**Lecture 6 (Exotics Markets) Assignment, MTH 9865**

Due start of class, October 21, 2015.

**Question 1 (6 marks)**

In class we looked at Gaussian copulas for pricing two-asset derivatives. We talked about using that model to price cross-pair options based on the two USD-pair option markets, calibrating the Gaussian copula correlation parameter such that the model reproduces the ATM cross option price.

When we do that, the model tends to underprice the butterfly – ie the implied volatility smile that comes out of the model is lower than the market. We said that was due to the fact that the model does not include any premium for stochastic correlation, which it should because correlation is not constant.

A stochastic correlation only affects the value of a derivative if its exposure to that correlation is non-linear. Let’s consider a specific case of the Gaussian copula to examine that: one where the RR and BF of the USD pairs is zero. In that case, their pricing is just Black-Scholes pricing, and if we assume a constant correlation, the pricing of the cross option is Black-Scholes with an implied volatility

Calculate the “gamma” of the cross option price with respect to the correlation parameter . Assume a market where the two USD-pair spots are 1, interest rates are zero, time to expiration is 0.5y, the USD-pair volatilities are both equal to 10%, and the correlation is +25%, and plot the correlation gamma as a function of strike for the cross-pair options.

Discuss the qualitative impact stochastic correlation should have on the cross-pair implied volatilities based on that plot.

**Question 2 (3 marks)**

Describe the market dynamic that is most important for knockout out pricing, and compare that to the market dynamic that is the most important for volatility swap pricing.

For each, explain why that market dynamic is important to the pricing.

**Question 3 (4 marks)**

Consider a dual digital option that pays $1 if EURUSD is above a strike K1 **and** GBPUSD is above a strike K2. All discount rates are zero. The price of the EURUSD European digital option (paying $1 if EURUSD is above K1) is 65% and the price of the GBPUSD European digital option (paying $1 if GBPUSD is above K2) is 30%.

Plot the price of the dual digital option priced under a Gaussian copula model, for correlation parameter ranging from -100% to +100%. Qualitatively explain the behavior of the price sensitivity to correlation.

**Question 4 (10 marks)**

Investigate knockout pricing under the LV/SV approximation model.

Let’s imagine a market environment with spot=1, zero denominated and asset discount rates, and implied volatilities by benchmark expiration time defined as

|  |  |  |  |
| --- | --- | --- | --- |
| Expiration Time (yr) | ATM Vol (%) | 25d Risk Reversal (%) | 25d Butterfly (%) |
| 1/12 | 7.50 | 0.25 | 0.25 |
| 1/4 | 7.80 | 0.00 | 0.30 |
| 1/2 | 8.00 | -0.50 | 0.32 |
| 3/4 | 8.05 | -1.00 | 0.30 |
| 1 | 8.20 | -1.25 | 0.30 |

The 10d risk reversal is equal to the 25d risk reversal multiplied by 1.8 for all expirations; the 10d butterfly is equal to the 25d butterfly multiplied by 3.0 for all expirations.

You should use a mean reversion parameter  equal to 1 in all cases. You should try eleven different values for : 0 through 1.0 in steps of 0.1. For numerical parameters, use nu=500, nt=150, and nsd=5.

The goal of this question is to investigate how sensitive a knockout price is to our choice of mixture parameter, which in this model is the value of . The knockout you’re pricing is a call option with one year to expiration, a strike price of 1.03, and a down-and-out barrier at 0.95.

For each value of , you should:

1. Calibrate the local volatilities in the model such that the model reproduces the market prices for all the benchmark options (for each of the five expiration dates and each of the five points in the strike direction). You’ll of course need to calculate the five benchmark implied volatilities from the ATM, RR, and BF marks; and you’ll need to calculate the strikes those benchmark deltas correspond to.
2. Print out a table showing the calibrated local volatilities.
3. Price the knockout option under the model, calculating a basis point price (knockout price divided by spot, multiplied by 104).
4. Price the vanilla option underlying the barrier option, again as a basis point price.

Plot the knockout price as a function of , and plot the vanilla option price as a function of . (The vanilla price should be only a weak function of .)

Remembering that the main dynamic impacting the knockout price is risk reversal beta, qualitatively explain the behavior of the knockout price with .