke^{-RT} - N_1(Se^{-QT_1} - K_1e^{-RT_1}) - N_2(Se^{-QT_2} - K_2e^{-RT_2}) \end{aligned} \tag{13}$$

Problem 6 [Solution] continued on next page...

In order to hedge two factors, in general, we need to take derivative of P with respect to z_1 and z_2, and set it to zero.
With some dirty rearrangement, We get following linear system.

$$Te^{-(\beta_1+Q)T} = N_1T_1e^{-(\beta_1+Q)T_1} + N_2T_2e^{-(\beta_1+Q)T_2} \tag{14}$$
$$Te^{-(\beta_2+Q)T} = N_1T_1e^{-(\beta_2+Q)T_1} + N_2T_2e^{-(\beta_2+Q)T_2} \tag{15}$$
$$\tag{16}$$

More rearrangement,

$$aN_1 + bN_2 = 1 \tag{17}$$
$$cN_1 + dN_2 = 2 \tag{18}$$

where $a,\ b,\ c,\ d$ are as follows:

$$a = \frac{T_1}{T}e^{-(\beta_1+Q)(T_1-T)} \tag{19}$$
$$b = \frac{T_2}{T}e^{-(\beta_1+Q)(T_2-T)} \tag{20}$$
$$c = \frac{T_1}{T}e^{-(\beta_2+Q)(T_1-T)} \tag{21}$$
$$d = \frac{T_2}{T}e^{-(\beta_2+Q)(T_2-T)} \tag{22}$$

Solved the linear system, we have:

$$N_1 = \frac{b-d}{bc-ad} \tag{23}$$
$$N_2 = \frac{c-a}{bc-ad} \tag{24}$$

When $T = T_1$, we let $N_1 = 1$ $N_2 = 0$
When $T = T_2$, we let $N_1 = 0$ $N_2 = 1$

Results:

Number of path is 10000

1. T = 0.1
   (a) UnHedged sd is 2.95967082838e-05
   (b) TriHedged sd is 1.89484182533e-06
   (c) FactorHedged sd is 1.62255991279e-07
2. T = 0.25
   (a) UnHedged sd is 7.21445381694e-05
   (b) TriHedged sd is 0.0
   (c) FactorHedged sd is 0.0
3. T = 0.5

   (a) UnHedged sd is 0.000131005261638
   (b) TriHedged sd is 1.70535609484e-06
   (c) FactorHedged sd is 1.33064738283e-07

4. T = 0.75
   (a) UnHedged sd is 0.000182866894417
   (b) TriHedged sd is 2.47868611631e-06
   (c) FactorHedged sd is 1.2329503197e-07

5. T = 1.0
   (a) UnHedged sd is 0.000226708062772
   (b) TriHedged sd is 0.0
   (c) FactorHedged sd is 0.0

6. T = 2.0
   (a) UnHedged sd is 0.00036205892176
   (b) TriHedged sd is 0.000131105439505
   (c) FactorHedged sd is 1.1320686058e-06

Discussion:
As we can see, when T is 0.25 or 1.0, both Triangle hedge and Two factor model hedge can do a perfect hedge, that is because T is the benchmark we use to hedge and then the program decided to just use the only respective benchmark to hedge the risk. And the result show that in general the factor models does a better hedge than the triangle hedge by around one order. Across different Tenor, T, it shows that when the Tenor come closer to one of the benchmark, the hedging performance is better, when Tenor, T, is getting further away than both hedging benchmarks, the hedging performance is getting worse.

Usage of code:
To replicate the result, go to the code directory, open a terminal, then type

```
python runHedgeEngine.py | tee log
```